On the Efficiency of Vote Buying when Voters have Common Interests*

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Abstract

We examine the conditions under which vote buying may promote efficiency in an environment where voters have identical preferences with respect to the behavior of their elected representatives (who are subject to both moral hazard and adverse selection). Our results suggest that permission of vote buying may prove beneficial in the market for corporate control and in some types of local political elections.

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1. Introduction

Vote buying has long met with suspicion. Three types of arguments are usually made against it. First, it is argued that because vote buying gives wealthier individuals an unfair advantage it violates the principle of equality. Second, it is sometimes argued that votes belong to the community as a whole, and should therefore not be alienable by individual voters. And third, there is a concern that vote buying may promote inefficiency. Hasen (2000) offers a critical analysis of these three arguments and concludes that although all three are justified in the context of general political election, they are all unjustified in the context of corporate elections, despite the apparent similarity between the two cases.[1] Perhaps for this reason, some states have indeed recently begun to allow for corporate vote buying in cases it passes a test of "intrinsic fairness" (Hasen, 2000; Levmore, 2000).

This sanguine view of corporate vote buying has recently been challenged by Cole (2001) who argued that with decreasing costs of obtaining and conveying information, corporate vote buying may seriously undermine efficiency by facilitating "looting" of the corporation. To appreciate the relative merits of the efficiency argument for and against vote buying, we focus our attention on a game-theoretic model that includes the following elements. A certain valuable position, such as, say, that of the CEO of a corporation, can be held by at most one player in any given period. The individual or player who holds the position (henceforth, the "incumbent") can either behave well or opportunistically. We assume that in any given period an incumbent earns a higher personal payoff by behaving opportunistically rather than well, whereas voters (say, the corporation's shareholders) are better off if the incumbent behaves well. In every period, the incumbent competes for the position in an election against another player (henceforth, the "challenger"). We allow the incumbent and the challenger to buy votes. The number of voters is assumed to be sufficiently large so as to make each single voter inconsequential. Hence all voters regard themselves as non-pivotal and the player who offers to pay more for any single vote is sure to get it. The player who obtains a majority of the votes wins the right to hold the position in the next period, upon which the whole process is repeated. The respective incumbent decides whether to behave well or not, election takes place, and so on.

Our analysis is motivated by the idea that permitting the buying of votes should generally help facilitate the replacement of the incumbent player. This is good in case the incumbent tends to behave opportunistically but bad when the incumbent behaves well. Moreover, there

[1] See also Levmore (2000) who agrees that in spite of their plausibility, the arguments against vote buying are "somewhat flimsy."

is a real concern that the possibility of buying votes may appeal particularly to players who intend to behave opportunistically and for this very reason are also willing to pay more for votes than players who intend to behave well. In other words, permitting vote buying may either lead to good or bad outcomes: since vote buying may be used as a disciplining device against opportunistic incumbents, permitting it may indeed lead to a "good" equilibrium where incumbents always behave well. The problem is that vote buying may also result in a "bad" equilibrium where incumbents always behave opportunistically. This concern about looting seems to be the motivation behind the general prohibition against the buying of votes in different contexts.

Whether vote buying can be plausibly said to promote or harm efficiency depends on the following conditions. Effective governance generally requires overcoming two major agency problems: *adverse selection*, or how to ensure that the incumbent is highly capable; and *moral hazard*, or how to ensure that the incumbent does indeed behave well. Our results indicate that in an environment where players' abilities are observable by voters, vote buying does not make a difference – a similar outcome is obtained both in case vote buying is permitted and when it is not. Vote buying makes a difference in an environment where players' abilities are not observable by voters, because in such an environment the players' bids for the votes depend on their abilities and so may serve as signals about the players' abilities. In such an environment, whether or not vote buying may promote efficiency depends on the relationship between the players' abilities, and the difference between the benefit that a player gets from holding the position and its outside option. If a higher ability translates into a smaller difference, then vote buying may be harmful, but if a higher ability implies a bigger difference, then, as we show, vote buying may indeed promote efficiency.[2]

This observation raises the question of when is it likely that the difference between an individual's benefit from holding the position and the value of its outside option would be increasing in the individual's ability. We believe that there are two such cases. One is where the position is in an organization that has a relative advantage in generating perks and other "ego rents." For example, if the organization is very notorious. Another, more mundane case, is when the notion of "ability" is interpreted as the ability to perform the specific task that is associated with the particular position that is considered. According to this interpretation, an individual's "ability" should not be thought of as some universal quality of the individual, but rather as an indicator of the "quality of the match" between the individual and the position or organization.

[2] We are grateful to an anonymous referee for the insights contained in this paragraph and the next.

It is important to emphasize that our analysis is limited to the case where voters have common interests with respect to the incumbent's behavior – they all prefer that the incumbent behaves well rather than opportunistically. This is certainly an inappropriate assumption in some cases, but it seems reasonable enough when the incumbent's position is that of the manager of a corporation, or of the mayor, chief of police, judge, or perhaps school commissioner in a small town. In the former case, the voters are shareholders who all want the manager of the corporation to do his best to maximize the profits of the corporation. In the latter cases, the voters are the residents of the town who all want the mayor, chief of police, judge, or school commissioner to govern effectively. Our conclusions do not apply when voters are likely to have conflicting preferences about the incumbent's behavior as would generally be the case whenever the position is contested in general political elections.[3] In this case, Neeman (1999) has shown that because the likelihood that any single voter is pivotal is negligible, it is unlikely that the price of a vote will reflect its true value. In other words, the "market for votes" is prone to market failure, and therefore vote buying may well lead to inefficient outcomes in this case (see also Dal Bo, 2000; and Snyder and Ting, 2002).

Another important qualification is that throughout our analysis we maintain the assumption that vote buying involves uniform and unrestricted offers. That is, the players are required to offer to buy all the votes and to pay the same price for each vote they buy, they are not allowed to discriminate among the voters. This is not a restrictive assumption if – as is the case in this paper – the purpose is to *design* a market for votes that would ensure efficient outcomes, because in this case, the designer is free to impose any reasonable restriction on the market for votes that would ensure efficiency. If our objective were to conduct positive, rather than normative, analysis, then the assumption that vote buying involves uniform and unrestricted offers would obviously be a strong assumption.

The assumption that vote buying involves uniform and unrestricted offers implies that buying the voters' *votes* is similar to buying the incumbent's *office* from the voters. That is, instead of offering to buy votes, we may think of the candidates as making a bid for the

[3]Levmore (2000) also recognizes the importance of the distinction between settings in which voters do and do not have the same preferences. This distinction mirrors the two types of game theoretic models of election. In one type of models, candidates compete for election by announcing different ideoligies, or policies they intend to implement once they are elected, and voters each vote for the candidate who is more appealing to them ideologically. In the other type of models, candidates are distinguished by their skill, and voters, who hold private information about the relative skill of the candidates, vote for the candidate who they believe to be more skillful. In the latter type of models, voters are assumed to hold the same preferences about the candidates – they all prefer a more skillful candidate to a less skillful one.

right to occupy the incumbent's office. The voters vote for the candidate to whom they prefer to sell the incumbent's office, the candidate who won the majority of the votes wins the office, and distributes his bid equally among all the voters.[4] The difference between office buying in this sense and vote buying is due to the free-rider problem that is facing the voters. Whereas with vote buying the free-rider problem implies that each voter sells her votes to the candidate who offers the highest price for them (see Proposition 1 below), with office buying the free-rider problem implies that voters have no incentive to vote.[5] Therefore, if voting entails a cost that exceeds its benefits, then the free-rider problem is more severe with office buying than with vote buying, as explained in Section 5 (in a somewhat different context) below. For this reason, and because vote buying seems "more realistic"—as well as more acceptable—than office buying, we consider the case of vote rather than office buying in our analysis. However, with only minor modifications, the analysis can be adapted to deal with the case of office buying.

Surprisingly enough, until recently, vote buying received almost no attention in the literature. Manne (1964), Clark (1979), and more recently Andre (1990), Hasen (2000) and Levmore (2000) presented informal arguments in favor of vote buying but they either ignored or otherwise dismissed the problem of looting or the possible existence of bad equilibria as discussed by Cole (2001) and others (see also the recent paper by Hu and Black, 2006). In 1985, in response to the brisk merger and acquisition activity of the early 1980s, a group of Wall Street professionals suggested decoupling shares and votes for short periods of time to facilitate vote buying transactions but did not submit a formal request to the Securities and Exchange Commission (Rent-A-Vote? *Wall Street Journal*, October 31, p. 1). In the more formal literature, Harris and Raviv (1988) considered a model that allows for the possibility of corporate vote buying, but their main concern was with the optimality of one share-one vote and simple majority rules.[6] They assumed vote buying only in order to simplify their

[4] In many, if not most, cases where office-buying can conceivably be arranged, the seller of the office would not be the voters, but rather some kind of government agency. In that case, it seems that such an agency should be able to contract directly with the incumbent by rewarding good behavior and penalizing opportunistic behavior so as to ensure that the incumbent behaves well. This would be much better than resorting to something as weak as our tie-breaking assumption (plus competition for office), which achieves the same outcome at the cost of forfeiting significant rents to the incumbent.

[5] This problem may be solved by distributing the winning candidate's payment only among those voters who actually voted, but this might be difficult to implement in practice, and besides, it can be argued that the money belongs to the entire group of voters, so that it is unfair to distribute it only to a subset of the voters.

[6] Allowing for a permanent separation between shareholders' voting rights and dividend claims is generally

analysis, and did not consider its possible disciplinary effects on incumbent managers as discussed here. Furthermore, their analysis presumes that managers may appropriate part of the firm's revenues as private benefits of control, and that more capable managers obtain higher private benefits of control. Consequently, in their analysis, managerial moral hazard harms shareholders, but also serves to solve the problem of managerial adverse selection. They show that under a one share-one vote rule, managerial competition for private benefits of control leads to the selection of a manager that generates the highest returns to shareholders. Blair et al. (1989) considered a static model in which they analyzed the question of whether vote buying can increase the efficiency of takeover contests. They concluded that if the tax treatment of realized and unrealized capital gains on shares is different, then vote buying can in fact improve the efficiency of takeover contests. Obviously, the reason that vote buying is beneficial in their model is very different from the reason described here.

The rest of the paper proceeds as follows. We first consider the case where vote buying is permitted. In the next section we present our main assumptions. In Section 3, we consider the case of pure moral hazard, and in Section 4, we consider the general case. Then, in Section 5 we consider the case where vote buying is not permitted, and compare the outcomes when vote buying is and is not permitted. Finally, in Section 6 we discuss the potential implications of our results in the context of corporate vote buying. All proofs are relegated to the Appendix.

2. The Model

A certain position is open for competition by popular vote. The voter population is composed of a large number (a continuum of measure one) of voters. Players who are interested in holding the respective position are infinitely lived and characterized by their ability $\alpha \in \{\alpha_1, ..., \alpha_H\}$, where $\alpha_1 < \alpha_2 < \cdots < \alpha_H$. The ability of player j is denoted by $\alpha(j)$. The higher is $\alpha(j)$, the more capable is player j. We assume that there is a large number of players in every level of ability, and denote the probability that a randomly chosen player has ability lower or equal to α by $F(\alpha)$. We assume that players know their abilities.

Every period, the player who holds the position has to decide whether to behave well or not. We denote the incumbent player in period t by i_t. The incumbent's choice of action in period t is denoted by $a_t \in \{0, 1\}$, where "1" is interpreted as behaving well and "0" as not. We assume that the incumbent's action is observable by the voters but is not verifiable in

believed to be inefficient. See the discussion in Hart (1995, chapter 8, pp. 205-206) and the references therein. Additional references can be found in Becht et al. (2002).

court.[7] We thus interpret opportunistic behavior as something that is still within the confines of what is considered legal behavior, but that is nevertheless unambiguously perceived as bad in the eyes of the voters. In the corporate context, for example, such opportunistic behavior may correspond to excessive indulgence in perquisites or diversion of corporate resources for personal gain. The reluctance of courts to interfere with the internal management of corporations in the absence of clear evidence for corruption (Easterbrook and Fischel, 1991, p. 110) suggests that the scope of such opportunistic behavior may be quite large.[8] Obviously, if the player's action was verifiable in court, then it would have been possible to write a contract in which the incumbent was obliged to behave well in every period, and the whole problem would have disappeared. Note also that although voters can tell if the incumbent behaved well or not, they might not be able to tell how capable the incumbent is, even after observing its performance while in office, because it is difficult to judge ability in the abstract, without comparing the incumbent's performance while in office to someone else's performance if she had been in office.

The benefit that the incumbent generates for the voters in period t is given by $v_t = v(\alpha(i_t), a_t)$, where i_t denotes the identity of the incumbent of period t. We assume that $v(\cdot, \cdot)$ is increasing in the incumbent's ability and action. That is, other things being equal, a more capable incumbent and an incumbent who behaves well generate a higher per-period benefit for the voters.

The per-period payoff of the incumbent player depends on his ability and action in that period and is given by $b(\alpha, a)$. We assume that $b(\cdot, \cdot)$ is increasing in the incumbent's ability but decreasing in his action. That is, other things being equal, a more capable incumbent and an incumbent who behaves opportunistically obtain a higher per-period payoff. The latter assumption reflects the moral hazard problem. The former is motivated by the fact

[7] This assumption may be relaxed. It is sufficient that the player's action is observed with noise by some fraction of the voters.

[8] Indeed, in reviewing the evidence that managers inefficiently choose to reinvest free cash within the company or use it for acquisitions rather than to return it to investors, Shleifer and Vishny (1997) note that "a considerable amount of evidence has documented the prevalence of managerial behavior that does not serve the interests of investors, particularly shareholders" (p. 746). They also point to evidence, that "is less direct, but perhaps as compelling. In one of the most macabre event studies ever performed, Johnson, Magee, Nagarajan, and Newman (1985) find that sudden executive deaths – in plane crashes or from heart attacks – are often accompanied by increases in share prices of the companies these executives managed. The price increases are largest for some major conglomerates, whose founders built vast empires without returning much to investors. A plausible interpretation of this evidence is that the flow of benefits of control diminishes after the death of powerful managers" (p. 747).

that higher ability usually translates into bigger bonuses and realized option packages, more favorable press coverage, more popular support, etc. We denote the per-period opportunity cost of a player with ability α by $r(\alpha)$, and assume that $b(\alpha, a) - r(\alpha)$ is positive for every level of ability α and action a, and increasing in the level of ability α.

We assume that players' and voters' payoffs are discounted according to a common discount factor $\delta < 1$.

The game proceeds as follows. In every period $t \geq 1$:

1. The incumbent i_t chooses whether to behave well or not.

2. At the end of the period, the payoffs to the incumbent and to the voters are realized.

3. Next, a challenger with a randomly chosen ability appears. The challenger may appeal for the voters' support by making them an unrestricted offer for their votes at a price $p_t^C \geq 0$. The challenger's bid is constrained to be an integer multiple of a smallest monetary unit $m > 0$.[9] Mounting a challenge requires the challenger to incur a small cost, μp_t^C, that is proportional to the challenger's bid.[10]

4. The incumbent may respond to the challenge by making voters an unrestricted counter-offer $p_t^I \geq 0$ at a cost μp_t^I. As before, the counter-offer p_t^I is constrained to be an integer multiple of m.

5. The voters decide whether to sell their votes to the incumbent at the price p_t^I or to the challenger at the price p_t^C. We denote the proportion of voters who vote for the incumbent and the challenger by n_t^I and n_t^C, respectively.[11]

6. A vote takes place and the player who has the majority of the votes wins the right to hold the position in the next period. If the challenger and the incumbent obtain the

[9] The smallest monetary unit m is introduced for the purpose of ensuring that maximization problems have well defined solutions.

[10] If challenging is costless, the incumbent may be challenged "just for fun." However, the specific form of the challenger's cost is irrelevant. For example, instead of assuming that bidding cost is proportional to the bid, we may assume that placing a positive bid requires to incur a small fixed cost $\bar{\mu} > 0$. The same holds for the incumbent's cost introduced below.

[11] Note that a voter can always benefit by selling her vote to the player for which she would have voted anyway (since all voters are equally informed, there is no advantage in abstention from voting). As a consequence, it is never in the voters' interest to refuse to sell their votes.

same number (measure) of votes, the incumbent has the right to hold the position in the next period.[12]

The following result is straightforward.

Proposition 1. *In every period, and after any history, if the incumbent and the challenger offer to pay different prices for voters' votes, then it is a dominant strategy for the voters to sell their votes to the player who offers them the highest price.*

The proof of the proposition follows from the fact that because there is a continuum of voters, each single voter has no effect on the outcome of the competition between the players and so may as well sell her vote to the player who offers to pay her more for it. This argument can be extended to the case where the number of voters is large but finite, provided it is assumed that there is some small uncertainty regarding voters' voting behavior (Neeman, 1999).

We make two important assumptions about the behavior and preferences of the players and voters.

Assumption 1 ("Players are patient"). *Players have low discount rates (i.e., δ is close to 1).*

The exact meaning of "low" depends on the presence (or absence) and on the "extent" of the adverse selection problem and will be specified below. Assumption 1 implies that players obtain a higher payoff from behaving well and holding the position forever than from behaving opportunistically but holding the position only for a "short" number of periods, where "short" will be given a precise meaning below.

The next assumption constrains voters' behavior. Proposition 1 implies that if the incumbent and the challenger offer different prices for the votes, voters have a dominant strategy to sell their votes to the player who offers them the higher price. But, if the incumbent and challenger offer the same price for the votes, or if vote buying is not permitted, then voters are indifferent as to whom to tender their votes. We make the following assumption about the way in which this indifference is resolved.

Assumption 2 ("Tie-Breaking Rule"). *If at (the end of) period t the challenger and the incumbent offer the same price $p_t^I = p_t^C$ for voters' votes, then voters sell their votes to*

[12] For completeness is it necessary to specify what happens in case of a tie. However, our results are independent of this specific assumption.

the incumbent if he behaved well in period t, and to the challenger if the incumbent did not behave well in period t.

This tie-breaking assumption is a rather weak assumption. In fact, to the extent that opportunistic behavior of the incumbent provokes the voters' ire, we believe it is plausible that voters would be willing to support the challenger even if he offers a slightly lower price for their votes than the incumbent. Furthermore, we also expect, but do not assume, that voters would also favor the challenger if the incumbent has behaved opportunistically in any one of the last couple of periods, not just the very last one.

Play in every period can be described by a vector that specifies the incumbent's action, the challenger's and incumbent's bids, and the proportion of voters that voted for either candidate. A player's strategy, which is a mapping from the set of all possible histories into a decision of how to behave and bid in every period for the incumbent, and how much to bid, and then behave as the incumbent for the challenger, can thus be a very complex object. We are interested in what the outcome of the game is when players adopt "simple" strategies, which are defined as follows.

Definition. *We say that a player's strategy is* ***simple*** *if it satisfies the following two restrictions:*

S1 **Anonymity.** *A player's decision about whether to challenge the incumbent and how much to bid for voters' votes may depend only on the incumbent's ability and performance, not on his name or identity, nor on what has happened before the incumbent assumed the position.*

S2 **Bounded Recall.** *In any given period, a player's decision about whether to challenge the incumbent and how much to bid for voters' votes is independent of whatever has happened more than k periods before for some finite $k \geq 1$.*[13]

Thus, a simple strategy treats incumbent players that are identical in everything except for their names identically (incumbent players do not receive "special" treatment just because of their name), and it ignores or "forgets" everything that has happened before the incumbent assumed the position or more than k periods ago. In this sense, simplicity imposes a restriction on what challenging players consider to be relevant history.

[13] Bounded recall is a standard "bounded rationality" type assumption. Note that the length of players' recall, k, can be arbitrarily large.

Importantly, when analyzing equilibria we do not assume that players consider only simple strategies – players in the game may consider whatever strategy they wish, especially when they examine the consequences of a deviation from their equilibrium strategies. However, it is easy to see that a simple strategy is a best response to other simple strategies. If all players expect other players to employ simple strategies, then the (subgame perfect) equilibrium will be in simple strategies. The reason we focus our attention on (subgame perfect) equilibria in which players' equilibrium strategies are simple is that we believe such equilibria to be more plausible.

3. The Pure Moral Hazard Problem

For simplicity, we consider first the case of pure moral hazard. This allows us to explain the equilibrium of the game with vote buying in a simpler context, where it can be more easily understood, before examining the more difficult general case in the next section.

Hence, we assume in this section that players do not differ by their ability; they all have the same ability α. We assume that players have discount rates that are sufficiently low to imply that they prefer the payoff associated with behaving well and holding the position forever to the payoff obtained from behaving opportunistically for one period.[14]

Proposition 2. *Suppose that Assumptions 1 and 2 are satisfied. The game with vote buying described above has a generically unique subgame perfect equilibrium in simple strategies. In this equilibrium, the incumbent behaves well in every period and is never challenged.*[15,16]

Proposition 2 establishes the existence of a unique "good" equilibrium that is distinguished by the fact that it is supported by simple strategies. "Bad" equilibria may exist, but the proposition implies that they must involve the use of non-simple strategies by at least some players. The proof is based on the following argument. An incumbent chooses to behave well since he knows that because of the tie-breaking rule described in Assumption 2, behaving opportunistically, while increasing his per-period payoff, would put him at a disadvantage relative to the next challenger. This reasoning does not explain, however, why this is the *unique* subgame perfect equilibrium. For example, the incumbent can choose to

[14] Thus, in this section the term "short" used in the context of Assumption 1 means one period.

[15] We use the term "generically" in the following sense: the players' payoff $b(\alpha(i_t), a_t)$ is generically not equal to some integer multiple of the smallest monetary unit m.

[16] This is also the unique Markov perfect equilibrium provided the "state" is defined as the history since either the incumbent assumed the position or k periods ago (whichever occured later).

behave opportunistically and yet the next challenger may be wary of challenging him because the challenger believes that if he becomes an incumbent himself, he would be challenged by future challengers while the current incumbent would not. As a consequence of not being challenged in the future, the incumbent has a higher future payoff than the challenger (conditional on winning the contest) and thus is willing to pay more for the votes. Expecting to be defeated, the challenger would simply decline to incur the cost required to mount a successful challenge. The proof of the proposition shows that if all players employ simple strategies, then such wariness on the part of the challenger is incompatible with the logic of subgame perfect equilibrium which requires that players' strategies be optimal with respect to each other in every possible subgame, or after every possible history. Intuitively, subgame perfect equilibria rule out the possibility of relying on "incredible threats" to support equilibrium play.

Because players employ simple strategies, the challenger at t, call him c, knows that if he survives k periods as the incumbent (i.e., he is still the incumbent at $t+k+1$) he will be treated thereafter exactly as the current incumbent i would. Therefore, k periods into the future (i.e., in period $t+k$), he would be willing to pay the same amount the incumbent would for the right to hold the position in period $t+k+1$. Simplicity of strategies again implies that the challenger that will appear k periods in the future (challenging the incumbent of period $t+k$ for the right to hold the position in period $t+k+1$) will only be judged according to his own performance while in office. He will therefore be indifferent between challenging challenger c or the incumbent i and will therefore treat both identically. Realizing this, challenger c would also realize that he faces the exact same future as the incumbent i, k rather than $k+1$ periods into the future. The same argument can be repeated to show that challenger c would be treated exactly as the incumbent i also $k-1$ periods into the future. Repeating this argument $k-2$ more times implies that challenger c and the incumbent i can expect to be treated identically by all future challengers. But, in this case, an incumbent who behaves opportunistically relinquishes his advantage as the incumbent and is surely going to be defeated by the next challenger. On the other hand, if he behaves well, he can expect the support of the voters and will win all future elections by matching challengers' bids. All challengers realize this and therefore, since mounting a challenge is costly, decline to challenge an incumbent who behaves well.

The following two examples illustrate the importance of the notion of subgame perfect as opposed to Nash equilibrium and of anonymity.[17] The fact that the equilibria described in

[17] Additional examples that demonstrate the necessity of our assumption about the tie-breaking rule, the

these examples are "pathological," lends further support to the robustness of our conclusions.

In all of the examples below, the statement "bid B" should be interpreted as "bid the highest integer multiple of m that is smaller or equal to B." Bids are to be interpreted as the respective player's total payment offer for all the votes. A player's bid for the votes depends on her discounted per period payoffs $b(\alpha, a)$ minus her per period opportunity cost $r(\alpha)$ (recall that α is assumed to be constant in this section). We denote the per-period payoff net of per period opportunity cost of a player who behaves well by $b_L = b(\alpha, 1) - r(\alpha)$, and of a player who behaves opportunistically by $b_H = b(\alpha, 0) - r(\alpha)$. By assumption, $b_H > b_L > 0$.

The following example shows that the stronger notion of rationality implied by subgame perfect as opposed to Nash equilibrium is necessary for the result of Proposition 2.

Example 1 (Subgame perfect vs. Nash equilibrium). Consider the following profile of strategies. The strategy of every challenger is to always challenge the incumbent by bidding $\frac{\delta}{1+\mu} b_H$. The strategy of every incumbent is to behave opportunistically after any history. When challenged, every incumbent bids m more that any bid smaller (by at least m) than $\frac{\delta}{1+\mu} b_H$ and declines to bid otherwise. Note that this is a Nash equilibrium of the game. Since the incumbent will be challenged regardless of how well he behaves, he may as well behave opportunistically. However, this equilibrium is not subgame perfect. If the incumbent behaves well, and matches any bid up to $\frac{1}{1+\mu} \frac{\delta}{1-\delta} b_L$ if challenged, then the next challenger is better off not challenging him since if he does, he will incur the cost of a challenge but because of the tie-breaking rule will lose.[18] But if the next challenger does not challenge, behaving well is better than behaving opportunistically which unravels the Nash equilibrium.

The next example shows that anonymity is important. That is, it is important that strategies do not depend on the incumbent's identity.

Example 2 (The importance of Anonymity). Consider the following profile of strategies. The incumbent at time 1, i_1, behaves opportunistically as long as he holds the position. Whenever challenged, he bids m more than any bid that is less (by at least m) than the discounted sum of payoffs from holding the position and behaving opportunistically, unchallenged, forever, taking the cost of bidding into account (i.e., he bids m more than any bid

part of S1 that requires players to ignore what has happened before the incumbent began holding the position, and S2 can be obtained from the authors upon request. The latter two examples also hinge on a failure of anonymity.

[18] Note that $\frac{1}{1+\mu} \frac{\delta}{1-\delta} b_L$ is the payoff obtained from behaving well while holding the position forever, taking the cost of bidding into account.

that is smaller or equal to $\frac{1}{1+\mu}b_H\left(\delta+\delta^2+\cdots\right)-m=\frac{1}{1+\mu}\frac{\delta}{1-\delta}b_H-m$, and declines to bid otherwise. The strategy of the challenger who appears in any period $t\geq 1$ is to decline to challenge the incumbent if it is the first incumbent i_1, but when facing any other incumbent, the challenger's strategy is to decline to bid if the incumbent behaved well, but to bid $\frac{1}{1+\mu}\frac{\delta}{1-\delta}b_L$ if the incumbent behaved opportunistically. As long as it holds the position, the challenger always behaves well and matches any bid that is smaller or equal to $\frac{1}{1+\mu}\frac{\delta}{1-\delta}b_L$. Along the equilibrium path, the first incumbent behaves opportunistically in every period and is never challenged. The reason for this is that future challengers treat him differently than they treat any other incumbent. They "allow" the first incumbent to behave opportunistically but they "demand" good behavior from anyone who replaces him. Since the payoff to a player who behaves well forever is smaller than that of a player who behaves opportunistically and holds the position forever, the first incumbent can never be defeated.

4. The General Problem

In this section we extend the analysis presented in the previous section to incorporate adverse selection. Namely, each player may have one out of a finite number of possible abilities $\{\alpha_1,...,\alpha_H\}$.

We introduce the following requirement which can also be thought of as capturing a notion of simplicity.

Definition. *We say that a challenger's strategy is* ***monotone*** *if after every history in which the challenger challenges an incumbent of ability* α, *he also challenges an incumbent of lower ability* $\alpha'<\alpha$.

Monotonicity can be motivated by considering the incentives that players have to reveal their true abilities. If challengers' strategies are not monotone in the incumbents' abilities and players can pretend to have a lower ability than they actually have, then incumbents would pretend that they are less capable than they really are. For the sake of simplicity we require strategies to be monotone rather than demonstrate that strategies would necessarily be monotone a more general model where players can pretend to have a lower ability than their true ability (but not a higher one).

The following proposition summarizes the important aspects of subgame perfect equilibrium behavior. For this proposition, the term "short" used in the context of Assumption 1 refers to the expected number of periods until a randomly selected challenger has the highest possible ability. Thus, when applied in this section, Assumption 1 implies that players have sufficiently low discount rates so as to prefer the payoff associated with behaving well forever to the payoff obtained from behaving opportunistically for a number of periods that is equal to the expected number of periods until a challenger of the highest possible ability appears.

Proposition 3. *Suppose that Assumptions 1 and 2 are satisfied. The game with vote buying described above has a generically unique subgame perfect equilibrium in simple and monotone strategies. In this equilibrium, with probability one, after some finite time a player with the highest possible ability assumes the position and behaves well forever. He is never replaced.*[19]

The equilibrium has the following structure. If a challenger is of a higher ability than the incumbent, he will always successfully challenge the incumbent and replace him. If a challenger is of a lower ability than the incumbent, he will not challenge the incumbent. If a challenger is of the same ability as the incumbent, he will challenge the incumbent if and only if the incumbent has behaved opportunistically in the preceding period, and his challenge will always be successful. Depending on his ability, an incumbent either behaves well in every period or behaves opportunistically in every period. A challenger's bid for the votes equals the incumbent's expected discounted sum of future benefits if the incumbent has behaved opportunistically, and is just above the incumbent's expected discounted sum of future benefits if the incumbent behaved well.[20] Thus, with probability one, after a finite number of periods a challenger who has the highest possible ability appears and becomes the incumbent. Since the tie-breaking rule favors good behavior, by behaving well a most capable incumbent can ensure that he will continue to hold the position forever. On the other hand, opportunistic behavior puts the incumbent at risk of being challenged and defeated by an equally able challenger. Assumption 1 implies that such a most capable incumbent prefers to behave well forever.

Note that, along the equilibrium path, a player that is not of the highest possible ability may behave opportunistically. This is because whenever a challenger of yet higher ability appears, the incumbent is replaced regardless of whether he acted well or not. It is only in

[19] Note that (i) the notion of genericity is identical to that used in Proposition 2; (ii) as before, this is also the unique Markov perfect equilibrium provided the "state" is defined appropriately; finally (iii) the reason we employ the notion of subgame perfet Nash equilibrium and not sequential or Bayesian equilibrium, is that our assumptions imply that players' beliefs about the type of the challenger play no role in the analysis anyway.

[20] The assumption that there exists a smallest monetary unit m guarantees the existence of a minimal bid that is strictly larger than any given incumbent's expected discounted sum of future benefits. This, in fact, is the reason for introducing this assumption.

the case that the challenger is of the same ability as the incumbent that good behavior makes a difference. Therefore, if the ratio of the probability that a challenger has a strictly higher ability than the incumbent's to the probability that the challenger has an ability larger or equal to the incumbent's is sufficiently large, the incumbent may well find it in his interest to behave opportunistically.[21]

The proof of Proposition 3 is similar to the proof of Proposition 2. Monotonicity implies that in every subgame perfect equilibrium, a more capable challenger (or, if the incumbent behaved opportunistically in the last period, a more or equally capable challenger) always successfully challenges a less able incumbent. Thus, if it is optimal for an incumbent to behave well (opportunistically) in some period, it is also optimal to behave well (opportunistically) in all other periods in which the player still holds the position. Moreover, the fact that in equilibrium a challenger always replaces a less capable incumbent implies that with probability one a player of the highest possible ability becomes the incumbent after a finite number of periods. The rest of the proof, or more specifically, the uniqueness of the equilibrium and the fact that a player of the highest possible ability prefers to behave well forever follow for the same reasons as those mentioned in the explanation of the proof of Proposition 2.[22]

5. The Advantages and Disadvantages of Vote Buying

What is the difference between the likely outcome in a model with and without vote buying? In order to answer this question, we consider a game that is identical to the game with vote buying presented in Section 2 above, except that vote buying is not permitted. Specifically, we assume that at the end of each period there is a voting contest between the incumbent and a challenger. Voters then have to decide, first, whether or not to vote, and second, for whom to vote (if they indeed decide to vote).

[21] More precisely, denote the incumbent's ability by a_i, let $\pi = \Pr(a \leq a_i)$, and let $\varphi = \Pr(a < a_i)$. The incumbent's expected payoff if he behaves well forever is $\delta b_L / (1 - \delta\pi)$, and $\delta b_H / (1 - \delta\varphi)$ if he behaves opportunistically forever. Thus, it is better for the incumbent to always behave opportunistically if and only if

$$\frac{b_L}{b_H} < \frac{1-\delta\pi}{1-\delta\varphi} \approx \frac{1-\pi}{1-\varphi} = \frac{\Pr(a > a_i)}{\Pr(a \geq a_i)}.$$

Since the incumbent's problem is stationary, it is optimal either to behave well in every period or to behave opportunistically in every period.

[22] An example that demonstrates that, absent any additional modifications to the model, monotonicity is necessary for Proposition 3 to hold can be obtained from the authors upon request.

Consider first the case where, perhaps because the cost of voting is zero or because voters derive a benefit from voting, all voters vote. In principle, a similar outcome to the one that can be obtained in a subgame equilibrium with vote buying, can also be obtained without vote buying, provided that voters behave according to the following rule, which is similar to the tie-breaking rule described in Assumption 2 above. Namely, "vote for the challenger at the end of period t if the incumbent does not have the highest possible ability, or if the incumbent has behaved opportunistically; otherwise vote for the incumbent."

This argument that a similar outcome can be obtained both with and without vote buying is subject to an important qualification. In order for voters to be able to follow the rule above and to sustain the good equilibrium outcome (in which an incumbent with the highest possible ability eventually assumes the position and behaves well thereafter) it has to be assumed that the voters can determine the ability of the incumbent, so that they can vote him out of office if he reveals himself to not have the highest possible ability, even if he behaved well. Compared to the case with vote buying, such voting behavior will lead to lower average ability of incumbents until an incumbent with the highest possible ability appears.[23] And, moreover, such voting behavior does not induce better behavior on part of the incumbent and may in fact lead to worse behavior of some incumbents.[24]

The advantage of vote buying becomes more pronounced if we consider the fact that voting entails a (small) positive cost. Because none of the voters is pivotal, the fact that voting is costly implies that none of the voters would vote and so the incumbent would keep his position regardless of whether or not he behaved well.[25] It is perhaps more plausible to consider a model in which a small number of voters who are associated with the incumbent would vote in his favor, and a few others would vote against him, but in any case, because the outcome of the competition between the incumbent and the challenger would boil down to

[23] This is due to the fact that when this is the case, incumbents with an intermediate ability (above the minimal ability but below the maximal ability) would be replaced by the challenger, regardless of whether their ability is higher or lower than the challenger's. In particular, such incumbents will be replaced by worse challengers with positive probability.

[24] This is due to the fact that with vote buying, incumbents behave well except when the expected time until a better challenger appears is too short. It therefore follows that an incumbent that behaves opportunistically when vote buying is allowed will (a fortiori) not change his behavior once he is certain to lose its position after one period. Conversely, an incumbent that behaves well when vote buying is allowed may behave opportunistically in the equilibrium of the game without vote buying because the expected duration of his incumbancy is shorter.

[25] In contrast, even if selling one's vote is costly, the equilibrium of the game with vote buying is unaffected as long as this cost is lower than the price paid for the vote.

who has a larger number of partisan supporters, it would be independent of the incumbent's behavior and ability in managing the office.

Our analysis shows that vote buying can be expected to be more efficient than no vote buying, if the difference between the benefit that a player gets from holding the position and its outside option, $b(\alpha, a) - r(\alpha)$, is increasing in the players' ability, α (recall the short discussion in the introduction of when this is likely to be the case). If $b(\alpha, a) - r(\alpha)$ is decreasing in the players' ability, then the argument presented here suggests that vote buying would be harmful, because in this case lower ability players would value the position more than high ability players. When this is the case, vote buying would allow lower ability players to bid more, win, and retain the position.[26]

Finally, even if vote buying is not permitted, the amount of resources spent in order to win voters' support in political and other campaigns (see the example of the loan-equity market in the next section) suggests that there is a sense in which it is nevertheless practiced, if only implicitly. Open vote buying may work better and avoid the negative effects on efficiency of sidestepping the laws.

6. Corporate Vote Buying

Probably the best example of a case in which voters are indeed likely to have identical preferences with respect to the behavior of their representatives and where $b(\alpha, a) - r(\alpha)$ is likely to be increasing in the players' ability, α, is that of the modern corporation. In this context, when a highly contentious issue is at stake, such as an acquisition or restructuring under new management, regular voting may turn into a proxy fight. Though potentially important, proxy fights are rare (Becht et al., 2002, pp. 89-90).[27]

A proxy mechanism that permits corporate vote buying on a per-issue basis and is further restricted to take place only before "control" as opposed to "issue" contests may improve upon the performance of the standard proxy mechanism at least in each of the following four ways: (1) Without allowing for vote buying, the proxy mechanism cannot encourage the replacement of incumbent managers of above average ability by yet higher ability managers (because the expected ability of a challenger would be lower than the incumbent's in this case, and there is no credible way in which better challengers could signal their superior ability).

[26] Note that because of Proposition 1, voters would sell their votes to the highest bidder in spite of the fact that in this case a higher bid indicates lower ability.

[27] For a comprehensive analysis of proxy contests see Mulherin and Poulsen (1998) and the references therein.

(2) As noted by Pound (1988), one of the reasons that the proxy mechanism does not perform well is that shareholders are wary of "crank" bids. Indeed, Bebchuk and Hart (2001), for example, consider the difficulty of persuading shareholders that a challenger's victory would be beneficial for them, to be the main weakness of the proxy mechanism. Allowing challengers to signal their seriousness by offering to pay shareholders for their votes may help solve this problem. (3) Corporate vote buying allows the shareholders to appropriate at least some of the private rents that would otherwise be captured entirely by incumbent managers. (4) Finally, there is an inefficiency in the system of proxy votes solicitation which, as Pound (1988) argues, gives incumbent management a vote-getting advantage.[28] Allowing both the challenger and the incumbent to bid openly for shareholders' votes may help place both contestants on an equal footage. However, as noted by Cole (2001), this advantage of vote buying may well disappear anyway with the decreasing costs of obtaining and conveying information.

Vote buying has also at least two distinct potential advantages vis-à-vis hostile takeovers, which is probably the most important mechanism used to discipline incumbent managements: (1) Because the price of a vote associated with a share can be expected to be substantially lower than the price of the share together with the associated vote, vote buying involves significantly lower transaction costs than takeovers and therefore suffers less from capital market imperfections.[29] (2) Corporate vote-trading does not distort shareholders' incentives in the way that a hostile takeover does. This distortion can take two forms: shareholders may inefficiently refuse to tender their shares to the raider in an attempt to "free-ride"

[28] See also Bebchuk and Kahan (1990).

[29] Zingales (1995) estimates the "voting premium," i.e., the difference in between the price of a share with and without a vote over the price of the share to be around 10.5% in US corporations (it may rise sharply when there is a "major and unexpected change in the ownership distribution that would facilitate a takeover." p. 1049). Here, since we only consider the trading of votes on a per-issue basis, their price is likely to be even lower. Christoffersen et al. (2002) analyse the implicit trade of votes by using security loans. They find a substantial effect of voting record dates on the *quantity* of security loans but only a trivial effect on the *price* of these loans (and thus implicitly on the price of votes). Shleifer and Vishny (1997, p. 747-748) review several studies that compare the prices of shares with identical dividend rights, but differential voting rights. The fact that the estimates of the voting premium are large for some continental European countries and for Israel may be related to higher benefits of control (as suggested by Zingales), which would be diminished by vote-trading, or to the observation that outside of the US the power of large shareholders vis-à-vis small ones is frequently a more significant problem than managerial moral hazard (see La Porta, Lopez-de-Silanes, Shleifer, and Vishny, 1998). Becht et al. (2002) also list several papers that estimate voting premia for different countries, but warn that "the studies at best imperfectly control for all the factors affecting the price differential" (p. 76).

on the subsequent increase in the value of the corporation (Grossman and Hart, 1980); or shareholders may be inefficiently pressured to tender their shares, fearing that if they don't, they will be left holding lower value shares (Bebchuk, 1985). As the analysis of Grossman and Hart (1980) shows, the free-rider problem associated with efficiency-enhancing takeovers implies that takeovers are unlikely to succeed unless raiders are able to capture some private rents from taking-over poorly managed corporations and installing better management teams. This argument is used to justify the introduction of modifications to corporations' charters that allow for the possibility of (inefficient) dilution of the corporations' assets after successful takeovers. In contrast, installing new and more efficient management through vote buying does not hinge on the possibility of dilution. Thus, permitting vote buying enables tightening the restrictions on dilution, which in turn can serve to increase the corporation's value by eliminating the moral hazard problem that arises when management can appropriate the corporation's assets through dilution (Burkart, Gromb, and Panunzi, 1998).

Interestingly, Christoffersen et al. (2005) document the existence of a market for corporate votes within the equity-loan market (see also Hu and Black, 2006, for other examples of markets for corporate votes). The equity-loan market is a market that facilitates, for a price, the borrowing of equity for short periods of time. The reason that this market facilitates vote buying is that a borrower may vote the shares it borrowed during the time in which they are in its possession, without being exposed to the economic risk associated with a change in the value of the borrowed equity. Christoffersen et al. show that the equity-loan market is especially active around proxy fights, which, as they say, implies the existence of competition for votes.

Appendix

Proof of Proposition 1. Fix a time t and a history $h_t = \left(l_\tau, r_\tau, b_\tau, p_\tau^C, p_\tau^I, n_\tau^C, n_\tau^I\right)_{\tau=1}^{t}$. Suppose that the incumbent's price offer is different from that of the challenger's. Since there is a continuum of voters, any single voter cannot affect who will hold the position in the next period. It is therefore a dominant strategy for her to sell her vote to the player who is willing to pay the higher price for it. ■

Proof of Proposition 2. We first describe the subgame perfect equilibrium and then prove that it is the generically unique equilibrium in which players employ simple strategies. Recall that the statement "bid b" should be interpreted as "bid the highest integer multiple of m smaller or equal to b." Denote the per-period payoff net of per period opportunity cost of a player who behaves well and opportunistically by $b_L = b(\alpha, 1) - r(\alpha)$ and $b_H = b(\alpha, 0) - r(\alpha)$, respectively. By our assumptions, $b_H > b_L > 0$. Consider the following profile of strategies. Every incumbent behaves well after every history. An incumbent who behaved well in the last period matches any challenge that is equal or below the highest integer multiple of m equal or below $\frac{1}{1+\mu}\frac{\delta}{1-\delta}b_L$, and declines to respond to higher challenging bids. An incumbent who behaved opportunistically in the last period responds to any challenge that is equal or below the highest integer multiple of m equal or below $\frac{1}{1+\mu}\frac{\delta}{1-\delta}b_L - m$ by bidding the smallest integer multiple of m above it, and declines to respond to higher challenging bids. Every challenger declines to challenge after any history in which the incumbent behaved well in the last period. A challenger challenges the incumbent whenever the latter behaved opportunistically in the previous period by bidding $\frac{1}{1+\mu}\frac{\delta}{1-\delta}b_L$. Voters sell their votes to the player who offers them a higher price. Ties are resolved according to the tie-breaking rule described in Assumption 2. It is immediate to verify that this profile of strategies constitutes a subgame perfect equilibrium. Notice that if the incumbent is ever disloyal, he is immediately replaced. Since by Assumption 1 he is sufficiently patient, he prefers the payoff associated with continuing to hold the position while behaving well forever, i.e. $\frac{\delta}{1-\delta}b_L$, to the payoff he would get by deviating and behaving opportunistically for one period, i.e. δb_H.

The proof of uniqueness follows from the next two lemmas.

Lemma 1. *Generically, in every subgame perfect equilibrium in simple strategies the incumbent is successfully challenged and replaced at the end of any period in which he has behaved opportunistically.*

Proof. Fix a subgame perfect equilibrium in simple strategies (SPESS). Suppose that at some period t, the incumbent I has behaved opportunistically. Denote the SPESS discounted sum of payoffs the incumbent expects to get from period $t+1$ onwards if he succeeds in deterring or defeating the challenger in period t by π^I_{t+1}. Similarly, let π^C_{t+1} denote the SPESS discounted sum of payoffs that the challenger in period t, denoted C, expects to get from period $t+1$ onwards if he succeeds in defeating the incumbent and assuming the position in period $t+1$. Note that since the incumbent in period $t+1$ can always choose to behave opportunistically at $t+1$ and decline to respond to challenges, both $\pi^I_{t+1}, \pi^C_{t+1} \geq \delta b_H > 0$.

Since the tie-breaking assumption favors the challenger, if $\pi^C_{t+1} \geq \pi^I_{t+1}$ the challenger can defeat the incumbent by bidding no more than the highest integer multiple of m smaller or equal to $\frac{1}{1+\mu}\pi^C_{t+1}$, and still generically obtain a positive payoff. Therefore, if $\pi^C_{t+1} \geq \pi^I_{t+1}$, then whenever the incumbent at t has behaved opportunistically at t, the challenger at t will successfully challenge and replace him.

We show that there is a strategy for C under which $\pi^C_{t+1} \geq \pi^I_{t+1}$. In a SPESS, since challenges are costly, if a challenge is mounted, then it is successful. We can therefore distinguish between the following two cases: (1) There exists some period $T \geq t$ where along the SPESS path, I is successfully challenged, and (2) I is never challenged on the SPESS path after and including period t.

Consider case (1) first. Since if $T = t$, then along the SPESS path C successfully challenges I at t, we may assume that $T > t$. Now, C knows that if he ever makes it to period $T-1$ as the incumbent, then starting from period T onwards, he can expect a larger or equal future discounted payoff than I since the fact that I is successfully challenged at (the end of) period T implies that $\pi^I_T \leq \delta b_H \leq \pi^C_T$. Therefore, if challenged at the end of period $T-1$, C would be willing to pay at least as much as I would in order to remain in power. S1 implies that the challenger that appears in period $T-1$ expects a future discounted payoff that depends only on how he himself performs while he holds the position. In particular, he is indifferent between challenging I or C. Consequently, the player that appears in period $T-1$ will (successfully) challenge C only if he will also successfully challenge I. Therefore, it must be the case that $\pi^C_{T-1} \geq \pi^I_{T-1}$. Repeating the same argument for period $T-2$ implies that $\pi^C_{T-2} \geq \pi^I_{T-2}$. Repeating the same argument $T-t-3$ more times, implies that it must also be the case that $\pi^C_{t+1} \geq \pi^I_{t+1}$.

Consider now case (2). By assumption, I is not challenged along the SPESS path. In particular, on the SPESS path, I is the incumbent in period $t+k+1$. Suppose now that C successfully challenges I in period t and then adopts the same strategy that I uses from time $t+1$ onwards. If C survives unchallenged to period $t+k+1$, then since he adopted I's strategy, the players' bounded recall implies that C will be treated thereafter exactly as I would, and thus $\pi^C_{t+k+1} \geq \pi^I_{t+k+1}$. The backwards induction argument presented in case (1) above can be then re-applied to imply that $\pi^C_{t+1} \geq \pi^I_{t+1}$. Therefore, to complete the proof of the lemma, we must show that C will indeed not be challenged between periods $t+1$ and $t+k$. This too is shown by backwards induction. Consider period $t+k$, and suppose that C has not been challenged between periods $t+1$ and $t+k-1$. If C defeats the challenger at $t+k$, then, because of the players' bounded recall, he can expect to be treated exactly as I would thereafter, and thus $\pi^C_{t+k+1} \geq \pi^I_{t+k+1}$. C would therefore be willing to pay as much as I would in order to defeat the challenger at $t+k$. S1 implies that the challenger that appears in period $t+k$ expects a future discounted payoff that depends only on how he himself performs while he holds the position. In particular, he is indifferent between challenging I or

C. By assumption, on the SPESS path, the challenger at $t+k$ refrained from challenging I, therefore, he must also refrain from challenging C. But this implies that C and I expect the same future discounted payoff starting from period $t+k$, and thus $\pi^C_{t+k} \geq \pi^I_{t+k}$. Repeating the same argument $k-1$ more times implies that C would not be challenged anytime between periods $t+1$ and $t+k$. This completes the proof of the lemma.[30] ■

Lemma 2. *In every subgame perfect equilibrium in simple strategies, the incumbent is never challenged (or replaced) at the end of a period in which he has behaved well.*

Proof. The proof is similar to the proof of the previous lemma. Fix a subgame perfect equilibrium in simple strategies (SPESS) and a time t. By the previous lemma, the incumbent I would have been replaced had he behaved opportunistically at t or before, therefore we may assume that the incumbent has behaved well since he started holding the position. Denote the SPESS expected discounted payoff to the incumbent at t if he keeps the position until period $t+1$ by π^I_{t+1}. Similarly, denote the SPESS expected discounted payoff to the challenger at t, denoted C, if he assumes the position at the end of period t by π^C_{t+1}. Suppose that contrary to what is claimed, the incumbent at t is successfully challenged and replaced. (Recall that in a SPESS, since challenging the incumbent is costly, if a challenge is mounted, then it is successful.) Since I has been loyal at t, he is favored by the tie-breaking rule, therefore the fact that he is successfully challenged implies that it must be the case that $\pi^C_{t+1} > \pi^I_{t+1}$. Repeating the backwards induction arguments in the proof of Lemma 1 with the roles of I and C reversed, shows that this is impossible. A contradiction. ■

Proof of Proposition 3. The proof utilizes the following lemma.

Lemma 3. *Suppose that Assumptions 1 and 2 are satisfied. In every subgame perfect equilibrium in simple and monotone strategies, a higher ability player always defeats a lower ability player.*

Proof. Monotonicity guarantees that higher ability players will be better off after k periods. The rest of the proof follows from the backwards induction arguments given in the proofs of Lemma 1 and Lemma 2. ■

[30] The argument bears a superficial resemblance to Jéhiel (1995) and Jéhiel and Moldovanu (1995). However, our argument is different. Among other things, the assumption of bounded recall, by itself, is not sufficient for our result, and, while in Jéhiel (1995) and Jéhiel and Moldovanu (1995) the players' forecasts or behavior strategies, respectively, are history-independent, this is obviously not the case here.

As a consequence of Lemma 3, Proposition 1 and the tie-breaking rule, in every subgame perfect equilibrium in simple and monotone strategies (SPESMS), an incumbent who has behaved well at t is successfully challenged and replaced at the end of period t if and only if the challenger has a higher ability than his; an incumbent who has behaved opportunistically at t is successfully challenged and replaced at the end of period t if and only if the challenger has a higher or equal ability to his. Thus, if it is optimal for a player to behave well (opportunistically) in some period t, it is also optimal to behave well (opportunistically) in all the following periods in which that player still holds the position.

Remember that $F(a)$ denotes the probability that any player has ability lower or equal to α. Let $B(\alpha_h, 1)$ denote the expected present discounted payoff net of opportunity costs of an incumbent of type $\alpha_h \in \{\alpha_1, ..., \alpha_H\}$ who behaves well in every period in which he holds the position (relative to its reservation payoff). Similarly, let $B(\alpha_h, 0)$ denote the expected present discounted payoff net of opportunity costs of an incumbent of type $\alpha_h \in \{\alpha_1, ..., \alpha_H\}$ who behaves opportunistically in every period in which he holds the position (relative to its reservation payoff). The tie-breaking rule together with the fact that, by Lemma 3, higher ability players always defeat lower ability players implies that,

$$\begin{aligned} B(\alpha_h, 1) &= (b(\alpha_h, 1) - r(\alpha_h)) \left[\delta + F(\alpha_h)\delta^2 + F(\alpha_h)^2 \delta^3 + ...\right] \\ &= \frac{\delta}{1 - F(\alpha_h)\delta} (b(\alpha_h, 1) - r(\alpha_h)) \end{aligned}$$

and

$$\begin{aligned} B(\alpha_h, 0) &= (b(\alpha_h, 0) - r(\alpha_h)) \left[\delta + F(\alpha_{h-1})\delta^2 + F(\alpha_{h-1})^2 \delta^3 + ...\right] \\ &= \frac{\delta}{1 - F(\alpha_{h-1})\delta} (b(\alpha_h, 0) - r(\alpha_h)). \end{aligned}$$

Generically, either $B(\alpha_h, 0) < B(\alpha_h, 1)$ or $B(\alpha_h, 0) > B(\alpha_h, 1)$. Thus, generically, in every SPESMS, a player of type α_h behaves well in every period in which he is the incumbent if and only if $B(\alpha_h, 0) < B(\alpha_h, 1)$. Otherwise, he behaves opportunistically in every period in which he holds the position. Consequently, in every SPESMS players must employ the following strategies. For an incumbent of ability α_h, if $B(\alpha_h, 0) < B(\alpha_h, 1)$ then the incumbent behaves well in every period; if $B(\alpha_h, 0) > B(\alpha_h, 1)$ then the incumbent behaves opportunistically in every period.

Incumbents who behave well match any offer below or equal to their expected discounted future payoffs, taking into account their bidding costs, and decline to bid if the offer is higher. Opportunistic incumbents bid m more than any offer that falls short by at least m of their expected discounted sum of future payoffs, taking into account their bidding costs. If the offer is higher, they decline to bid. If a challenger is more able than an incumbent who behaves well or if he is more or equally able than an opportunistic incumbent, he bids the lowest amount that implies that the incumbent declines to make a counter-bid.

Otherwise, the challenger declines to challenge. Finally, voters sell their votes to the player who offers them the higher price, and ties are resolved according to Assumption 2. Since, generically, only one profile of strategies satisfies all the requirements above, the SPESMS is generically unique. In this equilibrium, since higher ability players defeat lower ability ones, eventually the position is held by a player with the highest possible ability. Assumption 1 is taken to mean here that δ is sufficiently close to 1 to imply $B(\alpha_H, 1) > B(\alpha_H, 0)$. Consequently, the most able player that eventually assumes the position continues to behave well forever. ■

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