Slotting allowances and information gathering

by

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Abstract: This paper considers a mechanism design problem in which a downstream firm motivates an upstream firm to gather information on whether the demand for its new product is drawn from a high-profit or a low-profit distribution function. The model reveals that the mechanism involves upward or downward distortion in the quantity, depending on the gap between the two cumulative distribution functions. Moreover, the mechanism may involve negative fees in the form of slotting allowances. Slotting allowances increase or decrease social welfare, depending on the parameters of the model.

Keywords: mechanism design, asymmetric information, antitrust policy, productive information gathering

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1. Introduction

A buyer of an intermediate input may sometimes require a supplier to test a new and untried product that it is offering. Supermarkets and drugstores, for example, have limited shelf space and may ask a supplier to perform market research for a new product in order to determine whether to place it on the shelf instead of the old one. A manufacturer of an electronic or mechanical product such as a TV, laptop or a car may ask a supplier offering a new component, such as a new battery for a laptop or a new fuel pump for a car, to test its durability or safety before deciding whether to replace the old one.

This paper considers a mechanism design problem when a downstream firm, D, needs to motivate an upstream firm, U, to gather costly information concerning the demand for its new product. This information is of value to D in deciding whether to sell this new product to final consumers instead of an old one. U can only gather imperfect information: U can perform a costly test that reveals whether the state is H (L) in which case the demand is drawn from an expected high-profit (low-profit) distribution function. D cannot observe whether U indeed conducted a reliable test. If D chooses to sell the new product, D can privately observe the realization of demand, but cannot know the state from which the demand was drawn. D can then choose the quantity to be produced based on the actual realization of demand.

The paper reveals that the equilibrium contract for motivating U to gather information has two main characteristics. First, D distorts the equilibrium quantity away from the vertical-integration quantity. This distortion depends on the gap between the cumulative distribution functions in states L and H. If for a certain realization of demand this gap is positive (negative), then for this specific realization of demand D distorts the quantity downward (upward). Consequently, the contract may specify downward distortion in the quantity for some realizations of demand and upward distortion for others, depending on the shape of the two cumulative distribution functions. The second main characteristic of the equilibrium contract is that it may include negative fees, in which case U pays D. The intuition for these two results is that D distorts the equilibrium payment to U such that U's ex-post payoff depends on the state, making it profitable for U to accept the contract only after testing the new product and realizing that the state is H. However, by doing so, D has an ex-post incentive not to reveal to U the true realization of demand. Consequently, distorting the payment to U requires D to ex-ante distort the equilibrium quantity in order to motivate D to ex-post reveal the true demand, and the optimal contract balances between these two considerations.

The result that the contract may include negative fees can provide a new explanation for the use of slotting allowances, the upfront payments that manufacturers make to retailers such as supermarkets or drugstores for reserving shelf space. Slotting allowances are very common in the retail grocery industry. For example, the Federal Trade Commission (2003) found that
manufacturers paid slotting allowances for introducing bread, hot dogs, ice cream, pasta and salad dressing. Moreover, introducing a new grocery product requires, on average, paying $1.5 - $2 million in slotting allowances.

The paper finds that in the context of this model, slotting allowances can reduce social welfare even though they have the potential welfare enhancing property of enabling D to efficiently allocate its limited shelf space. Intuitively, slotting allowances involve distorting the quantity downward for some realizations of demand, and D does not internalize the negative effect that this has on consumers and welfare. This result indicates that antitrust authorities should not always tolerate potential anti-competitive effects of slotting allowances on the ground that they enable retailers to efficiently use their limited shelf space, because the latter effect can be yet another source of market inefficiency.

My paper is related to several areas of the economics literature. The first area is the literature on information gathering. Crémer and Khalil (1992), Lewis and Sappington (1997), Crémer, Khalil and Rochet (1998a,b), Shin (2008) and Szalay (forthcoming) consider a principal-agent problem when an agent can institute a costly gathering of information regarding a project. Finkle (2005) and Nosal (2006) consider the case when the principal can gather information and convey it to an agent. My paper is closely related to Crémer, Khalil and Rochet (1998a), Shin (2008) and Szalay (forthcoming) who consider an agent that can gather information concerning its marginal cost that is valuable for the principal. These papers show that the direction of the quantity distortion is affected by the average marginal cost because if the agent does not gather information, the agent chooses to produce the quantity that corresponds to the average costs. In contrast, in my paper it is the principal, namely D, that chooses the quantity to be produced and therefore the quantity distortion is not affected by the average but by the gap between the two cumulative distribution functions.

Eliaz and Spiegler (2007), Eliaz and Spiegler (2008a) and Eliaz and Spiegler (2008b) consider a mechanism design problem in which players have different priors over potential states. They show that contracts are essentially "bets" among players over the potential outcomes. Following their terminology, it is possible to interpret the equilibrium contract that D offers U in this paper as a "bet" over the potential realizations of demand, in that the contract assigns high (low) transfers for high (low) realizations of demand. The difference between this paper and the above studies is that while they assume that the players’ priors are exogenous, in this paper D uses the contract to manipulate U’s prior. That is, D's problem is to design a contract that includes a "bet" that will motivate U to carry out a costly updating of U’s prior and to accept the "bet" only in state H, as this state places high probabilities on high realizations of demand.

1 In a seller-buyer context, Matthews and Persico (2005) consider a seller that motivates consumers to gather information by offering a generous refund policy.
Matthews and Postlewaite (1985), Shavell (1994) and Polinsky and Shavell (2006) consider firms that can acquire information concerning the quality or safety of their products, and choose whether to credibly reveal it to consumers. This literature focuses on the question of whether mandatory disclosure increases or decreases the incentives of firms to acquire information, and the effect of mandatory disclosure on welfare. The informational structure in this paper differs from the above papers in that I assume that the seller, U, cannot credibly disclose its acquired information to the intermediate buyer, D. The paper reveals that if U can credibly disclose information, then D can write a contract that motivates U to do so without having to distort the quantity away from the vertical integration quantity.

My paper is also related to the literature on slotting allowances. Shaffer (1991), Kim and Staelin (1991), Shaffer (2005), Rey, Miklós-Thal and Vergé (2006), Innes and Hamilton (2006), Kuksov and Pazgal (2007) and Marx and Shaffer (2007) consider them to be the result of retail competition. Marx and Shaffer (2004) and Inderst (2005) show that a retailer may limit its shelf space and charge slotting allowances to increase the competition among suppliers for shelf space. Sullivan (1997) shows that slotting allowances emerge as part of an equilibrium clearance condition in a market with a set of competing manufacturers and retailers. Nocke and Thanassoulis (2009) consider vertical relations between credit constraint firms, and show that the optimal risk-sharing contract involves slotting allowances.

Another explanation for slotting allowances, which is closely related to my paper, is that they serve to convey manufacturers' private information regarding demand. Chu (1992) shows that a retailer may charge a slotting allowance to screen high demand manufacturers from low demand manufacturers. Lariviere and Padmanabhan (1997) show that a manufacturer will signal high demand through slotting allowances. Desai (2000) focuses on the tradeoff between slotting allowances and advertising in signaling quality to retailers. There are two main differences between the above literature and my paper. First, these papers assume that the manufacturer already knows the demand, thus ignoring the question of why manufacturers may perform a costly test to obtain this information. Second, Lariviere and Padmanabhan (1997) and Desai (2000) assume that the manufacturer has the bargaining power to set the slotting allowance and the wholesale price, and Chu (1992) assumes that the manufacturer has the bargaining power to set the wholesale price alone. In contrast, my paper focuses on the case where the retailer has full bargaining power to offer a non-linear tariff that may include a payment from U to D. The results of my model show that these two differences are crucial: if D has all the bargaining power, and U knows the true demand without testing the product, then the slotting allowance will not emerge in equilibrium. Therefore, this paper contributes to the above literature by explaining why retailers use slotting allowances in cases when they have significant bargaining power and manufacturers need to invest in costly information gathering.
The rest of the paper is organized as follows. The next section describes the model and the vertical integration benchmark. Section 3 considers the mechanism for motivating \(U\) to test the product when \(D\) cannot observe whether \(U\) has actually done so, and the conditions under which \(D\) will use such a mechanism. Section 4 evaluates the effect of slotting allowances on social welfare. Section 5 offers some concluding remarks. All the proofs are in the Appendix.

2. The model

Consider a market with a monopolistic downstream firm, \(D\), and an upstream firm, \(U\), both risk neutral. \(U\) can produce a new intermediate product at cost \(c(q)\), where \(q\) is the quantity and \(c(q) \geq 0\), \(c(q) \geq 0\), \(c(0) = 0\) and \(\lim_{q \to \infty} c(q) \to \infty\) for \(q \to \infty\). \(D\) can transform the new intermediate product into a final product at a one-to-one technology and sell it to final consumers at zero costs. The inverse demand for the new product is \(p(q; \theta)\), where \(p\) and \(q\) are the price and quantity respectively, and \(\theta\) measures consumers' willingness to pay for the new product. Suppose that \(p_q(q; \theta) < 0\) and \(p_\theta(q; \theta) > 0\). The parameter \(\theta\) can represent the new product's actual or perceived quality, from the viewpoint of consumers. For example, if \(D\) is a manufacturer of some electronic or mechanical device (say, a car, TV, computer, etc.), and \(U\) is a supplier that offers a new component (say, a new fuel pump for a can, a new battery for a laptop), then \(\theta\) can represent the quality of this component, such as its durability, compatibility with other components or safety.\(^2\) If \(D\) is a supermarket and \(U\) is a manufacturer that offers a new food product (a new yogurt flavor, for example), then \(\theta\) can represent the degree to which consumers find this new flavor tasty and appealing.

Let \(V(q; \theta) \equiv p(q; \theta)q\) denotes \(D\)'s payoff from selling the new product. Suppose that \(D\)'s marginal payoff is positive for low values of \(q\), decreasing with \(q\) and increasing with \(\theta\):

\[
V_{qq}(q; \theta) < 0, \quad V_q(q; \theta) > 0 \quad \text{and} \quad V_q(0; \theta) > 0. \quad (1)
\]

Let \(q^*(\theta)\) denote the quantity that maximizes vertical integration payoffs, \(V(q; \theta) - c(q)\), where \(q^*(\theta)\) is the solution to:

\[
V_q(q^*(\theta); \theta) - c_q(q^*(\theta)) = 0.
\]

It follows from the assumptions above that \(q^*(\theta)\), \(V(q^*(\theta); \theta)\) and \(V(q^*(\theta); \theta) - c(q^*(\theta))\) are increasing with \(\theta\).

\(^2\) In the latter interpretation it is reasonable to suspect that \(\theta\) affects \(D\)'s costs and not the actual demand. For example, suppose that each unit of the product can inflict on \(D\) a cost \(d(\theta)\), where \(d_\theta(\theta) < 0\), that measures the expected cost of paying liability in case the product cause damages, or the expected costs of a recall. In this scenario, given a demand function \(p(q)\), \(D\) earns on each unit \(\rho(q; \theta) = p(q) - d(\theta)\).

\(^3\) Notice that \(V_{qq}(q; \theta) = p_{qq}(q; \theta)q + 2p_q(q; \theta)\). Since \(p_q(q; \theta) < 0\), \(V_{qq}(q; \theta) < 0\) whenever \(p_{qq}(q; \theta)\) is either negative, or positive but sufficiently low. Also, \(V_q(q; \theta) = p_{qq}(q; \theta)q + p_q(q; \theta) > 0\) requires that \(p_{qq}(q; \theta)\) is positive, or negative but sufficiently low in absolute terms because \(p_q(q; \theta) > 0\). Finally, \(V_q(0; \theta) > 0\) only requires that \(p(0; \theta) > 0\).
Suppose that the parameter $\theta$ is initially unknown to both U and D. U and D believe that $\theta$ is distributed along the interval $[\theta_0, \theta_1]$ according to one of two probability functions. With probability $\gamma$, $0 < \gamma < 1$, the state is "H" and $\theta$ is drawn from a "high" probability distribution function, $f_H(\theta)$. With probability $1 - \gamma$, the state is "L" and $\theta$ is drawn from a "low" probability distribution function, $f_L(\theta)$. Suppose that $f_k(\theta) > 0$, $\forall \theta \in [\theta_0, \theta_1]$, $\forall k = \{H, L\}$. This assumption implies that any $\theta \in [\theta_0, \theta_1]$ can be drawn from both $f_H(\theta)$ and $f_L(\theta)$, and D cannot learn the state by observing $\theta$. The cumulative distribution function for $f_k(\theta)$, $k = H, L$, is $F_k(\theta)$, where $F_k(\theta_0) = 0$ and $F_k(\theta_1) = 1$. Let

$$E_k(V(q^*(\theta); \theta) - c(q^*(\theta))) = \int_{\theta_0}^{\theta_1} (V(q^*(\theta); \theta) - c(q^*(\theta))) f_k(\theta) d\theta, \quad k = \{H, L\},$$

(2)

denote the expected vertical integration payoff that can be obtained from the new product in state $k = \{H, L\}$, given that a vertically integrated firm can observe the realization of $\theta$ before setting $q^*(\theta)$. The condition for the difference between the two distributions is therefore that $E_H(V(q^*(\theta); \theta) - c(q^*(\theta))) > E_L(V(q^*(\theta); \theta) - c(q^*(\theta)))$. That is, from the viewpoint of maximizing the vertical integration profit, the new product is more profitable (in expectation) when the state is H. One obvious example is the case where $F_H(\theta)$ dominates $F_L(\theta)$ by First Order Stochastic Dominance (FOSD). However, the mechanism that I am considering is also sustainable for distribution functions that cannot be ranked according to FOSD, as shown in the examples provided below.

Since U offers a new intermediate product, D can choose not to carry it and instead offer an old product of known quality. For example, a manufacturer of an electronic or technical device can choose to use an old and already well-known components rather than using a new component that may either increase or decrease the total quality of the its product. A supermarket may choose to use its limited shelf space for selling well-known food products (say, a familiar yogurt flavor) rather than clearing shelf space for a new product that consumers may or may not find tasty. Suppose that the payoff that D can obtain from not using the new intermediate product is $V^*$, where $E_H(V(q^*(\theta); \theta) - c(q^*(\theta))) > V^* > E_L(V(q^*(\theta); \theta) - c(q^*(\theta)))$. That is, under vertical integration, it is profitable to sell the new product only in state H. Notice that I assume, without loss of generality, that the costs of the old product are zero.

Suppose that U can perform a test, at cost $C$, that reveals whether the state is H or L. For example, the supplier of an intermediate component such as a fuel pump for a car or a laptop battery can perform product tests to evaluate the durability, compatibility or safety of the new component. A yogurt manufacturer can perform market research to evaluate whether consumers are likely to find the new yogurt tasty, such as gathering marketing data through
trying the product on test groups or at selling points, as well as analyzing the marketing data in order to provide an estimate of the demand. Notice that the test is always inaccurate: it can only refine the prior probabilities of \( \theta \) but not fully reveals \( \theta \). Also, I assume that the test cannot refine the support of the distribution function, \( [\theta_0, \theta_1] \), as \( f_d(\theta) > 0, \forall \theta \in [\theta_0, \theta_1], \forall k = \{H,L\} \).

Suppose that D cannot perform its own test, or can do so at a very high cost, making it always unprofitable. In some cases, it is reasonable to expect that U, being the actual developer and producer of the intermediate product, is in a better position to use its acquired knowledge for testing it and evaluating its quality or safety. In the retail industry, supermarkets and drugstores usually carry a large variety of products and therefore cannot perform market research for each individual product.\(^4\)

Consider the following three-stage game, which I illustrate in Figure 1. In the first stage, D offers a take-it-or-leave-it contract to U. There are many potential contracts that D may offer U. I consider a general form of a menu, \((q(\theta), T(\theta))\). Accordingly, if U accepts the contract, D commits to use the new product instead of the old one, and then, upon observing the realization of the demand, \( \theta \), D commits to choose some \( q(\theta) \) from the menu and in return pay an amount \( T(\theta) \), where \( T(\theta) \) may be positive or negative. In the second stage, U observes the contract and chooses whether to perform the test or not. D cannot observe whether or not U has performed a reliable test, or the results of the test. After either observing the results of the test or not performing it, U decides whether to accept the contract or not. U's reservation profit from rejecting the contract is zero. In the third stage, if U has rejected the contract, or if D has chosen in the first stage not to make an offer to begin with, then D remains with the old intermediate product and earns \( V^* \). If D has made an offer that U has accepted, then D carries the new product, and then a learning process begins in which D learns \( \theta \). Following Chu (1992) in the context of the retail industry, I assume that this learning process is instantaneous, though D cannot learn \( \theta \) without forgoing the potential profit from the old intermediate product, \( V^* \).\(^5\) For example, many supermarkets use a barcode system that provides accurate, up-to-date data on actual sales. Also, suppose that only D can ex-post observe \( \theta \).\(^6\) Notice that since by assumption \( f_d(\theta) > 0, \forall k = H,L, \forall \theta \in [\theta_0, \theta_1] \), even though D observes \( \theta \) ex-post, D cannot infer from \( \theta \) if the test has been made. Had this been the case, D could have used this ex-post information in order to write a contract that motivates U to test the new product without having to abstract from the vertical integration outcome.

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\(^4\) The FTC (2001) reports that suppliers introduce 20,000 new items each year.

\(^5\) Based on several case studies, the FTC (2003) reports that retailers usually stock new products in the store for a "reasonable" period of at least four to six months, to give the new product a fair chance to get established.

\(^6\) An equivalent assumption is that both D and U can ex-post observe \( \theta \), but \( \theta \) is not verifiable and therefore not contractible.
From the viewpoint of maximizing total profits, a vertically integrated firm can choose between performing the test, selling the new product without testing it first, or selling the old product. Therefore, there is a cutoff,

\[
C^* = \begin{cases} 
\gamma \left( E_H \left( V(q^*(\theta); \theta) - c(q^*(\theta)) \right) - V^* \right), & \text{if } V^* > \bar{V}^*, \\
(1 - \gamma) \left( V^* - E_L \left( V(q^*(\theta); \theta) - c(q^*(\theta)) \right) \right), & \text{if } V^* \leq \bar{V}^*,
\end{cases}
\]

where \( \bar{V}^* \equiv \gamma E_H(V(q^*(\theta); \theta) - c(q^*(\theta))) + (1 - \gamma)E_L(V(q^*(\theta); \theta) - c(q^*(\theta))) \), such that the vertically integrated firm performs the test if and only if \( C < C^* \).

If U and D are vertically separated, but D can observe whether U tested the new product and the results of the test, then D will implement the vertical integration outcome and earn the vertical integration payoff by offering a menu \((q(\theta), T(\theta)) = (q^*(\theta), C/\gamma + c(q^*(\theta)))\) contingent on observing the result that the state is H. U's expected payoffs are zero, though U earns positive payoff in state H. Notice that D will truthfully report \( \theta \) because by setting \( T(\theta) = C/\gamma + c(q^*(\theta)) \) D fully internalizes U's production cost.

3. Asymmetric information

This section describes the case of asymmetric information, in which U has to perform the costly test in order to learn the state, and performing the test, as well as the results of the test, is U's private information. The main conclusion of this section is that under such an informational structure, the optimal mechanism for motivating U to test the new product includes downward or upward distortion in the quantity, depending on the two distribution functions. I will proceed as follows. First, I will define and solve for the optimal mechanism (from D's viewpoint) that induces U to perform the test. Then, I will characterize the features of the mechanism. Finally, I will move to the question of whether D will indeed find it optimal to use such a mechanism and how D can implement it.

3.1. The optimal mechanism for motivating U to gather information

Because D cannot observe whether U has performed the test and the results of the test, D faces the problem of moral hazard and adverse selection: D needs to motivate U to perform the test, and to reveal its outcome. D's problem is therefore:

\[
\max_{(q(\theta), T(\theta))} \ E_H \left( V(q(\theta); \theta) - T(\theta) \right),
\]

\[
\text{s.t.} \quad (IC_{U}^{\text{pre-post}}) \quad E_d \left( T(\theta) - c(q(\theta)) \right) \leq 0,
\]
\( (I_{U}^{\text{ex-post}}) \quad \mathbb{E}_{D}(T(\theta) - c(q(\theta))) \geq 0, \)
\( (I_{U}^{\text{ex-ante}}) \quad -C + \gamma \mathbb{E}_{D}(T(\theta) - c(q(\theta))) \geq \gamma \mathbb{E}_{D}(T(\theta) - c(q(\theta))) + (1 - \gamma) \mathbb{E}_{D}(T(\theta) - c(q(\theta))), \)
\( (I_{R}^{\text{ex-ante}}) \quad -C + \gamma \mathbb{E}_{D}(T(\theta) - c(q(\theta))) \geq 0, \)
\( (I_{C_{D}}) \quad \theta = \arg \max_{\tilde{\theta}} \left( V(q(\tilde{\theta}); \theta) - T(\tilde{\theta}) \right). \)

The first two constraints relate to U's ex-post behavior. After performing the test, \( I_{C_{D}}^{\text{ex-post}} \) ensures that U prefers not to accept the contract if the state is L, while \( I_{R}^{\text{ex-post}} \) ensures that U accepts the contract if the state is H. The next two constraints relate to U's ex-ante behavior. The \( I_{C_{D}}^{\text{ex-ante}} \) constraint ensures that U prefers to perform the test, given that afterwards it will accept the contract only if the state is H (which occurs with probability \( \gamma \)), over accepting the contract without testing the new product first. The \( I_{R}^{\text{ex-ante}} \) constraint ensures that U prefers to perform the test over not interacting with D. The first four constraints are derived under the assumption that U expects D to choose the line from the menu that corresponds to the true \( \theta \).

Therefore, a fifth constraint, \( I_{C_{D}} \), is D's incentive compatibility, which ensures that given that U has accepted the contract and D has observed the realization of \( \theta \), D will choose the corresponding line from the menu by truthfully reporting \( \theta \). This last constraint emerges because of D's ex-post private information and the inability to contract on the actual realization of \( \theta \).

To solve this problem, notice that \( I_{C_{D}}^{\text{ex-post}} \) and \( I_{R}^{\text{ex-post}} \) are not binding, as they are satisfied whenever \( I_{C_{D}}^{\text{ex-ante}} \) and \( I_{R}^{\text{ex-ante}} \) are satisfied. Next I turn to rewriting \( I_{C_{D}} \). Let \( U(\tilde{\theta}; \tilde{\theta}) = V(q(\tilde{\theta}); \theta - T(\tilde{\theta}) \) and let \( U(\theta) = U(\theta; \theta) \) denote D's ex-post payoff. We have:

**Lemma 1:** Suppose that \( q(\theta) \) is continuous and twice differentiable. Then, necessary and sufficient conditions for \( I_{C_{D}} \) are that \( q(\theta) \) is increasing with \( \theta \) and D earns:

\[
U(\theta) = U(\theta_{0}) + \int_{\theta_{0}}^{0} \nu_{\theta}(q(\tilde{\theta}); \tilde{\theta}) d\tilde{\theta}.
\]

Intuitively, in order to induce D to ex-post report \( \theta \), D needs to specify D's ex-post "information rents", defined in (5). These information rents differ from the usual information rents in the principal-agent literature in that here, the principal, D, leaves them not to the agent, but to itself. Using (5), \( I_{C_{D}} \) requires that:

\[
T(\theta) = V(q(\tilde{\theta}); \theta) - \int_{\theta_{0}}^{0} \nu_{\theta}(q(\tilde{\theta}); \tilde{\theta}) d\tilde{\theta} - U(\theta_{0}),
\]
Using (6) I can now rewrite D's problem as:

$$\max_{q(\theta)} E_H \left[ V(q(\theta); \theta) - c(q(\theta)) - \frac{C}{\gamma} \right],$$

s.t.

$$E_H \left( T(\theta) - c(q(\theta)) \right) - E_L \left( T(\theta) - c(q(\theta)) \right) \geq \frac{C}{\gamma(1-\gamma)},$$

$$q_0(\theta) > 0 \text{ and (6).}$$

Therefore, D's problem is to maximize the vertical integration profit, subject to the constraint that the gap between U’s expected payoffs in states H and L is sufficiently wide in comparison with the cost of the test. This constraint is binding, as the following Lemma shows:

**Lemma 2:** The full information quantity, q*(θ), does not satisfy (6) and (8).

I therefore maximize (7) given that (6) and (8) are binding, and then verify that the solution satisfies q_0(θ) > 0. Let q**(θ) and T**(θ) denote the equilibrium contract. Substituting (6) into (8), the first order condition with respect to q(θ) is:

$$V_q(q(\theta); \theta) - c_q(q(\theta)) = \lambda V_{q_0}(q(\theta); \theta) \frac{F_L(\theta) - F_H(\theta)}{f_H(\theta) + \lambda (f_H(\theta) - f_L(\theta))},$$

where the left-hand side is the first order condition under vertical integration (see (1)), and λ is the Lagrange multiplier. For λ = 0, right-hand side of (9) equals zero and therefore the solution is identical to the vertical integration quantity. If λ > 0 but not too high, then the term in the denominator in the right-hand side of (9) is positive because by assumption f_H(θ) > 0, ∀θ ∈ [θ₀, θ₁]. Moreover, recalling that by assumption V_q(q(θ); θ) > 0, the sign of the quantity distortion is negatively affected by the sign of the gap F_L(θ) – F_H(θ). The first Proposition shows that this is indeed the case:

**Proposition 1:** There is an interior solution to D's problem if C is sufficiently small and γ is intermediate. In this solution, for a given θ, q**(θ) < (>) q*(θ) if F_L(θ) > ( <) F_H(θ). The gap |q*(θ) – q**(θ)| equals 0 if C = 0 and is increasing (decreasing) with C, and decreasing (increasing) with γ for γ < (>) 1/2.

Proposition 1 indicates that the distortion in the equilibrium quantity at a given θ depends on the gap between the two cumulative distribution functions. Intuitively, recall that under full
information D sets $T(\theta) = C/\gamma + c(q^*(\theta))$ such that U's expected payoff is zero. The problem however is that in this case, U's payoff is independent of the realization of $\theta$, and therefore under asymmetric information U will not have an incentive to test the new product. D will therefore have to distort $T(\theta)$ away from $c(q(\theta)) + C/\gamma$. More precisely, in order to increase the gap between U's payoff in states H and L, D will have to set $T(\theta)$ such that U's payoff is increasing (decreasing) with $\theta$ if $F_L(\theta) > (<) F_H(\theta)$.

However, making U's payoff dependent on $\theta$ creates another problem for D. If this distortion in $T(\theta)$ is such that U's payoff is increasing with $\theta$ (the case where $F_L(\theta) > F_H(\theta)$), then ex-post, after D observes the realization of $\theta$, D has an incentive to understate $\theta$ in order to reduce U's payoff. Consequently, D will have to ex-ante distort the quantity that D specifies in the menu downwards in order to prevent D itself from ex-post understating $\theta$. Likewise, if the distortion in $T(\theta)$ is such that U's payoff is decreasing with $\theta$ (the case where $F_L(\theta) < F_H(\theta)$), D has an incentive to overstate $\theta$ ex-post, forcing D to ex-ante distort the equilibrium quantity upwards. It is possible to illustrate this point by substituting (6) into $T(\theta) - c(q(\theta))$ and differentiating with respect to $\theta$:

$$\frac{d}{d\theta}(T(\theta) - c(q(\theta))) = \left(V_q(q(\theta); \theta) - c_q(q(\theta))\right)q_0(\theta).$$

Since $q_0(\theta) > 0$ and since $V_q(q(\theta); \theta) - c_q(q(\theta)) = 0$ is the first order condition under vertical integration, it follows that if U's payoff is increasing (decreasing) with $\theta$, ICD forces D to distort the quantity downwards (upwards).

To conclude, under asymmetric information D distorts U's payoff, such that it will depend on $\theta$, in order to motivate U to test the new product, and distort the equilibrium quantity to motivate D to truthfully report $\theta$. The optimal solution balances between these two considerations. This result differs from Crémer Khalil and Rochet (1998), Shin (2008) and Szalay (forthcoming), which have shown that in order to motivate an agent to gather costly information concerning the agent's marginal cost, a principal distorts the equilibrium quantity (in comparison with the case were the agent has private information concerning the cost) upwards (downwards) if the cost is below (above) the average.\textsuperscript{7} Intuitively, in these papers the direction of the quantity distortion is determined by the average because it is the agent who decides the quantity to be produced and if the agent does not perform the test, the agent will

\textsuperscript{7}Szalay (forthcoming) assumes that as the agent invests more effort to acquire information, the agent receives a more accurate signal of the average cost. Therefore, the quantity distortion is not affected by the actual cost but by the signal the agent receives concerning the average cost. finds that to motivate the agent to gather more information (than the agent would like to acquire), the quantity is distorted downwards (upwards) as the signal indicates that the cost is above (below) the prior mean.
always choose to produce the quantity that corresponds to the average cost. In this paper however, while the agent performs the test, it is the principal that chooses the quantity, and the motivation for the quantity distortion is driven by the principal's ability to observe ex-post the actual realization of $\theta$ and by the need to motivate the principal to choose the right quantity; thus the quantity distortion is not necessarily related to the average.⁸

As for the effect of $C$ on the mechanism, the result that $D$ can implement the vertical integration outcome if $C = 0$ indicates that the assumption that $U$ needs to costly gather information is crucial in this model. As $C$ increases, a higher gap between $U$'s expected payoff in the two states is needed to motivate $U$ to test the new product, which in turn forces $D$ to increase the distortion in the quantity (either upwards or downwards). The effect of $\gamma$ on the equilibrium distortion is however non-monotone. For low values of $\gamma$, $U$ has a strong incentive not to perform the test and to reject the contract because it is most likely that the state will turn out to be $L$ anyway, making $IR^{ex-ante}_U$ more restrictive. For high values of $\gamma$, $U$ has a strong incentive to accept the contract without performing the test because the state is most likely to be $H$ anyway, making $IC^{ex-ante}_U$ more restrictive.

Notice that an interior solution to $D$'s problem can only be ensured if $C$ is not too high or $\gamma$ is intermediate. Intuitively, if $C$ is high enough or $\gamma$ is either close to zero or one, then the resulting distortion in $q(\theta)$ can potentially be too significant, such that there will be some realizations of $\theta$ in which $q_0^{**}(\theta) < 0$, in violation of the conditions of Lemma 1. Also, the proof of Proposition 1 shows that the second order condition for $D$'s problem can be ensured only if $C$ is low or $\gamma$ is close to 1/2. Otherwise, $D$'s problem as defined above does not have an interior solution. The critical values of $C$ and $\gamma$ that can give rise to such a problem, if at all, depend on the specification of the two distribution functions. To avoid making additional assumptions on the distribution functions, I will focus the discussion on the case where $C$ is low or $\gamma$ is intermediate, for which Proposition 1 ensures that there is an interior solution.

3.2. The characteristics of the optimal mechanism

Next I turn to specifying the characteristics of the equilibrium mechanism in more detail. As it is possible to think of a wide set of distribution functions that satisfy the condition that the vertical integration payoff is higher in state $H$ than in state $L$, the distortion in the equilibrium quantity, which is a direct result of the sign of $F_L(\theta) - F_H(\theta)$, may vary substantially. Moreover, as I will show below, the distortion is not directly related to the mean and variance

⁸ Notice that (9) is equivalent to the first order conditions (15), (5) and (24) in Crémer Khalil and Rochet (1998), Shin (2008) and Szalay (forthcoming) respectively. While in these papers the sign of the additional term that determines the direction of the quantity distortion is affected by the average, the sign of the additional term in (9) is affected by the gap between the two distribution functions.
of the two distributions. I will therefore offer some polar cases of distribution functions, and then move to characterizing the solution even further by making specific assumptions on the demand and cost.

The first and most natural polar case to think of is the case where \( F_H(\theta) \) dominates \( F_L(\theta) \) by FOSD: \( F_L(\theta) > F_H(\theta), \forall \theta \in (\theta_0, \theta_1) \). Since \( V(q^*(\theta); \theta) - c(q^*(\theta)) \) is increasing with \( \theta \), FOSD always satisfies the assumption \( E_H(V(q^*(\theta); \theta) - c(q^*(\theta))) > E_L(V(q^*(\theta); \theta) - c(q^*(\theta))). \) Applying Proposition 1 yields:

**Corollary 1:** Suppose that \( F_H(\theta) \) dominates \( F_L(\theta) \) by FOSD. Then, in the interior solution to D's problem, the equilibrium quantity is distorted downwards for \( \forall \theta \in (\theta_0, \theta_1) \), and equals the vertical integration quantity at the two extremes of the support, \( \theta_0 \) and \( \theta_1 \). Moreover, the equilibrium payment to U is increasing with \( \theta \) for \( \forall \theta \in [\theta_0, \theta_1] \).

The case of FOSD is illustrated in Panel (a) of Figure 2. For the case of FOSD, the quantity is distorted downwards regardless of the average. As for the equilibrium payment to U, \( F_L(\theta) > F_H(\theta) \) implies that \( T^*(\theta) - c(q^*(\theta)) \) is increasing with \( \theta \), and therefore \( T_0^*(\theta) > c(q^*(\theta))q_0^*(\theta) > 0 \). Notice that \( T^*(\theta) \) can therefore be negative for low realizations of \( \theta \). I elaborate on the implications of this result for public policy in the next section.

The second polar scenario concerns cases where the two distribution functions cannot be ranked according to FOSD. This implies that there is at least one interior intersection point between \( F_H(\theta) \) and \( F_L(\theta) \). To generate clean and intuitive predictions concerning the quantity distortion in this case, I follow Diamond and Stiglitz (1974) by assuming that \( F_H(\theta) \) and \( F_L(\theta) \) satisfy the single-crossing condition. Applying Proposition 1 yields:

**Corollary 2:** Suppose that there is exactly one \( \theta^C \in (\theta_0, \theta_1) \) such that \( F_H(\theta^C) = F_L(\theta^C) \). Then, in the interior solution to D's problem,

(i) \( f_L(\theta^C) < f_d(\theta^C) \), then the equilibrium quantity is distorted downwards (upwards) for \( \theta \in (\theta_0, \theta^C) (\theta \in (\theta^C, \theta_1)) \);

(ii) \( f_L(\theta^C) > f_d(\theta^C) \), then the equilibrium quantity is distorted upwards (downwards) for \( \theta \in (\theta_0, \theta^C) (\theta \in (\theta^C, \theta_1)) \).

The case where \( F_H(\theta) \) and \( F_L(\theta) \) satisfy the single-crossing condition is illustrated in panels (b) and (c) of Figure 2. Panel (b) illustrates part (i) of Corollary 2 in which \( F_H(\theta) \) intersects \( F_L(\theta) \) from below, such that the quantity is distorted downwards for low values of \( \theta \) and upwards for high values of \( \theta \). Panel (c) illustrates part (ii) where \( F_H(\theta) \) intersects \( F_L(\theta) \) from
above and the quantity distortion is completely reversed. Notice that if in addition to the single-crossing property, \( F_H(\theta) \) dominates \( F_L(\theta) \) by Second Order Stochastic Dominance (SOSD), then only the first scenario (part (i)) is possible.\(^9\) As for the payment to \( U \), here the effect of \( \theta \) on \( T^*(\theta) \) is inconclusive because \( T^*(\theta) - c(q^*(\theta)) \) is decreasing with \( \theta \) whenever \( F_L(\theta) < F_H(\theta) \).

The problem with the single-crossing condition, as well as with SOSD, is that these two features may not satisfy the assumption
\[
E_{iH}(V(q^*(\theta);\theta) - c(q^*(\theta))) > E_{iL}(V(q^*(\theta);\theta) - c(q^*(\theta))).
\]
Intuitively, SOSD ensures \( E_{iH}(V(q^*(\theta);\theta) - c(q^*(\theta))) > E_{iL}(V(q^*(\theta);\theta) - c(q^*(\theta))) \) only if \( V(q^*(\theta);\theta) - c(q^*(\theta)) \) is concave in \( \theta \). However,
\[
\frac{d^2}{d\theta^2} (V(q^*(\theta);\theta) - c(q^*(\theta))) = V_{\phi\theta}(q^*(\theta);\theta) + V_{\phi\phi}(q^*(\theta);\theta)q_0^*(\theta). \tag{11}
\]

Since by assumption \( V_{\phi\theta}(q;\theta) > 0 \) and since \( q_0^*(\theta) > 0 \), (11) is positive if \( V_{\phi\theta}(q^*(\theta);\theta) \) is either positive or negative but small in absolute terms. This raises the question of whether the two cases in Corollary 2 are possible under that assumption that \( E_{iH}(V(q^*(\theta);\theta) - c(q^*(\theta))) > E_{iL}(V(q^*(\theta);\theta) - c(q^*(\theta))) \), and how these two cases are affected by the shape of the distribution functions and their variance and mean.

The answer to the above question depends on the specific shape of the two distribution functions and on D's payoff. To show that indeed the two cases in Corollary 2 are feasible, and to characterize the solution even further, I turn to making more specific assumptions on the two distribution functions and the demand. Consider first the two distribution functions. Suppose that \( \theta \) is distributed in state \( \text{H} \) along the unit interval according to some probability distribution function \( f_H(\theta) \) with mean \( \mu_H = 1/2 \) and variance \( \sigma_H^2 \). In state \( \text{L} \), \( f_L(\theta) \) is a triangle transformation of \( f_H(\theta) \) in that
\[
f_L(\theta) = f_H(\theta) + g(\theta),
\]
and \( 0 < \alpha < 2, \ 0 < \beta < 1 \). Figure 3 illustrates the probability and cumulative distribution functions given \( g(\theta) \). For illustrative reasons only the figure shows the case where \( f_H(\theta) \) is the uniform distribution. The analysis below allows for any \( f_H(\theta) \), including those that are

\[^9\] This is because SOSD requires that \( \int_{\theta_0}^{\theta} F_L(\theta)d\theta \geq \int_{\theta_0}^{\theta} F_H(\theta)d\theta \), \( \forall \theta \in [\theta_0, \theta_1] \).
nonlinear in $\theta$, in which case $f_L(\theta)$ is also nonlinear. As the figure illustrates, the shape of the gap between $f_L(\theta)$ and $f_H(\theta)$ and between $F_L(\theta)$ and $F_H(\theta)$ is determined by the parameters $\alpha$ and $\beta$ in the following way. The parameter $\alpha$ determines the gap in the weight that the two distributions place on the extremes of the support. If $0 < \alpha < 1$ ($1 < \alpha < 2$), then state L places less (more) weight on the extremes of the support than state H, as illustrated in Panel (a) (Panel (b)) of Figure 3. Consequently, $F_L(\theta)$ and $F_H(\theta)$ satisfy the single-crossing condition and if $0 < \alpha < 1$ ($1 < \alpha < 2$), $F_L(\theta)$ crosses $F_H(\theta)$ from below (above). The parameter $\beta$ measures the skewness of the gap between the two probability distribution functions. If $\beta < (>) 1/2$, the gap is skewed to the left (right) side of the support. Notice that for any $F_H(\theta)$, $\alpha$ and $\beta$, the two cumulative distribution functions intersect exactly at $\theta^C = \beta$.

The two parameters $\alpha$ and $\beta$ also determine the mean and variance in state L. Using the simplifying assumption that $\mu_H = 1/2$, I can express $\mu_L$ and $\sigma_L^2$ only as a function of $\alpha$, $\beta$ and $\sigma_H^2$ for any given $f_H(\theta)$ in the following way:

$$
\mu_L = \left(2 + \alpha + 2\beta(1 - \alpha)\right)\frac{1}{\beta}, \quad \sigma_L^2 = \sigma_H^2 + (1 - \alpha)(\alpha(1 - 2\beta)^2 - 2\beta(1 - \beta) - 1)\frac{1}{\beta^2}. \tag{13}
$$

Next consider the demand function. Suppose that $p(q; \theta) = \theta - q$, and $c(q) = 0$. Therefore, $q^*(\theta) = \theta/2$ and $I(q^*(\theta); \theta) - c(q^*(\theta)) = \theta^2/4$. I can now explicitly write the equilibrium quantity, parameterized by the Lagrange multiplier, $\lambda$, as:

$$
q^*(\theta) = \begin{cases} 
\frac{\theta - \lambda(\alpha - 1)}{2} \left[\frac{(\beta - \theta)\theta}{2(\beta f_H(\theta) + \lambda(\alpha - 1)\theta)}\right], & \text{if } \theta \in [0, \beta], \\
\frac{\theta - \lambda(\alpha - 1)}{2} \left[\frac{(\theta - \beta)(1 - \theta)}{2(\beta f_H(\theta)(1 - \beta) + \lambda(1 - \alpha)(\theta - 1)(1 - \beta) - 2\theta)}\right], & \text{if } \theta \in [\beta, 1].
\end{cases} \tag{14}
$$

where the proof of Proposition 1 establishes that $\lambda > 0$ and is increasing with $C/\gamma(1 - \gamma)$.10 The first term in each line, $\theta/2$, is the full-information quantity. The term in the squared brackets in each line is positive for $\theta \in (\theta_0, \theta_1)$ and equal to zero at $\theta = \{\theta_0, \theta_1\}$ because the second-order condition requires that both denominators in the second terms are positive.11 Finally, I can use (12) and (13) to specify the conditions on $\alpha$, $\beta$, $\sigma_H^2$, $\mu_L$ and $\sigma_L^2$ (where recall that by assumption $\mu_H = 1/2$) that satisfy the assumption that $E_H(V(q^*(\theta); \theta) - c(q^*(\theta))) > E_L(V(q^*(\theta); \theta) - c(q^*(\theta)))$.

---

10 In this example $q^*(\theta)$ is not differentiable at $\theta = \beta$. However, $IC_D$ still holds in this case because $q^*(\theta)$ is continuous and increasing with $\theta$ at $\theta = \beta$ as long as $\lambda$ is sufficiently small.

11 The second order condition is always satisfied if $f_H(\theta)$ is sufficiently high or $\lambda$ is sufficiently low.
Figure 4 provides a full characterization of the example's parameters and their effect on the quantity distortion. Consider first the effect of $\alpha$ and $\beta$. The assumption that $E_{i}(V(q^*(\theta);\theta) - c(q^*(\theta))) > E_{i}(V(q^*(\theta);\theta) - c(q^*(\theta)))$ allows for two possibilities. The first is the case where $0 < \alpha < 1$ and $0 < \beta < 0.618$ (the lower left-hand side box in Figure 4). Intuitively, under linear demand $D$ is a "risk lover" in the sense that $D$'s payoff is convex in $\theta$. Consequently, given that $0 < \alpha < 1$ such that state $H$ places more weight on the extremes of the support than state $L$, $D$ prefers state $H$ over state $L$ for all $\beta < 1/2 = \mu_{H}$ and also for $\beta > 1/2$ as long as $\beta$ is not too high. In this case $F_{L}(\beta)$ intersects $F_{H}(\beta)$ from below and therefore, consistent with part (ii) of Corollary 2, (14) reveals that the contract admits upward (downward) distortion in the equilibrium quantity for $\theta \in [0, \beta], (\theta \in [\beta, 1])$. The second possibility is the case where $1 < \alpha < 2$ and $0.618 < \beta < 1$ (the upper right-hand side box in Figure 4). Now, state $H$ places less weight on the two extremes of the support than state $L$ and $D$ prefers state $H$ over state $L$ only if $\beta$ is sufficiently high. Consistent with part (i) of Corollary 2, (14) reveals that the contract admits downward (upward) distortion in the equilibrium quantity for $\theta \in [0, \beta], (\theta \in [\beta, 1])$. In both possibilities, $\alpha$ and $\beta$ fully characterize the direction of the quantity distortion. Notice that FOSD is a special case of this example, in which $\alpha = \beta = 1$ and $1 < \alpha < 2$. Consistent with Corollary 1, (14) reveals that the quantity is distorted downwards for all $\theta \in (\theta_{0}, \theta_{1})$.

Figure 4 also reveals that the effect of the mean and variance of the two distributions on the direction of the distortion is less conclusive than the effect of $\alpha$ and $\beta$. Notice that I can write $E_{i}(V(q^*(\theta);\theta) - c(q^*(\theta))) = E_{i}(\theta^{2}/4) = (E_{i}(\theta^{2}) + E_{i}(\theta - E_{i}(\theta)^{2})/4 = (\mu_{i}^{2} + \sigma_{i}^{2})/4, k = \{L, H\}$. It follows that with linear demand and no cost, $E_{i}(V(q^*(\theta);\theta) - c(q^*(\theta))) > E_{i}(V(q^*(\theta);\theta) - c(q^*(\theta)))$ only requires that $\mu_{H}^{2} + \sigma_{H}^{2} > \mu_{L}^{2} + \sigma_{L}^{2}$. Intuitively, since $V(q^*(\theta);\theta) - c(q^*(\theta))$ is convex in $\theta$, if $\mu_{H} = \mu_{L}$, $D$ prefers the state with the higher variance. Moreover, if $\sigma_{H}^{2} = \sigma_{L}^{2}$, $D$ prefers the state with the higher mean. Figure 4 reveals that if both $\mu_{H} > \mu_{L}$ and $\sigma_{H}^{2} > \sigma_{L}^{2}$, then the direction of the distortion is inconclusive, as $\mu_{H} > \mu_{L}$ and $\sigma_{H}^{2} > \sigma_{L}^{2}$ can emerge both in the lower left-hand side box where there is first upward and then downward distortion in the equilibrium quantity, or in the upper right-hand side box in which the quantity distortion is completely reversed. Intuitively, $\mu_{H} > \mu_{L}$ and $\sigma_{H}^{2} > \sigma_{L}^{2}$ can emerge when state $H$ places higher weight on the extremes of the support than state $L$ and the gap is skewed to left, or when state $H$ places lower weight on the extremes of the support than state $L$ and the gap is significantly skewed to right. If however both $\mu_{H} > \mu_{L}$ and $\sigma_{H}^{2} < \sigma_{L}^{2}$ then the quantity is distorted first downwards and then upwards (the upper right-hand side box). Alternatively, if both $\mu_{H} < \mu_{L}$ and $\sigma_{H}^{2} > \sigma_{L}^{2}$ then the quantity is distorted first upwards and then downwards (the lower left-hand side box). The model does not allow for the last possibility of both $\mu_{H} < \mu_{L}$ and $\sigma_{H}^{2} < \sigma_{L}^{2}$, as the assumption that state $H$ is the preferable one requires that $\mu_{H}^{2} + \sigma_{H}^{2} > \mu_{L}^{2} + \sigma_{L}^{2}$.
Finally, notice that the case where state L is a mean persevering spread of state H, \( \mu_H = \mu_L \), falls on the lower left-hand side box where quantity is distorted first upwards and then downwards.

To conclude, the main point of the above analysis is that the direction of the quantity distortion is directly affected by the gap between \( F_L(\theta) \) and \( F_H(\theta) \). Even though I considered only a single-crossing between \( F_L(\theta) \) and \( F_H(\theta) \), the results can be extended to any number of intersection points, \( \theta_1^C, \ldots, \theta_n^C \) that satisfy \( F_L(\theta_i^C) = F_H(\theta_i^C) \), \( i = \{1, \ldots, n\} \). In this case if \( f_L(\theta_i^C) < (>) f_H(\theta_i^C) \), then for \( \theta \) slightly below \( \theta_i^C \), there is downward (upward) distortion in the quantity while for \( \theta \) slightly above \( \theta_i^C \) there is upward (downward) distortion in the quantity. Finally, the same argument can also apply for the case where instead of intersection points, there are intervals of \( \theta \) in which \( F_L(\theta) = F_H(\theta) \).

3.3. When and how to implement the mechanism

The last step in solving for the optimal mechanism is to derive the conditions under which D indeed prefers to use the mechanism under asymmetric information. As under the full information benchmark, D’s alternatives for not using the mechanism are to sell the new product without motivating U to test it first, or to offer the old product and earn \( v^* \). Comparing the three options yields:

**Proposition 2:** Under asymmetric information there is a cutoff, \( C^{**} \), such that D uses the mechanism if and only if \( C < C^{**} \), where \( 0 < C^{**} < C^* \).

Notice that Proposition 2 does not depend on the shapes of \( F_H(\theta) \) and \( F_L(\theta) \). Intuitively, using the mechanism under asymmetric information forces D to distort its quantity away from the vertical integration quantity, either upwards or downwards, depending on the gap \( F_L(\theta) – F_H(\theta) \). In both cases, total industry profit is lower than under full information, making it less desirable for D to motivate U to perform the test.

I conclude this section by considering the issue of implementation. The optimal mechanism suffers from two implementation problems. First, it might involve negative fees such that U pays D. To see why, notice that \( IC_{\text{ex-ante}}^D \) requires that \( E_D(T^{**}(\theta) – c(q^{**}(\theta))) < -C/(1 - \gamma) \), implying that \( T^{**}(\theta) – c(q^{**}(\theta)) < 0 \) for some values of \( \theta \), which in turn implies that \( T^{**}(\theta) < 0 \) if \( c(q^{**}(\theta)) \) equals zero or if \( c(q^{**}(\theta)) \) is sufficiently low. In this case, U may clearly refuse both to produce a certain quantity for D, and to pay for doing so, instead of being compensated. Second, \( IC_D \) is derived under the assumption that D sells all the quantity it buys from U. Clearly, D can potentially buy a certain quantity and then offer consumers a lower quantity, while disposing of the rest. These two problems raise the question of how D can
implement the above mechanism. To answer this question, consider first the case of FOSD
where Corollary 1 indicates that $T^*(\theta)$ is strictly increasing with $\theta$. In this case D can solve
both problems by asking U to pay an upfront fee of $S = -T^*(\theta_0)$, and offer to pay U
$T(q) = -T^*(\theta_0) + T^*(\hat{\theta}(q)) > 0$ for any $q > q^*(\theta_0)$, where $\hat{\theta}(q)$ is the inverse function of $q^*(\theta)$. By doing so, D solves the enforcement problem associated with having $T^*(\theta) < 0$ for
some realizations of $\theta$. Moreover, under FOSD $T(q)$ is increasing with $q$, implying that D
will only buy the quantity that it intends to sell to consumers.

Next consider cases where $F_h(\theta)$ and $F_l(\theta)$ cannot be ranked according to FOSD. In such
cases $T^*(\theta)$ can still be negative, and therefore D can solve the commitment problem by
asking for upfront fees. However, the analysis above revealed that $T^*(\theta)$ and the equivalent
$T(q)$ can be decreasing with $\theta$ (and therefore with $q$). This means that D will have an
incentive to buy a high quantity in order to benefit from paying a lower $T(q)$, while
afterwards selling only a lower quantity than $q$, in violation of $IC_D$. D can solve this problem
by committing not only to buy the new product from U, but also to sell a certain quantity. Such a commitment is clearly difficult to enforce, though not entirely impossible. For
example, supermarkets can commit to having a certain quantity of the new product on the
shelf until this quantity is sold out.

4. Implication for antitrust policy
This section addresses the implications of the theory for public policy towards slotting
allowances. One of the features of the mechanism is that if production costs are small, then in
order to ensure the constraint $E_l(T^*(\theta) - c(q^*(\theta))) < -C/(1 - \gamma)$, D will set a negative
payment for some realizations of $\theta$, in which case it is U who pays D. Such negative fees are
common in the retail industry in supermarkets and drugstores and are referred to as slotting
allowances. Even if production costs are high, making $T^*(\theta)$ always positive, there are still
"silent" slotting allowances in that for some values of $\theta$, $T^*(\theta)$ does not cover all of U's
production costs. Indeed, there is some anecdotal evidence suggesting a link between slotting
allowances and market research, as predicted by my model. The report by the Federal Trade
Commission (2001), based on the testimonies of selected managers, says that "Some
participants stated that a manufacturer's willingness to pay an up-front slotting fee is a
tangible, credible statement of confidence … since the manufacturer is the party that has had
the best opportunity to study the potential of the new product – for example, as a result of
research and test marketing" (p. 13). Sudhir and Rao (2006) investigated the factors that
affected the probability of observing slotting allowances. They found that high opportunity
costs for shelf space increase the probability of observing slotting allowances. Moreover,
large suppliers that had low levels of credibility from the viewpoint of the retailer supplemented their test market data with slotting allowances, while large suppliers that had high levels of credibility used test market data as a substitute for slotting allowances. Small suppliers always complemented their marketing data with slotting allowances. Sudhir and Rao attribute this result to the possibility that suppliers that cannot credibly convey marketing data to the retailer (either because they are small, or not reliable enough from the retailer’s viewpoint) may not have the reputation for conducting reliable market research and therefore have to back up their data with slotting allowances. Sudhir and Rao concluded that their results support the signaling theory: market research indicates that the supplier has private information, which it signals by means of slotting allowances. My paper shows that the causality between market research and slotting allowances might be reversed. That is, it is not that market research motivates manufacturers to signal their private information through slotting allowances, but that slotting allowances may motivate manufacturers to perform market research.

I should note that there is some evidence against my argument on the link between market research and slotting allowances. The Food Marketing Institute (2002) argues that suppliers use slotting allowances in order to avoid the need to conduct market research. However the report does not offer any empirical investigation to support this claim and as stated above Sudhir and Rao (2006) found that slotting allowances and market research are substitutes only for suppliers of high repute. Bloom, Gundlach and Cannon (2000) found no evidence for the claim that slotting allowances convey suppliers’ private information to retailers. However, their research is based on views expressed by practitioners according to a questionnaire that did not include an explicit question on the relationship between market research and slotting allowances. Moreover, they found that, by and large, managers believe that slotting allowances emerge because of retailers' growing market power, which is consistent with the assumption in this paper that D has the bargaining power.

Antitrust authorities have investigated the potential effects of slotting allowances on competition, consumers and welfare. On one hand, following the theoretical literature by Shaffer (1991), Kim and Staelin (1991), Shaffer (2005), Rey, Miklós-Thal and Vergé (2006), Innes and Hamilton (2006), Kuksov and Pazgal (2007) and Marx and Shaffer (2007), slotting allowances can enable downstream firms to coordinate prices. Intuitively, competing downstream firms can relax price competition among themselves by committing to pay a high wholesale price to their suppliers, which in turn raises their retail cost. Doing so provides

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12 Sudhir and Rao (2006) also found some empirical evidence that slotting allowances serve to balance risk between retailers and manufacturers and to mitigate retail competition. At the same time they argue that the signaling theory is the most consistent with their data.

13 See for example the FTC (2001) and the Israeli Antitrust Authority (2003).
suppliers with profit margins, so the downstream firms can extract these profit margins by asking for upfront payments. Sloting allowances may also have the negative effect of discriminating against financially constraint suppliers that cannot afford to make high upfront payments.\footnote{The FTC (2001) found that small manufacturers described slotting allowances as "a major stumbling block for us to enter into any large distribution network" (p. 19). Also, they found that small manufacturers seeking to supply retail grocery markets have difficulties in finding sources of equity capital because of their size.} Indeed, the previous section shows that under FOSD, U earns negative payoff for low realizations of $\theta$. A financially constrained supplier may not be able to finance slotting allowances by means of loans, because if the realization of demand is low, the supplier will default. At the same time, slotting allowances may enhance social welfare by enabling supermarkets and drugstores to efficiently allocate scarce shelf space between products.

By the assumption that $D$ is a monopolist, my paper rules out the first anti-competitive argument against slotting allowances. I can therefore address the question of whether the latter argument is indeed valid. More precisely, I ask the following question. Suppose that slotting allowances do not have the anti-competitive effects of relaxing downstream competition and that $U$ is not financially constrained such that slotting allowances are used only for efficiently allocating limited shelf space. Are they indeed welfare enhancing? As this section reveals, slotting allowances can reduce social welfare even without anti-competitive effects. For antitrust policy, this result indicates that antitrust authorities should not always tolerate the presence of the anti-competitive effects of slotting allowances on the grounds that they are compensated by their welfare enhancing property of efficiently allocating shelf space, as the latter effect can be negligible, or even another source for inefficiency.

Thus, I turn to evaluating the effect of the mechanism on social welfare. Suppose that $c(q(\theta))$ is negligible such that $D$ cannot motivate $U$ to gather information without asking for negative fees. Consider a social planner that can prohibit $D$ from charging negative fees, but cannot regulate any other of $D$'s activities, such as the quantity that $D$ chooses to sell or $D$'s decision on whether to sell the old or the new product. If $D$ cannot motivate $U$ to gather information, $D$ will either sell the new product without testing it first (if $V^* \geq V^*$), or sell the old product (if $V^* < V^*$). Suppose that consumers' utilities are quasi-linear such that consumers' surplus is:

$$CS(q; \theta) = \int_0^q p(\hat{q}; \theta)d\hat{q} - p(q; \theta)q,$$  

and let $CS^*$ denote consumers' surplus from the old product. A first source of inefficiency in testing the new product can be that consumers' preferences concerning the old and the new products differ from those of $D$, in which case even under full information $D$ might prefer one
of the products while consumers prefer the other. To focus on the role of asymmetric information, I assume that $E_H CS(q^*(\theta);\theta) > CS^* > E_L CS(q^*(\theta);\theta)$. Total social welfare from the new product, gross of the cost of the test, is $W(q;\theta) = V(q;\theta) + CS(q;\theta) - c(q)$ and from the old one $W^* = V^* + CS^*$, where $E_H W(q^*(\theta);\theta) > W^* > E_L W(q^*(\theta);\theta)$.

Consider first the full-information case, in which $D$ can observe whether $U$ tested the new product and the result of the test. Under the assumptions above there is a cutoff,

$$C_{W^*} = \begin{cases} \gamma (E_H W(q^*(\theta);\theta) - W^*), & \text{if } V^* > \bar{V}^* , \\ (1-\gamma)(W^* - E_L W(q^*(\theta);\theta)), & \text{if } V^* \leq \bar{V}^* , \end{cases} \quad (16)$$

such that a social planner prefers that $D$ will ask $U$ to test the new product if $C < C_{W^*}$. Comparing (16) with (3) reveals that $C^* < C_{W^*}$. That is, under full-information $D$ motivates $U$ to test the new product less than is socially desirable, in that for $C^* < C < C_{W^*}$, $U$ will not test the new product even though it is welfare enhancing to do so. Intuitively, $D$ does not internalize the positive effect that testing the new product has on consumers.

Next consider asymmetric information. On one hand, as under full information $D$ does not internalize the positive effect that offering the better product has on consumers. At the same time, testing the new product now requires $D$ to distort its quantity downwards for at least some realizations of demand, and $D$ does not internalize the negative effect that doing so has on consumers. For the case of FOSD, the following proposition provides conditions under which the second effect offsets the first one:

**Proposition 3:** Suppose that $F_H(\theta)$ dominates $F_L(\theta)$ by FOSD. Then, under asymmetric information, there is a cutoff, $C_{W^**}$, such that a social planner prefers to test the new product if $C < C_{W^**}$. Moreover, let $q^{**}(C;\theta)$ denote the equilibrium quantity evaluated at $C$. Then:

(i) if $V^* < \bar{V}^*$ (without testing $D$ sells the new product), then $C_{W^**} < C^{**}$ iff

$$ (1-\gamma)(CS^* - E_L CS(q^*(\theta);\theta)) - \gamma (E_H CS(q^* (\theta);\theta) - E_H CS(q^{**}(C^{**};\theta);\theta)) < 0 \quad (17) $$

(ii) if $V^* > \bar{V}^*$ (without testing $D$ sells the new product), then $C_{W^**} < C^{**}$ iff

$$ E_H CS(q^{**}(C^{**};\theta);\theta) - CS^* < 0 \quad (18) $$

Proposition 3 provides conditions under which $D$ motivates $U$ to test the new product even if it is socially inefficient to do so, in which case slotting allowances reduce social welfare.
Consider first condition (17), which refers to the case where without the test D sells the new product. The first term in (17) represents the benefit that consumers gain from testing the new product. This benefit emerges because by testing the new product, D can choose to sell the old product instead of the new one in state L, and consumers gain \( CS^* \) instead of \( ELCS(q^*(\theta);\theta) \). The second term represents the harm to consumers that emerges because the quantity is distorted downwards and therefore in state H consumers gain \( E_HCS(q^{**}(\theta);\theta) \) instead of \( E_HCS(q^*(\theta);\theta) \). Notice that, other things being equal, condition (17) is always satisfied when \( CS^* \) is sufficiently close to \( ELCS(q^*(\theta);\theta) \). Next consider condition (18), which refers to the case where without the test D sells the old product. On one hand, testing the new product benefits consumers as they buy the new product instead of the old one in state H. At the same time, the downward quantity distortion offsets this benefit if \( q^{**}(C^*,\theta) \) is sufficiently small. Notice that, other things being equal, condition (18) is always satisfied when \( CS^* \) is sufficiently close to \( E_HCS(q^*(\theta);\theta) \).

The case where the two distribution functions cannot be ranked according to FOSD is somewhat ambiguous. Now, the quantity is distorted upwards for some values of \( \theta \), which may enhance social welfare. However, it is clear that in order to maintain the assumption that \( E_U(V(q^*(\theta);\theta) - c(q^*(\theta))) > E_U(V(q^*(\theta);\theta) - c(q^*(\theta))) \), it has to be that \( F_L(\theta) > F_H(\theta) \) for some values of \( \theta \) for which there is downward distortion. I can therefore apply a simple argument of continuity by stating that the results of Proposition 3 still hold without FOSD as long as by and large there is more downward distortion in the equilibrium quantity than upward distortion.

To conclude, this section shows that slotting allowances can reduce social welfare even without any anti-competitive effects, implying that antitrust authorities should not always tolerate such anti-competitive effects on the grounds that slotting allowances enhance the efficiency of allocating shelf space. This result differs from Chu (1992), who shows that slotting allowances are welfare enhancing when an upstream firm pays slotting allowances to convey its private information concerning demand to a retailer. In Chu's paper, under slotting allowances the retailer sells the full information quantity because the upstream firm sets the wholesale price after agreeing to pay the slotting allowances and after the retailer is convinced that the demand is high. Moreover, in Chu's paper there are only two realizations of demand and since the retailer sells the new product only if the demand is high, the contract does not need to provide incentives for the retailer to report the true realization of demand as in my paper. This section corroborates Lariviere and Padmanabhan (1997) and Desai (2000), who show that when the upstream firm sets both slotting allowances and a wholesale price to signal high demand, the wholesale price is above its full information level. The result of my paper that slotting allowances induce D to distort the quantity downwards is consistent with
their findings, for the case where D has all the bargaining power. Moreover, my paper extends Lariviere and Padmanabhan (1997) and Desai (2000) by showing that slotting allowances may induce D to distort the quantity downwards for some realizations of demand, and upwards for others (as in Chu (1992), Lariviere and Padmanabhan (1997) and Desai (2000) assume that there are only two realizations of demand and that the retailer sells the new product only if the demand is high). Finally, my paper extends Lariviere and Padmanabhan (1997) and Desai (2000) by considering the effect of slotting allowances on social welfare, when slotting allowances indeed distort the equilibrium quantity.

5. Conclusion

This paper considers the problem of motivating an upstream firm to gather information concerning the demand for its new intermediate product. The paper shows that the equilibrium contract has two main characteristics. First it includes downward (upward) distortion in the equilibrium quantity if the gap between the distribution functions in states L and H is positive (negative). Second, the payment to D can be negative, in the form of slotting allowances. For antitrust policy, the paper shows that slotting allowances may decrease social welfare even without potential anti-competitive effects. This result may call for a more restrictive approach by antitrust authorities toward slotting allowances.

The paper makes two simplifying assumptions that are worth noting. First, I assume that gathering information reveals to U that the demand is drawn from one of only two potential distribution functions, and that D wants to sell the new product for only one of these two distribution functions. In this case, D does not need to leave U with ex-ante information rents. It is possible to think of a more general setting in which there are $n > 3$ potential distribution functions, and that D wants to sell the new product for $m$ distribution functions, where $n > m > 1$. Under such a setting D may have to leave U with ex-ante information rents. In other words, U’s ex-post incentive compatibility constraints, which are not binding in my model, may now bind. This in turn may create another motivation for D to distort its quantity – to reduce U's information rents. For future research, I think that it would be interesting to investigate the direction of the quantity distortion in such a case. A second simplifying assumption is that U and D are risk neutral. Risk aversion may, by itself, create a motivation for D to use slotting allowances in order to share the risk between the two firms. Also, risk aversion may affect D's preferences concerning which of the two states is the preferable one. This in turn may affect the direction of the quantity distortion. For future research, I think that it would be interesting to introduce risk aversion into this mechanism design problem.
Appendix

Following are the proofs of Lemmas 1 and 2 and Propositions 1 – 3.

**Proof of Lemma 1:**

Differentiating $U(\theta, \hat{\theta})$ and using the envelop theorem,

$$U'_0(\theta) = \frac{\partial}{\partial \theta} U(\theta; \hat{\theta}) \bigg|_{\hat{\theta}=0} = V_0(q(\theta); \theta). \quad \text{(A-1)}$$

Integrating (A-1) yields (5). To see that $q_0(\theta) > 0$ satisfies $IC_D$, using (5), D's ex-post profit is:

$$U(\theta; \hat{\theta}) = V(q(\hat{\theta}); \hat{\theta}) - \left( V_0(q(\hat{\theta}); \hat{\theta}) - \int_{\theta_0}^{\hat{\theta}} V_0(q(\theta); \theta) d\theta - U(\theta_0) \right). \quad \text{(A-2)}$$

Differentiating (A-2) with respect to $\hat{\theta}$ yields:

$$\frac{\partial}{\partial \hat{\theta}} U(\theta; \hat{\theta}) = \left( V_0(q(\hat{\theta}); \hat{\theta}) - V_0(q(\hat{\theta}); \hat{\theta}) \right) q_0(\hat{\theta}) = 0. \quad \text{(A-3)}$$

Thus D sets $\hat{\theta} = \theta$. The second order condition is:

$$\frac{\partial^2}{\partial \theta^2} U(\theta; \hat{\theta}) \bigg|_{\hat{\theta}=0} = -q_0(\theta) V_{q0}(q(\theta); \theta) < 0. \quad \text{(A-4)}$$

Since $V_{q0}(q(\theta); \theta) > 0$, the second order condition is satisfied if $q_0(\theta) > 0$.

**Proof of Lemma 2:**

Substituting (6) into (8) yields

$$\int_{\theta_0}^{\theta} \left( V(q(\hat{\theta}); \hat{\theta}) - c(q(\hat{\theta})) \right) - \int_{\theta_0}^{\hat{\theta}} V_0(q(\hat{\theta}); \hat{\theta}) d\hat{\theta} \left( f_{HL}(\hat{\theta}) - f_{HL}(\hat{\theta}) \right) d\hat{\theta} \geq \frac{C}{\gamma(1-\gamma)}. \quad \text{(A-5)}$$

Integrating by parts the left hand side of (A-5) and evaluating at $q(\theta) = q^*(\theta)$ yields:
\[
\int_{\theta_0}^{0} \left[ V(q^*(\theta);\tilde{\theta}) - c(q^*(\tilde{\theta})) \right] - \int_{\theta_0}^{0} \left[ V^*_0(q^*(\theta);\tilde{\theta}) d\tilde{\theta} \right] \left( f_H(\tilde{\theta}) - f_L(\tilde{\theta}) \right) d\tilde{\theta} \\
= \int_{\theta_0}^{0} \left( V_q(q^*(\theta);\tilde{\theta}) - c_q(q^*(\tilde{\theta})) \right) q^*_0(\tilde{\theta}) \left( F_L(\tilde{\theta}) - F_H(\tilde{\theta}) \right) d\tilde{\theta} \\
= 0 < \frac{C}{\gamma(1-\gamma)},
\]

where the second equality follows because \( V_q(q^*(\theta);\theta) = c_q(q^*(\theta)) \).

**Proof of Proposition 1:**

The plan of the proof is the following. First, I will solve for the optimal mechanism ignoring the constraint \( q_0(\theta) > 0 \). Second, I will prove the properties of the optimal mechanism. Third, I will show that the solution satisfies the second order condition and the condition that \( q_0(\theta) > 0 \) if \( C \) is low enough or if \( \gamma \) is not too close to either 0 or 1.

First, I start by solving for the optimal mechanism. To this end, substituting (6) into (8) and rearranging yields that the Lagrangian is:

\[
L(q(\theta), \lambda) = \int_{\theta_0}^{0} \left[ V(q(\theta);0) - c(q(\theta)) - \frac{C}{\gamma} f_H(\theta) \right] d\theta
\]

\[
+ \lambda \left[ \int_{\theta_0}^{0} \left( V(q(\theta);0) - c(q(\theta)) - V^*_0(q(\theta);0) \right) \left( \frac{1-F_H(\theta)}{f_H(\theta)} \right) d\theta \right]
\]

\[
- \frac{C}{\gamma(1-\gamma)}
\]

Differentiating (A-7) with respect to \( q(\theta) \) and \( \lambda \) and rearranging yields that \( (q^{**}(\theta), T^{**}(\theta)) \), and \( \lambda^{**} \) are the solution to:

\[
V_q(q(\theta);0) - c_q(q(\theta)) = \lambda V_{q0}(q(\theta);0) \frac{F_L(\theta) - F_H(\theta)}{(f_H(\theta) + \lambda(f_H(\theta) - f_L(\theta)))},
\]

\[
E_H \left( V(q(\theta);0) - c(q(\theta)) - V^*_0(q(\theta);0) \frac{1-F_H(\theta)}{f_H(\theta)} \right)
\]

\[
E_L \left( V(q(\theta);0) - c(q(\theta)) - V^*_0(q(\theta);0) \frac{1-F_L(\theta)}{f_L(\theta)} \right) = \frac{C}{\gamma(1-\gamma)}.
\]
\[ T(\theta) = V(q(\theta); \theta) - \int_{\theta_0}^{\theta} V(q(\hat{\theta}); \hat{\theta}) d\hat{\theta} \]
\[ -E_H \left( V(q(0); 0) - c(q(0)) - V_0(q(0); 0) \frac{1 - F_H(\theta)}{f_H(\theta)} - C \right), \quad (A-10) \]

where (A-8) and (A-9) are the first order conditions for (A-7) with respect to \( q(\theta) \) and \( \lambda \) respectively, and (A-10) is derived from (6) (notice that \( U(\theta_0) \) is the last term in (A-10)). The second order condition for (A-7) with respect to \( q(\theta) \) requires that

\[ \left( f_H(\theta) + \lambda(f_H(\theta) - f_L(\theta)) \right) \left( V_{qq}(q(\theta); \theta) - c_{qq}(q(\theta)) \right) \]
\[ + \lambda V_{q0}(q(\theta); \theta) \left( F_H(\theta) - F_L(\theta) \right) \]
\[ < 0. \]

Next, I turn to the second part of showing the properties of the optimal solution. Let \( q(\lambda; \theta) \) denotes the solution to (A-8) given \( \lambda \) and let \( T(\lambda; \theta) \) denotes the right-hand side of (A-10) evaluated at \( q(\lambda; \theta) \). \( \lambda^{**} \) is therefore the solution to

\[ E_H \left( T(\lambda; \theta) - c(q(\lambda; \theta)) \right) - E_L \left( T(\lambda; \theta) - c(q(\lambda; \theta)) \right) = \frac{C}{\gamma(1 - \gamma)}, \quad (A-12) \]

and \( q^{**}(\theta) = q(\lambda^{**}; \theta) \). Now, if \( C = 0 \), then \( \lambda^{**} = 0 \) and \( q^{**}(\theta) = q^{*}(\theta) \). To see why, notice that (A-6) is satisfied in equality for \( q(\lambda; \theta) = q^{*}(\theta) \) and \( C = 0 \). Substituting \( q(\lambda; \theta) = q^{*}(\theta) \) into (A-8), the left hand side in (A-8) equals to zero implying \( \lambda^{**} = 0 \). Next suppose that \( C\gamma(1 - \gamma) > 0 \). Given that the second order condition holds, the derivative of the left hand side of (A-12) is with respect to \( \lambda^{**} \) is

\[ \frac{\partial}{\partial \lambda} \left( E_H \left( T(\lambda; \theta) - c(q(\lambda; \theta)) \right) - E_L \left( T(\lambda; \theta) - c(q(\lambda; \theta)) \right) \right) \bigg|_{\lambda = \lambda^{**}} \]
\[ = \int_{\theta_0}^{\theta} \frac{f_H(\theta) V_{0q}(q(\theta); 0)}{f_H(\theta) + \lambda^{**}(f_H(\theta) - f_L(\theta))} \left( F_H(0) - F_L(0) \right) \frac{\partial q(\lambda^{**}; \theta)}{\partial \lambda} d\theta > 0. \]

Since \( f_{\theta}(\theta) - \lambda^{**}(f_H(\theta) - f_L(\theta)) \) is always positive for low values of \( \lambda^{**} \) and since by assumption, \( V_{0q}(q; \theta) > 0 \), the sign of (A-13) is determined according to the sign of the term in the squared brackets. If \( F_H(\theta) > F_L(\theta) \), then the right hand side of (A-8) is decreasing with \( \lambda \), implying that \( q(\lambda; \theta) \) is increasing with \( \lambda \), and the term in the squared brackets in (A-13) is positive. Likewise, if \( F_H(\theta) < F_L(\theta) \), then the right hand side of (A-8) is increasing with \( \lambda \),
implying that \( q(\lambda; \theta) \) is decreasing with \( \lambda \), and the term in the squared brackets in (A-13) is positive. Consequently, the term in the left hand side of (A-12) is increasing with \( \lambda \). This in turn implies that \( \lambda^{**} \) is increasing with \( C/\gamma(1 - \gamma) \), and the term in the squared brackets in (A-13) is positive. Consequently, the term in the left hand side of (A-12) is increasing with \( \lambda \). This in turn implies that \( \lambda^{**} \) is increasing with \( C/\gamma(1 - \gamma) \), and \( \lambda^{**} > 0 \) if \( C/\gamma(1 - \gamma) > 0 \). As for \( q^{**}(\theta) \), since \( \lambda^{**} > 0 \), it follows from (A-8), that \( q^{*}(\theta) - q^{**}(\theta) > (>) = 0 \) if \( F_\lambda(\theta) > (<) = F_{\lambda ^*}(\theta) \) and the gap \(|q^{*}(\theta) - q^{**}(\theta)|\) is increasing with \( C/\gamma(1 - \gamma) \).

Next I turn to the third part of showing that the above solution satisfies the condition \( q^{**}(\theta) > 0 \) and the second order condition if \( C \) is low and \( \gamma \) is intermediate. Notice first that \( q^{**}(\theta) \) is increasing with \( \theta \). Since \( q(0; \theta) = q^{*}(\theta) \) and \( q(\lambda; \theta) \) is continuous with \( \lambda \), it follows that \( q^{**}(\theta) \) is increasing with \( \theta \) if \( \lambda^{**} \) is not too high, which in turn holds if \( C/\gamma(1 - \gamma) \) is not too high. As for the second order condition in (A-11), it always holds for \( \lambda = 0 \), and therefore for \( C = 0 \). As (A-11) is continuous in \( \lambda \), it holds for \( \lambda > 0 \) but not too high or if \( C/\gamma(1 - \gamma) \) is not too high. The exact condition on \( C/\gamma(1 - \gamma) \) from which on these two conditions do not hold depends on the two distribution functions.

**Proof of Proposition 2:**

Suppose first that \( V^* > V^{**} \). In this case D will use the mechanism if

\[
-C + \gamma E_{\mu}(V(q^{**}(\theta); \theta) - c(q^{**}(\theta))) + (1 - \gamma)V^* \geq V^*. \tag{A-14}
\]

From Proposition 1, the gap \(|q^{*}(\theta) - q^{**}(\theta)|\) is increasing with \( C \) and equals to zero for \( C = 0 \). Therefore, \( V(q^{**}(\theta); \theta) - c(q^{**}(\theta)) \) is decreasing with \( C \) and equals to \( V(q^{*}(\theta); \theta) - c(q^{*}(\theta)) \) for \( C = 0 \). Thus the left hand side of (A-14) is decreasing with \( C \), and the inequality holds for \( C < C^{**} \), where \( C^{**} \) is the solution to

\[
C^{**} = \gamma (E_{\mu} V(q^{**}(\theta); \theta) - c(q^{**}(\theta)) - V^*). \tag{A-15}
\]

Since \( V(q^{**}(\theta); \theta) - c(q^{**}(\theta)) < V(q^{*}(\theta); \theta) - c(q^{*}(\theta)) \) for \( C > 0 \), the right hand side in (A-15) is lower than the term in the first line in (3), implying that \( C^{**} < C^{*} \). Also, if \( C = 0 \), (A-14) always holds because \( V(q^{**}(\theta); \theta) - c(q^{**}(\theta)) = V(q^{*}(\theta); \theta) - c(q^{*}(\theta)) \) and by assumption, \( E_{\mu}(V(q^{*}(\theta); \theta) - c(q^{*}(\theta))) > V^* \). Next suppose that \( V^* < V^{**} \). In this case D will use the mechanism if

\[
-C + \gamma E_{\mu}(V(q^{**}(\theta); \theta) - c(q^{**}(\theta))) + (1 - \gamma)V^* \geq \gamma E_{\mu}(V(q^{*}(\theta); \theta) - c(q^{*}(\theta))) + (1 - \gamma)E_{\mu}(V(q^{*}(\theta); \theta) - c(q^{*}(\theta))). \tag{A-16}
\]
Since \( V(q^*(\theta); \theta) - c(q^*(\theta)) \) is decreasing with \( C \), the left hand side of (A-16) is decreasing with \( C \), implying that \( D \) will use the mechanism if \( C < C^* \), where \( C^* \) is the solution to:

\[
C^* = (1 - \gamma) \left( V^* - E_L \left( V(q^*(\theta); \theta) - c(q^*(\theta)) \right) \right) - \gamma E_H \left( V(q^*(\theta); \theta) - c(q^*(\theta)) \right) - E_H \left( V(q^*(\theta); \theta) - c(q^*(\theta)) \right).
\]  

(A-17)

Since \( V(q^*(\theta); \theta) - c(q^*(\theta)) < V(q^*(\theta); \theta) - c(q^*(\theta)) \) for \( C > 0 \), the right hand side of (A-17) is lower than the second line in (3), implying that \( C^* < C^* \). Also, if \( C = 0 \), (A-16) always holds because \( V(q^*(\theta); \theta) - c(q^*(\theta)) = V(q^*(\theta); \theta) - c(q^*(\theta)) \) and by assumption, \( E_L(V(q^*(\theta); \theta) - c(q^*(\theta))) < V^* \).

**Proof of Proposition 3:**

Suppose first that \( V^* > V^* \). In this case it socially optimal to use the mechanism if

\[
-C + \gamma E_H W(q^*(\theta); \theta) + (1 - \gamma) W^* \geq W^*.
\]  

(A-18)

Condition (A-18) always holds if \( C = 0 \). Under FOSD, the left-hand side of (A-18) is strictly decreasing with \( C \) and therefore (A-18) holds iff \( C < C_H^* \), where \( C_H^* \) solves (A-18) in equality. Moreover, substituting (A-15) into (A-18) and evaluating (A-18) at \( C = C^* \) yields that \( C_H^* < C^* \) if (17) holds. Next suppose that \( V^* < V^* \). In this case it is socially optimal to use the mechanism if

\[
-C + \gamma E_H W(q^*(\theta); \theta) + (1 - \gamma) W^* \geq \gamma E_H W(q^*(\theta); \theta) + (1 - \gamma) E_L W(q^*(\theta); \theta).
\]  

(A-19)

Condition (A-19) always holds if \( C = 0 \) and under FOSD the left-hand side of (A-19) is decreasing with \( C \) so again (A-19) holds iff \( C < C_H^* \) where \( C_H^* \) solves (A-19) in equality. Substituting (A-17) into (A-19) and evaluating at \( C = C^* \) yields that \( C_H^* < C^* \) if (18) holds.
Figure 1: The timing of the game
Figure 2: The optimal mechanism under asymmetric information
Figure 3: The distribution functions given $g(\theta)$ (when $f_{H}(\theta)$ is the uniform distribution)

Panel (a): $0 < \alpha < 1$

Panel (b): $1 < \alpha < 2$
Figure 4: The effect of $\alpha$, $\beta$, $\mu_H$, $\mu_L$, $\sigma_H^2$ and $\sigma_L^2$ on the direction of the quantity distortion

For $\mu_H = 1/2$. The areas in the bold line satisfy the assumption $E[q^*(\theta); \theta] - c(q^*(\theta)) > E[q^*(\theta); \theta] - c(q^*(\theta))$. 

Downwards (upwards) quantity distortion for $\theta \in (0, \beta) (\theta \in (\beta, 1))$

Upwards (downwards) quantity distortion for $\theta \in (0, \beta) (\theta \in (\beta, 1))$
References


