The value of information in dynamic vertical relations*

By Yaron Yehezkel†

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Abstract

This paper asks whether a privately informed retailer may have an incentive to share its marketing data with the manufacturer, in a way that would enable the manufacturer to gain ex-post, but non-contractible information. I consider an infinitely repeated dynamic vertical relations with adverse selection. In every period, a retailer (an agent) has private information concerning the demand while a manufacturer (the principal) can observe this information only at the end of the period. The paper finds that ex-post information motivates the manufacturer to leave the retailer with higher (lower) one-period profits when the retailer is short-sighted (forward looking).

Keywords: vertical relations, adverse selection, relational contract

JEL Classification Numbers: L22, L42, D82

1 Introduction

When a manufacturer signs a contract with a retailer, the retailer may have private information concerning the valuation from interacting with the manufacturer. For example, when a grocery supplier signs a vertical contract with a supermarket, the supermarket may have superior information than the grocery supplier concerning the demand for a food product, due to the supermarket's direct interaction with end consumers.

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†Coller School of Management, Tel Aviv University (email: yehezkel@post.tau.ac.il)

Yet, recent advancements in Big Data technologies provide manufacturers with ex-post information. A manufacturer can gain access to such information when the retailer agrees to share its IT with the manufacturer. For example, at the time when the grocery supplier negotiates the distribution contract with the supermarket, the grocery supplier may not know the expected demand and relay entirety on the supermarket's report. But, at the end of the period, the grocery supplier can collect data on the supermarket's actual retail prices for this period, as well as volumes of orders and marketing and economic data, and use this data for estimating the demand for this particular period. Indeed, in recent years, the supermarkets' willingness to share their Big Data analysis with their suppliers is gradually increasing.[1] Consulting firms emphasize the importance of applying Big Data analysis in the supply chain.[2]

In such cases, however, the manufacturer collects the data ex-post: after the retailer signed the contract and sold the products, and the data may not be contractible. Consequently, the manufacturer cannot sign a contract ex-ante, that depends on the manufacturer's ex-post information. Moreover, the manufacturer's ability to collect date may depend on the retailer's willingness to reveal its private information. This raises the questions of how the manufacturer can use such ex-post data? What is the economic value of ex-post data to the manufacturer and to the retailer? When does a retailer benefit from agreeing to reveal private information ex-post? Can ex-post data enhance efficiency?

In a one-period, unverifiable and i.i.d information is of course meaningless. Yet, manufacturers and retailers typically engage in long-term relationships. Thus, ex-post information may help them to facilitate relational contracts based on trust.

This paper considers an infinitely repeated principal - agent problem between a manufacturer (principal) and a retailer (agent). At the beginning of each period, the retailer has private information concerning the demand. The manufacturer offers menus of non-leaner tariffs that the retailer can choose from, based on its private information. The contracts are valid for one period. I consider two informational structures. First, the case where at the end of each period, the manufacturer can observe the demand of the current period. The manufacturer cannot contract on

[1] For reports on how supermarkets share Big Data with their suppliers, see for example:
https://www.v3.co.uk/v3-uk/feature/2475098/food-for-thought-supermarket-data-boom-ripe-for-providing-fresh-market-insights-to-suppliers
and:
https://www.manthan.com/cpg-solutions/insights/504-top-3-insights-that-suppliers-gain-from-downstream-data

[2] See for example:
https://assets.kpmg.com/content/dam/kpmg/au/pdf/2017/big-data-shaping-supply-chains-of-tomorrow.pdf

this ex-post information, and the state of demand is i.i.d. between periods. Second, the case where the manufacturer cannot observe the demand.

The comparison between the two information sets reveals that the manufacturer's ex-post information increases the retailer's expected one-period profits when the retailer is short-sighted (the retailer's discount factor is below some threshold) and decreases the retailer's expected one-period profits when the retailer is forward -looking (the retailer's discount factor is above some threshold). For intermediate discount factors, the retailer prefers that the manufacturer learns the state ex-post with some probability. Moreover, ex-post information always benefits the manufacturer and increases social welfare. The equilibrium quantity increases with the retailer's discount factor, and converges to the first-best quantity.

The intuition for these results is that ex-post information has two conflicting effects on the manufacturer's incentive to leave positive profit to the retailer. First, ex-post information enables the manufacturer to detect a retailer's deviation. When the retailer understate the demand, the manufacturer can detect it at the end of the period and then "punish" the retailer by offering the static contract in all future periods. This effect reduces the retailer's incentive to understate the demand, and enables the manufacturer to increase the quantity towards its first-best level without having to pay the retailer high information rents. The second, opposite effect is that offering the static contract instead of the dynamic mechanism inflicts a punishment on the retailer only if the dynamic contract offers the retailer a sufficiently higher expected profit than the static one. This effect increases the manufacturer's incentive to leave the retailer a high expected profit, because by so doing, the retailer has more to loose by understating the demand.

The size of the two effects depends on the level of quantity distortion, which is indirectly affected by the retailer's discount factor. When the retailer is sufficiently short-sighted, the quantity distortion is substantial. Then, if the manufacturer can learn the demand ex-post, the manufacturer prefers using this information for increasing the quantity and by so doing increasing the retailer's information rents. When the retailer is forward-looking, quantity distortion is minor, and the manufacturer prefers to take advantage of its ex-post information for collecting higher rents from the retailer.

These results have two implications. First, they can explain why, in some cases, supermarkets can benefit from sharing IT systems and Big Data technologies with their suppliers. Second, they can explain how a manufacturer and a retailer can engage in an implicit resale price maintenance (RPM). When all the relevant information concerning the demand is embedded in the retail price, then the dynamic mechanism with ex-post observable information is equivalent to an implicit RPM arrangement. The manufacturer expects to observe a certain retail price at the end of each period, and continues offering the dynamic contract as long as the retailer sets the "right" price. This

This paper relates to two fields of economic literature. First, it is related to Gal-Or (1991a) and (1991b), Martimort (1996), Yehezkel (2008) and Acconcia, Martina and Piccolo (2008) that consider vertical relations with asymmetric information in a static game. The paper contributes to this literature by showing how dynamic considerations can resolve problems of asymmetric information and replace vertical restraints. Second, this paper is related to Levin (2003), Halac (2012), Akifumi (2016) Calzolari and Spagnolo (2017) and Martimort, Semenov and Stole (2017) that consider a repeated principal - agent game when the agent has some private information and in addition, the agent can choose an uncontractible, though publicly observable action. This literature assume that the agent's private information remains private throughout the game. The current paper contributes to this literature by showing how the principal and potentially the agent can benefit from sharing ex-post information concerning the agent's private information.

The rest of the paper is organized as follows. Section 2 describes the model and the static benchmark. Section 3 solves for the dynamic mechanism with ex-post information. Section 4 solves for the dynamic mechanism without ex-post information. Section 5 compares between the two mechanisms. Section 6 extends the model to a partial degree of manufacturer's ex-post information. Section 7 offers several extensions.

2 The Model

Consider an upstream manufacturer and a downstream retailer. The manufacturer and the retailer interact for an infinite number of periods, and discount future profits by δ $(0 \leq \delta \leq 1)$.

In every period, there are two states of demand, high (H) and low (L), with probabilities p and $1-p$, respectively (the demand realization is i.i.d between periods). The retailer's profit from selling a quantity q of the manufacturer's product in the two states are $\pi_H(q)$ and $\pi_L(q)$, where $\pi_H(q) > \pi_L(q)$ and $\pi'_H(q) - \pi'_L(q) > 0$, $\forall q \geq 0$. Suppose that $\pi_H(q)$ and $\pi_L(q)$ are concave in q and have a unique maximization at the first-best quantities, q_H^{fb} and q_L^{fb} respectively, defined by $\pi'_H(q_H^{fb}) = 0$ and $\pi'_L(q_L^{fb}) = 0$. From the above assumptions, $q_H^{fb} > q_L^{fb}$. The first-best expected profit is $\Pi^{fb} \equiv p\pi_H(q_H^{fb}) + (1-p)\pi_L(q_L^{fb})$.

It is possible to think of several interpretations to $\pi_H(q)$ and $\pi_L(q)$. For example, suppose that given a quantity q, the retailer charges end consumers the price, P, according to the consumes' inverse demand function: $P(q;\theta)$, where $\theta \in \{H, L\}$ is the state of demand. The retailer's profit is therefore $\pi_\theta(q) = P(q;\theta)q$. Alternatively, the retailer may have retail costs c_θ, where $c_L > c_H$. Under this interpretation, $\pi_\theta(q) = (P(q) - c_\theta)q$, where $P(q)$ is the inverse demand.

The timing and information structure of each period is the following. At the beginning of the period, the retailer privately observes whether the demand is H or L in the current period. The manufacturer offers a take-it-or-leave-it menu $\{(q_H, T_H), (q_L, T_L)\}$ from which the retailer chooses a contract, where T is a fixed payment. If the retailer rejects both contracts, there is no trade in the current period. [3] Otherwise, the retailer chooses a contract from the menu. I compare between two information sets. First, ex-post information, where the manufacturer observes the state of demand at the end of the period. Yet, the manufacturer's ex-post information is non-verifiable and non-contractible. This informational structure can emerge because the retailer agreed to share its IT systems or marketing data with the manufacturer. Alternatively, this structure can represent an implicit RPM arrangement, where the manufacturer monitors the retail price at the end of each period, though this price is uncontractible. In the base model, I assume for simplicity that the manufacturer perfectly observes the state. In section 6, I extend the analysis to the case where the manufacturer observes the state with probability α which measures the degree to which the retailer shares information with the manufacture, or the degree to which the manufacturer can monitor the retail price for implementing an implicit RPM arrangement. The second informational structure is ex-post asymmetric information, where the manufacturer never observes the true realization of demand.

As a benchmark, consider the (already standard) static equilibrium benchmark, where firms are short-sighted, or when firms believe that the outcome of the current period has no effect of the future behavior. The model becomes a simple principal-agent problem under adverse selection (see, for example, Martimort (2006)). As is well-known, the manufacturer sets a menu $\{(q_H, T_H), (q_L, T_L)\}$ that satisfies:

$$IR_L^S: \qquad \pi_L(q_L) - T_L \geq 0, \tag{1}$$

$$IC_H^S: \quad \pi_H(q_H) - T_H \geq \pi_H(q_L) - T_L. \tag{2}$$

[3] The results qualitatively follow to the case where the manufacturer has several retailers to choose from, such that is the retailer rejects the menu, the manufacturer can turn in the current period to another retailer.

The constraints ensure that the retailer agrees to accept the contract (q_L, T_L) in state L (IR_L^S), and prefers (q_H, T_H) over (q_L, T_L) in state H (IC_H^S). Let T_H^S and T_L^S denote the solutions to IR_L^S and IC_H^S in equality. The manufacturer's expected profit, $pT_H^S + (1-p)T_L^S$, is:

$$\Pi_M^S(q_H, q_L) = p\pi_H(q_H) + (1-p)\pi_L(q_L) - p\big[\pi_H(q_L) - \pi_L(q_L)\big]. \quad (3)$$

The first two terms are the expected industry profits. The term in the squared brackets, $\pi_H(q_L) - \pi_L(q_L)$, is the "information rents" that the manufacturer needs to leave to the retailer in state H, for motivating the retailer to reveal the type by choosing (q_H, T_H). Let q_H^S and q_L^S denote the quantities in the static, asymmetric case, where q_H^S and q_L^S are the solutions to $p\pi'_H(q_H^S) = 0$ and:

$$\frac{\partial \Pi_M^S(q_H, q_L)}{\partial q_L} = \pi'_L(q_L) - p\pi'_H(q_L) = 0. \quad (4)$$

It is straightforward to show that $q_H^S = q_H^{fb}$ and $q_L^S < q_L^{fb}$.[4] This result is due to the manufacturer's incentive to decrease q_L below its first-best level in order to reduce the retailer's incentive to mimic L in state H, which enable the manufacturer to reduce the retailer's information rents. I conclude that in the static equilibrium, the manufacturer earns the expected profit $\Pi_M^S \equiv \Pi_M^S(q_H^S, q_L^S)$. The retailer earns in the two states, H and L, $\Pi_R^S(H) \equiv \pi_H(q_H^S) - T_H^S = \pi_H(q_L^S) - \pi_L(q_L^S)$ and $\Pi_R^S(L) \equiv \pi_L(q_L^S) - T_L^S = 0$, respectively. The retailer's expected profit is: $\Pi_R^S \equiv p[\pi_H(q_L^S) - \pi_L(q_L^S)]$.

3 Repeated game with ex-post information

This section studies the infinitely repeated game, where the manufacturer observes the realization of demand at the end of each period. The main conclusions of this section are that although the ex-post information is non-verifiable, both the manufacturer and the retailer benefits from the manufacturer's ex-post information, in comparison with the static contract. Yet, the manufacturer's benefit from ex-post information is increasing with δ while the retailer's benefit is non-monotonic in δ.

[4] I assume that $q_L^S > 0$, which occurs if p and $\pi'_H(q) - \pi'_L(q)$ are not too high.

3.1 The derivation of the dynamic, ex-post information mechanism

The static equilibrium in section 2 is an equilibrium in the dynamic case as well. It is supported by the firms' beliefs that the manufacturer offers in every period the static contract regardless of the manufacturer's ex-post information. Yet, the presence of dynamics and ex-post information support other equilibria. Consider the following mechanism.[5] In every period, the manufacturer offers a *dynamic, ex-post information* menu that does not necessarily satisfy the static constraints in section 2. Yet, the retailer accepts the menu and reveals the state truthfully, because the retailer expects that by deviating, the manufacturer, who can detect the deviation ex-post, will offer the static menu considered in section 2 in all future periods, while continue to offer the dynamic incentive menu otherwise. In equilibrium, the manufacturer believes that the menu motivates the retailer to reveal the truth, and these beliefs are reassured at the end of each period. Likewise, the retailer believes that the manufacturer will continue to offer the menu as long as the retailer does not deviate, and again these beliefs are reassured at the beginning of the next period.

The menu has to satisfy three constraints. The first constraint, IR_L^D, is the retailer's participation constraint in state L. It ensures that the retailer prefers accepting the contract (q_L, T_L) in state L given that doing so maintains the equilibrium, over rejecting the contract and receiving the static menu in all future periods:

$$IR_L^D\colon \qquad \pi_L(q_L) - T_L + \frac{\delta}{1-\delta}\big[p(\pi_H(q_H) - T_H) + (1-p)(\pi_L(q_L) - T_L)\big] \geq \tag{5}$$

$$0 + \frac{\delta}{1-\delta}\big[p((\pi_H(q_L^S) - \pi_L(q_L^S))\big].$$

The second constraint, IC_H^D, is the retailer's incentive compatibility constraint in state H. It ensures that the retailer prefers accepting the contract (q_H, T_H) in state H given that doing so maintains the equilibrium, over accepting the contract (q_L, T_L). In the latter case the manufacturer detects the deviation at the end of the period and then offers that static equilibrium in all future periods: [6]

$$IC_H^D\colon \; \pi_H(q_H) - T_H + \frac{\delta}{1-\delta}\big[p(\pi_H(q_H) - T_H) + (1-p)(\pi_L(q_L) - T_L)\big] \geq \tag{6}$$

$$\pi_H(q_L) - T_L + \frac{\delta}{1-\delta}\big[p((\pi_H(q_L^S) - \pi_L(q_L^S))\big].$$

[5] I focus on stationary mechanisms because it is a repeated game with i.i.d type.

[6] It is straightforward to show that the retailer's participation constraint in state H and incentive compatibility constraint in state L are not binding.

The third constraint, IR_M^D, is the manufacturer's participation constraint. Given that the manufacturer believes that the dynamic menu motivates the retailer to reveal the state of demand, the manufacturer prefers it over offering the static menu:

$$IR_M^D\colon\ \ pT_H + (1-p)T_L \geq p\pi_H(q_H^S) + (1-p)\pi_L(q_L^S) - p\big[\pi_H(q_L^S) - \pi_L(q_L^S)\big]. \quad (7)$$

Any menu satisfying conditions (5) - (7) can be a dynamic equilibrium. Notice that these menus differ from the "*self-enforcing*" contracts introduced by Levin (2003). Here, all the elements of the contract are verifiable and the manufacturer does not offer voluntary bonuses. It is self-enforcing because the retailer will agree to report the state even though the short-run IC_H^S is not satisfied, and at the same time the manufacturer agree to offer it because of condition (7).

There are multiple equilibria that can be supported by conditions (5) - (7). I focus on the menu that maximizes the manufacturer's expected profit. The logic for doing so is that the manufacturer has the market power to offer a take-it-or-leave menu to the retailer. Any menu that satisfies (5) - (7) makes it profitable for the retailer to accept, given the beliefs that rejecting it will motivate the manufacturer to offer the static menu in future periods. Thus the manufacturer can choose out of the set of self-enforcing menu the menu that maximizes the manufacturer's own payoff. Also, since I ask whether the retailer would like to provide the manufacturer with ex-post information, this question becomes relevant when the the manufacturer's ability to observe the state ex-post affects the manufacturer's ability to collect rents from the retailer.[7]

The menu that maximizes the manufacturer's profit requires that (5) and (6) hold in equality. Solving (5) and (6) for T_L and T_H, the manufacturer's expected one-period profit is:

$$\begin{aligned}\Pi_M^{D,e}(q_H, q_L) = {} & p\pi_H(q_H) + (1-p)\pi_L(q_L) - p\big[\pi_H(q_L) - \pi_L(q_L)\big] \\ & + \frac{\delta p(1+p)\big(\pi_H(q_L) - \pi_L(q_L) - (\pi_H(q_L^S) - \pi_L(q_L^S))\big)}{1+\delta p}.\end{aligned} \quad (8)$$

The term in the first line in (8) is identical to the static profit: the total expected profit minus the retailer's static information rents. The term in the second line is the dynamic information rents, due to the retailer's dynamic considerations. Notice that now the static information rents, $\pi_H(q_L) - \pi_L(q_L)$, also appears in the second line in (8), but with a positive sign. The intuition for this result is that the manufacturer can reduce the retailer's incentive to misrepresent the type by increasing the retailer's

[7]Levin (2003) focuses on the self-enforcing contracts that maximize the parties' joint profit.

expected future profits from the dynamic contract. As the retailer knows that misrepresenting the type in the current period will result in the static equilibrium in all future periods, the higher are the retailer's expected future profits, the lower is the retailer's incentive to misrepresent the type in the current period.

The first order conditions with respect to q_H and q_L are:

$$\frac{\partial \Pi_M^{D,e}(q_H, q_L)}{\partial q_L} = \pi'_L(q_L) - p\pi'_H(q_L) + \frac{\delta p(1+p)\big(\pi'_H(q_L) - \pi'_L(q_L)\big)}{1+\delta p} = 0, \tag{9}$$

$$\frac{\partial \Pi^{D,e}(q_H, q_L)}{\partial q_H} = p\pi'_H(q_H) = 0.$$

Let $q_L^{D,e}$ and $q_H^{D,e}$ denote the solutions to (9). The following lemma shows that such an equilibrium exists, i.e., satisfy condition IR_M^D:

Lemma 1 *The menu that maximizes the manufacturer's profit subject to conditions IR_L^D and IC_H^D in equality, provides the manufacturer with strictly higher profits than the in the static equilibrium.*

Proof. See the Appendix. ■

Intuitively, the manufacturer can implement that static contract in the dynamic setting, and therefore can only benefit from adjusting the menu to the presence of ex-post information. Notice however that condition IR_M^D may be binding in the equilibria that do not maximize the manufacturer's profit. [8]

To conclude, in the dynamic equilibrium, the manufacturer offers in every period the menu $\{(q_H^{D,e}, T_H^{D,e}), (q_L^{D,e}, T_L^{D,e})\}$, where $T_H^{D,e}$ and $T_L^{D,e}$ are the solutions to (5) and (6) in equality, evaluated at $q_H^{D,e}$ and $q_L^{D,e}$. The retailer accepts the menu and chooses the contract that corresponds to the true state. The manufacturer earns in every period $\Pi_M^{D,e} \equiv \Pi_M^{D,e}(q_H^{D,e}, q_L^{D,e})$ and the retailer earns $\Pi_R^{D,e} \equiv p(\pi_H(q_H^{D,e}) - T_H^{D,e}) + (1-p)(\pi_L(q_L^{D,e}) - T_L^{D,e})$, or:

$$\Pi_R^{D,e} = \frac{p\big((1+\delta)(\pi_H(q_L^{D,e}) - \pi_L(q_L^{D,e})) + \delta(1+p)(\pi_H(q_L^S) - \pi_L(q_L^S))\big)}{1+\delta p}. \tag{10}$$

[8] It is possible to show that there is a dynamic equilibrium in which the manufacturer offers the full-information, joint-profit maximizing quantities. Yet, this equilibrium does not maximize the manufacturer's profit and condition IR_M^D is binding and restrict the set of parameters that enable this equilibrium. In particular, due to the IR_M^D constraint, such equilibrium exists only if δ is above some threshold.

The retailer's one-period profits in states H and L are:

$$\Pi_R^{D,e}(H) \equiv \pi_H(q_H^{D,e}) - T_H^{D,e} \tag{11}$$
$$= \frac{(1-p\delta)\big((\pi_H(q_L^{D,e}) - \pi_L(q_L^{D,e})\big) + 2\delta\big((\pi_H(q_L^S) - \pi_L(q_L^S))\big)}{1+\delta p},$$

and:

$$\Pi_R^{D,e}(L) \equiv \pi_L(q_L^{D,e}) - T_L^{D,e} \tag{12}$$
$$= -\frac{p\delta\big((\pi_H(q_L^{D,e}) - \pi_L(q_L^{D,e})\big) - \big((\pi_H(q_L^S) - \pi_L(q_L^S))\big)}{1+\delta p}.$$

3.2 The features of the dynamic, ex-post mechanism

In this subsection I turn to study the features of the dynamic, ex-post mechanism. Consider first the effect of dynamics on the quantity distortion. As is usually the case in a two-state model, the quantity at the high state is identical to the first-best level: (9) implies that $q_H^{D,e} = q_H^S$ $(= q_H^{fb})$.[9] I therefore focus on the quantity distortion in state L:[10] Figure 1 illustrates the results of the following proposition:

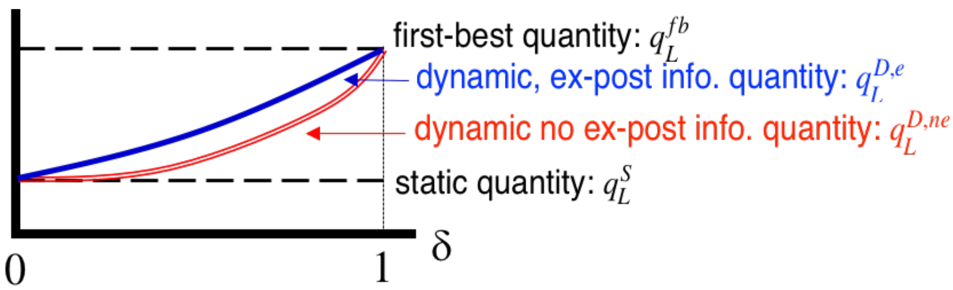


Figure 1: Effect of ex-post information on quantities

Proposition 1 ***(The effect of*** δ ***on the quantity distortion):*** *In the dynamic, ex-post information mechanism, the quantity distortion due to the retailer's private information decreases with* δ *and vanishes at* $\delta \to 1$*. That is,* $q_L^{D,e} = q_L^S$ *when* $\delta = 0$*,* $q_L^{D,e}$ *is increasing with* δ *and* $q_L^{D,e} \to q_L^{fb}$ *as* $\delta \to 1$*.*

Proof. See the Appendix. ■

[9]When the state is a continuous variable, the quantity at the highest state possible is identical to the first-best level, while all other quantities are distorted downwards.

[10] Proofs of all lemmas' and propositions will be available in the next version of this paper.

Proposition 1 shows that manufacturer takes advantage of the ex-post information for reducing the quantity distortion associated with the retailer's ex-ante information, even though the ex-post information is non-verifiable. The quantity distortion decreases as the retailer becomes more forward looking, and vanishes in the extreme case where the $\delta \to 1$.

Before describing the intuition for this result, it would be useful to compare between the firms' profits in the static and the dynamic mechanisms:

Proposition 2 ***(The effect of δ on the firms' profits):***

(i) The manufacturer's expected profit in the dynamic, ex-post information mechanism, $\Pi_M^{D,e}$, is increasing with δ and converges to $\Pi^{fb} - \Pi_R^S$, as $\delta \to 1$.

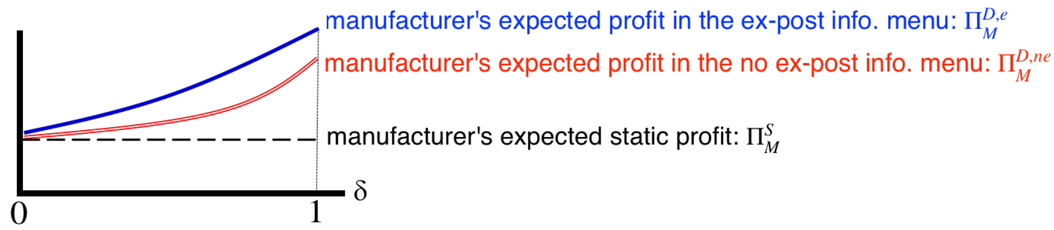
(ii) The retailer's expected profit in the dynamic, ex-post information mechanism, $\Pi_R^{D,e}$, is higher than in the static mechanism, Π_R^S, for $\forall \delta \in (0,1)$ and equals to the static profits at $\delta = 0$ and $\delta = 1$. Moreover, $\Pi_R^{D,e}$ is increasing (decreasing) with δ for δ close to 0 (1), and is concave with δ if $\pi_H''(q_L^{D,e}) - \pi_L''(q_L^{D,e}) - \left[(\pi_H(q_L^S) - \pi_L(q_L^S))\right]$ is not too high.

(iii) The retailer's profit in state H (state L) are higher (lower) in the dynamic, ex-post information mechanism than in the static mechanism, $\Pi_R^{D,e}(H) > \Pi_R^S(H)$ and $\Pi_R^{D,e}(L) < \Pi_R^S(L)$.

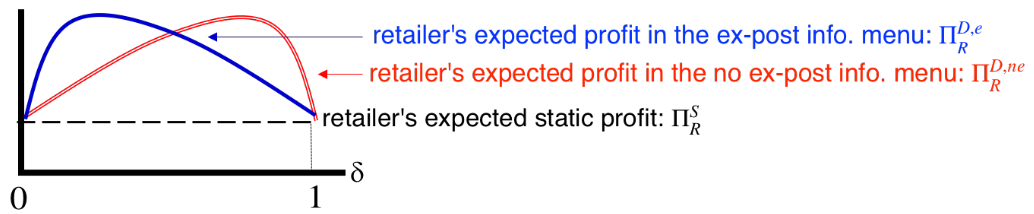
Proof. See the Appendix. ■

Figure 2 illustrates the results of proposition 2. Part (i) of proposition 2 shows that the manufacturer always benefits from ex-post information, and this benefit increases with δ. Part (ii) reveals that although ex-post information reduces the retailer's informational advantage over the manufacturer, the retailer's expected profits in the dynamic, ex-post information mechanism are higher than in the static case. This result holds despite of the manufacturer's superior bargaining power that should have enabled the manufacture to collect the retailer's rents. Yet, the retailer's benefit is small when the retailer is either very short-sighted or very forward- looking, and high for intermediate values of δ. Moreover, part (iii) shows that the retailer benefits from ex-post information in high demands, and is hurt in low demands (though again benefit on average).

The intuition for propositions 1 and 2 is that dynamic considerations enter through both the IR_H^D and IC_H^D constraints. I start by describing the effect of dynamics on the IR_H^D constraint. Suppose that δ only enters into the IR_H^D constraint.



Panel (a): the effect of ex-post information on the manufacturer's expected one-period profit



Panel (b): the effect of ex-post information on the retailer's expected one-period profit

Figure 2: Effect of ex-post information on one-period expected profits

The manufacturer can do the following. First, reduce the quantity distortion, in comparison with the static contract. Doing so increases the joint profits of the manufacturer and the retailer, increases the retailer's information rents in state H, while in state L the retailer continues to earn 0. Second, the manufacturer can charge a higher fixed fees, such that the retailer's profits in state H (L) are higher (lower) than in the static case. In total, the manufacturer needs to leave the retailer with a higher expected profit than in the static case, because otherwise the retailer does not agree to the contract when the state is L. Yet, the combination of the two changes enables the manufacturer to increase the joint profit, and collect from the retailer higher rents than in the static case. That is, the manufacturer gains a smaller fraction of a larger pie.

Dynamic considerations also affect the IC_H^D. For a given q_L, the manufacturer's ability to observe the state ex-post reduces the rents that the manufacturer needs to leave the retailer. This in turn reduces the manufacturer's marginal "informational costs" of offering q_L. Notice that this effect holds only if the manufacturer offers the retailer a higher expected rents in the dynamic contract than in the static one. Otherwise, the retailer does not care whether the manufacturer observes the state ex-post or not. Consequently, leaving the retailer higher information rents enables the manufacturer to reduce the informational costs of offering q_L. This in turn provide an incentive for the manufacturer to both raise the retailer's expected profit and reduce the quantity distortion. Again, doing so enables the manufacturer gain a smaller fraction of a larger pie.

As δ increases, the positive effect of dynamics on both the IR_H^D and IC_H^D constraints reduces the quantity distortion, and eliminates it as $\delta \to 1$. Moreover, it enables the manufacturer to increase its profits. Notice though that the manufacturer does not earn the full-information profits as $\delta \to 1$. While the quantity converges to the full-information level, the retailer's profit converges to the static, asymmetric information case, which is higher than the retailer's full information profits.

The retailer's expected profit is an inverse U-shape function of δ because of the manufacturer's tradeoff between increasing the joint profit and collecting a higher share of these profits. For small values of δ, the quantity distortion, $q_L^{fb} - q_L^{D,e}$, is high. Since $\pi_L(q)$ is concave in q, the manufacturer can substantially increase the joint profit, $\pi_L(q_L^{D,e})$, by increasing $q_L^{D,e}$. Therefore, as δ increases, the manufacturer substantially increases $q_L^{D,e}$ and prefers to collect a smaller share of higher joint profit. Doing so benefits the retailer who earns a higher expected profit. For high values δ, $q_L^{D,e}$ is very close to $q_L fb$, and a further increasing $q_L^{D,e}$ does not substantially increase $\pi_L(q_L^{D,e})$. Now, if δ increases, the manufacturer prefers to exploit this increase for collecting a higher share of the joint profit, rather than further increasing it by substantially increasing $q_L^{D,e}$. Consequently, the retailer's expected profit decreases with δ.

4 Dynamic contract without ex-post information

The results of section 3 show that both the manufacturer and the retailer prefer the dynamic contract with ex-post information over the static contract. Yet, recall that dynamic consideration affects the retailer's participation and incentive compatibility constraints. This raises the question of how ex-post information affects the comparison between the dynamic and static contract. That is, does the retailer benefit from sharing ex-post information with the manufacturer?

This section considers the case where the manufacturer and the retailer engage in a dynamic, infinitely repeated game, but do not share ex-post information. Such a scenario will take place if the retailer refuses to provide the manufacturer with sales and marketing information.

4.1 The derivation of the dynamic, no ex-post information mechanism

Consider the following *dynamic, no ex-post information mechanism.* In every period, the manufacturer offers a menu of contracts, $\{(q_H^{D,ne}, T_H^{D,ne}), (q_L^{D,ne}, T_L^{D,ne})\}$, and the retailer accepts and choose the contract that corresponds to the true state. As long as the retailer accepts the dynamic contract, the manufacturer continues to offer it in future periods. If the retailer rejects the contract, there is no trade in the current period and in all future periods, the manufacturer offers the static contract, $\{(q_H^S, T_H^S), (q_L^S, T_L^S)\}$. The manufacturer does not observe the state at the end of the period, and continue to offer the dynamic, no ex-post menu when the retailer deviates by choosing the "wrong" contract. As before, there can be multiple equilibria, and I focus on the menu that maximizes the manufacturer's profit.

The optimal menu for the manufacturer needs to satisfy the following constraints. First, since the retailer and the manufacturer engage in a dynamic game, the dynamic participation constraint, IR_L^D (as in equation (5)), continues to hold. Yet, the manufacturer does not observe the realization of the state at the end of the period, so the static incentive compatibility constraint, IC_H^S (as given in equation (6)) also holds. The third condition is the manufacturer's participation constraint, IR_M^D, as in (7).[11]

Let $T_H^{D,ne}(q_H, q_L)$ and $T_L^{D,ne}(q_H, q_L)$ denote the solution to IR_L^D and IC_H^S for T_H and T_L. The manufacturer's profit as a function of the quantities is $pT_H^{D,ne}(q_H, q_L) + (1-p)T_L^{D,ne}(q_H, q_L)$, or:

$$\Pi_M^{D,ne}(q_H, q_L) = p\pi_H(q_H) + (1-p)\pi_L(q_L) - p\big[\pi_H(q_L) - \pi_L(q_L)\big] \qquad (13)$$

$$+\delta p\big(\pi_H(q_L) - \pi_L(q_L) - (\pi_H(q_L^S) - \pi_L(q_L^S))\big).$$

As before, the first line in (13) is the static profit and the second line is the additional profit due to the dynamic interaction between the two firms.

As in the dynamic game with ex-post information, it is possible to show that the IR_M^D constraint does not bind. The manufacturer can always implement the static contract. Consequently, the manufacturer's optimal dynamic contract can only improves the manufacturer's profit.

The firs-order conditions are:

[11]It is straightforward to show that the retailer's participation constraint in state H and the the retailer's incentive compatibility constraint in state L do not bind.

$$\frac{\partial \Pi_M^{D,ne}(q_H, q_L)}{\partial q_L} = -p\pi'_H(q_L) + \pi'_L(q_L) + \delta p\big(\pi'_H(q_L) - \pi'_L(q_L)\big) = 0, \qquad (14)$$

$$\frac{\partial \Pi^{D,e}(q_H, q_L)}{\partial q_H} = p\pi'_H(q_H) = 0.$$

Let $q_H^{D,ne}$ and $q_L^{D,ne}$ denote the solution to (14) and $T_L^{D,ne} \equiv T_L^{D,ne}(q_H^{D,ne}, q_L^{D,ne})$, $T_H^{D,ne} \equiv T_H^{D,ne}(q_H^{D,ne}, q_L^{D,ne})$. The manufacturer's profit is: $\Pi_M^{D,ne} \equiv \Pi_M^{D,ne}(q_H^{D,ne}, q_L^{D,ne})$, and the retailer earns $\Pi_R^{D,ne} \equiv p(\pi_H(q_H^{D,ne}) - T_H^{D,ne}) + (1-p)(\pi_L(q_L^{D,ne}) - T_L^{D,ne})$, or:

$$\Pi_R^{D,ne} = (1-\delta)\big(\pi_H(q_L^{D,ne}) - \pi_L(q_L^{D,ne})\big) + \delta p(\pi_H(q_L^S) - \pi_L(q_L^S)). \qquad (15)$$

Notice that even though the manufacturer does not gain ex-post information, the quantities are different than in the static, asymmetric information case. As explained in section 3, the manufacturer can take advantage of the retailer's attitude towards the future, as reflected by the IR_R^D constraint, for increasing the quantity in state H, which increases the retailer's information rents in comparison with the static case. At the same time, the manufacturer increases the tariffs in both states, hence decreases the retailer's rents in state L, in comparison with the static case. The retailer agrees to pay the high tariff in state L because of the expectation to gain a high rents in future periods when the state is H.

The comparison between the features of the dynamic, no ex-post mechanism and the static, asymmetric case is therefore qualitatively similar to the the comparison in section 3. In particular, both mechanisms yield higher quantity, manufacturer's expected profits and retailer's expected profits in comparison with the static mechanism (as shown in the next section). I therefore move in the next section to compare between the dynamic mechanisms with and without ex-post information.

5 Comparison between the dynamic ex-post and no ex-post information mechanisms

The previous sections raise the question of who benefits from ex-post information? Naturally, the manufacturer cannot be worst off by ex-post information because I focus on the mechanisms that maximize the manufacturer's profits. The manufacturer can always ignore ex-post information. Yet, exchange of information should involve both the manufacturer and the retailer. If the retailer is worst off by manufacturer's ex-post information, then the retailer will commit to collaborate with the manufacturer on sharing marketing and sales data. The following proposition compares between the profits of the manufacturer and the retailer in the dynamic ex-post and no ex-post mechanisms:

Proposition 3 ***(comparison between the ex-post and no ex-post information mechanisms):***

(ii) *For* $\delta = 0$, $q_L^{D,e} = q_L^{D,ne} = q_L^S$. *Moreover,* $q_L^{fb} > q_L^{D,e} > q_L^{D,ne} > q_L^S$ *for* $\forall\delta \in (0,1)$, *and for* $\delta = 1$, $q_L^{D,e} = q_L^{D,ne} = q_L^{fb} > q_L^S$.

(ii) $\Pi_M^{D,e} > \Pi_M^{D,ne}$ *for* $\forall\delta > 0$.

(iii) *For* $\delta = \{0,1\}$, $\Pi_R^{D,e} = \Pi_R^{D,ne} = \Pi_R^S$. *Moreover,* $\Pi_R^{D,e} > \Pi_R^{D,ne}$ *when* δ *is sufficiently close to* 0 *and* $\Pi_R^{D,ne} > \Pi_R^{D,e}$ *when* δ *is sufficiently close to* 1.

Proof. For a proof of part (*iii*) see the Appendix. ■

The first part of proposition 3 shows that ex-post information increases output, in comparison with the no ex-post information case. The second part shows that the manufacturer always benefit from ex-post information. Yet, the third part shows that the effect of ex-post information on the retailer's expected profit depends on δ. In particular, the manufacturer chooses to leave a short- sighted retailer (i.e., a retailer with a small δ) a higher expected profit in the ex-post information case than in the no ex-post information case. In contrast, a forward - looking retailer gains a higher expected profits when the manufacturer does not gain ex-post information.

The intuition for this result is the following. When the retailer enables ex-post information, the retailer's incentive compatibility constraint shifts from IC_H^S to IC_H^D. Doing so has two conflicting effects on the manufacturer's incentive to collect rents from the retailer. The first effect is that the manufacturer can detect at the end of the period a retailer's deviation in state H and then "punish" the retailer in all future periods. This effect reduces the information rents that a the manufacturer needs to leave the retailer in state H. An increase in δ makes the IC_H^D constraint less binding (given the quantities), and hence allows the manufacturer to reduce the retailer's information rents. The second effect is that the manufacturer benefits from observing the state ex-post only if the retailer gains a higher expected rents in the dynamic mechanism than in the static one. The higher is the gap between the retailer's profit in the dynamic and the static mechanism, the stronger is the retailer's incentive to reveal the true state, because the retailer has more to lose by understating the state. As δ increases, the gap between the dynamic and the static profit provides the retailer with a stronger incentive to reveal the true type.

Consequently, as δ increases, this second effect increases the manufacturer's incentive to leave the retailer with a higher expected profit.

When δ is low, the quantity distortion is significant (i.e., $q_L^{fb} - q_L^D$ is high), and the manufacturer would like to take advantage of an increase in δ for increasing q_L^D. Consequently, under both the ex-post and no ex-post mechanisms the manufacturer would like to raise the retailer's expected profit. Yet, in the ex-post information mechanism, the IC_H^D constraint provide the manufacturer with an additional incentive to do so, because an increase in the retailer's expected profit makes the IC_H^D less binding. In this case, the ex-post information mechanism provides the retailer a higher expected profit than the no ex-post information case.

The opposite case occurs when δ is high. Now, the quantity distortion is insignificant, and the manufacturer would like to take advantage of an increase in δ for collecting a higher rents from the retailer. The IC_H^D constraint provides the manufacturer with a stronger incentive to do so, than in the no ex-post information case. Consequently, the ex-post information case provides the retailer with a lower expected profit than the no ex-post information case.

These results indicate that a short-sighted retailer has more of an incentive to enable the manufacturer access to its marketing data. Doing so incentivize the manufacturer to leave the retailer with higher expected information rents. Yet, a forward-looking retailer does not benefit from a manufacturer's ex-post information, and may choose not to share its IT systems and collaborate on sharing data with the manufacturer.

6 Ex-post asymmetric information with probability α

Since the retailer strictly benefit (hurt) by the manufacturer's ex-post private information when δ is low (high), the question is whether, for intermediate values of δ, the retailer would rather allow the manufacturer to learn the state ex-post with some probability.

Suppose now that in each period, the manufacturer observes a perfect signal to the true state in this particular period with probability α ($0 \geq \alpha \geq 1$). The parameter α measures the degree to which the retailer shares information with the supplier. Suppose also that the demand functions by end consumers in states H and L are $P_H(q) = \theta_H - q$ and $P_L(q) = \theta_L - q$, respectively, where $\theta_H > \theta_L > p\theta_H$. The last inequality ensures that there is an interior solution to the static quantities.

Therefore, $\pi_H(q) = q(\theta_H - q)$ and $\pi_L(q) = q(\theta_L - q)$.

When the retailer accepts the "wrong" contract (q_L, T_L) in state H, the manufacturer detects it with probability α and then offers the static contract in all periods. With probability $1-\alpha$, the manufacturer does not detect the deviation and the manufacturer continues to offer the dynamic mechanism. Hence, the IC_H^D becomes:

$$IC_H^{D,\alpha}: \quad \pi_H(q_H) - T_H + \frac{\delta}{1-\delta}\big[p(\pi_H(q_H) - T_H) + (1-p)(\pi_L(q_L) - T_L)\big] \geq \tag{16}$$

$$\pi_H(q_L) - T_L + \frac{\delta}{1-\delta}\alpha\big[p(\pi_H(q_L^S) - \pi_L(q_L^S)\big]$$

$$+\frac{\delta}{1-\delta}(1-\alpha)\big[p(\pi_H(q_H) - T_H) + (1-p)(\pi_L(q_L) - T_L)\big].$$

The manufacturer's binding constraints are $IC_H^{D,\alpha}$ and IR_L^D. Solving (16) and (5) for T_H and T_L defines $T_L^D(q_H, q_L; \alpha)$ and $T_H^D(q_H, q_L; \alpha)$. As before, the manufacturer's profit is $\Pi_M^D(q_H, q_L; \alpha) = pT_H^D(q_H, q_L; \alpha) + (1-p)T_L^D(q_H, q_L; \alpha)$. Substituting into the manufacturer's profit, yields that the optimal quantities for the manufacturer are $q_H^{D,\alpha} = \frac{\theta_H}{2}$ and:

$$q_L^{D,\alpha} = \frac{\theta_L - p\theta_H}{2(1-p)} + \frac{\delta p(1+\alpha p)(\theta_H - \theta_L)}{2(1-p)(1+\alpha\delta p)}. \tag{17}$$

The first term is the static quantity, q_L^S, and the second term is due to the dynamic feature of this model. I can now turn to evaluating how α affects the firm's profits:

Proposition 4 ***(The effect of*** α ***on the firms' profits:)*** *The manufacturer's profits are increasing with* α. *Yet, the retailer's optimal level of* α *is* $\hat{\alpha}(\delta)$*:*

$$\hat{\alpha}(\delta) = \begin{cases} 1; & \text{if } \delta \in (0, \frac{1}{2+p}]; \\ \frac{1-2\delta}{\delta p}; & \text{if } \delta \in (\frac{1}{2+p}, \frac{1}{2}); \\ 0; & \text{if } \delta \in \left[\frac{1}{2}, 1\right], \end{cases}$$

where $\hat{\alpha}(\delta)$ *is decreasing with* δ *when* $\delta \in (\frac{1}{2+p}, \frac{1}{2})$.

Intuitively, proposition 4 shows that if the retailer can choose how much information to reveal to the manufacturer ex-post, the retailer will choose an intermediate level of α when δ is intermediate. On one hand, an increase in α increases the manufacturer's ability to detect a deviation by the retailer (and "punish" the retailer by going back to the static contract), which reduces that retailer's profit. At the same time, an increase in α provides an incentive for the manufacturer to offer the retailer a higher expected profit, because otherwise the "punishment" of going back to the static contract is meaningless.

Following the observations in the introduction that supermarkets are sometimes reluctant to share their data with their manufacturers, this result can explain why a supermarket may choose to share information to only a certain degree. Doing so can balance between the positive and negative effects of information sharing.

7 Extensions

7.1 Limited liability

[To be Continue...]

7.2 Voluntary bonuses

[To be Continue...]

8 Conclusion

This paper considers long-term relationship between a manufacturer and a retailer, when in every period, the retailer has ex-ante private information concerning the demand that the manufacturer can only learn ex-post. The paper identifies the features of the optimal dynamic mechanism that takes into account ex-post information and finds that ex-post information mitigates the asymmetric information problem and increases the quantity and manufacturer's profit. Moreover, ex-post information has two effects on the retailer's profits. First, it reduces the retailer's incentive to understate the demand because then the manufacturer will be able to detect it at the end of the period, and terminate the dynamic mechanism. Second, it increases the manufacturer's incentive to leave the retailer with higher expected profit, because a retailer's expectation to gain high expected profits in future periods reduces the retailer's incentive to understate the demand. When the retailer is forward-looking (short-sighted), the first (second) effect is stronger and the retailer benefits (hurts) by ex-post information. For intermediate discount factors, the retailer prefers that the manufacturer would be able to detect a misrepresentation of the demand with some probability. These results can explain why retailers sometimes agree to collaborate with their suppliers in revealing ex-post information.

Appendix

Below are the proofs of Lemma 1 and propositions 1, 2, and 3.

Proof of Lemma 1:

In the dynamic mechanism with ex-post information, the manufacturer earns in every period:

$$\Pi_M^{D,e}(q_H^{D,e}, q_L^{D,e}) \geq \Pi_M^{D,e}(q_H^S, q_L^S) = p\pi_H(q_H^S) + (1-p)\pi_L(q_L^S) - p\big[\pi_H(q_L^S) - \pi_L(q_L^S)\big]$$
$$+ \frac{\delta p(1+p)\big(\pi_H(q_L^S) - \pi_L(q_L^S) - (\pi_H(q_L^S) - \pi_L(q_L^S))\big)}{1+\delta p}$$
$$= p\pi_H(q_H^S) + (1-p)\pi_L(q_L^S) - p\big[\pi_H(q_L^S) - \pi_L(q_L^S)\big] = \Pi_M^{D,e}(q_H^S, q_L^S),$$

where the first inequality follows from revealed preferences and the last equality follows from the definition of $\Pi_M^{D,e}(q_H^S, q_L^S)$. ■

Proof of Proposition1:

At $\delta = 0$, the first order condition (9) is identical to (4), hence $q_L^{D,e} = q_L^S$. Also:

$$\frac{\partial^2 \Pi_M^{D,e}(q_H, q_L)}{\partial\delta\partial q_L} = \frac{p(1+p)\big(\pi'_H(q_L) - \pi'_L(q_L)\big)}{(1+\delta p)^2} > 0,$$

hence, $q_L^{D,e}$ is increasing with δ. Finally, at $\delta = 1$, the first order condition (9) becomes:

$$\left.\frac{\partial \Pi_M^{D,e}(q_H, q_L)}{\partial q_L}\right|_{\delta=1} = (1-p)\pi'_L(q_L),$$

which is identical to the first order condition under full information, hence $q_L^{D,e} \to q_L^{fb}$ as $\delta \to 1$. ■

Proof of Proposition2:

Proof of part (i): By the envelope theorem,

$$\frac{\partial \Pi_M^{D,e}(q_H^{D,e}, q_L^{D,e})}{\partial \delta} = \frac{p(1+p)\big(\pi_H(q_L^{D,e}) - \pi_L(q_L^{D,e}) - (\pi_H(q_L^S) - \pi_L(q_L^S))\big)}{(1+\delta p)^2} > 0,$$

where the inequality follows because $q_L^{D,e} > q_L^S$ and because $\pi_H(q) - \pi_L(q)$ is increasing in q. Evaluated at $\delta = 1$:

$$\begin{aligned}\Pi_M^{D,e}(q_H^{D,e}, q_L^{D,e})\big|_{\delta=1} &= p\pi_H(q_H^{D,e}) + (1-p)\pi_L(q_L^{D,e}) - p\big[\pi_H(q_L^S) - \pi_L(q_L^S)\big] \\ &= p\pi_H(q_H^{fb}) + (1-p)\pi_L(q_L^{fb}) - p\big[\pi_H(q_L^S) - \pi_L(q_L^S)\big] \\ &= \Pi^{fb} - \Pi_R^S,\end{aligned}$$

where the second equality follows because $q_H^{D,e} = q_H^{fb}$ and at $\delta = 1$, $q_L^{D,e} = q_H^{fb}$.

Proof of part (ii): The gap between the retailer's expected profit in the dynamic, ex-post mechanisms and the static contract is:

$$\Pi_R^{D,e} - \Pi_R^S = \frac{(1-\delta)pX(\delta)}{1+\delta p}, \tag{18}$$

where

$$X(\delta) \equiv \pi_H(q_L^{D,e}) - \pi_L(q_L^{D,e}) - (\pi_H(q_L^S) - \pi_L(q_L^S)).$$

Notice that $X(0) = 0$, because $q_L^{D,e} = q_L^S$ at $\delta = 0$. Moreover, $X'(\delta) > 0$ because $q_L^{D,e}$ is increasing with δ and $\pi_H(q) - \pi_L(q)$ is increasing with q. Finally, $X(\delta) > 0$ for all positive values of δ (including $\delta = 1$).

At $\delta = 0$, (18) equals 0 because $X(0) = 0$, and at $\delta = 1$, (18) equals 0 because $1 - \delta = 0$. Moreover,

$$\frac{d\big(\Pi_R^{D,e} - \Pi_R^S\big)}{d\delta} = \frac{p\Big((1-\delta)(1+\delta p)X'(\delta) - (1+p)X(\delta)\Big)}{(1+\delta p)^2}.$$

Therefore,

$$\frac{d\left(\Pi_R^{D,e} - \Pi_R^S\right)}{d\delta}\Big|_{\delta=0} = pX'(0) > 0,$$

and

$$\frac{d\left(\Pi_R^{D,e} - \Pi_R^S\right)}{d\delta}\Big|_{\delta=1} = -\frac{pX(1)}{1+p} < 0.$$

Finally, the gap $\Pi_R^{D,e} - \Pi_R^S$ is strictly concave in δ iff:

$$\frac{d^2\left(\Pi_R^{D,e} - \Pi_R^S\right)}{d^2\delta} < 0 \iff X''(\delta) < \frac{2(1+p)\left(X'(\delta) + X'(\delta)\delta p - pX(\delta)\right)}{(1+\delta)(1+\delta p)^2}.$$

Proof of part (iii): The proof that $\Pi_R^{D,e}(L) < \Pi_R^S(L)$ follows directly from evaluating (12) at $q_L^{D,e} > q_L^S$. The proof that $\Pi_R^{D,e}(H) > \Pi_R^S(H)$ follows because part (*ii*) shows that $\Pi_R^{D,e} > \Pi_R^S$ while $\Pi_R^{D,e}(L) < \Pi_R^S(L)$. ■

Proof of Proposition 3: The proof of part (*i*) follows from the comparison between (9) and (14). The proof of part (*ii*) follows directly from the comparison between (13) and (8) and from revealed preferences. Turning to the proof of part (*iii*), recall that proposition 2 shows that at $\delta = 0$, $\Pi_R^{D,e} = \Pi_R^S$ and $\Pi_R^{D,e}$ is increasing with δ. It is possible to show that the same feature follows to $\Pi_R^{D,e}$, i.e., at $\delta = 0$, $\Pi_R^{D,ne} = \Pi_R^S$ and $\Pi_R^{D,ne}$ is increasing with δ (a formal proof can be provided upon request). Therefore, I turn to show that $\frac{d\Pi_R^{D,e}}{d\delta}\Big|_{\delta=0} > \frac{d\Pi_R^{D,ne}}{d\delta}\Big|_{\delta=0}$ which implies that for δ close to 0, $\Pi_R^{D,e} > \Pi_R^{D,ne}$.

The gap between the retailer's expected profit in the ex-post and no ex-post mechanisms is:

$$\Pi_R^{D,e} - \Pi_R^{D,ne} = (1-\delta)p\left(\frac{Y(q_L^{D,e}) + \delta p Y(q_L^S)}{1+\delta p} - Y(q_L^{D,ne})\right), \quad (19)$$

where $Y(q) \equiv \pi_H(q) - \pi_L(q)$. Consider first the case where $\delta = 0$:

$$\Pi_R^{D,e} - \Pi_R^{D,ne}\big|_{\delta=0} = p\left(Y(q_L^{D,e}) - Y(q_L^{D,ne})\right) = 0,$$

where the last equality follows because at $\delta = 0$, $q_L^{D,e} = q_L^{D,ne} = q_L^S$. Moreover:

$$\left.\frac{d\left(\Pi_R^{D,e} - \Pi_R^{D,ne}\right)}{d\delta}\right|_{\delta=0} = p\left(\frac{\partial Y(q_L^{D,e})}{\partial q}\frac{\partial q_L^{D,e}}{\partial \delta} - \frac{\partial Y(q_L^{D,ne})}{\partial q}\frac{\partial q_L^{D,ne}}{\partial \delta}\right)\Bigg|_{\delta=0}$$

$$= p\frac{\partial Y(q_L^S)}{\partial q}\left[\frac{\partial q_L^{D,e}}{\partial \delta} - \frac{\partial q_L^{D,ne}}{\partial \delta}\right]\Bigg|_{\delta=0},$$

where the first equality follows from differentiating $\Pi_R^{D,e} - \Pi_R^{D,ne}$ with respect to δ and then substituting $\delta = 0$, $q_L^{D,e} = q_L^S$ and $q_L^{D,ne} = q_L^S$. Recall that $Y'(q) > 0$. To evaluate the sign of the squared brackets, define $\varphi^{D,e}(q_L)$ as the FOC defined by (9) and $\varphi^{D,ne}(q_L)$ as the FOC defined by (14). Then,

$$\left[\frac{\partial q_L^{D,e}}{\partial \delta} - \frac{\partial q_L^{D,ne}}{\partial \delta}\right]\Bigg|_{\delta=0} = \left[-\frac{\frac{\partial \varphi^{D,e}(q_L)}{\partial \delta}}{\frac{\partial \varphi^{D,e}(q_L)}{\partial q_L}} - \left(-\frac{\frac{\partial \varphi^{D,ne}(q_L)}{\partial \delta}}{\frac{\partial \varphi^{D,ne}(q_L)}{\partial q_L}}\right)\right]\Bigg|_{\delta=0} = -\frac{p^2 Y'(q_L^S)}{\frac{\partial \varphi^{D,ne}(q_L^S)}{\partial q_L^S}} > 0.$$

The last inequality follows because at $\delta = 0$, $\varphi^{D,e}(q_L) = \varphi^{D,ne}(q_L)$, hence, $\frac{\partial \varphi^{D,e}(q_L)}{\partial q_L}\big|_{\delta=0} = \frac{\partial \varphi^{D,ne}(q_L)}{\partial q_L}\big|_{\delta=0}$ are the SOC of (9) and (14) and are negative. Hence, $\frac{d(\Pi_R^{D,e} - \Pi_R^{D,ne})}{d\delta}\big|_{\delta=0} > 0$. Since for small values of δ both $\Pi_R^{D,e}$ and $\Pi_R^{D,ne}$ are increasing with δ, this implies that $\Pi_R^{D,e} > \Pi_R^{D,ne}$ for a δ close enough to 0.

Consider now the case where $\delta = 1$. Recall that proposition 2 shows that as $\delta \to 1$, $\Pi_R^{D,e}$ is decreasing with δ and $\Pi_R^{D,e} \to \Pi_R^S$. It is possible to show that the same feature follows to $\Pi_R^{D,ne}$, i.e., at $\delta = 1$, $\Pi_R^{D,ne} = \Pi_R^S$ and $\Pi_R^{D,ne}$ is decreasing with δ (a formal proof can be provided upon request). I therefore show that $\frac{d\Pi_R^{D,e}}{d\delta}\big|_{\delta=1} < \frac{d\Pi_R^{D,ne}}{d\delta}\big|_{\delta=1}$, implying that when δ is close to 1, $\Pi_R^{D,e} < \Pi_R^{D,ne}$.

At $\delta = 1$, $\left(\Pi_R^{D,e} - \Pi_R^{D,ne}\right)\big|_{\delta=1} = 0$. Moreover,

$$\left.\frac{d\left(\Pi_R^{D,e} - \Pi_R^{D,ne}\right)}{d\delta}\right|_{\delta=1} = \frac{p^2\left(Y(q_L^{fb}) - Y(q_L^S)\right)}{1+p} > 0,$$

where the first equality follows differentiating $\Pi_R^{D,e} - \Pi_R^{D,ne}$ with respect to δ and then substituting $\delta = 1$, $q_L^{D,e} = q_L^{fb}$ and $q_L^{D,ne} = q_L^{fb}$. The last inequality follows because $q_L^{fb} > q_L^S$ and $Y'(q) > 0$. Therefore, when δ decreases slightly below 1, $\frac{d\Pi_R^{D,e}}{d\delta}\big|_{\delta=1} > \frac{d\Pi_R^{D,ne}}{d\delta}\big|_{\delta=1}$ implies that $\Pi_R^{D,e} < \Pi_R^{D,ne}$. ■

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