The Relation between Ground Motion, Earthquake Source Parameters and Attenuation: Implications for Source Parameter Inversion and Ground Motion Prediction Equations

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Key Points:

- Theoretical equations for the displacement, velocity and acceleration root-mean-square are derived, relying on the Brune omega-squared model.
- Using these relations, source parameter inversion is performed in the time domain, producing stable and robust estimates.
- Physics-based Ground Motion Prediction Equations are presented, exhibiting good agreement with previous empirical ones for all magnitudes.
Abstract

Theoretical equations relating the root-mean-square (rms) of the far-field ground motions with earthquake source parameters and attenuation are derived for Brune’s omega-squared model that is subject to attenuation. This set of model-based predictions paves the way for a completely new approach for earthquake source parameter inversion, and forms the basis for new physics-based Ground Motion Prediction Equations (GMPEs). The equations for ground displacement, velocity and acceleration constitute a set of three independent equations with three unknowns: the seismic moment, the stress drop and the attenuation parameter. These are used for source parameter inversion that circumvents the time-to-frequency transformation. Initially, the two source parameters and the attenuation constant are solved simultaneously for each seismogram. Sometimes, however, this one-step inversion results in ambiguous solutions. Under such circumstances, the procedure proceeds to a two-step approach, in which a station-specific attenuation parameter is first determined by averaging the set of attenuation parameters obtained from seismograms whose one-step inversion yields well constrained solutions. Subsequently, the two source parameters are solved using the averaged attenuation parameter. It is concluded that the new scheme is more stable than a frequency-domain method, resulting in considerably less within-event source parameter variability. The above results together with rms-to-peak ground motion relations are combined to give first-order GMPEs for acceleration, velocity and displacement. In contrast to empirically-based GMPEs, the ones introduced here are extremely simple and readily implementable, even in low seismicity regions, where the earthquake catalog lacks strong ground motion records.

1 Introduction

Resolving earthquake source parameters is key for addressing fundamental questions in earthquake science. The determination of earthquake source parameters is model-based, and is commonly done in the frequency-domain using far-field records. The most widely adopted earthquake source model, describing the far-field body-wave radiation, is that of Brune (Brune, 1970). According to Brune’s model, the far-field ground motions read as:

\[ \frac{d^n}{dt^n} \Omega(f) = (2\pi f)^n \frac{\Omega_0}{1 + \left( \frac{f}{f_0} \right)^2}, \]

where \( f \) is the frequency, \( f_0 \) is a corner frequency, and \( n = 0, 1 \) or 2 for displacement, velocity or acceleration spectra, respectively. According to (1), the displacement spectra are constant and equal \( \Omega_0 \) at frequencies well below the corner frequency, and decay as \( f^{-2} \) above it (solid curves in top panels of Figure 1). The velocity spectra increase proportionally to \( f \) below the corner frequency, and decrease as \( f^{-1} \) above it (solid curves in middle panels of Figure 1). Finally, the acceleration spectra rise as \( f^{2} \) up to the corner frequency, and are flat above it (solid curves in bottom panels of Figure 1). The spectral parameters \( \Omega_0 \) and \( f_0 \) are related to the seismic moment, \( M_0 \), and the stress drop, \( \Delta \tau \), as follows (Eshelby, 1957):

\[ \Omega_0 = \frac{M_0 U_{\phi\theta} F_s}{4\pi \rho C_s^2 R}, \]

and:

\[ f_0 = k C_s \left( \frac{16 \Delta \tau}{7 M_0} \right)^{1/3}, \]

where \( U_{\phi\theta} \) is the radiation pattern, \( F_s \) is the free-surface correction factor, \( C_s \) is the S-wave velocity, \( R \) is the hypocentral distance, \( \rho \) is the density and \( k \) is a constant (Brune, 1970;
Madariaga, 1976, Sato & Hirasawa, 1973). Relation (2b) is valid for a circular crack embedded within an infinite homogeneous isotropic Poissonian medium. The effect of site and path attenuation can be modeled by multiplying the source spectra by an exponent function:

\[ \frac{d^n}{dt^n} \Omega(f) = (2\pi f)^n \frac{\Omega_0}{1+(f/f_0)^n} \exp(-\pi \kappa f), \]

where \( \kappa \) is an attenuation parameter that embodies both anelastic and near surface attenuations (Anderson & Hough, 1984). Positive \( \kappa \) results in a high frequency decay of the spectra, and effectively produces an additional corner frequency, \( f_\kappa \) (dashed curves in Figure 1, \( f_\kappa \) is referred to as \( f_{\text{max}} \) in Hanks, 1982):

\[ f_\kappa = \frac{1}{\pi \kappa} \]

In general, it is more difficult to resolve the spectral attributes when \( f_0/f_\kappa > 1 \) than \( f_0/f_\kappa < 1 \). To see why, it is useful to compare attenuated and non-attenuated earthquake spectra, with fixed \( \Omega_0 \) and \( f_0 \) and varying \( f_0 \) to \( f_\kappa \) ratio (Figure 1). When \( f_0 \) is well below \( f_\kappa \), the source spectra are only slightly attenuated (dotted curves in left-hand panels of Figure 1), and the source corner frequencies are well resolved. In contrast, when \( f_0 \) is larger than \( f_\kappa \), the source spectra are more strongly distorted (right-hand panels of Figure 1), and consequently the source corner frequencies can only be resolved if the attenuation parameters may be resolved as well. The \( f_0 \) to \( f_\kappa \) ratio is thus an important parameter affecting ground motion intensity and limiting the resolution of \( f_0 \). Hereafter this ratio is labeled as \( \alpha_0 \):

\[ \alpha_0 = f_0/f_\kappa = \pi \kappa f_0. \]

In this study, a new approach for seismic moment and stress drop inversion is introduced that circumvents the time-to-frequency data transformation. This is advantageous, since the modeling of ground motion spectra is non-trivial and introduces elements of subjectivity into the process. For example, some researchers resample the power spectrum coefficients at constant log of frequency units (e.g., Allmann & Shearer, 2009; Lior & Ziv, 2017; Ziv & Lior, 2016), some weight them proportionally to \( 1/\log(f) \) (e.g., Kaneko & Shearer, 2015; Trugman & Shearer, 2017), and some model them as is (e.g., Chen & Shearer, 2011; Prieto et al., 2004; Shearer et al., 2006). Because different schemes yield notably different results (Figure 2), seeking ways to bypass the biases inherited in the modeling of ground motion spectra is instructive.

Next, theoretical equations relating the root-mean-square (rms) of the far-field ground motions (displacement, velocity and acceleration) to their spectral attributes are derived using the omega-squared model (Brune, 1970). These results, which constitute a set of three independent equations with three unknowns, are then used to invert for \( \Omega_0, f_0 \) and \( \kappa \). The trade-off between the effects of the source corner frequency and the attenuation parameter is quantified and addressed. The new inversion scheme is validated and assessed using an extensive dataset of seismograms. It is shown that the new approach is more stable than a frequency-domain inversion. Empirical relations between the peak ground motion and the rms of the ground motions are established. Finally, these empirical relations and the theoretical results for the ground motion rms are combined to give first order ground motion prediction equations.
2 Far-Field Model

2.1 The relation between ground motion and source parameters

Lior and Ziv (2017) (hereafter referred to as LZ17) obtained expressions relating the rms of ground acceleration with the source parameters and $\kappa$ (presented below for completeness). Their approach is now used to obtain equivalent expressions for the rms of the ground displacement and velocity. From Parseval’s theorem, the ground motion rms can be calculated in both the time and the frequency domains:

$$y_{\text{rms}} = \sqrt{\frac{\int_0^T |y(t)|^2 dt}{T}} = \sqrt{\frac{\int_0^\infty |Y(f)|^2 df}{T}},$$  \hspace{1cm} (6)

where $y(t)$ is the ground motion time series, $Y(f)$ is the ground motion spectra and $T$ is the record interval. The displacement, velocity and acceleration ground motion rms are obtained by substituting $Y(f)$ in (6) with Equation (3):

$$\left(\frac{d^n}{dt^n}D\right)_{\text{rms}} = \Omega_0 \sqrt{\frac{2}{T}} \int_0^\infty \frac{(2\pi f)^n}{(1 + (f/f_0)^\kappa)} \exp(-2\pi\kappa f) \, df.$$ \hspace{1cm} (7)

The solutions of these integrals are:

$$D_{\text{rms}}^{\text{exact}} = \Omega_0 \frac{a_0}{\pi^{3/2} kT} \sqrt{G_{1,1}^{3,1} \left( \frac{1}{2}, \frac{3}{2}, \left| a_0^2 \right| \right)},$$  \hspace{1cm} (8a)

$$v_{\text{rms}}^{\text{exact}} = 2\pi \Omega_0 \left( \frac{a_0}{(2\pi)^{1/4}(\pi k)^{3/4}} \right)^{3/2} \frac{\pi c_i(2a_0)[2a_0 \cos(2a_0) + \sin(2a_0)] + \sqrt{\pi - 2\pi i(2a_0)[\cos(2a_0) - 2a_0 \sin(2a_0)]}}{\sqrt{\pi a_0 - 2a_0 \pi i(2a_0)[2a_0 \sin(2a_0) - 3 \cos(2a_0)]}},$$  \hspace{1cm} (8b)

$$a_{\text{rms}}^{\text{exact}} = (2\pi)^2 \Omega_0 \left( \frac{a_0}{(2\pi)^{1/4}(\pi k)^{3/4}} \right)^2 \frac{\pi c_s(2a_0)[2a_0 \cos(2a_0) + 3 \sin(2a_0)] + \sqrt{\pi a_0 - 2a_0 \pi i(2a_0)[2a_0 \sin(2a_0) - 3 \cos(2a_0)]}}{\sqrt{\pi a_0 - 2a_0 \pi i(2a_0)[2a_0 \sin(2a_0) - 3 \cos(2a_0)]}},$$  \hspace{1cm} (8c)

where $D_{\text{rms}}, V_{\text{rms}}$ and $A_{\text{rms}}$ are the displacement, velocity and acceleration rms, respectively, the superscript exact signifies exact solutions, $G_{\text{sp}}^{\text{pq}}$ is the Meijer G-function, and $c_i$ and $s_i$ are the cosine and the sine integral functions, respectively. As the rms of the ground motion may be calculated directly from the seismograms in the time-domain, these three independent expressions can be used to invert for the spectral parameters $\Omega_0$, $f_0$ and $\kappa$, without having to transform the data from the time to the frequency domain. Approximate expressions that match the exact solutions when $f_0 \ll f_\kappa$ and $f_0 > f_\kappa$ are as follows (the approach for approximating (8) is detailed in LZ17):

$$D_{\text{rms}}^{\text{approx}} = \Omega_0 \frac{\pi f_0}{2 \pi T^{1 + 0.5 \pi^2 k f_0}},$$  \hspace{1cm} (9a)

$$v_{\text{rms}}^{\text{approx}} = 2\pi \Omega_0 \frac{\pi f_0}{2 \pi T^{1 + 4/3 \pi^2 k f_0}},$$  \hspace{1cm} (9b)

$$A_{\text{rms}}^{\text{approx}} = (2\pi)^2 \Omega_0 \frac{f_0^2}{\sqrt{\pi T^{1 + 1.5^{1/4} \pi^2 f_0^2}}},$$  \hspace{1cm} (9c)

where the superscript approx signifies analytic approximations. Plots of exact and approximate solutions as a function of $a_0$ are shown in Figure 3 for $D_{\text{rms}}$ and $V_{\text{rms}}$ (see Figure 2 in LZ17 for $A_{\text{rms}}$). The discrepancies between the two solutions reach a few percent when $f_0 \approx f_\kappa$. The approximate expressions in (9) may be expressed in terms of the physical source parameters $M_0$ and $\Delta \tau$, by substituting Equations (2) into (9):
\[ D_{\text{rms}}^{\text{approx}} = M_0^{5/6} \Delta \tau^{1/6} \frac{\beta_D}{\sqrt{\tau R}} \left[ 1 + 0.5 \pi^2 kC_s^2 \left( \frac{16 \Delta \tau}{7 M_0} \right)^{1/3} \right]^{1/2}, \]  
\[ V_{\text{rms}}^{\text{approx}} = \sqrt{M_0 \Delta \tau} \frac{\beta_V}{\sqrt{\tau R}} \left[ 1 + \pi^4/3 kC_s^2 \left( \frac{16 \Delta \tau}{7 M_0} \right)^{1/3} \right]^{3/2}, \]  
\[ A_{\text{rms}}^{\text{approx}} = M_0^{1/3} \Delta \tau^{2/3} \frac{\beta_A}{\sqrt{\tau R}} \left[ 1 + 1.5^{-1/4} \pi^4 kC_s^2 \left( \frac{16 \Delta \tau}{7 M_0} \right)^{1/3} \right]^{2/3}, \]

in which \( \beta_D = U_{\phi \theta} F_5(16/7)^{1/6} / \sqrt{kC_s} / (\sqrt{2} \pi^4 \rho C_s^2) \), \( \beta_V = 2 \pi U_{\phi \theta} F_s(\sqrt{16/7})^{3/2} / (\sqrt{2} \pi^4 \rho C_s^2) \) and \( \beta_A = 4 \pi U_{\phi \theta} F_s(16/7)^{2/3} (kC_s)^2 / (\sqrt{2} \pi^4 \rho C_s^2) \). For large magnitudes, the second terms in the square brackets are much smaller than unity, and the above expressions simplify to:

\[ \lim_{a_0 \to 0} D_{\text{rms}} = M_0^{5/6} \Delta \tau^{1/6} \frac{\beta_D}{\sqrt{\tau R}}, \]  
\[ \lim_{a_0 \to 0} V_{\text{rms}} = \sqrt{M_0 \Delta \tau} \frac{\beta_V}{\sqrt{\tau R}}, \]  
\[ \lim_{a_0 \to 0} A_{\text{rms}} = M_0^{1/3} \Delta \tau^{2/3} \frac{\beta_A}{\sqrt{\tau R}}. \]

In this regime, \( D_{\text{rms}} \) and \( V_{\text{rms}} \) (but not \( A_{\text{rms}} \)) are insensitive to the attenuation. In contrast, for small magnitudes, the second terms in the square brackets are much larger than unity, and Equations (10) simplify to:

\[ \lim_{a_0 \to \infty} D_{\text{rms}} = M_0 \frac{\beta_D}{\sqrt{\tau R}} [0.5 \pi^2 kC_s^2 (16/7)^{1/3}]^{1/2}, \]  
\[ \lim_{a_0 \to \infty} V_{\text{rms}} = M_0 \frac{\beta_V}{\sqrt{\tau R}} [\pi^4/3 kC_s^2 (16/7)^{1/3}]^{3/2}, \]  
\[ \lim_{a_0 \to \infty} A_{\text{rms}} = M_0 \frac{\beta_A}{\sqrt{\tau R}} [1.5^{-1/4} \pi^4 kC_s^2 (16/7)^{1/3}]^{2/3}. \]

In that case, the ground motion is insensitive to the stress drop. In a later section, the conditions described by Equations (11) and (12) are referred to as the \( \Delta \tau \)-dependent and \( \Delta \tau \)-independent regimes, respectively.

### 2.2 Low frequency \( D_{\text{rms}} \) correction

The finiteness of the data interval, the frequency above which the seismograms are high-passed and the sensor’s dynamic range set a lower limit on the signals’ frequency content. In this study, the values of this cut-off frequency, denoted as \( f_l \), are set to be equal to the largest of the following: the reciprocal of the data interval and the frequency above which the seismograms are high-passed. Among the three ground-motion rms in (9) and (10), \( D_{\text{rms}} \) is the one that is most affected by \( f_l \). Thus, observed \( D_{\text{rms}} \) should be corrected for the missing low frequency content:

\[ D_{\text{rms}}^{\text{obs+}} = \sqrt{(D_{\text{rms}}^{\text{obs}})^2 + (D_{\text{rms}}^{\text{corr}})^2}, \]

with \( D_{\text{rms}}^{\text{obs+}} \) and \( D_{\text{rms}}^{\text{corr}} \) being the observed rms and the correction term, respectively. Because under normal circumstances \( f_l \ll f_c \), attenuation does not affect frequencies below \( f_l \), and the displacement correction term is obtained via solution of (1) and (6) as:

\[ D_{\text{rms}}^{\text{corr}} = \Omega_0 \frac{1}{\sqrt{\frac{\pi}{2} \int_0^f \frac{1}{1 + \left( \frac{f}{f_0} \right)^2} df}} = \Omega_0 \frac{\sqrt{\pi}}{\pi} \left[ \frac{f_0 f_l}{f_0^2 + f_l^2} + \tan^{-1} \left( \frac{f_l}{f_0} \right) \right]. \]
Finally, for \( f_t \ll f_0 \), the above expression may be approximated as:

\[
D_{rms}^{corr} = \Omega_0 \sqrt{\frac{f_t}{f_0}}.
\]  

(15)

3 Validation of model predictions

Model predictions are validated using a composite dataset of 6320 seismograms compiled by LZ17. Data selection criteria and processing steps are as in LZ17, except that here the high-pass filter is set to 0.06 instead of 0.02 Hz. Hypocentral distances and magnitudes are limited to 60 km and 3 to 7.6, respectively. As in LZ17, the data interval used in this study is set as: \( T = 1/f_0 + R/C_s \). The first term is a liberal estimate of the rupture duration (Hanks & McGuire, 1981), with \( f_0 \) obtained from Equation (2b) using the catalog magnitude and a stress drop of 1 MPa (this is merely used for setting the data interval, whereas the actual source corner frequency is later on calculated differently). The second term accounts for the spreading of the wave-packet with distance from the earthquake source (Boore & Thompson, 2014). Here and in a later part of this study, results obtained using the new approach are compared to those calculated from the ground motion spectra. The frequency-domain source parameter inversion is carried out in two steps. First, \( \kappa \) is obtained by fitting the high-frequency acceleration spectra to:

\[
\ln \left( \hat{\Omega}(f) \right) = a - \pi \kappa f \quad ; \quad 10 \text{ Hz} < f < 25 \text{ Hz},
\]

(16)

with \( \hat{\Omega}(f) \) being the acceleration amplitudes, and \( a \) and \( \kappa \) being the fitting coefficients (Anderson & Hough, 1984). Subsequently, \( \Omega_0 \) and \( f_0 \) are obtained by fitting the logarithm of the acceleration spectra, resampled in equal log-of-frequency bins, with Equation (3) using \( \kappa \) obtained in the first step.

The validation of the model prediction consists of comparing the observed ground motion rms, computed directly from the seismograms in the time-domain, with Equations (9) using \( \Omega_0 \), \( f_0 \) and \( \kappa \) obtained from the frequency-domain source parameter inversion. The results of this comparison for \( A_{rms} \) are shown and discussed in LZ17, and the results for the velocity, and the corrected and uncorrected displacements are presented in Figure 4 (top panels). Also shown in Figure 4 (bottom panels) are the discrepancies between observed and modeled rms as a function of the ratio between the source corner frequency and the attenuation corner frequency, \( \alpha_0 \). Inspection of Figure 4 reveals good agreement between modeled and observed ground motion rms. While the discrepancy of the uncorrected displacement rms increases with decreasing \( \alpha_0 \) (bottom-left panel), that of the corrected displacement rms does not (bottom-middle panel). It is thus concluded that the displacement correction term is imperative, especially for low \( \alpha_0 \) seismograms, which are generally rich in low frequencies.

4 Source parameter inversion circumventing the time-to-frequency data transformation

4.1 General

The results detailed in the previous section pave the way for a source parameter inversion method that circumvents the time-to-frequency data transformation. Specifically, Equations (8a)-(8c) constitute a set of three independent equations with three unknowns, in which the data vector is \( \mathbf{D}_{rms}^{obs+}, \mathbf{V}_{rms}^{obs+}, \mathbf{A}_{rms}^{obs+} \), obtained in the time domain, and the model vector is \( (\Omega_0, f_0, \kappa) \). The objective function, \( OF \), used in this study is a measure of the discrepancy between the observed and the predicted ground motion rms:
The three model parameters are solved simultaneously for each seismogram. Sometimes, however, this one-step inversion results in ambiguous solutions for \( f_0 \) and \( \kappa \). Under such circumstances, the procedure proceeds to a two-step approach, in which a station-specific \( \kappa \) is first determined by averaging the set of \( \kappa \)s obtained from seismograms whose one-step inversion yields well constrained solutions, and then \( \Omega_0 \) and \( f_0 \) are solved using the averaged \( \kappa \). The decision whether to proceed from a one-step to a two-step inversion is based on the quality of the one-step solution (further explained in the next section).

4.2 Single-step inversion

For each seismogram, a 3D grid-search algorithm is employed that finds \( \Omega_0 \), \( f_0 \) and \( \kappa \) corresponding to the smallest \( OF \) (Equation 17). To assess the robustness of this approach, it is useful to examine contour diagrams of the objective function in \( \log(f_0)-\log(f_\kappa) \) space (Figure 5). In many cases, this approach yields one unique solution which may be either well-constrained (panel a of Figure 5) or poorly-constrained (panel b of Figure 5). In some cases, however, the solutions are ambiguous with \( OF \) minima on either side of the \( f_0 = f_\kappa \) line (panel c of Figure 5). In these situations, because the two solutions share similar \( \Omega_0 \), their spectra are nearly identical up to the lowest corner frequency \( \approx \min(f_0, f_\kappa) \), at which they both peak, but differ in the rate of decay beyond that frequency (panel f of Figure 5). Thus, in addition to minimizing \( OF \), it is also vital to quantify the degree of uncertainty in the \( f_0-f_\kappa \) space.

Poorly-constrained solutions are identified using an uncertainty measure, \( \delta \), that is proportional to the normalized area enclosed within the \( OF = 5\% \) contour in Figure 5 (top diagrams). The distribution of \( \delta \) is bimodal, with the largest magnitudes residing mostly within the narrow peak around \( \delta \approx 2\% \) and the smallest magnitude earthquakes residing mostly within the wider peak for which \( \delta > 6\% \) (Figure 6). Yet, there are quite a few exceptions, and each record should be examined individually regardless of its magnitude. Based on visual inspection of a large number of \( OF \) maps, such as those shown in Figure 5, it is concluded that single-step solutions are well-constrained and unique if \( \delta < 6\% \). It is interesting to compare \( \Omega_0 \) and \( f_0 \) obtained using the new single-step approach with those obtained using the frequency-domain approach (Figure 7). The new single-step inversion and the frequency-domain inversion yield similar \( \Omega_0 \) estimates for any \( \delta \) and similar \( f_0 \) estimates when \( \delta < 6\% \). In cases where \( \delta > 6\% \), further analysis is needed, and the inversion proceeds to a two-step approach.

4.3 Two-step inversion

A two-step inversion is invoked when the one-step approach returns poorly-constrained solutions. The method rests on the premise that at short hypocentral distances, such as those considered in this study, the effect of anelastic path-dependent attenuation is negligible (e.g., Lior & Ziv, 2017; Lior et al., 2015; Wu et al., 2005; Wu & Zhao, 2006), and the observed attenuation is entirely due to near-site effects. Hereafter, in order to emphasize the confinement to near-field attenuation, \( \kappa \) is replaced by \( \kappa_0 \) (e.g., Ktenidou et al., 2014). Thus, site-specific \( \kappa_0 \) parameters are determined by averaging well-constrained \( \kappa_0 \) values, i.e. those corresponding to \( \delta < 6\% \). In the second step, a grid-search over the \( \Omega_0-f_0 \) space is performed for all available records (regardless of their \( \delta \)), with the values of \( \kappa_0 \) held fixed at the site-specific values. Thus, in addition to circumventing the time- to frequency-domain data transformation, the two-step inversion also addresses the ambiguity in the \( f_0-f_\kappa \) space.
A necessary condition for the well-posedness of any inverse problem is that its solution be stable. In the context of source parameter inversion, the solution stability may be assessed by means of within-event variability analysis. Specifically, the inversion is stable if the source parameter estimates of different seismic records corresponding to the same earthquake yields similar solutions. A comparison between the within-event variability of the two-step inversion described above (top panels in Figure 8) and that of the frequency-domain inversion (bottom panels in Figure 8) indicates an overall smaller within-event variability for the first. The reduction in the within-event variability is more pronounced for \( f_0 \) than for \( M_0 \), and is also more pronounced for the smallest magnitudes than for the largest ones. The stress drops, being proportional to the product of \( M_0 \) and \( f_0^3 \) (Equation 2b), exhibit the largest within-event variability, and the greatest improvement when switching from the frequency-domain to the new two-step inversion. It is concluded that the new approach is more stable than a frequency-domain inversion, and its within-event variability is well within the limits of the expected variability resulting from take-off angle (Kaneko & Shearer, 2015; Madariaga, 1976; Sato & Hirasawa, 1973), radiation pattern (Dong & Papageorgiou, 2002; Kaneko & Shearer, 2015), irregular source geometry (Dong & Papageorgiou, 2002; Kaneko & Shearer, 2015) and propagation effects (Kaneko & Shearer, 2015; Ross & Ben Zion, 2016).

A plot of the corner frequency as a function of the seismic moment is shown in Figure 9, along with lines of best fit for California and Japan (solid lines) that indicate a slight increase of stress drop with magnitude in both regions (using \( k = 0.37 \), as in Brune, 1970). The following stress drops versus seismic moment relations are inferred for California:

\[
\log(\Delta \tau) = 4.57 + 0.14 \log(M_0) (\pm 0.32)
\]

(18a)

and Japan:

\[
\log(\Delta \tau) = 4.37 + 0.17 \log(M_0) (\pm 0.45),
\]

(18b)

with standard deviations reported in the parentheses. This trend is at odd with previous claims for self-similarity (e.g., Baltay et al., 2011; Ide & Beroza, 2001; Ide et al., 2003; Oth et al., 2010; Shearer et al., 2006), but is in line with recent studies from California and Japan (e.g., Drouet et al., 2011; Izutani & Kanamori, 2001; Mayeda et al., 2007; Takahashi et al., 2005; Trugman & Shearer, 2017). Furthermore, the stress drops are on average 85\% larger in Japan than in California. As the dataset from Japan is restricted to intraplate earthquakes, this result may be attributed to the generally higher stress drops observed in intraplate than in interplate earthquakes (e.g., Kanamori & Anderson, 1975; Leyton et al., 2009; Scholz et al., 1986). Given the strong dependency of the ground acceleration on the stress drop (Equations 10), these results are of importance for hazard assessment, as well as for understanding earthquake source physics in general.

The distribution of station-specific \( \kappa_0 \) arising from the new approach differs markedly from the one obtained using the frequency-domain approach (Figure 10). While the first is of log-normal distribution with a median \( \kappa_0 \) around 0.028 seconds (mean \( \log(\kappa_0) = -1.72 \) and \( \sigma = 0.39 \)), the latter is close to a normal distribution with a larger median value of 0.043 seconds (mean \( \kappa_0 = 0.043 \) s and \( \sigma = 0.019 \) s). Thus, the results of this study imply weaker near-site attenuation than previously estimated (Lior & Ziv, 2017; Oth et al., 2011; Van Houtte et al., 2011). The disparity between the two \( \kappa_0 \) distributions in Figure 10 is attributed mainly to differences in the bandwidths used by the two methods (e.g., Edwards et al., 2015; Mayor et al., 2018); while in Anderson and Hough (1984) \( \kappa \) is inferred from the acceleration spectra in a limited high-frequency band, here it is inferred from the full bandwidth of the three ground motion measures, i.e., \( D_{rms}, V_{rms} \) and \( A_{rms} \).
The effect of frequency-dependent site amplification is not accounted for in Equation (3), and consequently is neither explicitly addressed by the source parameter inversion introduced here, nor by the standard frequency-domain approach. The performance of the inversion in the presence of such amplification is examined at two stations, MYG004 of K-net and FKSH10 of KiK-net, where strong high-frequency amplification has been reported (Nakano et al., 2015). A comparison between single-station \( \log(f_0) \) estimates at these stations and multiple-station event-average \( \log(f_0) \) is presented in Figure 11 for 14 earthquakes recorded at 11 and more stations. Good agreement between single- and multiple-station estimates are observed for the two-step approach. In contrast, single-station \( f_0 \) estimates resulting from the frequency-domain inversions are clearly biased toward higher frequencies. In addition to underlining the importance of using as many stations as possible, this result again indicates higher stability for the two-step time-domain inversion than for the frequency-domain inversion. It is further concluded that the ability of the two-step inversion to resolve the \( f_K f_0 \) ambiguity is not limited to favorable site conditions.

5 New ground motion prediction equations

Ground Motion Prediction Equations (GMPEs), relating peak ground motions (especially acceleration and velocity) with earthquake magnitude, distance and various other attributes of the earthquake, are key for seismic hazard analysis. Equations (10) together with an rms-to-peak relation may be combined to give first-order GMPEs for acceleration, velocity and displacement. A previously adopted statistical theory, relating the signal’s rms to its peak value, is restricted to \( f_0 \ll f_K \) (e.g., Baltay & Hanks, 2014; Vanmarcke & Lai, 1980). Because, however, this condition is not always met, an empirical approach is taken. Log-log diagrams of \( PGD \), \( PGV \) and \( PGA \) as functions of \( D_{\text{rms}} \), \( V_{\text{rms}} \) and \( A_{\text{rms}} \), respectively, are shown in Figure 12. The following empirical relations are obtained:

\[
PGD = 2.1(\pm 0.7)D_{\text{rms}}, \quad (19a)
\]

\[
PGV = 2.9(\pm 0.9)V_{\text{rms}}, \quad (19b)
\]

and:

\[
PGA = 3.3(\pm 0.9)A_{\text{rms}}, \quad (19c)
\]

in which the values inside the parentheses indicate one standard deviation. It is emphasized that because the rms of the ground motions are subject to the data intervals, so do the peak-to-rms proportions, and different schemes for setting the data intervals would result in different relations (Hanks & McGuire, 1981). The empirical relations in (19) combined with the theoretical results in (10) and the above expression for the data interval, constitute semi-theoretical GMPEs for \( PGD \), \( PGV \) and \( PGA \):

\[
PGD = 2.1M_0^{2/6}\Delta \tau^{1/6} \frac{\beta_D}{R \left[ \frac{1}{kCS}\left(\frac{7M_0}{16\Delta \tau}\right)^{1/3} + R/C_S \left[ 1 + 0.5\pi^2kckCS\left(\frac{16\Delta \tau}{7M_0}\right)^{1/3} \right]^{1/2} \right]^2}, \quad (20a)
\]

\[
PGV = 2.9\sqrt{M_0}\Delta \tau \frac{\beta_V}{R \left[ \frac{1}{kCS}\left(\frac{7M_0}{16\Delta \tau}\right)^{1/3} + R/C_S \left[ 1 + \pi^2kckCS\left(\frac{16\Delta \tau}{7M_0}\right)^{1/3} \right]^{3/2} \right]^2}, \quad (20b)
\]

\[
PGA = 3.3M_0^{1/3} \Delta \tau^{2/3} \frac{\beta_A}{R \left[ \frac{1}{kCS}\left(\frac{7M_0}{16\Delta \tau}\right)^{1/3} + R/C_S \left[ 1 + 1.5^{-1/4}\pi^2kckCS\left(\frac{16\Delta \tau}{7M_0}\right)^{1/3} \right]^{2} \right]^2}. \quad (20c)
\]

For hazard assessment, the stress drop in (20) may be substituted with a magnitude-dependent
and/or region-specific functions, such as Equations (18) (e.g., Archuleta & Ji, 2016; Trugman & Shearer, 2017).

PGA and PGV as a function of magnitude are shown in Figure 13 for hypocentral distances of 10 and 50 km, and for stress drops of 1 and 10 MPa. The arrows adjacent to the horizontal axes mark the transition from the \( \Delta r \)-independent to the \( \Delta r \)-dependent regimes (see discussion in reference to Equations 11 and 12). As already shown using stochastic simulations (Baltay & Hanks, 2014; Douglas & Jousset, 2011), the switch from one regime to another results in the peak shaking versus magnitude curves being steeper under the \( \Delta r \)-independent regime than under the \( \Delta r \)-dependent regime. This slope change is smaller for PGV as it is more magnitude-dependent than PGA (Equation 11). Additionally, the vertical separation between curves corresponding to different stress drops increases with increasing earthquake magnitude. Consequently, the PGA of sites located 10 km away from of a \( M_w 7 \) and \( \Delta r = 1 \) \( \text{MPa} \) is the same as that experienced at a site that is 50 km away from an earthquake of the same magnitude, but a stress drop that is 10 times larger. A comparison between the new GMPE for PGA (Equation 20c) and those of Cua & Heaton (2009) and Abrahamson et al. (2014) reveals good agreement (Figure 14). The main advantage of the new GMPE with respect to empirical ones is that it is based on a simple widely adopted physical source model (attenuated Omega-square spectra). As such, it accounts for the three most important source parameters affecting ground motion intensity: the seismic moment, the hypocentral distance and the stress drop. The effect of the latter has only recently been integrated into GMPEs (e.g., Ameri et al., 2017). The discrepancies between the (natural) logarithm of the observed and predicted PGA are shown in Figure 15 as a function of magnitude and distance, with predicted PGA calculated using \( \Delta r, M_0 \) and \( \kappa_0 \) obtained via the two-step inversion. That the discrepancy diagrams show no magnitude or distance dependencies indicate that the PGA to magnitude and distance relations are properly captured by the new GMPE, and that the neglect of anelastic attenuation is justified for the hypocentral distances considered in this study. The smallness of the between-event discrepancies (panel c of Figure 15) highlights the benefit of accounting for stress drop effects in GMPEs. Because the between-event discrepancies are small, the within-event and the total discrepancies are nearly identical (therefore the within-event discrepancies are not shown here). The total discrepancies of the new GMPE are comparable to recent empirically-based GMPEs (e.g., Abrahamson et al., 2014; Boore et al., 2014; Bora et al., 2015; Campbell & Bozorgnia, 2014; Chiou & Youngs, 2014; Idriss, 2014), which account for numerous additional effects that are not considered here.

6 Conclusions

Theoretical equations relating the rms of the far-field ground motions (displacement, velocity and acceleration) with earthquake source parameters and attenuation were derived using the omega-squared model (Brune, 1970). Apart from providing useful insight into how source and site parameters control earthquake ground motions, these model-based predictions may also be exploited for earthquake source parameters inversion and ground motion prediction.

A new approach for inverting the source spectral parameters is introduced. The main advantage of this method is the circumventing of the time- to frequency-domain transformation, and consequently also the spectral modeling intricacies. An additional advantage of the new scheme is that it yields more robust and more stable source parameters than those obtained using the frequency-domain approach (Figure 8).
Finally, a set of physics-based GMPEs for displacement, velocity and acceleration is derived (Equations 20) that is shown to be in good agreement with recent empirical GMPEs (Figure 14). Unlike previous region-specific GMPEs, containing numerous empirically-tuned coefficients, the new GMPEs are extremely simple, and are readily implementable worldwide, even in low seismicity regions, where the dataset available for setting the many degrees of freedom in the GMPE is of limited size.

Acknowledgements and Data

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References


Figure 1. A set of diagrams illustrating the effect of near-site attenuation on the displacement (top row), velocity (middle row) and acceleration (bottom row) spectra of a synthetic earthquake ($M_W = 5.5$, $\Delta \tau = 10$ MPa and $R = 10$ km). The source and the near-site attenuation models are indicated by solid and dashed lines, respectively, and their combined effect is indicated by dotted lines. Situations where $f_0 < f_K$, $f_0 = f_K$ and $f_0 > f_K$ are shown in the left, middle and right columns, respectively.
Figure 2. Comparison between different least-square fitting schemes. In all, the data (gray curve) are fitted to Equation (3), with κ that is obtained from the acceleration spectra as in Anderson and Hough [1984]. The dotted blue curve is the result of fitting the spectrum as-is. The red dashed curve is the result of fitting the model to a spectrum that is resampled at constant log(f) units. The green solid curve is the result of fitting the spectrum using a weighted least-square fit, with weights that are inversely proportional to log(f).
Figure 3. Comparison between the exact (Equation 8, solid curves) and approximated solutions (Equation 9, dashed curves). (a) The logarithm of the normalized displacement, \( \langle D_{\text{rms}} \rangle = D_{\text{rms}} \sqrt{2\kappa T}/\Omega_0 \), as a function of the logarithm of \( \alpha_0 \). (b) The logarithm of the normalized velocity, \( \langle V_{\text{rms}} \rangle = V_{\text{rms}} \sqrt{T}/(\pi \kappa)^{1.5}/(\sqrt{2\pi} \pi \Omega_0) \), as a function of the logarithm of \( \alpha_0 \).
Figure 4. Observed versus predicted displacement and velocity rms. (a) The logarithms of $D_{\text{rms}}^{\text{obs}}$ as a function of the logarithm of $D_{\text{rms}}^{\text{approx}}$. (b) The logarithms of $D_{\text{rms}}^{\text{obs}+}$ as a function of the logarithm of $D_{\text{rms}}^{\text{approx}}$. (c) The logarithm of $V_{\text{rms}}^{\text{obs}}$ as a function of $V_{\text{rms}}^{\text{approx}}$. (d) $\log(D_{\text{rms}}^{\text{obs}}) - \log(D_{\text{rms}}^{\text{approx}})$ as a function of $\alpha_0$. (e) $\log(D_{\text{rms}}^{\text{obs}+}) - \log(D_{\text{rms}}^{\text{approx}})$ as a function of $\alpha_0$. (f) $\log(V_{\text{rms}}^{\text{obs}}) - \log(V_{\text{rms}}^{\text{approx}})$ as a function of $\alpha_0$. 
Figure 5. Plots of objective function and acceleration spectra corresponding to three example seismograms. (a-c) Contour diagrams of $OF$ in $\log(f_0)$-$\log(f_k)$ space. The color code indicates optimal $\Omega_0$, and gray stars indicate $OF$ minima. The uncertainty $\delta$-parameter, reported at the top-right corner of each panel, is the area enclosed within the $\delta = 5\%$ contour (thick contour), normalized by the gray shaded area shown in panel (a). Dashed lines indicate $f_0 = f_k$. (d-f) The corresponding acceleration spectra are plotted in grey, along with the single-step solution (or solutions). The dashed red and dotted blue curves correspond to solutions for which $f_0 > f_k$ and $f_0 < f_k$, respectively.
Figure 6. The distribution of seismogram-specific $\delta$ values according to magnitude bins.
Figure 7. Comparison between the result of the frequency-domain and the single-step schemes. (a) $\Omega_0$ and (b) $f_0$ estimates. Grey circles and black ‘x’ symbols correspond to seismograms with $\delta$ smaller than or greater than 6%, respectively.
Figure 8. Comparison between within-event variability of $f_0$, $M_0$ and $\Delta \tau$ for the two-step algorithm (top panels) and the frequency-domain scheme (bottom panels). The distributions of within-event variabilities are shown according to three different magnitude bins for 444 earthquakes recorded by 4 or more stations. (left panels) the distribution of the logarithm of the corner frequency normalized by event-averaged corner frequency. (middle panels) The distribution of the logarithm of the seismic moment normalized by event-averaged seismic moment. (right panels) The distribution of the logarithm of the stress drop normalized by event-averaged stress drop. Standard deviations are reported on each panel.
Figure 9. Log-log diagram of $f_0$ as a function of $M_0$. Gray and black dots correspond to seismogram-specific and event-average results, respectively. Least-square fits to the data for California and Japan are shown by red and blue curves, respectively.
Figure 10. The distributions of station specific $\kappa_0$ for: (a) the two-step inversion, and (b) the frequency-domain approach. Seismograms with $\delta > 6\%$ are excluded from these diagrams. Dark, medium and light grey correspond to stations whose $\kappa$ values were averaged using 1, 2 or more than 2 seismograms, respectively.
Figure 11. Comparison between single-station log($f_0$) estimates at stations MYG004 (circles) and FKSH10 (triangles), where strong frequency-dependent site amplification has been reported (Nakano et al., 2015), and multiple-station event-average log($f_0$). Empty and solid symbols indicate results of the frequency-domain approach and the new two-step inversion, respectively.
Figure 12. Peak versus rms ground motions. (a) log($PGD$) as a function of log($D_{rms}$). (b) log($PGV$) as a function of log($V_{rms}$). (c) log($PGA$) as a function of log($A_{rms}$). For consistency with common practices in earthquake engineering, the peak ground motions are the geometric mean of the peak motion of the two horizontal components. Gray line indicates a linear regression to: log($PG$) = $a$ + log ($rms$). The fitting coefficient, $a$, and the standard deviation are reported at the top left corner of each panel.
Figure 13. Peak ground acceleration (a) and velocity (b) as a function of magnitude (i.e., Equations 18), with $\kappa = 0.03$ s, and different values of stress drop and distance. The gray and black arrows adjacent to the magnitude axes mark the transition from the $\Delta \tau$-independent and the $\Delta \tau$-dependent regimes for $\Delta \tau = 1 \text{ MPa}$ and $\Delta \tau = 10 \text{ MPa}$, respectively.
Figure 14. Comparison between the new peak acceleration GMPE (Equation 18c, with $\Delta \tau = 5 \text{ MPa}$, $\kappa = 0.03 \text{ s}$, and $R = 10 \text{ km}$, solid grey curve) and those of Cua & Heaton (2009) (CU09, solid black curve), and Abrahamson et al. (2014) (ASK14, dashed black curve) with $R = 10 \text{ km}$. 
Figure 15. The discrepancies between the natural logarithm of the observed and predicted $PGA$. (a) The total discrepancies as a function of magnitude. (b) The total discrepancies as a function of distance. (c) The between-event discrepancies as a function of magnitude. The dashed lines indicate zero discrepancy, and the solid lines indicate one standard deviation, estimated per bin of 300 seismograms (panels a and b) or 40 events (panel c). Standard deviations are reported at the top-left corner of each panel.