Interaction in Linear Versus Logistic Models: A Substantive Illustration Using the Relationship Between Motivation, Ability, and Performance

YOAV GANZACH
ISHAK SAPORTA
Tel Aviv University

YAACOV WEBER
The Hebrew University of Jerusalem

A binary performance measure (high school graduation) is examined as a function of motivation (educational goal), ability (scores in an intelligence test), and their interaction. The interaction was positive when a logistic model was used and negative when a linear probability model was used. The reason for the difference in the results of the two models is examined, and the conditions under which this difference occurs are discussed.

Many of the important dependent variables in organizational research are binary. For example, between 1995 and 1997 about 5% of the articles in the *Journal of Applied Psychology* and about 8% of the articles in the *Academy of Management Journal* used binary dependent variables. Nevertheless, methodological issues associated with the modeling of such variables, and, in particular, the modeling of interaction effects among their determinants, have received little attention in the literature. The purpose of this article is to examine issues associated with the modeling of interaction when the dependent variable is binary within the context of an important, yet little researched, substantive issue: The relationship between motivation and ability in the determination of performance. A binary performance measure (high school graduation) is examined. Scores in a cognitive ability test are used as a measure of ability, and educational goal is used as a measure of motivation.

The article is organized as follows. We first review the relevant literature regarding (a) the modeling of binary dependent variables using linear and logistic models, and (b) the relationship between motivation, ability, and performance. We then present data showing that modeling performance as a function of motivation and ability by a linear probability model and modeling it by a logistic model lead to diametrically opposite conclusions about the interaction between motivation and ability. In the last

Organizational Research Methods, Vol. 3 No. 3, July 2000 237-253

section we discuss the reasons for the difference in the results of the two models, and the conditions under which this difference occurs.

Linear Versus Logistic Models

The logistic model is often used to model the relationship between a binary dependent variable and nominal or continuous independent variables. In this model, the natural logarithm of the odds of being in a category is modeled as a linear function of the independent variables. If P is the probability of being in the category, X_is are the independent variables and k the number of the independent variables the logistic model is given by the following:

$$\ln(\mathrm{P} / 1 - \mathrm{P}) = \alpha + \sum_{i=1}^{k} \beta_i X_i.$$

An alternative to the logistic model is the linear probability model. In this model, 1 is the code for being in the category and not being in the category is coded as 0. If Y represents this binary dependent variable, the linear probability model is given by the following:

$$Y = \alpha + \sum_{i=1}^{k} \beta_i X_i.$$

Although both the logistic model and the linear probability model depict the probability of belonging to a category as a function of the independent variables, the linear probability model has some disadvantages when compared to the logistic model. First, whereas in the logistic model the predicted probability of belonging to a category is bounded between 0 and 1, in the linear probability model this predicted probability could be greater than 1 and less the 0. Second, the logistic model assumes that a binomial distribution describes the distribution of the error term, whereas the linear probability model assumes a normal distribution. Because the distribution of the error term of a binary variable is clearly non-normal, hypothesis testing in this model may be inaccurate (Maddala, 1983). Nevertheless, it is commonly argued that although significance tests in multiple regression are inaccurate when the dependent variable is binary, the parameter estimates are unbiased (e.g., Aldrich & Nelson, 1984). For example, Pindyck and Rubinfeld (1981) argue that " . . . the signs (and frequently the relative magnitude) of the estimated parameters obtained from linear probability models and the maximum likelihood logit estimators are usually the same. This provides an additional rationalization for the use of the linear probability model." Indeed, many textbooks describe the linear probability model as a good modeling technique for the case of a binary dependent variable (e.g., Cohen & Cohen, 1983; Pedhazur, 1982).

However, all these assertions were made regarding linear probability models that include only main effects. For models that include interactions, important differences between the linear probability model and the logistic model may arise. So far, however, only two studies have compared the logistic model to the linear probability model with regard to detecting interactions in organizational research or related areas. In one study, Huselid and Day (1991) examined how the interaction between job-commitment and job-involvement influences turnover, and found that it was significant in the linear probability model and nonsignificant in the logistic model. They concluded that previous research that relied on linear probability models in assessing this interaction was faulty, and cast serious doubt on the theory that predicted this interaction. Note, however, that the differences on which Huselid and Day base their conclusions are very small. In both the logistic model and the linear probability model the interaction coefficient was positive; in the former case it was about two standard errors larger than zero, and in the latter about 1.75 standard errors larger than zero (see Huselid & Day, 1991, Tables 3 and 4, pp. 388-389).

In the other study, Landerman, George, Campbell, and Blazer (1989) examined the effect of the interaction between number of stressful life events and social support on depression. They too found that this interaction was significant when a linear probability model was used and nonsignificant when a logistic model was used. However, unlike Huselid and Day (1991), their conclusion was that previous research that relied on logistic models in assessing this interaction was insufficient, and that the doubt cast earlier on the theory that predicted this interaction was unjustified.

In the present article we reexamine the use of linear and logistic models in the context of the multiplicative theory of the relationship between motivation, ability, and performance. In comparison to previous research, the sample examined is much larger. This allows for a more sensitive comparison between the linear and the logistic models.

Motivation, Ability, and Performance

A commonly accepted view is that performance is positively related to the product of motivation and ability (Vroom, 1964); that is, that there exists a positive interaction between motivation and ability in the determination of performance. This view is based on the notion that people will not act if their actions do not serve relevant goals, and that when the ability to act is absent, the existence of a goal cannot lead to the intended result; and it suggests that the relationship between motivation and performance is stronger the higher the ability level.[1]

Although a positive interaction between motivation and ability is often considered axiomatic (e.g., Locke & Latham, 1990, p. 206), the evidence is scant and controversial. Some studies have found support for the interaction (French, 1958; Lawler, 1966; Locke, Mento, & Katcher, 1978), others have not (e.g. Locke, 1965; Tziner & Eden, 1985); and one study even found a negative interaction (Kipnis, 1962). In fact, Campbell and Pritchard (1976), Korman, Greenhaus, and Badin, (1977), and Tziner and Eden (1985) conclude that the existing evidence for an interaction between motivation and ability in determining performance is extremely weak. For example, Campbell and Pritchard (1976) write "The attempts to account for additional variance in performance by some multiplicative combination of motivational and ability variables have been singularly unsuccessful" (p. 91). Although in a recent statement, Tubbs (1994) cites a number of publications (most of them books and review articles) and concludes that " . . . motivation and ability are thought to be interactive in their effect on performance, a conclusion supported by much research and theory" (p. 808), an investigation of these citations indicates that the arguments they bring in support of interactive relationships are primarily theoretical, and rarely empirical. In fact, except for Locke (1982), these citations do not include empirical evidence obtained after the Campbell and Pritchard (1976) review.

One reason for these inconsistent findings is the low statistical power in detecting interaction, which is associated primarily with the low reliability of the product term (Dunlap & Kemery, 1988; MacCallum & Marr, 1995) and with the small residual variance of this term when the main effects are controlled for (Bobko, 1986; Dawes & Corrigan, 1974). These problems are particularly serious in the modeling of non-experimental field data in which the predictors are correlated (McClelland & Judd, 1993), as is typical of the data generally used in studying the relationship between motivation, ability, and performance. Thus, one purpose of the present article is to examine for interaction between motivation and ability in field data on the basis of a very large sample that provides considerable statistical power.

Data

The data were taken from the National Longitudinal Survey of Youth (NLSY), conducted with a probability sample of 12,686 participants (oversampling of African Americans, Hispanics, and economically disadvantaged Whites) born between 1957 and 1964. Participants who were in grade 12 and above in 1979, and participants who had missing data were not included in the analysis. This resulted in a sample size of 5,690. The sample was roughly equally distributed among participants age 15 through 18, and 19 through 22. It was also roughly equally distributed among participants with 0 to 11 years of education.

The dependent variable was high school graduation, defined as completing 12 years of education in 1993. The independent variables were educational motivation and cognitive ability. Educational motivation was derived from the answers to two questions asked in the 1979 survey, one about educational expectations (the number of years of education the participant expects to complete) and the other about educational aspirations (the number of years of education the participant would like to complete). Because the answers to these two questions were highly correlated ($r = .85$), they were averaged, and their mean was taken as a measure of educational motivation.[2]

Cognitive ability was measured by the Armed Forces Qualifying Test (AFQT). This test was administered to groups of 5 to 10 members of the NLSY during the period June through October of 1980; respondents were compensated, and the overall completion rate was 94%. The AFQT score in the NLSY is the sum of the standardized scores of four tests: Arithmetic reasoning, paragraph comprehension, word knowledge, and mathematics knowledge.

The percentage of high school graduates in the sample was 79.2%. The mean educational motivation was 13.3 and its standard deviation was 2.4. The correlation between educational motivation and cognitive ability was .45.

Results

Continuous Representation of the Independent Variables

The estimated logistic and linear probability models in which high school graduation (coded as 1 for graduating and 0 for not graduating) is the dependent variable, and educational motivation, ability, and their interaction are the predictors where (numbers in parenthesis are standard errors of the estimates):

$$\log(P/1 - P) = \underset{(0.072)}{2.288} + \underset{(0.085)}{1.303}{*}EM + \underset{(0.075)}{1.362}{*}CA + \underset{(0.084)}{0.325}{*}EM{*}CA$$

$$Y = \underset{(0.005)}{0.829} + \underset{(0.005)}{0.106}{*}EM + \underset{(0.005)}{0.132}{*}CA - \underset{(0.005)}{0.075}{*}EM{*}CA,$$

where EM stands for educational motivation, CA for cognitive ability, P for the probability of graduating from high school, and *Y* is a binary variable that has the value of 1 for graduating and 0 for not graduating. To obtain meaningful coefficients for the main effects, the independent variables in these models were standardized. By doing so, the coefficient of each of the independent variables represents its typical effect on the dependent variable—its effect when the other independent variable is at its mean, and the magnitude of the coefficient represents the change in the dependent variable associated with a change of one standard deviation in the independent variable.

The results of these two models are quite similar with regard to the main effects. Both educational motivation and cognitive ability have a strong positive effect on the probability of graduating from high school. However, the results are diametrically opposite with regard to the interaction. The logistic model indicates that the interaction between motivation and ability is positive, whereas the linear model indicates that it is negative. From a substantive point of view, the logistic model suggests that the effect of motivation on performance is stronger when cognitive ability is high, whereas the linear model suggests that the effect of motivation on performance is stronger when cognitive ability is low. Note also that the interaction suggested by the logistic model is consistent with the common theoretical view about the relationship between motivation, ability, and performance, whereas the interaction suggested by the linear probability model is contrary to this theory.

These interaction effects, although opposite in sign, are highly significant. For the logistic model, the null hypothesis that the interaction coefficient is equal to zero is rejected, $\chi^{2(1)} = 23.9$, $p < .0001$ using a likelihood ratio test. For the linear model this hypothesis cannot be tested directly, because significance tests in the linear probability model are inaccurate. However, in the next section this hypothesis is examined indirectly.

Robustness checks. Our data are characterized by a strong multicollinearity between the two independent variables. Because multicollinearity could lead to considerable bias in estimated interaction coefficients when quadratic terms are omitted (Cortina, 1993; Ganzach, 1997, 1998; Lubinski & Humphreys, 1990), it could be asked whether this multicollinearity is related to the change in signs reported in the article. To examine for this, we added quadratic terms to both our linear probability and logistic regression models (see columns 2 and 3 of Table 1, as well as the cubic terms—columns 4 and 5). However, the pattern of the interactions in these models was quite similar to the pattern in the models reported above, indicating that the overlap between multiplicative and quadratic terms when multicollinearity is high is not the reason for the change in sign of the interaction coefficient in our study.[3]

Another robustness check involved adding a number of covariates to the basic model (the model that includes intelligence, educational motivation and their interaction). These covariates included age, ethnic background (coded as 1 for Black and 0 for non-Black) and gender (0 for males and 1 for females), and the interactions between

Table 1
Robustness Check

	Quadratic Terms		*Quadratic and Cubic Terms*		*Control Variables*	
	Linear	*Logistic*	*Linear*	*Logistic*	*Linear*	*Logistic*
Cognitive Ability (CA)	0.134	1.396	0.139	1.407	0.145	1.516
	(0.005)	(0.089)	(0.011)	(0.130)	(0.006)	(0.079)
Educational Motivation (EM)	0.109	1.218	0.138	1.083	0.081	1.043
	(0.005)	(0.083)	(0.009)	(0.115)	(0.005)	(0.086)
CA × EM	–0.042	0.373	–0.034	0.347	–0.060	0.254
	(0.007)	(0.084)	(0.007)	(0.113)	(0.005)	(0.084)
CA^2	–0.038	–0.030	–0.038	–0.024		
	(0.006)	(0.064)	(0.006)	(0.091)		
EM^2	–0.016	–0.252	–0.037	–0.132		
	(0.004)	(0.056)	(0.006)	(0.104)		
CA × EM^2			–0.005	0.006		
			(0.006)	(0.091)		
EM × CA^2			–0.007	–0.052		
			(0.007)	(0.095)		
CA^3			–0.002	–0.027		
			(0.005)	(0.061)		
EM^3			–0.006	0.126		
			(0.002)	(0.056)		
Age					–0.034	–0.290
					(0.003)	(0.036)
Sex					0.028	0.204
					(0.009)	(0.081)
Black					0.081	0.647
					(0.011)	(0.090)
Age × CA					0.005	–0.086
					(0.003)	(0.031)
Age 8M´ EM					0.011	0.036
					(0.003)	(0.030)

Note. Numbers in parentheses are standard errors.

intelligence and age. In particular, age is expected to be negatively related to the probability of obtaining 12 years of education, because, other things being equal, participants who did not obtain this level of education by the age of 18 are less likely to obtain it than younger participants. In addition, because the effects of cognitive ability and motivation may differ for various age groups, their interactions with age were also added to the basic model. The results of the linear probability and logistic versions of this model are reported in columns 6 and 7 of Table 1, respectively. It is clear from these results that the covariates did not change the results of the basic model reported above, and in particular, they did not change the pattern of positive interaction in the logistic model and negative interaction in the linear probability model. Note that the two models were similar with regard to the main effects of the covariates (other things being equal, females, Blacks,[4] and younger participants had a higher probability of graduating from high school) but were not similar with regard to the interactions. This pattern of results is another indication that main effects, but not interactions, are robust with regard to the type of model that is used.

Finally, to provide the reader with the entire set of relationships between the variables of the models, the correlation matrix among these variables is provided in Table 2.

Table 2
Correlation Matrix of the Variables in the Various Models

	CA	*EM*	*CA × EM*	*CA²*	*EM²*	*CA × EM²*	*EM × CA²*	*CA³*	*EM³*	*Age*	*Sex*	*Black*	*Age × CA*	*Age × EM*
CA	1.00													
EM	0.50	1.00												
CA×EM	0.14	0.05	1.00											
CA²	0.20	0.14	0.58	1.00										
EM²	0.04	0.00	0.57	0.22	1.00									
CA×EM²	0.58	0.57	–0.03	0.11	–0.40	1.00								
EM×CA²	0.62	0.73	0.13	0.25	–0.08	0.80	1.00							
CA³	0.87	0.47	0.21	0.30	0.03	0.60	0.75	1.00						
EM³	0.25	0.55	–0.22	–0.01	–0.58	0.82	0.57	0.27	1.00					
Age	–0.16	–0.26	0.06	0.04	0.11	–0.18	–0.20	–0.15	–0.18	1.00				
Sex	–0.00	0.02	–0.03	–0.06	0.00	–0.02	–0.01	–0.02	–0.01	–0.01	1.00			
Black	–0.37	–0.01	–0.01	–0.06	–0.06	–0.13	–0.13	–0.29	0.03	0.05	–0.02	1.00		
Age×CA	–0.02	0.03	–0.27	–0.21	–0.18	0.11	0.06	0.02	0.13	–0.31	–0.03	0.03	1.00	
Age×EM	0.03	0.08	–0.26	–0.10	–0.46	0.28	–0.14	0.06	0.36	–0.30	–0.03	0.04	0.48	1.00

Note. CA = Cognitive Ability; EM = Educational Motivation.

Table 3
Probability, Log Odds, and Odds of Graduating
From High School as a Function of Cognitive Ability and Motivation (real data)

	Cognitive Ability	
Motivation	*High*	*Low*
Probability		
High	.981	.834
	(1,877)	(984)
Low	.815	.569
	(956)	(1,873)
Log odds		
High	3.94	1.61
	(1877)	(984)
Low	1.48	.278
	(956)	(1,873)
Odds		
High	51.6	5.02
	(1,877)	(984)
Low	4.41	1.32
	(956)	(1,873)

Note. Numbers in parentheses are the cell's *n*.

Binary Representation of the Independent Variables

To gain a better understanding of the change in sign, it is helpful to dichotomize the independent variables by a median split. The probability of high school graduation (i.e., the percentage of high school graduates) as a function of these binary variables is given in the probability section of Table 3. This representation is a 2×2 equivalent of a continuous linear probability model. And indeed, the data in this table are consistent with the results of the continuous linear model of the previous section: The effect of motivation is stronger when cognitive ability is low than when it is high. When cognitive ability is low, the probability of high school graduation increases from .569 to .834 when motivation rises from low to high, whereas when cognitive ability is high, the probability of high school graduation increases only from .815 to .981. Thus, although the difference in the probability between the two levels of motivation is .265 when ability is low, it is only .166 when ability is high. This gap between the two differences implies a negative interaction, and it corresponds to the negative interaction coefficient in the linear probability model in our analysis above.

Table 3 also presents the data in the form of the (natural) log of the odds of graduating from high school. This representation is a 2×2 equivalent of a continuous logistic model. The interaction that emerges when the data are presented in this way is quite different from the interaction that emerges when the data are presented as probabilities: The effect of motivation is stronger when cognitive ability is high than when it is low. When ability is high, the log odds of high school graduation increase from 1.48 to 3.94 when motivation rises from low to high, whereas when ability is low, it increases only from .278 to 1.61. Thus, although the difference in the log odds of graduation is 2.46 when ability is high, it is only 1.33 when ability is low. This gap between the two differences implies a positive interaction, and it corresponds to the positive interaction coefficient in the linear probability model in our analysis above.

A more concrete (and dramatic) illustration of the difference between the probability representation and the odds representation can be obtained by considering the section of Table 3 that presents the odds (instead of the log odds) of graduating as a function of motivation and ability. It is clear from this table that when ability is high, the odds of graduating increase about 10 times when motivation rises from low to high, whereas when ability is low the odds increase only about 4 times. Again, this interaction is in sharp contrast to the interaction that emerges when probabilities, rather than odds, are examined (see Table 3).

The difference in the sign of the interaction coefficient in the binary representation can also be demonstrated by estimating both a linear probability model and a logistic model on the data in Table 3. The estimated models (in which the independent variables are coded as 1 for high values and 0 for low values) are, respectively,

$$Y = .567 + .276*\text{EM} + .248*\text{CA} - 0.113*\text{EM}*\text{CA}$$

$$\log(\text{P}/1 - \text{P}) = .271 + 1.41*\text{EM} + 1.21*\text{CA} + .88*\text{EM}*\text{CA}.$$

It is clear from these models that the interaction coefficients of the two models have opposite signs: the coefficient of the logistic model is positive, whereas the coefficient of the linear model is negative.

Note also that the effects of the interactions in the two models, although opposite in sign, are highly significant. For the logistic model, the null hypothesis that the interaction is equal to zero is rejected, $X^2(1) = 19.0$, $p < .0001$. For the linear model, this null hypothesis can be tested indirectly, by using the normal approximation of the sampling distribution of proportions, and comparing the difference in the proportion of high school graduates when motivation is low (.834 – .569 = .265) to the difference when motivation is high (.981 – .815 = .166). The gap between these two differences is significant $Z = 14.6$, $p < .0001$ (see appendix for further details).

Discussion

The article provides a dramatic illustration of a difference that may occur between the linear probability model and the logistic model in estimating the coefficients of interaction terms. What is the reason for these dramatic differences? Using the binary representation of the independent variables in Table 3 provides a good answer to this question. It is clear from this table that when the data are presented in terms of probabilities (see Table 3), the difference between low and high level of motivation in the probability of graduating from high school is higher when ability is low than when ability is high. This corresponds to a negative interaction in a linear probability model. On the other hand, when the data are presented in terms of the log odds of graduating from high school (see Table 3), this difference is higher when ability is high than when ability is low. Because the log of the odds is the dependent variable in the logistic model, this corresponds to a positive interaction in a logistic regression.

A close look at the data in Table 3 reveals that the cell associated with high ability and high motivation plays a major role in creating this sign reversal of the interaction coefficient. Because the probability of graduation in this cell is so high, the denomina-

Table 4
Reduced Probabilities, Reduced Log Odds, and Reduced Odds of Graduating From High School

	Cognitive Ability	
Motivation	*High*	*Low*
Reduced probabilities		
High	.681	.534
	(1,877)	(984)
Low	.515	.269
	(956)	(1,873)
Reduced log odds		
High	.758	.136
	(1,877)	(984)
Low	.060	–1.00
	(956)	(1,873)
Reduced odds		
High	2.13	1.14
	(1,877)	(984)
Low	1.06	.368
	(956)	(1,873)

Note. Numbers in parentheses are the cell's *n*.

tor of the odds becomes very close to zero, the odds become very high, resulting in a large difference between the log odds of the two cells in which ability is high, a difference much larger than the corresponding difference between the log odds of the cells in which ability is low.

This analysis suggests that the change in the sign of the interaction occurs because of the existence of domains in the variable space (e.g., cells) for which the probability of one of the two binary outcomes is very high (and different than the probabilities of the other cells). To illustrate, consider a situation in which for each of the cells the probabilities of graduating from high school are smaller by .3 from the probabilities in Table 3. These probabilities are presented and the corresponding log odds are presented in Table 4. It is clear from these two tables that there is no discrepancy between the sign of the interaction in the probability representation and its sign in the log odds representation (and therefore, no difference in the sign of the interaction of the linear and logistic models). Here, in both representations the interaction is associated with a large difference between the cells in which ability is low (a difference of .265 and 1.36 in the probability and log odds representation, respectively) and a small difference between the cells in which ability is high (a difference of .166 and .698, respectively).

It is often argued that the results of the linear probability model are quite similar to the results of the logistic model as long as the probability of one of the two binary outcomes does not exceed .70. Our analyses, however, suggest that this is true only for main effects but not for interactions (in contrast to the interaction coefficients, there is no disparity between the coefficients of the main effects in our models). For example, consider the hypothetical data in Table 5. In these data, the probability of graduating from high school is only 64.7%. Nevertheless, the interaction coefficient of the linear probability model of these data is negative whereas the interaction coefficient of the logistic model is positive (coding high values as 1 and low values as 0, these coeffi-

Table 5
Simulated Probabilities and Log Odds of High School Graduation

	Cognitive Ability	
Motivation	*High*	*Low*
Probabilities		
High	.99	.60
	(2,000)	(2,000)
Low	.80	.20
	(2,000)	(2,000)
Log odds		
High	4.60	0.41
	(2,000)	(2,000)
Low	1.39	–1.39
	(2,000)	(2,000)

Note. Numbers in parentheses are the cell's *n*.

cients are –.23 and +1.38, respectively). The former coefficient is associated with a large difference of .60 – .20 = .40 between the two motivation levels when ability is low, as opposed to a small difference of only .99 – .80 = .19 when ability is high. The latter coefficient is associated with a small difference of 41 – (–1.39) = 1.80 between the two motivation levels when ability is low, as opposed to a large difference of 4.60 – 1.39 = 3.21 when ability is high. Thus, a base rate close to 0.5 is not enough to assure equivalence between the logistic model and the linear probability model, if there are still domains in the variables' space in which there are sharp deviations from this base rate.

The data in Table 5 also demonstrate the robustness of the change in sign to the correlation between the independent variables. Though the correlation between the independent variables in Table 5 is zero (the numbers of observations in the four cells is equal), there is a clear change in the sign of the interaction coefficients (as reported above, the interaction coefficients are –.23 and +1.38 for the linear and logistic models, respectively). Indeed, if the number of observations within each cell in our "real data" (Table 3) are changed to create zero correlation between motivation and ability, keeping the probabilities of graduating within each of the cells constant, the change in sign of the interaction coefficient still occurs. Thus, the reason for the change in sign is the rapid increase in the value of the logistic transformation when probability increases, rather than the multicollinearity between independent variables.

Although our analysis of the change in sign of the interaction coefficient focused on 2×2 tables, this analysis has more general implications. First, a similar analysis could be applied to explain the change in sign of the interaction coefficient in the continuous case. In the logistic model, small changes in the probability of the outcome near the endpoints of the empirical range of the independent variables are associated with large changes of the independent variables. In particular, when the probability approaches 1, and the independent variables are high, large changes in the independent variables result in small changes in probability (we assume in this discussion that the independent variable is scaled so that higher values correspond with higher probability). Therefore, similar changes in probability (e.g., the probability of graduating) as a function of one variable (e.g., motivation) represent a stronger effect of this variable when the value of the other independent variable (e.g., ability) is high than when it is low (because when this second independent variable is high, the probabilities are much closer to 1). On the other hand, in the linear model, changes in probability represent a similar effect of the independent variables across the entire probability scale.

Second, the existence of domains in the variable space in which the probability of one of the two outcomes is very high leads not only to a change in sign of interaction coefficients, but also to a change in sign of quadratic coefficients. Consider, for example, a situation in which the probabilities of a binary dependent variable (e.g., high school graduation) are 0.5, 0.8, and 0.99, respectively, for each of three levels—low middle and high, respectively—of a trichotomous independent variable (e.g., motivation). The corresponding log odds are 0.0, 1.4, and 4.6, respectively. It is clear that in this case, the relationship between the independent and dependent variables is concave in the probability representation (the difference in probabilities is larger between the top and middle levels of the independent variable than between the middle and bottom levels), but convex in the log odds representation (the difference in probabilities is smaller between the top and middle levels of the independent variable than between the middle and bottom levels). In regression, the probability representation will yield a negative quadratic coefficient, whereas the log odds representation will yield a positive quadratic coefficient.

The explanation for this reversal in sign of the quadratic term is similar to the explanation for the reversal in sign of the interaction coefficient discussed above. Again, the cell associated with the top level of the independent variable plays a major role in this reversal. Because the probability in this cell is very high, the denominator of the odds is close to zero, the odds are very high, resulting in a large difference between the log odds of the top and middle level of the independent variable, a difference much larger than the corresponding difference between the log odds of the middle and bottom levels. Indeed, when each of the probabilities is smaller by 0.3 (0.2, 0.5, and 0.68 for the bottom, middle, and top level of the independent variable, respectively), and the probability of the top cell is not very high, there is no discrepancy in the relationships between the dependent and independent variables in the probability representation and in the log odds representation (–1.4, 0.0, and 0.8, respectively). In both representations this relationship is concave.

A natural question to ask is, which of the two models—the logistic or the linear model—is the appropriate one to choose? Quite often, comparison between two alternative models is done in terms of model fit. However, this answer may not necessarily be appropriate in our case. For example, in 2×2 tables, the fit of the logistic model is exactly equal to the fit of the linear probability model when main effects and interaction are estimated—both models precisely reconstruct the data (e.g., by substituting the values of the independent variables in the estimated models, the data of the tables are obtained).

What then is the appropriate choice in this case? In our view, priority should still be given to the logistic model, because on an a priori basis it is more likely to represent the specific features of a probability scale, especially the fact that it is constrained by a ceiling of 1 and a floor of 0. However, other aspects of the study, such as the goals of the analysis and fit to the theory should be considered as well. These aspects are discussed below.

Two important goals of data analysis are theory testing and policy development. From a policy point of view, the negative interaction between motivation and ability suggested by the linear model is the appropriate interaction. If an intervention that could increase educational motivation is to be implemented, it will be more beneficial to implement it among youngsters whose cognitive ability is low rather than among youngsters whose cognitive ability is high. For example, if the utility of not graduating from high school is set to 0, and the utility of graduating from high school is U_g, then, on the basis of Table 3, the utility of increasing the educational motivation of a youngster from low to high is $.834U_g - .569U_g = .265U_g$ when cognitive ability is low and $.981U_g - .815U_g = .166U_g$ when cognitive ability is high.

However, from a theoretical point of view, the positive interaction between motivation and ability suggested by the logistic model is the appropriate one. First, from a Bayesian perspective, when the available theory and data are considered, it is the positive, rather than the negative interaction between motivation and ability that has the higher prior probability. Second, as discussed above, the logistic model is more appropriate in that it supplies a better representation of the probability scale. This feature of the logistic model results in an interaction that is sensitive to the relative effects of the dependent variable. Thus, the interaction suggested by the logistic model is consistent with the idea that boosting motivation will lead to a higher effect when ability is high than when it is low in the following way. When ability is high, boosting motivation increases the proportion of high school graduates by .113 out of a maximum possible increase of 1.00 – .873 = .127, but when ability is low the increase in this proportion is only .300 out of a maximum of 1.00 – .529 = .471. Thus, we view the results of the current study as supporting the theory that performance is positively related to the product of motivation and performance (Vroom, 1964).

This distinction between the role of theory and policy in determining the appropriateness of a model is obviously relevant to other areas of applied psychology. For example, even if one accepts the Huselid and Day (1991) view that the interaction between job-commitment and job-involvement in linear models of turnover does not support previous theories about the antecedents of turnover (e.g., Blau & Boal, 1987), this interaction has practical implications. Because, from a practical point of view, managers are interested in the probability of turnover (rather than its odds), this interaction suggests that the effectiveness in increasing job involvement (e.g., through job redesign) will be higher when commitment is low than when it is high.

It has been suggested in the epidemiological literature that logistic models are often appropriate for examining questions regarding the etiology of a disease, whereas linear models are appropriate for examining questions regarding public health policy (e.g. Rothman, Greenland, & Walker, 1980). A common situation in this research is when there are two factors (e.g., smoking and exposure to asbestos) that predispose toward developing a disease (e.g., cancer), and the researcher is interested in whether there is synergism between them (Saracci, 1977); that is, whether the two factors in combination produce an effect greater than their additive effects. To estimate this interaction effect, a variety of measures were developed (see Greenland, 1994, for a recent review), some consistent with a logistic model (e.g., Walter, 1976), and others consistent with a linear model (e.g. Hogan, Kupper, Most, & Hasmean, 1978). These measures may also be useful for applied psychologists. For example, when behavior is measured on a dichotomous scale, research about the relationship between the behavior (e.g., attaining a required level of performance; staying on the job) and its antecedents (e.g., motivation and ability; commitment and involvement) may be conceptualized in terms of the “risk” of not displaying the behavior, making it methodologically similar to research concerning the influence of predisposing factors on the risk of developing a disease.

What are the implications of the current study vis-à-vis substantive conclusions in previous research examining binary dependent variables? Because there are quite a few binary variables that are of interest to behavioral scientists in general and organizational researchers in particular, we limit our discussion here to research about turnover, the most heavily studied binary dependent variable in organizational research. A review of the literature indicates that before 1990, reliance on the linear probability model in the study of turnover was widespread. Of the 33 articles published in the *Journal of Applied Psychology* (*JAP*) and in the *Academy of Management Journal* (*AMJ*) between 1981 and 1989 which included actual turnover as dependent variable, 25 used the linear probability model as a method of analysis, whereas none (!) used the logistic model. (Two articles—one that used LISREL and one that used a partial correlation technique, were categorized as relying on the linear probability model. The rest of the articles used other methods such as *t* tests and frequency tables. Only one article used the hazard rate model). Can we trust the results of this research? As the analysis of the current study suggests, we can be more confident regarding some of the results and less confident regarding others. In particular, main effects obtained by using linear probability models are more likely to hold when logistic regression is used, whereas interaction effects obtained by using linear probability models are less likely to hold when logistic regression is used. Thus, whereas many of the substantive conclusions in turnover research conducted in the 1980s (and earlier) regarding interaction effects are suspicious (e.g., Caldwell & Oreilly, 1985; Lee & Mowday, 1987; Oldham & Fried, 1987; Spencer & Steers, 1981; Werbel & Gould, 1984), most of the results regarding main effects are not (e.g., Abelson, 1987; Mowday, Koberg, & McArthur, 1984; Rusbult, Farrell, Rogers, & Mainous, 1988).

The situation is quite different with regard to research published in the 1990s. In this period, reliance on the logistic model became much more widespread in turnover research, perhaps because of the inclusion of logistic regression procedures in the major statistical packages such as SAS or SPSS (for example, SAS introduced a logistic regression program in 1990). Indeed, in this period, only a minority of the 23 *AMJ* and *JAP* studies that used actual turnover as the dependent variable relied on the linear probability model (seven studies), whereas most of the studies used either the logistic model (eight studies), or the hazard rate model (four studies), which is essentially a development of the logistic model allowing for incorporating time into the model (of the remaining four articles, two used other methods and two used both the linear probability model and the logistic model). But even the eight turnover studies, which were published during the 1990's and relied on the linear probability model, are not likely to lead to biased conclusions, because none of them tested for interaction effects.

Finally, the results of the current article can also explain why some recent studies that used both the linear probability model and the logistic regression found that the two yield the same results model (e.g., Gerhart & Rynes, 1990; Sommers & Birnbaum, 1999), whereas others found that they yield different results (e.g., Huselid & Day, 1991; Landerman et al., 1989): The former studies focused on main effects, whereas the latter focused on interaction effects.

APPENDIX

When n is large, under the null hypothesis of no interaction, the following statistic has a Z distribution:

$$\frac{(P_{LH} - P_{LL}) - (P_{HL} - P_{HH})}{\sqrt{S_{E^2_{LH}} + S_{E^2_{LL}} + S_{E^2_{HL}} + S_{E^2_{HH}}}},$$

where P_{HL}, P_{HH}, P_{LL}, P_{LH} are, respectively, the proportions of high school graduates for high motivation/low ability, high motivation/high ability, low motivation/low ability, and low motivation/high ability (i.e., the first index represents the motivation level and the second the ability level). SE_{HL}, SE_{HH}, SE_{LL}, SE_{LH} are the standard errors of the corresponding proportions, where SE_{ij} is given by

$$SE_{ij} = \sqrt{\frac{P_{ij}(1 - P_{ij})}{n}}.$$

Notes

1. Recently Kanfer and Ackerman (1989) suggested that under certain circumstances the relationship between motivation and performance is stronger the lower the ability level. However, this relationship is limited to the training phase of acquiring complex skills.

2. Although educational aspirations may appear as more closely related to the concept of educational motivation, educational expectation is also likely to be a good measure for this concept because of the strong relationships between goals, self-set goals, and motivation. See, for example, Locke & Latham, 1990, for a discussion.

3. Interestingly enough, the quadratic models (columns 2 and 3 in Table 1) show a significant curvilinear (concave) effect on motivation and performance and perhaps even a concave effect of cognitive ability. For a discussion of such effects see Lubinski and Humphreys (1990).

4. Although the finding that Blacks have a higher probability of graduating from high school may appear surprising, it is consistent with previous analysis of the National Longitudinal Survey of Youth (see, for example, Herrnstein and Murray, 1996, p. 319).

References

Abelson, M. (1987). Examination of avoidable and unavoidable turnover, *Journal of Applied Psychology*, *72*, 382-386.

Aldrich, H. H., & Nelson, F. D. (1984). *Linear probability, logit and probit models*. London: Sage.

Blau, G. J., & Boal, K. B. (1987). Conceptualizing how job involvement and organizational commitment affect turnover and absenteeism. *Academy of Management Review*, *12*, 288-300.

Bobko, A. (1986). A solution to some dilemmas when testing hypotheses about ordinal interaction. *Journal of Applied Psychology*, *71*, 323-326.

Caldwell, D., & Oreilly, C. (1985). The impact of information on job choice and turnover. *Academy of Management Journal*, *28*, 934-943.

Campbell, J. P., & Pritchard, R. D., (1976). Motivation theory in industrial and organizational psychology. In M. D. Dunnette (Ed.), *Handbook of industrial and organizational psychology* (pp. 84-95). Chicago: Rand McNally.

Cohen, J., & Cohen, P. (1983). *Applied multiple regression/correlation analysis for the behavioral science*. London: Lawrence Erlbaum.

Cortina, J. M. (1993). Interaction, nonlinearity, and multicollinearity : Implications for multiple regression. *Journal of Management*, *19*, 915-922.

Dawes, R. M., & Corrigan, B. (1974). Linear models in decision making. *Psychological Bulletin*, *81*, 95-106.

Dunlap, W. P., & Kemery, E. R. (1988). Effects of predictor intercorrelations and reliabilities on moderated multiple regression. *Organizational Behavior and Human Decision Processes*, *41*, 248-258.

French, E. G. (1958). The interaction of achievement motivation and ability in problem-solving success. *Journal of Abnormal and Social Psychology*, *57*, 306-309.

Ganzach, Y. (1997). Misleading interaction and curvilinear terms. *Psychological Methods*, *3*, 235-247.

Ganzach, Y. (1998) Nonlinearity, multicollinearity and the probability of type II error in detecting interaction. *Journal of Management*, *24*, 615-622.

Gerhart, B., & Rynes, S. (1990). Determinants and consequences of salary negotiations by male and female MBA graduates. *Journal of Applied Psychology*, *76*, 256-262.

Greenland, S. (1994). Basic problems in interaction assessment. *Environmental Health Perspectives*, *101* (Suppl. 4), 59-66.

Herrnstein, R., & Murray, C. (1996). *The bell curve*. New York: Simon & Schuster.

Hogan, M. D., Kupper, L. L., Most, B. M., & Hasmean, J. K. (1978). Alternatives to Rothman's approach for assessing synergism (or antagonism) in cohort studies. *American Journal of Epidemiology*, *108*, 60-67.

Huselid, M. A., & Day, N. E. (1991). Organizational commitment, job involvement, and turnover: A substantive and methodological analysis. *Journal of Applied Psychology*, *76*, 380-391.

Kanfer, R., & Ackerman, P. L. (1989). Motivation and cognitive abilities: An integrative/aptitude-treatment interaction approach to skill acquisition. *Journal of Applied Psychology—Monograph*, *74*, 657-690.

Kipnis, D. A. (1962). Noncognitive correlates of performance among lower aptitude men. *Journal of Applied Psychology*, *46*, 76-80.

Korman, A. K., Greenhaus, J. H., & Badin, I. J. (1977). Personnel attitudes and motivation. *Annual Review of Psychology*, *28*, 175-196.

Landerman, R. L., George, L. K., Campbell, R. T., & Blazer, D. G. (1989). Alternative models of the stress buffering hypothesis. *American Journal of Community Psychology*, *17*, 625-642.

Lawler, E. E. (1966). Ability as a moderator of the relationship between job attitudes and job performance. *Personnel Psychology*, *19*, 153-164.

Lee, T., & Mowday, R. (1987). Voluntarily leaving an organization: An empirical investigation of Steers and Mowday's model of turnover. *Academy of Management Journal*, *30*, 721-743.

Locke, E. A. (1965). Interaction of ability and motivation in performance. *Perceptual and Motor Skills*, *21*, 719-725.

Locke, E. A. (1982). Relation of goal level to performance with a short work period and multiple goal levels. *Journal of Applied Psychology*, *67*, 512-514.

Locke, E. A., & Latham, G. P. (1990). *A theory of goal setting and task performance*. Engelwood, NJ: Prentice Hall.

Locke, E. A., Mento, A. J., & Katcher, B. L. (1978). The interaction of ability and motivation in performance: An exploration of the meaning of moderators. *Personnel Psychology*, *31*, 269-280.

Lubinski, D., & Humphreys, L. G. (1990). Assessing spurious "moderator effects": Illustrated substantively with the hypothesized ("synergistic") relation between spatial and mathematical ability. *Psychological Bulletin*, *107*, 385-393.

MacCallum, R. C., & Marr, C. M. (1995). Distinguishing between moderator and quadratic effects in multiple regression. *Psychological Bulletin*, *118*, 405-421.

Maddala, G. S. (1983). *Limited-dependent and qualitative variables in econometrics*. Cambridge, UK: Cambridge University Press.

McClelland, G. H., & Judd, C. M. (1993). Statistical difficulties of detecting interactions and moderator effects. *Psychological Bulletin*, *114*, 376-390.

Mowday, R., Koberg, C., & McArthur, A. (1984). The psychology of Withdrawal Process: A cross-validation test of Mobley's intermediate linkage model of turnover in two samples. *Academy of Management Journal*, *27*, 79-94.

Oldham, R. & Fried, Y. (1987). Employee reactions to workplace characteristics. *Journal of Applied Psychology*, *72*, 75-80.

Pedhazur, E. J. (1982). *Multiple regression in behavioral science*. New York: Holt, Rinehart & Winston.

Pindyck, R., & Rubinfeld, D. (1981). *Econometric models and economic forecasts*. New York: McGraw-Hill.

Rothman, K. J., Greenland, S., & Walker, A. M. (1980). Concepts of interaction. *American Journal of Epidemiology*, *112*, 467-470.

Rusbult, C., Farrell, D., Rogers, G., & Mainous, A. (1988). Impact of exchange variables on exit, voice, loyalty, and neglect: An integrative model of responses to declining job satisfaction. *Academy of Management Journal*, *31*, 599-627.

Saracci, R. (1977). Asbestos and lung cancer: An analysis of the epidemiological evidence on the asbestos-smoking interaction. *International Journal of Cancer*, *20*, 323-331.

Sommers, M. J., & Birnbaum, D. (1999). Survival versus traditional methodologies for studying employee turnover: Differences, divergences and directions for future research. *Journal of Organizational Behavior*, *20*, 273-284.

Spencer, D. G., & Steers, R. M. (1981). Performance as a moderator of the job satisfaction—turnover relationship. *Journal of Applied Psychology*, *66*, 511-514.

Tubbs, M. (1994). Commitment and the role of ability in motivation: Comment on Wright, O'Leary-Kelly, Cortina, Klein, and Hollenbeck (1994). *Journal of Applied Psychology*, *79*, 804-811.

Tziner, A., & Eden, D. (1985). Effects of crew composition on crew performance: Does the whole equal the sum of its parts? *Journal of Applied Psychology*, *70*, 85-93.

Vroom, V. (1964). *Work and Motivation*. New York: Wiley.

Walter, S. D. (1976). The estimation and interpretation of attributable risk in health research. *Biometrics*, *32*, 829-849.

Werbel, J., & Gould, S. (1984). A comparison of relationships of commitment to turnover in recent hires and tenured employees. *Journal of Applied Psychology*, *69*, 687-690.

Yoav Ganzach is an associate professor in the Faculty of Management at Tel Aviv University. He received his Ph.D. from Columbia University and his research interests are in organizational behavior and behavioral decision making. His methods research focuses on nonlinear regression models of judgments and decisions and on the estimation of interaction effects.

Ishak Saporta is a lecturer in the Faculty of Management at Tel Aviv University. He received his Ph.D. from University of California at Berkeley. His research interests are in industrial relations and organizational theory.

Yaacov Weber is a senior lecturer in the Faculty of Management at Ben Gurion University. He received his Ph.D. from the University of South Carolina. His research interests are in corporate strategy.