Vertical Relations and Dynamic Exclusion of Product Improvement

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Abstract

We consider infinitely repeated vertical relations when a retailer can sell an established product and a new product that is initially inferior but can improve over time. We find that the retailer has an incentive to sell the new product more than what maximizes industry profits. The manufacturer of the established product excludes the new product by setting a below-cost wholesale price, combined with a fixed fee, in all periods, and accommodates the new product more than under vertical integration. If exclusive dealing is allowed, the manufacturer imposes it and replicates the vertically integrated outcome. Such exclusive dealing can either increase or decrease social welfare.

Keywords: vertical relations, product improvement, learning, predatory pricing, exclusive dealing.

JEL Classification Numbers: L41, L42, K21,D8

1 Introduction

It is often the case that an established product is challenged by a new product, which is currently inferior to the established product. Nevertheless, this new product may often have the potential for improvement, if it is supplied and sold to consumers. Hence vendors

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face a trade-off between the current losses from selling the inferior product and the future-benefits from improving it. For example, consumers may not be aware of the inferior product's virtues, and only if it is sold, and consumers try it, their demand for it increases (Guadagni and Little 1983; Seetharaman et al 1999). This is particularly the case in online markets, where consumers can rate new products and online retailers can publish how many consumers had bought the new product, both of which can enhance consumers' awareness to the product and its perceived quality (Danaher et al 2003).[1] Another prominent case is that of learning by doing: when the inferior product is produced in high enough quantities, its cost of production may decline and/or its quality may improve (e.g., Glocka et al 2019; Löbberding and Madlener 2019).

When both the established product and the inferior product are supplied by the same firm, which also resells it to consumers, the decision whether to sell the inferior product along-side the established product involves a quite straightforward calculation: The more this vertically integrated entity discounts future profits, the less likely it is to sell the inferior product. The sacrifice in current profits involved in selling an inferior product is too great to be offset by the future benefits from improving it. As this article reveals, the strategies and equilibrium behavior change when firms are vertically separated and operate in a dynamic setting. When the established manufacturer is not allowed to impose an explicit exclusive dealing agreement on the independent retailer, the vertically separated industry accommodates the inferior product more than does a vertically integrated firm and thus more than what maximizes industry profits.

In this paper we study a manufacturer of an established product, selling to an independent (monopoly) retailer. The retailer can buy the established product, but it can also buy a competing product that is currently inferior, and can improve if sold at some threshold quantity. When the inferior product improves, it becomes more profitable than the established product. The firms interact for an infinite number of periods and are equally patient (i.e., they have the same discount factor). Each period, the manufacturer of the established product offers the retailer a contract involving a two-part tariff – with a wholesale price and a fixed fee. We also explore the possibility that the manufacturer of the established product imposes exclusive dealing that explicitly forbids the retailer

[1] Jabr and Rahman (2019) have documented surveys showing that 63% percent of customers consider it extremely or very important to read reviews online before buying an unfamiliar product, and that 93% of customers read online reviews before choosing what to buy.

to buy the inferior product.

The paper shows that when the manufacturer of the established product is not allowed to impose explicit exclusive dealing, and the parties are patient enough, a resulting equilibrium diverges from the behavior of a vertically integrated firm. In particular, at a certain discount factor, if the manufacturer attempts to implement the vertically integrated outcome and charge a wholesale price equal to marginal cost, the retailer has an incentive to accommodate the inferior product. This is while at this level of the discount factor, a vertically integrated firm would still want to exclude the inferior product. In order to induce the retailer to nevertheless avoid the inferior product, the manufacturer of the established product charges a below-cost wholesale price. Despite this form of predatory pricing, the manufacturer of the established product makes positive profits each period, thanks to a positive fixed fee. At a somewhat higher discount factor, however, this predatory strategy becomes prohibitively costly to the manufacturer of the established product, so it prefers to accommodate the inferior product and focus on current profits. The accommodation of the inferior product occurs for strictly lower discount factors than under vertical integration. Hence the vertically separated industry accommodates the inferior product more than a vertically integrated firm and fails to maximize industry profits.

When the manufacturer of the established product is allowed to impose explicit exclusive dealing, however, it does so and replicates the vertically integrated outcome: the inferior product is accommodated for the same discount factor as a vertically integrated firm, and there is no predatory pricing.

The distortion we revealed can be compared to a different distortion that arises with vertical separation, due to double marginalization. With double marginalization, a vertically separated industry fails to implement the vertically integrated optimum, because the supplier charges an above-cost price, on top of which the retailer also charges an above-cost price. Both with double marginalization and with the distortion we reveal, when vertically separated, the parties fail to implement the vertically integrated outcome. Unlike double marginalization, however, the distortion identified here exists even in the presence of fixed fees. Another important difference from double marginalization is that the latter harms both the firms and consumers, while the distortion we identify harms the firms' joint profits, but may benefit consumers and social welfare.

Our results have welfare implications along two different dimensions: the total quantity offered to consumers, and the level of accommodation or exclusion of the inferior (but improvable) product. Also, the results introduce implications for two renowned practices: predatory pricing and exclusive dealing. Our analysis implies that banning exclusive dealing not only promotes more accommodation of the rival product, but also induces the manufacturer to engage in (socially beneficial) predatory pricing. It is often claimed in the antitrust case law and legal literature that exclusive dealing has the benefit of inducing pro-competitive price discounts.[2] We show the contrary: in our model, when exclusive dealing is banned, the manufacturer excludes via below-cost pricing. When exclusive dealing is allowed, the manufacturer raises the wholesale price because it can exclude solely by imposing exclusive dealing on the retailer. This exacerbates the monopoly distortion.

The results further show that if exclusive dealing is allowed, the inferior product is always over-excluded compared to the social optimum. Conversely, if exclusive dealing is banned, the industry may over-accommodate the inferior product or over-exclude it, depending on market circumstances. Exclusive dealing may therefore reduce or enhance social welfare, depending on market conditions. A ban on both predatory pricing and exclusive dealing involves more accommodation of the inferior product, but, again, this level of accommodation may be either higher or lower than what is socially optimal. Furthermore, if predatory pricing is banned, then the retailer's monopoly pricing persists, whereas if predatory pricing by the manufacturer is allowed, consumers benefit from lower retail prices. In the kind of predatory pricing that occurs in our model, there is no need to show long-term recoupment of predatory losses. The manufacturer recoups its losses from predatory pricing immediately, via the fixed fee, and makes positive overall profits each period. Accordingly, we shall call this "persistent predatory pricing". As for vertical mergers, our model demonstrates that they cause the market outcome to change in two ways: they cause over-exclusion of the inferior (improvable) product, and raise prices. The price-increasing effect of vertical mergers that we expose stands in stark contrast to the conventional notion that vertical mergers eliminate double margins and in this sense tend to reduce prices. In our framework, with vertical separation, there is no double margin, but rather pricing below the monopoly price, while a vertical merger restores

[2]See, e.g., Bork 1993 at p. 304; Knight and Windell 2001; Moore and Wright 2015.

monopoly pricing.

This article contributes to several strands of the literature, including vertical foreclosure and competition in dynamic settings, vertical relations and product quality, and exclusive dealing.

With respect to vertical foreclosure and competition in dynamic settings, Carlton and Waldman (2002) study tying by a dominant incumbent in order to block entry. In their model, the new entrant can first enter the complementary product market, with a superior product, and then enter the primary product market, or a newly emerging market. Cabral and Riordan (1994) study one duopolist gaining dominance over it's rival in a dynamic learning environment. They find that one firm may price aggressively in order to prevent its rival from going down its learning curve. Villas-Boas (2006) studies a dynamic model of competition with experience goods. The consumer in Villas-Boas's model may have a good experience with the product, which increases demand, or a bad experience, which reduces demand. In all of these papers, firms sell directly to end-consumers and consumers buy only one of the products each period. We contribute to these papers by studying vertical relations with an independent retailer, who can buy either one of the products or both. We find that the incentives to sell the initially inferior product under vertical separation differ from the vertically integrated case.

In the context of vertical relations, Bergès-Sennou and Waterson (2005) study a dynamic model in which a retailer can buy a reputable product from a manufacturer or a product of quality unknown to consumers from a competitive fringe. They find that the retailer may choose to sell the second product at poor quality, so as to extract surplus from end consumers who expect quality to be higher. Like in our model, the retailer can either sell only one of the products or both. They focus, though, on the retailer's exploitation of consumers' lack of information about the second product, which is always of poorer quality than the reputable product. By contrast, we focus on the manufacturer's strategies of excluding or accommodating the second product, which always improves so as to be better than the manufacturer's own product if sold by the retailer. This motivates our analysis of the manufacturer's practices, including predatory pricing or exclusive dealing, and their effects, and the divergence between the vertically integrated outcome and the vertically separated one. Argenton (2009) shows that an incumbent with an inferior product can exclude new entrants selling a superior product by offering retailers upfront payments in exchange for exclusivity. Our paper differs from Argenton's paper in that we study a dynamic setting, where the rival is initially inferior to the dominant firm, but can improve in the future if sold by the retailer. Also, after one period of co-existence of the rival product and the established product, the established product in our framework is no longer sold. Hence it is not the reduction of post-entry rents that drives exclusionary contracts, as they do in his paper, but the retailer's impatience, combined with the predatory pricing on the part of the manufacturer of the established product.

Fumagalli and Motta (2018) show that a vertically integrated firm may want to foreclose a more efficient downstream entrant in order to block entry of efficient upstream firms or extract more rents from them. We contribute to their paper by considering the incentives of a vertically separated retailer to accommodate an initially inferior product. Because our results depend on the impatience of the parties, we can analyze how the discount factor affects exclusionary behavior and welfare. In their model, the incumbent forecloses more efficient entrants. By contrast, in our model, the dominant firm may accommodate a competing product even though a vertically integrated firm would not have accommodated it.

Our article also contributes to literature on exclusive dealing used to exclude rival firms. Aghion and Bolton (1987) show that the incumbent can use its first-mover advantage to offer the buyer a contract that excludes some efficient entrants. Fumagalli et al (2012), show that exclusive dealing that promotes investment by the incumbent may foreclose a more efficient entrant.

In Rasmusen, Ramseyer, and Wiley (1991), Segal and Whinston (2000), and Fumagalli and Motta (2006), the incumbent abuses a coordination problem among buyers to foreclose entry. Spector (2011) extends these results to the exclusion of rivals that are already present in the market.

Our article contributes to this literature by showing that exclusive dealing can be used to foreclose a rival even absent coordination problems among buyers and with full information. A similarity between our article and some of this literature is that in both cases the excluded rival needs to operate at some minimum scale in order to enable entry. In this literature, however, the excluding supplier's advantage comes from buyers' coordination problems, while in our paper the excluding supplier has the advantage of initially having a superior product, while utilizing the retailer's impatience.

We also contribute to literature that distinguishes between explicit exclusive dealing contracts and de facto exclusive dealing. Mathewson and Winter (1987) show that a dominant firm may wish to impose explicit exclusive dealing when firms can use only linear pricing, while O'Brien and Shaffer (1997) and Bernheim and Whinston (1998) show that the manufacturer can use non-linear contracts to obtain de facto exclusivity. Yehezkel (2008) shows that when the retailer is privately informed about demand, a supplier of a high-quality product may use exclusive dealing to exclude a low-quality substitute even when the low-quality substitute would not be excluded under vertical integration. All of the above-mentioned exclusive dealing literature deals with static models of exclusion. By contrast, our article deals with a dynamic environment, in which a rival product that is initially inferior improves if sold by the retailer. The dynamic nature of the game in our framework highlights the role of the retailer's discount factor in affecting the parties' strategies and their effect on welfare. It stresses the divergence between the vertically integrated outcome and the vertically separated outcome.

2 The model

Consider an upstream manufacturer that supplies product 1 to a monopolistic retailer. A second product, denoted product 2, is also available to the retailer. The marginal costs of producing product i is c_i, where $c_2 > c_1$. Without loss of generality, we assume that product 2 is sold by a perfectly competitive fringe and is available to the retailer at marginal costs.[3] The manufacturer and retailer play an infinitely repeated game and discount future profits by δ ($0 \leq \delta \leq 1$).

In the first period, consumers view products 1 and 2 as perfect substitutes. [4] The inverse demand function facing the retailer is $p(Q)$, where the price p and marginal revenue are decreasing with the total quantity $Q = q_1 + q_2$. Let q_i^{VI} denote the quantity that maximizes the one-period vertically integrated (monopoly) profits when selling product i, $(p(q_i) - c_i)q_i$, and let $\pi_i^{VI} \equiv (p(q_i^{VI}) - c_i)q_i^{VI}$ denote the monopoly profits. Because $c_2 > c_1$, the one-period profits of selling product 1 are higher than selling product 2:

[3]Our results qualitatively extend to a model in which product 2 is sold by a strategic player.

[4]Our results carry over to the case where the two products are imperfect substitutes and $c_2 > c_1$ makes product 2 less profitable than product 1.

$q_1^{VI} > q_2^{VI}$ and $\pi_1^{VI} > \pi_2^{VI}$. Yet, suppose that stand alone, product 2 provides positive monopoly profits: $\pi_2^{VI} > 0$.

Product 2 can improve over time. Such improvement can take several forms, all of which can be captured by our analysis. For example, when consumers are exposed to the product, consumers may learn to like it and change their preferences. Alternatively, because of learning-by-doing, the quality (cost) of product 2 may increase (decrease) over time, when the product is produced and sold to consumers. In particular, suppose that if the retailer sells at least the quantity $q_2 = \underline{q} < q_2^{VI}$ for one period, in all future periods product 2 substantially improves and the monopoly profit from selling product 2 is π_{2H}^{VI}, where $\pi_{2H}^{VI} > \pi_1^{VI}$. Without loss of generality, we assume that if product 2 is improved, it is profitable to offer only product 2 instead of offering both products.[5] It is possible to improve product 2 in any period. As long as product 2 is not improved, product 1 remains the superior product.

The two firms play an infinitely repeated game. In each period, the manufacturer offers a two-part-tariff contract (w_1, T_1) valid for the current period, where w_1 is a wholesale price per-unit and T_1 is a fixed fee. The retailer chooses whether to sell products 1, 2 or both and sets the total quantity to consumers. We assume that the manufacturer remains active in the market in all periods.[6]

We first solve for a dynamic vertical integration benchmark. We ask when it is profitable for a vertically integrated firm to carry product 2 in the first period in order to improve it in the following periods. When the integrated firm does not carry product 2 in any period, it earns π_1^{VI} in each period. When the integrated firm chooses to carry product 2, it sets $q_2 = \underline{q}$ and chooses q_1 as to maximize:

$$\pi_{12}^{VI}(q_1) \equiv p(q_1 + \underline{q})(q_1 + \underline{q}) - c_1 q_1 - c_2 \underline{q}.$$

Let $\widehat{q}_1$ denote the quantity that maximizes $\pi_{12}^{VI}(q_1)$. It is straightforward to show that $\widehat{q}_1 = q_1^{VI} - \underline{q}$. Intuitively, the retailer chooses the total quantity based on the marginal cost of the last unit, c_1 because $\underline{q} < q_2^{VI} < q_1^{VI}$. We note that the crucial feature of $\widehat{q}_1$ for our results is that $\widehat{q}_1 < q_1^{VI}$, which holds even if the two products are differentiated.

[5]The qualitative results would carry over to a case where after product 2 is improved, it is profitable to offer only a smaller quantity of product 1.

[6]Our results follow to the case where a manufacturer that does not make positive sales exits the market and then in the next period an identical manufacturer enters.

If $\widehat{q}_1 < q_1^{VI}$, there is a short-term sacrifice in selling product 2 along-side product 1. When selling both products, the vertically integrated firm earns $\pi_{12}^{VI}(\widehat{q}_1) < \pi_1^{VI}$ in the current period, followed by $\pi_{2H}^{VI} > \pi_1^{VI}$ in all future periods. Hence, it is profitable for a vertically integrated firm to sell product 2 in the first period in order to improve it in the subsequent periods iff:

$$\pi_{12}^{VI}(\widehat{q}_1) + \frac{\delta}{1-\delta}\pi_{2H}^{VI} \geq \frac{\pi_1^{VI}}{1-\delta}. \tag{1}$$

Let δ^{VI} denote the solution to (1) in equality. Because $\pi_{2H}^{VI} > \pi_1^{VI} > \pi_{12}^{VI}(\widehat{q}_1)$, it is profitable to sell product 2 in the first period iff $\delta > \delta^{VI}$, where $0 < \delta^{VI} < 1$. Intuitively, it is profitable for the vertically integrated firm to sell both products (despite product 2's inferiority) if it cares more about future profits and is willing to sacrifice current profits to increase future profits (thanks to product 2's improvement). It is straightforward to show that it is more profitable to sell products 1 and 2 as $\underline{q}$ and $c_2 - c_1$ decrease. Intuitively, the smaller is $\underline{q}$, the less of a short-term sacrifice it is to hold a quantity $\underline{q}$ of inferior product 2. Similarly, the smaller is the cost-difference between products 1 and 2, the smaller this short-term sacrifice. In the following sections, we shall assume that the manufacturer and the retailer are vertically separated.

3 Vertical-separation, non-exclusive contract

In this section we assume that the manufacturer and retailer are vertically separated and the manufacturer cannot impose explicit exclusive dealing. That is, he cannot offer the retailer a contract that forbids the retailer from selling product 2 or penalizes the retailer explicitly for selling product 2. All the manufacturer can do is set the two part tariff (t_1, w_1). Consider first an equilibrium in which the manufacturer sets (t_1, w_1) so as to motivate the retailer not to sell product 2. In this equilibrium, the manufacturer offers (t_1, w_1) in every period and the retailer accepts and sells only product 1. As we show below, the manufacturer's two part tariff can motivate the retailer to avoid product 2 (we shall call such a two part tariff an exclusionary offer) in two ways: The first way is to set a low (below cost) w_1 that reduces the retailer's marginal profit from selling product 2 alongside product 1. The second way is to reduce the fixed fee, t_1. Although a reduction in t_1 in a particular period does not affect the retailer's profit from selling product 2 in that period, it implicitly offers the retailer a reduced t_1 also in future periods. This reduced t_1 is relevant, however, only if the retailer continues to accept the manufacturer's exclusionary offer in future periods as well. If the retailer rejects the manufacturer's exclusionary offer in a certain period, it will sell a quantity $\underline{q}$ of product 2, and product 2 will improve. In the following periods, the retailer would choose to sell only the improved product 2, so it would not benefit from the reduced t_1 offered by the manufacturer.

We start by considering the manufacturer's optimal exclusionary offer: the contract that motivates the retailer to sell only product 1. Then we move to the manufacturer's optimal accommodating contract, which induces the retailer to sell product 2 alongside product 1. Finally, we compare the two in order to identify when the manufacturer prefers accommodation to exclusion.

The manufacturer's optimal exclusionary contract

After accepting the manufacturer's exclusionary offer (t_1, w_1), the retailer sets the quantity $q_1(w_1)$ that maximizes $(p(q_1) - w_1)q_1(w_1)$ and earns $\pi_1^R(w_1) - t_1$, where $\pi_1^R(w_1) \equiv (p(q_1(w_1)) - w_1)q_1(w_1)$. We have that $\pi_1^R(c_1) = \pi_1^{VI}$ and $\pi_1^R(w_1) < \pi_1^{VI}$ for $w_1 \neq c_1$. The manufacturer earns in every period $\pi_1^M(w_1) + t_1$, where $\pi_1^M(w_1) \equiv (w_1 - c_1)q_1(w_1)$.

The retailer can deviate from this equilibrium in two ways. First, the retailer can reject the manufacturer's offer completely and sell only product 2. In this case, the retailer sells the quantity q_2^{VI} of product 2, earns π_2^{VI} in the current period, and then earns π_{2H}^{VI} in all future periods. Alternatively, the retailer may choose to accept the manufacturer's offer of (t_1, w_1), but deviate from the exclusionary equilibrium by selling both products instead of only product 1. In this case, the retailer sells $q_2 = \underline{q}$ and $q_1 = \widehat{q}_1(w_1)$ where $\widehat{q}_1(w_1)$ maximizes:

$$\pi_{12}^{VI}(q_1, w_1) \equiv p(q_1 + \underline{q})(q_1 + \underline{q}) - w_1 q_1 - c_2 \underline{q}.$$

We have that $\widehat{q}_1(w_1) = q_1(w_1) - \underline{q}$. Hence, $\widehat{q}_1(w_1)$ is decreasing with w_1 and $\widehat{q}_1(c_1) = \widehat{q}_1$ when $w_1 = c_1$.[7] That is, a below cost w_1 induces the retailer to increase the quantity of

[7]Note that $q_1(w_1)$ solves $p'(q_1)q_1 + p(q_1) - w_1 = 0$ while $\widehat{q}_1(w_1)$ solves $p'(q_1 + \underline{q})(q_1 + \underline{q}) + p(q_1 + \underline{q}) - w_1 =$

product 1 that it sells, an above-cost w_1 induces the retailer to decrease this quantity, and a w_1 equal to cost induces the retailer to sell the same quantity of product 1 as a vertically integrated firm selling both products. The retailer's current period profit from accepting the manufacturer's two part tariff and also selling product 2 is $\pi_{12}^R(w_1) \equiv \pi_{12}^{VI}(q_1(w_1), w_1)$, where $\pi_{12}^R(c_1) = \pi_{12}^{VI}(\widehat{q}_1)$. Then, in all future periods, the retailer earns π_{2H}^{VI}. Therefore, given the manufacturer's offer (w_1, t_1), the retailer does not deviate from the equilibrium in which product 2 is excluded if:

$$\frac{\pi_1^R(w_1) - t_1}{1-\delta} \geq \max\Big\{ \underbrace{\pi_2^{VI} + \frac{\delta}{1-\delta}\pi_{2H}^{VI}}_{\text{selling 2}} \; , \; \underbrace{\pi_{12}^R(w_1) - t_1 + \frac{\delta}{1-\delta}\pi_{2H}^{VI}}_{\text{selling 1+2}} \Big\}. \tag{2}$$

The left-hand side is the retailer's profit from selling only product 1 in all periods. The right hand side contains the retailer's two options of deviating from the exclusionary equilibrium. We call these two options "selling 2" and "selling 1+2", as denoted in the right-hand-side of (2). If the retailer deviates, whether it prefers selling 2 to selling 1+2 depends on the manufacturer's proposed contract (w_1, t_1). To see how (w_1, t_1) affects the comparison between selling 2 and selling 1+2, notice first that selling 2 is higher than selling 1+2 if:

$$t_1 \geq \underline{T}(w_1) \equiv \pi_{12}^R(w_1) - \pi_2^{VI}. \tag{3}$$

The fee $\underline{T}(w_1)$ is the difference between the retailer's profits from selling 2 and its profits from selling 1+2. It boils down to the difference between the retailer's first-period profits from selling 2 and 1+2, because the profits in the subsequent periods from selling only the improved product 2 are the same in both cases. Intuitively, for selling 2 to bind, the manufacturer's offer has to be oppressive enough so that the retailer would rather sell only inferior product 2 than sell both products. Accordingly, when only selling 2 binds, then solving (2) (while ignoring the option of selling 1+2) for t_1 , the manufacturer can set t_1 as high as:

$$t_1 \leq T_2(w_1, \delta) \equiv \pi_1^R(w_1) - \pi_2^{VI} - \delta\big(\pi_{2H}^{VI} - \pi_2^{VI}\big). \tag{4}$$

Accordingly, in order to induce the retailer to avoid product 2 when it's binding

0. Hence $\widehat{q}_1(w_1) = q_1(w_1) - \underline{q}$. The crucial feature of $\widehat{q}_1(w_1)$ for our results is that $\widehat{q}_1(w_1) < q_1(w_1)$.

constraint is selling 2, it must be that $\underline{T}(w_1) < t_1 \leq T_2(w_1, \delta)$. Such a t_1 exists only if $\underline{T}(w_1) < T_2(w_1, \delta)$. This further implies that if $\underline{T}(w_1) \geq T_2(w_1, \delta)$ the binding constraint for an exclusionary equilibrium must be selling 1+2. In such a case, in order to induce the retailer to sell only product 1 rather than 1+2, the manufacturer must set t_1 so that:

$$t_1 < T_{12}(w_1, \delta) \equiv \frac{1}{\delta}\Big[\pi_1^R(w_1) - \pi_{12}^R(w_1) - \delta\big(\pi_{2H}^{VI} - \pi_{12}^R(w_1)\big)\Big]. \tag{5}$$

It is straightforward to see that $\underline{T}(w_1) - T_{12}(w_1, \delta) = -\frac{1}{\delta}[T_2(w_1, \delta) - \underline{T}(w_1)]$. Hence, when $\underline{T}(w_1) - T_{12}(w_1, \delta) \geq 0$, selling 1+2 binds and the manufacturer sets $t_1 = T_{12}(w_1, \delta)$, because $\underline{T}(w_1) > T_2(w_1, \delta) \geq T_{12}(w_1, \delta)$. Likewise, when $\underline{T}(w_1) - T_{12}(w_1, \delta) \leq 0$, selling 2 binds and and the manufacturer sets $t_1 = T_2(w_1, \delta)$, because $T_{12}(w_1, \delta) \geq T_2(w_1, \delta) \geq \underline{T}(w_1)$.

We can use this feature for comparing the incentives of the vertically separated retailer to sell 1+2 with those of a vertically integrated firm. The level of δ above which a vertically integrated firm prefers selling 1+2 is the solution to (after rewriting (1):

$$\delta\big(\pi_{2H}^{VI} - \pi_{12}^{VI}\big) \geq \pi_1^{VI} - \pi_{12}^{VI}. \tag{6}$$

Likewise, the level of δ above which, under vertical separation, the binding constraint is selling 1+2 is the solution to:

$$\underline{T}(w_1) \geq T_2(w_1, \delta) \iff \delta\big(\pi_{2H}^{VI} - \pi_2^{VI}\big) \geq \pi_1^R(w_1) - \pi_{12}^R(w_1). \tag{7}$$

At $w_1 = c_1$, the right hand side of (6) and (7) are identical. Yet, when $\delta > 0$, the left-hand side of (7) is almost identical to the left-hand side of (6), except that the term $-\pi_{12}^{VI}$ from (6) replaces the term $-\pi_2^{VI}$ from (7). Since $\pi_{12}^{VI} > \pi_2^{VI}$, it is clear that at $w_1 = c_1$, the left-hand side of (6) is lower. This implies that if, under vertical separation, the manufacturer sets $w_1 = c_1$, the retailer has an incentive to sell 1+2 for discount factors below $\delta = \delta^{VI}$. These are discount factors in which a vertically integrated firm (with an implicit wholesale price of $w_1 = c_1$) would prefer to sell only product 1. The intuition for this divergence between the vertically integrated outcome and the vertical separation case is as follows: At $\delta = \delta^{VI}$ and $w_1 = c_1$, a vertically integrated firm, according to (6), still prefers avoiding product 2, while a vertically separated retailer, according to (7), strictly prefers to sell also product 2, precisely because, as noted, $\pi_{12}^{VI} > \pi_2^{VI}$.

The term π_2^{VI} (the retailer's profit from selling only product 2) is unique to vertical separation. A vertically integrated firm would never consider selling only product 2. A vertically separated retailer, on the other hand, considers selling only product 2 if the manufacturer's offer, (t_1, w_1) is too oppressive. This gives the retailer more leeway in its negotiations with the manufacturer. The retailer can threaten to either sell only product 2, or sell 1+2. As we shall see below, this opens the door to equilibrium contracts that diverge from the vertically integrated outcome.

Hence the term $\underline{T}(w_1) - T_2(w_1, \delta)$ plays a crucial role in determining which constraint is binding, and following this, the equilibrium strategies, under vertical separation. The following lemma summarizes the main features of $\underline{T}(w_1) - T_2(w_1, \delta)$ (all proofs are in the Appendix):

Lemma 1. ***(When does selling 1+2 bind under vertical separation)*** *Under vertical separation, selling 1+2 binds for $\underline{T}(c_1) - T_2(c_1, \delta^{VI}) \geq 0$. This occurs for values of δ under which a vertically integrated firm would sell only product 1. As w_1 and δ increase, it becomes more attractive for the retailer to sell 1+2. That is, $\underline{T}(w_1) - T_2(w_1, \delta)$ is increasing in w_1 and δ.*

It follows from lemma 1 that selling 1+2 binds if w_1 is high enough. Let $\widetilde{w}_1(\delta)$ denote the level of w_1 in which $\underline{T}(w_1) = T_2(w_1, \delta)$. Selling 1+2 binds iff $w_1 > \widetilde{w}_1(\delta)$. The following lemma characterizes the features of $\widetilde{w}_1(\delta)$:

Lemma 2. *(i) Selling 1+2 binds iff $w_1 \geq \widetilde{w}_1(\delta)$. For $\delta = 0$, $\widetilde{w}_1(0) = c_2$, $\widetilde{w}_1(\delta)$ is decreasing with δ, and crosses c_1 at $\widetilde{\delta}$, where $\widetilde{\delta}$ is the solution to $\widetilde{w}_1(\widetilde{\delta}) = c_1$. (ii) Evaluated at $w_1 = c_1$, selling 1+2 under vertical separation binds more than under vertical integration: $\widetilde{\delta} < \delta^{VI}$.*

The next step is to solve for the manufacturer's optimal contract given that the manufacturer chooses to exclude product 2. The manufacturer earns $\Pi_1^M(w_1) \equiv \pi_1^M(w_1) + t_1$. From Lemmas' 1 - 2, $t_1 = T_2(w_1, \delta)$ if $w_1 < \widetilde{w}_1(\delta)$ (where the retailer's binding constraint is selling 2) and $t_1 = T_{12}(w_1, \delta)$ if $w_1 > \widetilde{w}_1(\delta)$ (where the retailer's binding constraint is selling 1+2). It will be useful to write the manufacturer's exclusionary profits in the different ranges of w_1 in terms of the manufacturer's and retailer's joint profits from sales (excluding the fixed fees): $\pi_1^{VI}(w_1) = \pi_1^M(w_1) + \pi_1^R(w_1)$. Accordingly, the manufacturer's per-period exclusionary profits, including the fixed fees, can be written as:

$$\Pi_1^M(w_1) = \begin{cases} \pi_1^{VI}(w_1) - \pi_2^{VI} - \delta\Big(\pi_{2H}^{VI} - \pi_2^{VI}\Big); & w_1 < \widetilde{w}_1(\delta); \\ \frac{1}{\delta}\Big[\pi_1^{VI}(w_1) - (1-\delta)\Big(\pi_1^M(w_1) + \pi_{12}^R(w_1)\Big)\Big] - \pi_{2H}^{VI}; & w_1 \geq \widetilde{w}_1(\delta). \end{cases} \tag{8}$$

It is clear from (8) that its first line is maximized at $w_1 = c_1$. Let $w_{12}(\delta)$ denote the w_1 that maximizes the second line in (8). We therefore have:

Lemma 3. *The exclusionary wholesale price involves persistent predatory pricing (i.e., predatory pricing in all periods). The exclusionary w_1 that maximizes the manufacturer's profit is:*

$$w_1^E(\delta) = \begin{cases} c_1; & \delta \in [0, \widetilde{\delta}]; \\ \widetilde{w}_1(\delta); & \delta \in [\widetilde{\delta}, \widetilde{\widetilde{\delta}}]; \\ w_{12}(\delta); & \delta \in [\widetilde{\widetilde{\delta}}, 1]; \end{cases} \tag{9}$$

where $\widetilde{\delta}$ is the solution to $\widetilde{w}_1(\delta) = c_1$ and $\widetilde{\widetilde{\delta}}$ is the solution to $\widetilde{w}_1(\delta) = w_{12}(\delta)$. $w_1^E(\delta)$ is constant for $\delta \in [0, \widetilde{\delta}]$, is decreasing with δ and predatory (i.e., below c_1) for $\delta \in [\widetilde{\delta}, \widetilde{\widetilde{\delta}}]$, is increasing with δ, but still predatory, at $\delta \in [\widetilde{\widetilde{\delta}}, 1]$ and converges back to c_1 as $\delta \to 1$.

Figure 1 illustrates $w_1^E(\delta)$, the manufacturer's exclusionary wholesale price. The intuition for this result is as follows: As noted above, the manufacturer has two tools to exclude product 2. First, lowering t_1, as an implicit promise for future benefits the retailer reaps if it avoids product 2 in the current period. This tool becomes more useful the more patient the retailer is, because the retailer receives the "reward" of the low t_1 only in future periods. The second tool the manufacturer has to exclude product 2 is lowering w_1 to predatory levels. A reduced w_1 affects both the retailer's current and future profits. Furthermore, a predatory w_1 reduces the retailer's marginal profits from selling both products 1 and 2 simultaneously. The manufacturer's profitability from each of these two tools depends on the binding constraint. For low discount factors ($\delta \in [0, \widetilde{\delta}]$), when $w_1 = c_1$, only selling 2 binds. Since selling 1+2 is not considered by the retailer, a predatory w_1 cannot affect the retailer's marginal profits from selling both products. Hence in these low ranges of δ, the manufacturer merely needs to convince the retailer to sell only product 1 rather than only product 2. While this can be done via a predatory w_1, this would reduce the parties' joint profits. Hence the manufacturer

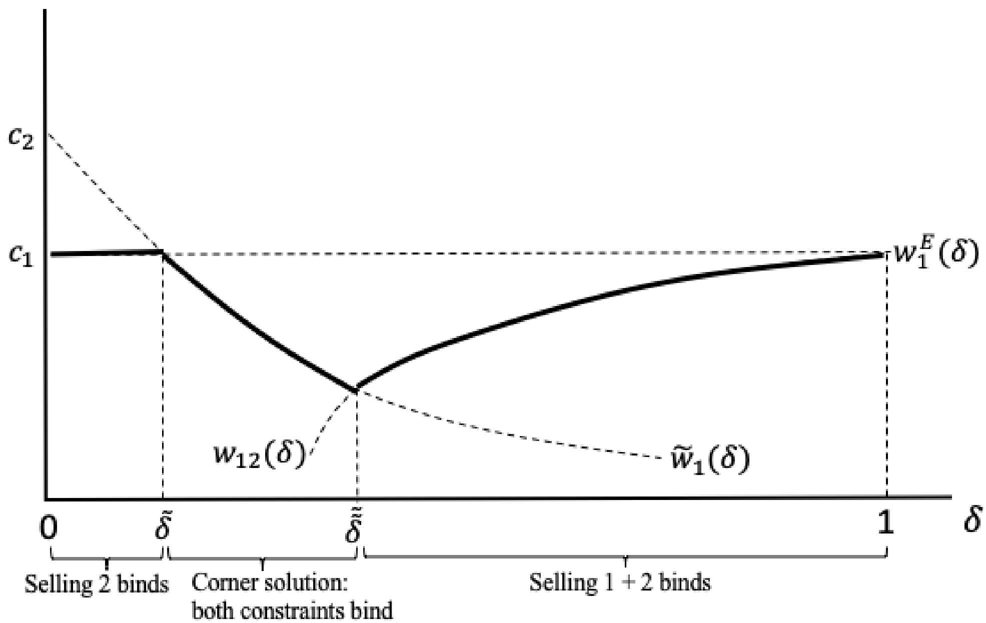

Figure 1: The manufacturer's optimal w_1, $w_1^E(\delta)$, under exclusion as a function of δ

prefers to maximize joint profits with $w_1 = c_1$, as depicted in figure 1, and exclude product 2 with a reduced fee $t_1 = T_2(c_1, \delta)$. This fee decreases with δ, to adjust for the retailer placing more value on improving product 2.

For medium values of δ ($\delta \in [\widetilde{\delta}, \widetilde{\widetilde{\delta}}]$), the manufacturer maximizes its profits by charging a predatory wholesale price of $\widetilde{w}_1(\delta)$. In this range of δ, the fee is $\overline{T}(w_1) = T_2(w_1, \delta) = T_{12}(w_1, \delta)$ and $\widetilde{w}_1(\delta)$ is a corner solution. Since $\overline{T}(w_1)$ is decreasing in w_1 (see (3)), and $\widetilde{w}_1(\delta)$ is decreasing with δ, the fee in the middle range is increasing with δ. Intuitively, since the discount factor is relatively low, the fee in this range is less effective than a predatory wholesale price as an exclusionary tool. A reduced fee affects the retailer's profits only due to future period earnings, while a predatory wholesale price affects the retailer's current profits as well. This is why, for $\delta \in [\widetilde{\delta}, \widetilde{\widetilde{\delta}}]$, the manufacturer prefers predatory pricing as the sole exclusionary tool and uses the fee for rent extraction. As the retailer becomes more forward-looking within this range, the retailer has a stronger incentive to offer product 2 alongside product 1, and the wholesale price becomes more predatory, while the fixed fee increases.[8]

[8]We should note that the manufacturer's optimal strategy may involve $w_{12}(\widetilde{\widetilde{\delta}}) = \widetilde{w}_1(\widetilde{\widetilde{\delta}}) < 0$. If a negative wholesale price is impossible to implement, then there are two cutoffs of δ, $\delta' < \widetilde{\widetilde{\delta}} < \delta''$ such that the manufacturer sets $w_1(\delta) = 0$ for $\delta \in [\delta', \delta'']$. This will not qualitatively change our results.

For high levels of δ ($\delta \in [\tilde{\tilde{\delta}}, 1]$), the manufacturer finds it profitable to combine the two exclusionary tools, of predatory pricing and a reduced fixed fee. It sets a predatory price of $w_1 = w_{12}(\delta)$ so that only 1+2 binds, combined with a reduced fixed fee $t_1 = T_{12}(w_1, \delta)$. Here the wholesale price and fixed fee evolve in opposite directions compared to the middle range of δ: The wholesale price becomes less and less predatory as δ increases (so as to reduce losses) while the fixed fee shrinks, to exclude product 2. Intuitively, the fixed fee (which is entirely forward looking) becomes a more and more useful tool for exclusion compared with a predatory wholesale price (which also affects current profits) as the retailer becomes more forward-looking. As δ approaches 1, $w_{12}(\delta)$ approaches c_1 so that for $\delta = 1$ exclusion is accomplished solely by the reduced fixed fee and w_1 is set to maximize joint profits.

The manufacturer's optimal accommodation contract

Next, we turn to the question whether the manufacturer prefers to exclude product 2 or finds it more profitable to accommodate product 2. Under accommodation, the highest t_1 that the manufacturer can set needs to satisfy:

$$\pi_{12}^R(w_1) - t_1 + \frac{\delta}{1-\delta}\pi_{2H}^{VI} \geq \pi_2^{VI} + \frac{\delta}{1-\delta}\pi_{2H}^{VI}. \tag{10}$$

Hence, $t_1 = \underline{T}(w_1)$. The manufacturer earns:

$$\Pi_{12}^M(w_1) = (w_1 - c_1)\widehat{q}_1(w_1) + \underline{T}(w_1) = \pi_{12}^{VI}(w_1) - \pi_2^{VI}. \tag{11}$$

Under accommodation, the manufacturer sets $w_1 = c_1$ to maximize $\pi_{12}^{VI}(w_1)$, and earns $\Pi_{12}^M(c_1)$.

When does the manufacturer accommodate product 2?

When the manufacturer accommodates product 2, it maximizes its first period profit from being sold together with product 2 (a quantity of $\widehat{q}_1(w_1)$), knowing that in all subsequent periods, only the improved product 2 will be sold, so the manufacturer would make no profits in subsequent periods. Accordingly, the manufacturer's profit when accommodating product 2 is independent of δ. Comparing the manufacturer's profit under exclusion and accommodation, we obtain the following result:

Proposition 1. *(The manufacturer's decision whether to exclude product 2 in all periods or accommodate product 2) There is a cutoff* δ^{VS} *where* $\widetilde{\delta} < \delta^{VS} < \delta^{VI}$*, such that:*

1. *For* $\delta \in [0, \widetilde{\delta}]$*, the manufacturer excludes product 2 by setting* $w_1 = c_1$ *and charging* $t_1 = T_2(w_1, \delta)$ *in all periods;*

2. *For* $\delta \in [\widetilde{\delta}, \delta^{VS}]$*, the manufacturer excludes product 2 by setting* $w_1 < c_1$ *(and* $t_1 > 0$*), corresponding to the results of Lemma 3,in all periods;*

3. *For* $\delta \in [\delta^{VS}, 1]$*, the manufacturer accommodates product 2 by setting* $w_1 = c_1$ *and* $t_1 = \underline{T}(c_1)$*in the first period . The retailer sells 1+2 and then only the improved product 2 in all future periods.*

4. *Under vertical separation, the market accommodates product 2 more than under vertical integration:* $\delta^{VS} < \delta^{VI}$.

Proposition 1 shows that under vertical separation, in the low range of $\delta \in [0, \widetilde{\delta}]$, the manufacturer would never want to accommodate product 2. In this range, because the retailer is impatient, the manufacturer can set $w_1 = c_1$ to maximize joint profits, and a fixed fee that makes the retailer indifferent between selling only 1 and selling only 2. Since the wholesale price equals marginal cost and product 2 is excluded, the vertically separated industry acts the same as a vertically integrated firm in this region (recall from lemma 2 that $\delta^{VI} > \widetilde{\delta}$). As δ increases beyond $\widetilde{\delta}$, however, the manufacturer has to engage in predatory pricing to exclude product 2. When δ approaches δ^{VS}, this predatory pricing becomes prohibitively costly to the manufacturer, so that it prefers to accommodate product 2 for discount factors below δ^{VI}. This is while a vertically integrated firm would decide to accommodate product 2 only for $\delta > \delta^{VI}$. Accordingly, under vertical separation, the manufacturer and retailer cannot maximize their joint profits. The intuition for this result is that in the middle and high regions of δ, if the manufacturer tries to implement the vertically integrated result and set $w_1 = c_1$, the retailer prefers to sell 1+2 over selling 1 (see figure 1). The retailer sacrifices joint profits by accepting the manufacturer's two part tariff but also selling product 2, so as to increase its own profits. The best the manufacturer can do in this region to induce the retailer to nevertheless sell only 1 is to set a predatory w_1. Notice that this practice is somewhat different than standard predatory pricing, where a manufacturer sells at a loss in the short run in order to eliminate a competitor and raise prices once the competitor is excluded. Here, the manufacturer sets a below-cost wholesale price indefinitely, and recovers its losses immediately, via the fixed fee. The manufacturer's overall profits each period are positive. Such predatory pricing by the manufacturer further sacrifices joint profits, since production is higher than vertically integrated production, but maximizes the manufacturer's own profits. Yet, for $\delta > \delta^{VS}$, such predatory pricing becomes prohibitively costly for the manufacturer, and it prefers to accommodate. Such accommodation is again not optimal for the firms' joint profits, but the manufacturer individually prefers it, so as to stop pricing below cost. A vertically integrated firm, on the other hand, does not have to worry about the retailer holding product 2 more than jointly optimal, because the firm controls the retailer, and forces it to accommodate only for $\delta > \delta^{VS}$. [9]

Suppose now that the manufacturer cannot set a wholesale price below marginal costs, for example, due to fear of antitrust intervention. Then product 2 is accommodated for an even wider set of discount factors:

Corollary 1. *Suppose that the manufacturer cannot set a wholesale price below marginal costs. Then, the retailer sells products 1+2 for lower values of δ than when a below-cost wholesale price is possible. The manufacturer sets $w_1 = c_1$ and the retailer avoids product 2 for $\delta \in [0, \widetilde{\delta}]$ and sells products 1+2 for $\delta \in [\widetilde{\delta}, 1]$.*

This result can be easily demonstrated via Figure 1. If the wholesale price is forced to be no lower than c_1, the manufacturer cannot charge the predatory prices that would maximize its exclusionary profits for $\delta > \widetilde{\delta}$. For such wholesale prices, the only exclusionary tool left to the manufacturer is reduction of the fixed fee. But for $\delta > \widetilde{\delta}$, using such an exclusionary tool yields less profit than the profits from accommodating product 2.

[9]It should be noted that their may be multiple solutions to δ^{VS}. Yet, it is always the case that $\delta^{VS} < \delta^{VI}$. In cases where $\delta^{VI} < \widetilde{\widetilde{\delta}}$, there is a unique δ^{VS}.

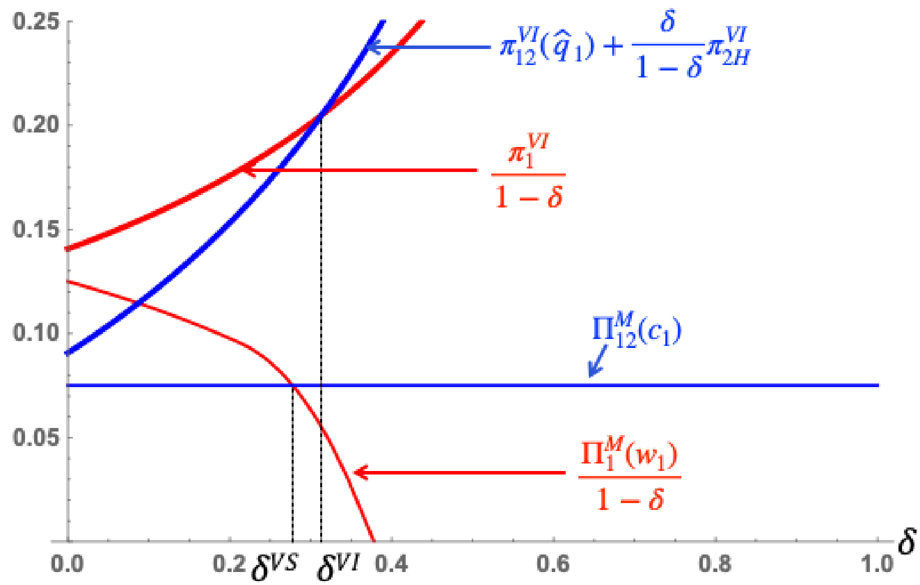

Figure 2: The threshold values δ^{VI} and δ^{VS} (for $c_1 = 1/4$, $c_2 = 3/4$ and $\underline{q} = 0.1$)

A linear demand example

To illustrate our results in a concrete example, consider linear demand, which is initially $P(Q) = 1 - Q$ for both products, where $Q = q_1 + q_2$. Suppose that $1 > c_2 > c_1 > 0$, and if in a certain period $q_2 > \underline{q}$ (where $0 < \underline{q} < \frac{1-c_2}{2}$) the demand for product 2 becomes $P(q_2) = 1 + c_2 - q_2$, whereas the demand for product 1 remains $P(q_1) = 1 - q_1$. That is, demand for product 2 rises by c_2, from the next period and on. This implies that it is more profitable to sell only the improved product 2 (and earn $\frac{1}{4}$ per period) rather than product 1, and earn only $\frac{(1-c_1)^2}{4}$.

Figure 2 illustrates the threshold values δ^{VI} and δ^{VS}. The bold lines represent the vertically integrated profits from selling 1 (red) and selling 1+2 (blue), while the thin lines represent the manufacturer's profits under vertical separation. The vertically separated manufacturer's low exclusionary profits, $\frac{\Pi_1^M(w_1)}{1-\delta}$, caused by the need to charge a predatory wholesale price, induce it to accommodate product 2 more than a vertically integrated firm.

4 Exclusive dealing

Suppose now that the manufacturer can explicitly impose exclusive dealing, by prohibiting the retailer from selling product 2. The manufacturer offers a two-part-tariff contract (with or without an exclusive dealing clause) that is valid for the current period only. The retailer can accept or reject, in which case the retailer sells only product 2.

Given that the manufacturer imposes exclusive dealing, the retailer agrees to the contract if:

$$\frac{\pi_1^R(w_1) - t_1}{1-\delta} \geq \pi_2^{VI} + \frac{\delta}{1-\delta}\pi_{2H}^{VI}. \tag{12}$$

Hence, under exclusive dealing, the manufacturer sets $t_1 = T_2(w_1, \delta)$ and earns:

$$\Pi_1^{M,ED}(w_1) = (w_1 - c_1)q_1(w_1) + T_2(w_1, \delta) = \pi_1^{VI}(w_1) - \pi_2^{VI} - \delta\big(\pi_{2H}^{VI} - \pi_2^{VI}\big). \tag{13}$$

Notice that the manufacturer sets $w_1 = c_1$ so as to maximize joint profits. Comparing the manufacturer's profits under the exclusive and the non-exclusive contracts described in Proposition 1, we have:

Proposition 2. *(exclusive dealing implements the vertically-integrated outcome) Suppose that the manufacturer can impose exclusive dealing. Then, for $\delta \in [0, \widetilde{\delta}]$, exclusive dealing is redundant, so the manufacturer excludes product 2 by offering $w_1 = c_1$ and $t_1 = T_2(c_1, \delta)$. For $\delta \in [\widetilde{\delta}, \delta^{VI}]$, the manufacturer imposes explicit exclusive dealing and again sets $w_1 = c_1$ and $t_1 = T_2(c_1, \delta)$. For $\delta \in [\delta^{VI}, 1]$, the manufacturer accommodates product 2 and sets $w_1 = c_1$ and $t_1 = \underline{T}(c_1)$.*

Proposition 2 shows that when the manufacturer can impose exclusive dealing, it finds it optimal to implement the vertically integrated outcome. The intuition for proposition 2 is that if the manufacturer can explicitly forbid the retailer from selling product 2, the retailer's incentives to buy product 2 in addition to product 1 after accepting the manufacturer's two part tariff is eliminated. The explicit prohibition to sell product 2 allows the manufacturer to set the wholesale price that maximizes industry profits, $w_1 = c_1$, regardless of δ, without the concern that the retailer would accept the manufacturer's contract and then also sell product 2. Hence, the manufacturer's and retailer's joint profits are the same as a vertically integrated firm, which is maximized by accommodating product 2 only if $\delta > \delta^{VI}$.

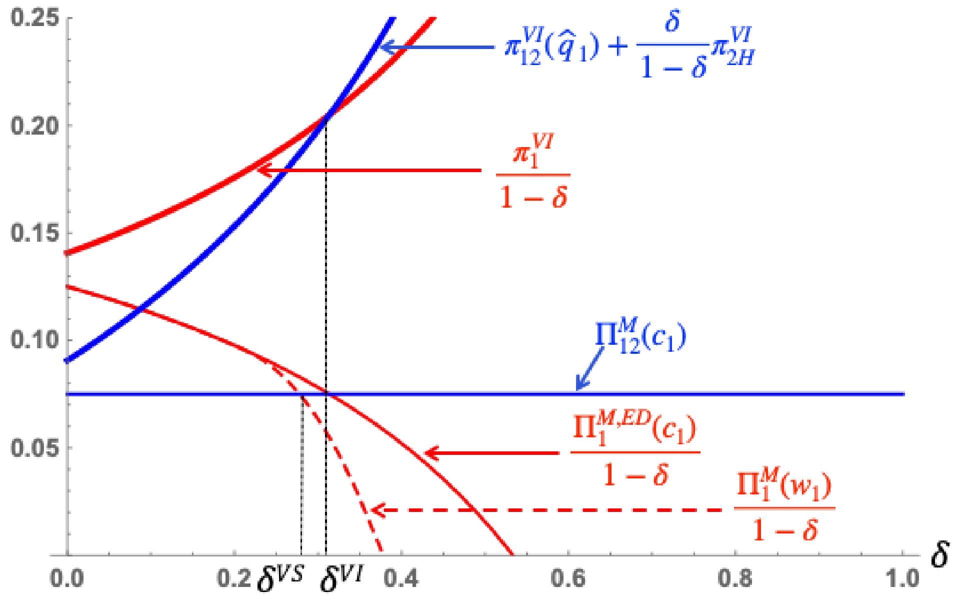

Figure 3: The threshold values δ^{VI} and δ^{VS} under exclusive dealing (for $c_1 = 1/4$, $c_2 = 3/4$ and $\underline{q} = 0.1$)

Figure 3 illustrates how exclusive dealing implements the vertically integrated outcome in our linear demand example. Because under exclusive dealing the manufacturer can exclude product 2 by setting a wholesale price equal to marginal cost, the manufacturer's profit from exclusion increases, from the dashed red line (representing $\frac{\Pi_1^M(w_1)}{1-\delta}$) to the solid red line (representing $\frac{\Pi_1^{M,ED}(c_1)}{1-\delta}$). As shown, the manufacturer's incentives whether to exclude product 2 are aligned with the vertically integrated outcome.

5 Welfare analysis and antitrust implications

Let's now examine the welfare implications of our results from sections 3 and 4. Our results are critically affected by two renowned practices: predatory pricing and exclusive dealing. In particular, in the context of our paper, an antitrust court or agency has three regimes to consider:

(i) Banning exclusive dealing but allowing predatory pricing ("No ED,Yes pred");

(ii) Banning both exclusive dealing and predatory pricing ("No ED, No pred"), and

(iii) Allowing exclusive dealing ("ED").

When exclusive dealing is allowed, it matters not in our model whether predatory pricing is allowed or not, because the manufacturer would not want to charge a predatory price if exclusive dealing is allowed. The following table summarizes the outcomes of the three regimes:

Regime	$[0, \widetilde{\delta}]$	$[\widetilde{\delta}, \delta^{VS}]$	$[\delta^{VS}, \delta^{VI}]$	$[\delta^{VI}, 1]$
(i) No ED, Yes pred	$w_1 = c_1$ exclusion	$w_1 < c_1$ exclusion	$w_1 = c_1$ accommodation	$w_1 = c_1$ accommodation
(ii) No ED, No pred	$w_1 = c_1$ exclusion	$w_1 = c_1$ accommodation	$w_1 = c_1$ accommodation	$w_1 = c_1$ accommodation
(iii) ED	$w_1 = c_1$ exclusion	$w_1 = c_1$ exclusion	$w_1 = c_1$ exclusion	$w_1 = c_1$ accommodation

Table 1: the three antitrust regimes

As shown in table 1, in the bottom range of δ ($[0, \widetilde{\delta}]$) and in the top range ($[\delta^{VI}, 1]$) the three antitrust regimes are equivalent. The differences between them lie in the middle range, of $\widetilde{\delta} < \delta < \delta^{VI}$. In what follows, we focus our comparison on this range.

The table reveals that the welfare implications of our results can be assessed along two dimensions: the first dimension is whether the inferior (but improvable) product, product 2, is accommodated by the industry too much or too little from a welfare perspective. As we highlight below, a vertically separated industry may either over-accommodate or over-exclude the inferior product compared to the social optimum. The second dimension concerns the quantity the industry sells (and the corresponding price). In other words, does the industry supply a monopoly quantity, or is the monopoly distortion alleviated?

The importance of these two dimensions in a particular case depends, among other things, on the firms' discount factor. Yet an antitrust agency or court would find it hard to verify in what range of the discount factor an industry is. Hence the antitrust agency or court needs to determine which of these two dimensions is more important. For an antitrust court or agency mainly concerned with monopoly pricing, regime (i), of banning exclusive dealing and allowing predatory pricing, is the most attractive, since only it induces pro-consumer predatory pricing, that helps shrink the market's monopoly distortion.

Turning to the other dimension-of accommodation versus exclusion, when accommodation of a new product is the main concern, regime (ii) (banning both predatory pricing and exclusive dealing) is the most attractive regime. This regime motivates the industry to accommodate the new product for all $\delta \in [\widetilde{\delta}, \delta^{VI}]$. At the same time, regime (iii) (allowing exclusive dealing) is particularly harmful in this respect, because as we shall see, it always involves over-exclusion of product 2 compared with the social optimum. This is while the other regimes only involve over-accommodation of product 2 for some parameter values. Conversely, if the main concern is from the over-accommodation of an (initially) inferior product, then regime (iii) may actually be favored, as this regime never involves over-accommodation of product 2 and excludes it for all $\delta \in [\widetilde{\delta}, \delta^{VI}]$.

Our analysis similarly provides antitrust implications for vertical mergers. Suppose that we apply regime (i) (exclusive dealing is banned and predatory pricing is allowed) to the vertically separated industry. This implies that a vertical merger between the retailer and manufacturer would shift the industry to the behavior corresponding to regime (iii). That is, it would be as if we allowed exclusive dealing: The effect of a vertical merger in our model is to raise prices to monopoly levels (because the manufacturer no longer needs to engage in predatory pricing), and over-exclusion of the improvable product. The upside of the vertical merger in our model would be that it could prevent over-accommodation of the inferior product, for some parameter values. Especially striking is the unambiguous price-hike that a vertical merger would cause in our setting. Conventional wisdom is that vertical mergers tend to cause price-reductions, due to elimination of double margins. But in our framework, there is no double margin under vertical separation. On the contrary, under regime (i), the manufacturer charges a below-cost wholesale price, and this helps reduce the industry's monopoly distortion.

For the reader interested in seeing how the discount factor affects the welfare comparison among the three regimes, we elaborate further below. Let us first compare regime (i) (exclusive dealing is banned and predatory pricing is allowed) with regime (iii) (exclusive dealing is allowed). One clear observation is that, when $\widetilde{\delta} < \delta < \delta^{VS}$, regime (i) is unambiguously better for social welfare than regime (iii). While in both regimes product 2 is excluded, in regime (i) the manufacturer engages in (welfare enhancing) predatory pricing, while in regime (iii) the manufacturer charges $w_1 = c_1$. Intuitively, when the manufacturer cannot explicitly restrain the retailer with an exclusive dealing prohibition, the manufacturer needs to exclude product 2 via price reduction. This reduction in prices, although predatory, alleviates the market's monopoly distortion and expands output, to the benefit of end consumers.

Turning to the range $\delta^{VS} < \delta < \delta^{VI}$, in both regimes (i) and (iii) $w_1 = c_1$, but in regime (i), without exclusive dealing, product 2 is accommodated, while in regime (iii), with exclusive dealing, it is excluded. Thus, in this range, the comparison between the two regimes hinges only on whether the accommodation of product 2 increases or decreases social welfare. To this end, let $SW_1(w_1)$ denote the per-period social welfare when the retailer sells only product 1 (the quantity is $q_1(w_1)$). Similarly, $SW_{12}(w_1)$ denotes social welfare in a period in which the retailer sells both products. In the latter case, the retailer sells the same total quantity $q_1(w_1)$ (recall that $\widehat{q}_1(w_1) = q_1(w_1) - \underline{q}$) hence:

$$SW_1(w_1) = \int_0^{q_1(w_1)} (p(q) - c_1)dq, \quad SW_{12}(w_1) = \int_0^{q_1(w_1)} p(q)dq - c_2\underline{q} - c_1(q_1(w_1) - \underline{q}).$$

As $c_1 < c_2$, we have that $SW_1(w_1) > SW_{12}(w_1)$. This stems from the short-run downside, in terms of social welfare, to selling the inferior product in the current period. Turning to the up-side, of improving product 2 in the future, let SW_{2H} denote per-period social welfare when the retailer sells the improved product 2. We assume that the improved product 2 offers not only higher vertically integrated profits (i.e., $\pi_{2H}^{VI} > \pi_1^{VI}$), but also (weakly) higher consumer surplus: $SW_{2H} - \pi_{2H}^{VI} \geq SW_1(c_1) - \pi_1^{VI}$. This would be the case when the improved product 2 is of higher quality than product 1, or when its marginal costs are lower than those of product 1, and hence its quantity is higher.

Under exclusive dealing (regime (iii)), recall that product 2 is excluded in the range we are considering, $\delta \in [\delta^{VS}, \delta^{VI}]$, and furthermore $w_1 = c_1$. Conversely, under regime (i) product 2 is accommodated in this range, while the wholesale price is the same as in regime (iii) ($w_1 = c_1$). Accordingly, we assess the socially optimal level of accommodation while taking the pricing in these two regimes as given: $w_1 = c_1$. In other words, since in this range of δ, both regimes involve similar (monopoly) quantities and prices, we derive the socially optimal level of accommodation given that monopoly pricing in the industry persists. Accommodation of product 2 improves welfare iff $\delta > \delta^{SW}$, where δ^{SW} is the solution to:

$$SW_{12}(c_1) + \frac{\delta}{1-\delta} SW_{2H} \geq \frac{SW_1(c_1)}{1-\delta}. \tag{14}$$

Because $SW_{2H} - \pi_{2H}^{VI} \geq SW_1(c_1) - \pi_1^{VI}$, exclusive dealing (regime (iii)) always involves over-exclusion of product 2 compared to what is socially optimal. Recall from Section 4 that under exclusive dealing, the industry replicates the vertically integrated outcome. This further implies that a vertically integrated firm over-excludes product 2 compared to what is socially desirable: $\delta^{SW} < \delta^{VI}$. Intuitively, the vertically integrated monopoly does not internalize the benefits that improving product 2 has on consumers, and hence "under-improves" product 2.

While exclusive dealing (regime (iii)) always involves over-exclusion of product 2, regime (i) may involve over accommodation of product 2 compared to the social optimum. In particular, δ^{SW} can be higher or lower than δ^{VS}, depending on the model's parameters. Intuitively, $\delta^{SW} < \delta^{VS}$ if there are welfare-enhancing advantages to accommodating product 2 that the manufacturer and retailer do not fully internalize. This occurs if c_2 and $\underline{q}$ are not too high, so that the short-term sacrifice to social welfare of selling the inferior product 2 in a certain period is small enough. The converse is true ($\delta^{SW} > \delta^{VS}$) if c_2 is high enough: here the vertically separated industry accommodates product 2 more than what is socially optimal. Recall that the manufacturer's losses from predatory pricing become prohibitively costly to him for $\delta > \delta^{VS}$. The social planner is indifferent to these losses, so the parties might over-accommodate product 2.[10]

Suppose first that $\delta^{SW} < \delta^{VS}$, so that even regime (i), when exclusive dealing is banned, involves over-exclusion of product 2 compared to the social optimum. The corollary below follows directly from Table 5:

Corollary 2. *Suppose that $\delta^{SW} < \delta^{VS}$. Then regime (i) (banning exclusive dealing and allowing predatory pricing), is better for social welfare than regime (iii) (allowing exclusive dealing).*

The intuition for this result is that for $\delta^{SW} < \delta^{VS}$, regime (i) dominates regime (iii) along both dimensions, of quantities supplied and accommodation of product 2. With respect to the first dimension, regime (i) is superior to regime (iii), because it involves persistent predatory pricing by the manufacturer (for $\widetilde{\delta} < \delta < \delta^{VS}$). This persistent

[10] In our linear demand example in Section 3, we find that given $c_1 = 1/4$, if $c_2 < 0.46$ then $\delta^{SW} < \delta^{VS}$ for all values of $\underline{q}$. For $0.46 < c_2 < 0.91$, there is a threshold of $\underline{q}$ such that $\delta^{SW} < \delta^{VS}$ if $\underline{q}$ is below this threshold. The threshold is increasing in c_2. On the other hand, for $0.91 < c_2 < 1$, it is always the case that $\delta^{SW} > \delta^{VS}$.

predatory pricing helps alleviate the industry's monopoly distortion.[11] As for the second dimension, even regime (i) involves over-exclusion of product 2 compared to the social optimum. Hence regime (iii), which allows exclusive dealing, must have even more severe over exclusion ($\delta^{SW} < \delta^{VS} < \delta^{VI}$).

Next, consider the case where $\delta^{VS} < \delta^{SW}$ (hence, $\widetilde{\delta} < \delta^{VS} < \delta^{SW} < \delta^{VI}$). We have:

Corollary 3. *Suppose that $\delta^{SW} > \delta^{VS}$. Then, regime (iii) (allowing exclusive dealing) is socially superior to regime (i) (banning exclusive dealing and allowing predatory pricing) iff $\delta \in [\delta^{VS}, \delta^{SW}]$.*

The intuition for this result is that when $\delta \in [\delta^{VS}, \delta^{SW}]$, from a social perspective it is optimal to exclude product 2 ($\delta < \delta^{SW}$), yet the industry under regime (i) accommodates product 2 ($\delta > \delta^{VS}$). This is while in regime (iii) the industry excludes product 2 ($\delta < \delta^{VI}$), conforming to what is socially optimal. With respect to the other dimension, of pricing, in both cases the manufacturer sets $w_1 = c_1$ for $\delta \in [\delta^{VS}, \delta^{SW}]$. The welfare-enhancing effect of regime (i)'s predatory pricing is not relevant. This results in higher social welfare in this range under regime (iii), of allowing exclusive dealing.

The converse is true for $\delta \notin [\delta^{VS}, \delta^{SW}]$: here regime (i) is socially superior to regime (iii). In particular, if $\delta \in [\delta^{SW}, \delta^{VI}]$, it is socially optimal to accommodate product 2, as regime (i) achieves in this range. Regime (iii) excludes product 2 for such discount factors, thereby over-excluding from a welfare perspective.

Next, we turn to evaluate the effects of regime (ii): both predatory pricing and exclusive dealing are banned. As noted in Table 5, this regime triggers accommodation when $\delta > \widetilde{\delta}$ – more than in regime (i) ($\delta > \delta^{VS}$) and regime (iii) ($\delta > \delta^{VI}$). At the same time, the ban on predatory pricing of the wholesale price increases the retail price to monopoly levels. Consider first the comparison between regimes (ii) and (i). When $\delta \in [\delta^{VS}, \delta^{VI}]$, both regimes are identical. However, when $\delta \in [\widetilde{\delta}, \delta^{VS}]$ the comparison between regimes (i) and (ii) is inconclusive. Regime (i) in this range involves predatory pricing with exclusion of product 2, while regime (ii) involves a wholesale price equal to marginal cost and accommodation of product 2. Hence, along the dimension of

[11]Notice that a predatory wholesale price, $w_1 < c_1$, can never result in selling too much of product 1 in comparison with the quantity that maximizes social welfare. This is because the manufacturer would never set $w_1 < c_1$ so low such that $p(q_1(w_1)) < c_1$, as doing so involves negative joint profits. For any $w_1 < c_1$ for which $p(q_1(w_1)) > c_1$, social welfare is decreasing in p, and hence in w_1.

pricing and quantity, regime (i) dominates regime (ii), because regime (i) involves welfare-enhancing predatory pricing, while regime (ii) involves monopoly pricing.

The other dimension, of accommodating product 2, may point to the opposite direction, depending on market circumstances. We have seen above that δ^{SW}, the socially optimal cutoff for accommodation given monopoly pricing, may be either below or above δ^{VS}, the corresponding cutoff under regime (i). But market conditions could also be such that δ^{SW} is either above or below $\widetilde{\delta}$, the cutoff for accommodation under regime (ii).[12] This implies that the welfare comparison between regimes (i) and (ii) is affected by both the industry quantity dimension and the over-accommodation or over-exclusion dimension in a way that critically depends on market parameters.

Next consider the comparison between regimes (ii) and (iii). For $\delta \in [\widetilde{\delta}, \delta^{VI}]$, both regimes include a wholesale price equal to marginal cost, though regime (ii) accommodates product 2 and regime (iii) excludes it. Hence, the comparison between the two regimes depends only on whether accommodating product 2 is socially superior to exclusion. As noted, regime (iii) always involves over-exclusion of product 2, while regime (ii) may over-exclude or over-accommodate product 2, depending on whether δ^{SW} lies above or below $\widetilde{\delta}$. The following corollaries summarize the results:

Corollary 4. *Suppose that $\delta^{SW} < \widetilde{\delta}$. Then regime (ii) (banning both exclusive dealing and predatory pricing), is better for social welfare than regime (iii) (allowing exclusive dealing).*

The intuition is that if $\delta^{SW} < \widetilde{\delta}$, even regime (ii) over-excludes product 2 compared to what is socially optimal, so regime (iii) involves even more harmful exclusion.

Corollary 5. *Suppose that $\delta^{SW} > \widetilde{\delta}$. Then, regime (iii) (allowing exclusive dealing) is socially superior to regime (ii) (banning both exclusive dealing and predatory pricing) iff $\delta \in [\widetilde{\delta}, \delta^{SW}]$.*

This implies a scenario where allowing exclusive dealing could be socially beneficial. Note, though, that for $\delta \notin [\widetilde{\delta}, \delta^{SW}]$, regime (ii) dominates regime (iii): it is socially better to ban both exclusive dealing and predatory pricing than to allow exclusive dealing.

[12] In our linear example, if $c_1 = 1/4$, and $c_2 < 0.72$, then there is a cutoff in $\underline{q}$ such that $\delta^{SW} < \widetilde{\delta}$ iff $\underline{q}$ is below this threshold. When $c_1 = 1/4$, and $c_2 > 0.72$, then $\delta^{SW} < \widetilde{\delta}$ for all values of $\underline{q}$.

6 Finite game

In this section we extend the analysis to the case where the parties interact for a finite number of periods. TBC.

7 Conclusion

This paper revealed that a vertically separated industry fails to maximize industry profits when the setting is dynamic and a new product is improvable. The failure to maximize industry profits stems from the fact that the retailer has “two many” options: it can sell only the established product, only the new product, or both. This multitude of options grants the retailer a stronger bargaining position facing the manufacturer of the established product. When the manufacturer is banned from imposing an exclusive dealing agreement, the manufacturer cannot replicate the vertically integrated outcome with a simple two part tariff: If the manufacturer attempts to set the wholesale price equal to marginal cost, to maximize industry profits, a patient enough retailer prefers to sell both products. Hence, the manufacturer of the established product is compelled to charge a predatory wholesale price to exclude the new product. But for a high enough discount factor, this predatory strategy becomes too costly to the manufacturer, so it accommodates the new product even though this implies no future sales for the manufacturer. This occurs for discount factors lower than the discount factor in which a vertically integrated firm would accommodate the new product. The results introduce welfare implications for the antitrust rules dealing with predatory pricing and exclusive dealing.

Appendix

Below are the proofs of lemmas 1-3 and propositions 1-2.

Proof of Lemma 1:

Let:

$$\underline{T}(w_1) - T_2(w_1, \delta) = \delta\big(\pi_{2H}^{VI} - \pi_2^{VI}\big) - \big(\pi_1^R(w_1) - \pi_{12}^R(w_1)\big).$$

The result that $\underline{T}(c_1) - T_2(c_1, \delta^{VI}) > 0$ is straightforward from comparing (6) with (7) and recalling that $\pi_1^R(c_1) = \pi_1^{VI}$, $\pi_{12}^R(c_1) = \pi_{12}^{VI}$ and $\pi_{12}^{VI} > \pi_2^{VI}$. The gap $\underline{T}(w_1) - T_2(w_1, \delta)$ is increasing with w_1 because from the envelope theorem:

$$\frac{\partial\big(\underline{T}(w_1) - T_2(w_1, \delta)\big)}{\partial w_1} = q_1(w_1) - \widehat{q}_1(w_1) > 0,$$

where the inequality follows because $q_1(w_1) > \widehat{q}_1(w_1)$. Moreover, $\underline{T}(w_1) - T_2(w_1, \delta)$ is increasing with δ because $\pi_{2H}^{VI} > \pi_2^{VI}$. ■

Proof of Lemma 2:

The features of $\widetilde{w}_1(\delta)$ are as follows. Starting with $\delta = 0$, we have $\underline{T}(c_2) - T_2(c_2, 0) = 0$ because at $w_1 = c_2$, $\pi_1^R(c_2) = \pi_{12}^R(c_2)$ and because $\delta = 0$. This implies that $\widetilde{w}_1(0) = c_2$. Because $\underline{T}(w_1) - T_2(w_1, \delta)$ is increasing with w_1 and δ, $\widetilde{w}_1(\delta)$ is decreasing with δ. Next we show that $\widetilde{w}_1(\widetilde{\delta}) = c_1$. To this end, we have that at $w_1 = c_1$ and $\delta = 0$, $\underline{T}(c_1) - T_2(c_1, 0) = -(\pi_1^R(c_1) - \pi_{12}^R(c_1)) = -(c_2 - c_1)\underline{q} < 0$. Because $\underline{T}(c_1) - T_2(c_1, \delta^{VI}) > 0$ and $\underline{T}(c_1) - T_2(c_1, \delta)$ is increasing in δ, there is a cutoff, $\widetilde{\delta}$, such that $\widetilde{w}_1(\widetilde{\delta}) = c_1$, where $\widetilde{\delta} < \delta^{VI}$. ■

Proof of Lemma 3:

We first establish the features of $w_{12}(\delta)$ (we established the features of $\widetilde{w}_1(\delta)$ in the proof to Lemma **2**). Differentiating the term in the squared brackets of the second line of (8) with respect to w_1, we have that $w_{12}(\delta)$ is the solution to:

$$\left.\frac{d\Pi(w_1)}{dw_1}\right|_{w_1 \geq \widetilde{w}_1(\delta)} = \frac{\partial \pi_1^{VI}(w_1)}{\partial w_1} - (1-\delta)\Big[q_1(w_1) - \widehat{q}_1(w_1) + (w_1 - c_1)q_1'(w_1)\Big] = 0. \quad (15)$$

The following lemma derives the features of $w_{12}(\delta)$:

Lemma. *At $\delta \to 0$, $w_{12}(\delta) \to -\infty$. Moreover, $w_{12}(\delta)$ is increasing with δ and $w_{12}(\delta) = c_1$ at $\delta = 1$.*

Proof: For $\delta = 1$, the second term in (15) vanishes, hence $w_{12}(\delta) = c_1$. For any $\delta < 1$, $w_{12}(\delta) < c_1$. To see why, evaluating (15) at $w_{12}(\delta) = c_1$, the first term vanishes, but the term in the squared brackets is positive because $q_1(w_1) > \widehat{q}_1(w_1)$ (note that $(w_1 - c_1)q_1'(w_1)$ becomes 0), hence (15) is negative. Moreover, $w_{12}(\delta)$ is increasing with δ because for $w_{12}(\delta) < c_1$, the term in the squared brackets is positive ($q_1(w_1) > \widehat{q}_1(w_1)$, and $(w_1 - c_1)q_1'(w_1) > 0$ when $w_{12}(\delta) < c_1$and $q_1'(w_1) < 0$). Finally, to show that when $\delta \to 0$, $w_{12}(\delta) \to -\infty$, we have that (15) evaluated at $\delta = 0$ is:

$$\begin{aligned}
\left.\frac{d\Pi(w_1)}{dw_1}\right|_{w_1 \geq \widetilde{w}_1(\delta),\, \delta=0} &= q_1'(w_1)\left(\frac{dp(q_1(w_1))}{dq_1}q_1(w_1) + (p(q_1(w_1)) - c_1)\right) \\
&\quad -q_1(w_1) + \widehat{q}(w_1) - (w_1 - c_1)q_1'(w_1) \\
&= q_1'(w_1)\left[\frac{dp(q_1(w_1))}{dq_1}q_1(w_1) + (p(q_1(w_1)) - w_1)\right] - q_1(w_1) + \widehat{q}(w_1) \\
&= -q_1(w_1) + \widehat{q}(w_1) < 0,
\end{aligned}$$

where the last inequality follows because the term in the squared brackets of the third line is the retailer's first order condition (and hence equals 0) and $q_1(w_1) > \widehat{q}_1(w_1)$. This completes the proof of the intermediate Lemma.

Going back to the proof of Lemma 3, we have from the features of $\widetilde{w}_1(\delta)$ and $w_{12}(\delta)$ that there are two cutoffs $\widetilde{\delta}$ (the solution to $\widetilde{w}_1(\delta) = c_1$) and $\widetilde{\widetilde{\delta}}$ (the solution to $w_{12}(\delta) = \widetilde{w}(\delta)$). For $\delta \in [0, \widetilde{\delta}]$, $\widetilde{w}_1(\delta) > c_1 > w_{12}(\delta)$. In this range, the manufacturer maximizes the first line of (8) and sets $w_1 = c_1$. For $\delta \in [\widetilde{\delta}, \widetilde{\widetilde{\delta}}]$, $c_1 > \widetilde{w}_1(\delta) > w_{12}(\delta)$. In this range, if the manufacturer sets $w_1 = c_1$, the retailer deviates from the exclusionary equilibrium and sells 1+2. Also, the manufacturer in this range can do better than setting $w_1 = w_{12}(\delta)$ by setting a higher wholesale price of $w_1 = \widetilde{w}_1(\delta)$ instead. In this corner solution, both the retailer's constraints bind, as $t_1 = T(w_1, \delta) = T_2(w_1, \delta) = \underline{T}(w_1)$. Finally, for $\delta \in [\widetilde{\widetilde{\delta}}, 1]$, $c_1 > w_{12}(\delta) > \widetilde{w}(\delta)$, so the manufacturer maximizes the second line of (8) by setting $w_1 = w_{12}(\delta)$. This establishes $w_1^E(\delta)$ as the w_1 that maximizes (8). Note that following the above definitions of $\widetilde{\delta}$ and $\widetilde{\widetilde{\delta}}$, $w_1^E(\delta)$ is continuous in δ. The manufacturer's exclusionary profit (8) too is continuous in δ. In particular, at $w_1 = \widetilde{w}_1(\delta)$, it is continuous as at this point $t_1 = T(w_1, \delta) = T_2(w_1, \delta) = \underline{T}(w_1)$. ■

Proof or Proposition 1:

Suppose first that $\delta \in [0, \widetilde{\delta}]$. The manufacturer sets $w_1 = c_1$ and hence the retailer does not offer product 2. This is optimal for the manufacturer because the gap in the manufacturer's profits from exclusion and accommodation is:

$$\frac{\Pi_1^M(c_1)}{1-\delta} - \Pi_{12}^M(c_1) = \frac{1}{1-\delta}\left[\pi_1^{VI} - \pi_{12}^{VI} - \delta(\pi_{2H}^{VI} - \pi_{12}^{VI})\right] > 0, \tag{16}$$

where $\Pi_1^M(c_1)$ is the first line of (8), $\Pi_{12}^M(c_1)$ is given by (11), and the inequality follows because the term in squared brackets is equivalent to (6), which is positive for $\delta < \delta^{VI}$. For $\delta \in [\widetilde{\delta}, \widetilde{\widetilde{\delta}}]$, the gap in the manufacturer's profits from exclusion and accommodation is:

$$\frac{\Pi_1^M(\widetilde{w}_1(\delta))}{1-\delta} - \Pi_{12}^M(c_1) = \frac{1}{1-\delta}\left[\pi_1^{VI}(\widetilde{w}_1(\delta)) - \pi_{12}^{VI} - \delta(\pi_{2H}^{VI} - \pi_{12}^{VI})\right], \tag{17}$$

where $\Pi_1^M(\widetilde{w}_1(\delta))$ is the second line in (8), evaluated at $w_1 = \widetilde{w}_1(\delta)$, and $\Pi_{12}^M(c_1)$ is given by (11). The term in the squared brackets is positive at $\delta = \widetilde{\delta}$ because (8) is continuous at w_1 and $\widetilde{w}_1(\widetilde{\delta}) = c_1$. Moreover, the term in the squared brackets is decreasing in δ because the derivative is:

$$\frac{\partial \pi_1^{VI}(\widetilde{w}_1(\delta))}{\partial w_1} \times \frac{\partial \widetilde{w}_1(\delta)}{\partial \delta} - \left(\pi_{2H}^{VI} - \pi_{12}^{VI}\right) < 0,$$

where the inequality follows because when $w_1 = \widetilde{w}_1(\delta) < c_1$, $\frac{\partial \pi_1^{VI}(\widetilde{w}_1(\delta))}{\partial w_1} > 0$, $\frac{\partial \widetilde{w}_1(\delta)}{\partial \delta} < 0$ and $\pi_{2H}^{VI} - \pi_{12}^{VI} > 0$.

For $\delta \in [\widetilde{\widetilde{\delta}}, 1]$, the gap in the manufacturer's profits from accommodation and exclusion is:

$$\begin{aligned}
\tfrac{\Pi_1^M(w_{12}(\delta))}{1-\delta} - \Pi_{12}^M(c_1) &= \tfrac{1}{1-\delta}\left[\pi_1^M(w_{12}(\delta)) + T_{12}(w_{12}(\delta),\delta)\right] - \Pi_{12}^M(c_1) \\
&< \tfrac{1}{1-\delta}\left[\pi_1^M(w_{12}(\delta)) + T_2(w_{12}(\delta),\delta)\right] - \Pi_{12}^M(c_1) \\
&= \tfrac{1}{1-\delta}\left[\pi_1^{VI}(w_{12}(\delta)) - \pi_2^{VI} - \delta(\pi_{2H}^{VI} - \pi_{12}^{VI})\right] - \Pi_{12}^M(c_1) \\
&< \tfrac{1}{1-\delta}\left[\pi_1^{VI}(c_1) - \pi_2^{VI} - \delta(\pi_{2H}^{VI} - \pi_{12}^{VI})\right] - \Pi_{12}^M(c_1) \\
&= \tfrac{1}{1-\delta}\left[\pi_1^{VI} - \pi_{12}^{VI} - \delta(\pi_{2H}^{VI} - \pi_{12}^{VI})\right],
\end{aligned} \tag{18}$$

where the first inequality follows because $T_{12}(w_{12}(\delta),\delta) < T_2(w_{12}(\delta),\delta)$ and the last inequality follows because $w_1 = c_1$ maximizes $\pi_1^{VI}(w)$. Recall from (6) that $\pi_1^{VI} - \pi_{12}^{VI} - \delta(\pi_{2H}^{VI} - \pi_{12}^{VI})$ is decreasing in δ and equals 0 at δ^{VI}.

The rest of the proof depends on the comparison between δ^{VI} and $\widetilde{\widetilde{\delta}}$. Suppose first

that $\delta^{VI} < \widetilde{\widetilde{\delta}}$. Then, evaluated at $\delta = \delta^{VI}$, the term in the squared brackets in (16) is negative, which follows from comparing it with the definition of δ^{VI} in (6) and because $\pi_1^{VI}(c_1) > \pi_1^{VI}(\widetilde{w}_1(\delta))$. As $\pi_1^{VI} - \pi_{12}^{VI} - \delta(\pi_{2H}^{VI} - \pi_{12}^{VI})$ is always decreasing in δ, it follows that if $\delta^{VI} < \widetilde{\widetilde{\delta}}$, there is a unique cutoff, δ^{VS}, ($\widetilde{\delta} < \delta^{VS} < \delta^{VI}$), where δ^{VS} solves $\pi_1^{VI}(\widetilde{w}_1(\delta)) - \pi_{12}^{VI} = \delta(\pi_{2H}^{VI} - \pi_{12}^{VI})$, such that the manufacturer excludes (accommodates) product 2 if $\delta < \delta^{VS}$ ($\delta > \delta^{VS}$). Suppose now that $\delta^{VI} > \widetilde{\widetilde{\delta}}$. We cannot determine that $\frac{\Pi_1^M(w_{12}(\delta))}{1-\delta} - \Pi_{12}^M(c_1)$ is always decreasing in δ. Yet, from (18) it follows that $\pi_1^{VI} - \pi_{12}^{VI} - \delta(\pi_{2H}^{VI} - \pi_{12}^{VI})$ places an upper bound on $\frac{\Pi_1^M(w_{12}(\delta))}{1-\delta} - \Pi_{12}^M(c_1)$, and because $\pi_1^{VI} - \pi_{12}^{VI} - \delta^{VI}(\pi_{2H}^{VI} - \pi_{12}^{VI}) = 0$, we have $\frac{\Pi_1^M(w_{12}(\delta))}{1-\delta} - \Pi_{12}^M(c_1) < 0$ for $\delta \to \delta^{VI}$. This implies that there is a cutoff $\delta^{VS} < \delta^{VI}$ such that the manufacturer accommodates product 2 for $\delta > \delta^{VS}$ (δ^{VS} can be higher or lower than $\widetilde{\widetilde{\delta}}$). If $\frac{\Pi_1^M(w_{12}(\delta))}{1-\delta} - \Pi_{12}^M(c_1)$ is decreasing with δ, there is a unique δ^{VS} which is the solution to $\frac{\Pi_1^M(w_{12}(\delta))}{1-\delta} - \Pi_{12}^M(c_1) = 0$. Otherwise, there can me multiple solutions, but there is always accommodation for δ slightly below δ^{VI}.
■

Proof or Proposition 2:

The result that for $\delta \in [0, \widetilde{\delta}]$, the manufacturer excludes product 2 without exclusive dealing follows directly from the proof to Proposition 1. Suppose now that $\delta > \widetilde{\delta}$. The gap in the manufacturer's profits when imposing exclusive dealing and accommodating product 2 is:

$$\begin{aligned} \frac{\Pi_1^{M,ED}(c_1)}{1-\delta} - \Pi_{12}^M(c_1) &= \frac{\pi_1^{VI} - \pi_2^{VI} - \delta\big(\pi_{2H}^{VI} - \pi_2^{VI}\big)}{1-\delta} - \big(\pi_{12}^{VI} - \pi_2^{VI}\big) \qquad (19) \\ &= \frac{\pi_1^{VI} - \pi_{12}^{VI} - \delta\big(\pi_{2H}^{VI} - \pi_{12}^{VI}\big)}{1-\delta}, \end{aligned}$$

where the first equality follows from substituting (13) into $\Pi_1^{M,ED}(c_1)$ and (11) into $\Pi_{12}^M(c_1)$. Comparing the second line of (19) with the definition of δ^{VI} in (6) yields that $\frac{\Pi_1^{M,ED}(c_1)}{1-\delta} - \Pi_{12}^M(c_1) > 0$ iff $\delta < \delta^{VI}$. ■

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