Collusion between supply chains under asymmetric information*

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Abstract

This paper considers an infinitely repeated competition between manufacturer-retailer supply chains. In every period, retailers privately observe the demand and manufacturers pay retailers "information rents". I study collusive equilibria between the supply chains that may or may not involve the retailers. I find that including forward-looking retailers in the collusive scheme may facilitate or hinder collusion, depending on the likelihood of a high demand and the gap between a high and a low demand. Moreover, collusion on monopoly profits can be easier or more difficult to implement than collusion on upstream profits.

Keywords: vertical relations, collusion, asymmetric information

JEL Classification Numbers: L22, L42, D82

1 Introduction

Competition between vertical manufacturer-retailer supply chains may involve both repeated interaction between the supply chains and long-term relationship within each supply chain. Manufacturers typically engage in an on-going competition with other manufacturers. Such repeated interaction enables manufacturers to horizontally collude on restricting competition.

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At the same time, manufactures may also engage in long-term relationships with their retailers and may use the retailers' help to facilitate collusion.

The possibility of collusion in a market with repeat interaction between supply chains and within each supply chain raises two main questions. First, are retailers helpful or harmful to such collusion? Manufacturers may exclude retailers from the collusive scheme by dealing with myopic retailers, switching between retailers in each period, or by ignoring retailers' deviations from collusion. Alternatively, manufacturers may include forward-looking retailers in the collusive scheme, such that collusion breaks following a retailer's deviation. In such a case, collusion is also "vertical" because it includes the retailers. This raises the question of whether retailers facilitate or hinder collusion. The second question is whether collusion on the monopoly outcome is easier to maintain than collusion on maximizing upstream profits. Because manufacturers share some of the collusive profits with the retailers, manufacturers prefer to collude on maximizing upstream profits rather than the total monopoly profits. This raises the question of what are the features of the collusive outcomes on upstream profits.

The answers to these questions can explain why in recent years, some collusion cases involved the active participation of the retailers, such that retailers were part of the collusive scheme and were able to break collusion. Yet, other collusion cases involved retailers' passive adherence to the manufacturers' collusion, or even attempts to break the collusive scheme. For example, in 2021, the Germany's Federal Cartel Office (FCO) fined leading music instruments manufacturers and their retailers for limiting price competition. According to the FCO: "For years, manufacturers and retailers of musical instruments have systematically endeavoured to restrict price competition for the end consumer,.." Accordingly, manufacturers asked retailers "...not to undercut fixed minimum sales prices, which they did in many cases."[1] The collusive scheme involved the active collaboration of retailers, who where closely monitored by their suppliers. This implies that a retailer's deviation from collusion could break the collusive scheme, though retailers chose to support and facilitate collusion. As an opposite example, a federal appeals court in San Francisco ruled in 2022 against two leading canned tuna manufacturer for alleged collusive scheme to inflate prices to restaurants and retailers. In this case, retailers were not part of the collusive scheme and in fact attempted to stop it by suing their suppliers.[2]

[1] See *Bundeskartellamt*, 2021.
[2] See *Competition Policy International*, 2022.

This paper considers an infinitely repeated competition between two manufacturer-retailer supply chains. At the beginning of each period, retailers privately observe the demand, which is i.i.d between periods. Each manufacturer offers its retailer a menu of contracts valid for this particular period and each retailer chooses a contract from the menu. Manufacturers design a menu that solicits retailers to reveal their private information by their contract selections. To so so, manufactures need to share some of their collusive profits with their retailers, in the form of "information rents". At the end of the period, all information becomes common knowledge. In particular, each manufacturer observes the demand realization as well as the menu offer of the competing supply chain and quantities.

I study collusive equilibria in which the two manufacturers offer the same collusive menu in all periods and retailers choose the contract from the menu that corresponds to the true state of demand. An observable deviation from the collusive equilibrium triggers the competitive, static equilibrium in all future periods. In the context of this model, collusive equilibria can vary in two dimensions. The first dimension is whether retailers are included or excluded from the collusive scheme. In the former case, the collusive equilibrium breaks once a retailer deviates by choosing a contract that corresponds to the wrong state of demand or by rejecting the menu all together. In the latter case, manufacturers ignore a retailer's deviation from the collusive path and continue to collude. The second dimension concerns the profit that firms collude on. Firms may collude on maximizing monopoly profits: the joint profit of the four firms. Alternatively, firms may collude on maximizing the joint upstream profits only.

The paper establishes the following results. First, retailers hinder collusion when the probability of a high demand is low, and may facilitate collusion otherwise. In the latter case, the lowest discount factor that enables firms to collude when retailers take part of the collusive scheme is lower than the equivalent discount factor when retailers are myopic or when manufacturers ignore retailers' deviations. The intuition for this result is that when the retailers' expected information rents given the collusive quantities are higher than given the static outcome, including retailers in the collusive scheme motivates them to truthfully reveal their private information. This is because retailers expect that should they misrepresent the state of demand in a certain period, collusion stops in all future periods as retailers take part in the collusive scheme. This enables manufacturers to reduce the information rents, in comparison with the case in which a retailer's deviation does not stop collusion, and to facilitate collusion. In contrast, when retailers' expected information rents in the static case are higher than under collusion, retailers have an incentive to stop collusion, in which case collusion is easier to maintain when retailers are excluded. For antitrust policy, this result identifies how forward-looking retailers can facilitate or hinder horizontal collusion. In the former case, manufacturers prefer to deal with the same retailers with whom they can establish long-term relationship and trust. Yet, counterintuitively, in the latter case it becomes easier to maintain collusion by dealing with myopic retailers, by rapidly replacing retailers in each period, or by ignoring a retailer's deviation from collusion. Hence, short-term relationships between manufacturers and retailers may actually be indicative of the manufacturers' incentive to collude without the interference of their retailers. In both cases, when evaluating the possibility of collusion, antitrust authorities should take into account the retailers' potential role in facilitating collusion as well as the manufacturers' choice of retailers to interact with.

The second main result is that when manufacturers collude on the outcome that maximizes their upstream profits, collusion involves a quantity above (below) the monopoly quantity in periods of high (low) demand. The intuition for this result is that increasing the gap between the quantities in high and low demand reduces the retailers' incentive to miss-represent a high demand as low, which in turn reduces their information rents. Hence, manufacturers collude on reducing their retailers' information rents. For antitrust policy, it is typically expected that low quantities may raise the concern of collusion. Yet, this result indicates that under asymmetric information, when manufacturers offer menus of contracts, collusion on upstream profits take a more complex form, with a high quantity in periods of high demand and a low quantity otherwise. As a result, antitrust authorities should not look at the short-term quantity when investigating the possibility of collusion, and instead look at the long term quantities and how they vary along time. Wide variations in quantities between periods can be indicative of collusion on the quantities that maximize upstream profits.

The third main result is that collusion on the monopoly quantities is easier to maintain than collusion on upstream profits when the probability of high demand and the gap between the demand in the two states is sufficiently small. Intuitively, manufacturers gain higher profits when colluding on upstream profits than when colluding on monopoly profits. Yet, manufacturers' short-run benefit from defecting from collusion may also be higher when colluding on upstream profits. This is because upstream collusion involves a lower quantity than the monopoly quantity when demand is low and a higher quantity otherwise. Hence, when low demand is more likely, each manufacturer has a stronger incentive to defect from the low quantity by rising its own quantity, although doing so breaks collusion. For antitrust policy, this result identifies when antitrust authorities should anticipate collusion on upstream profits and when they should anticipate collusion on the monopoly outcome.

To the best of my knowledge, this is the first paper that shows how asymmetric information between a manufacturer and its retailer affects collusion, in a market when retailers can be included or excluded from the collusive scheme. The paper relates to three fields of economic literature. First, it is related to Gal-Or (1991a) and (1991b), Caillaud, Jullien and Picard (1995), Martimort (1996), Yehezkel (2008), Acconcia, Martina and Piccolo (2008) and Yehezkel (2014) that consider static vertical relations with asymmetric information. My paper contributes to this literature by showing how dynamic considerations can solve problems of asymmetric information between manufacturers and retailers, when retailers take part in the collusive scheme.[3]

Second, this paper is related to the literature on repeated vertical relations and anti-competitive behavior. In the context of tacit collusion, Jullien and Rey (2007) consider a closely related paper, with two competing vertical supply chains facing demand uncertainty. This paper focuses instead on asymmetric information concerning the demand. Piccolo and Reisinger (2011) show how exclusive territories agreements can facilitate upstream collusion. Piccolo and Miklós-Thal (2012) consider collusion between competing retailers that offer myopic suppliers a high wholesale price and negative fixed fees. Reisinger and Thomes (2017) compares upstream collusion with a joint and a separate retailer. Gilo and Yehezkel (2020) show how retailers can use a forward-looking supplier for maintaining collusion, when retailers are too shortsighted to collude by themselves. Gieselmann et al (2023) consider a repeated game between two vertical chains, when retailers are myopic and when vertical contracts are secret. They show how different retailers' beliefs concerning the contract of the competing retailer affect the manufacturers' ability to collude.

This line of literature also looked at how vertical integration affects the firms' ability to collude. Nocke and White (2007), Normann (2009) and Nocke and White (2010) show how vertical integration affects upstream collusion. Mendi (2009) considers the effect of vertical integration on downstream collusion when downstream firms have asymmetric costs.

[3]Athey and Bagwell ((2001) and (2008)) consider asymmetric information between horizontal competitors, in the context of horizontal collusion in an infinitely repeated game. I contribute to these papers by considering the role of privately informed retailers in facilitating collusion.

The paper shows how input payments can serve as a side payment among colluding firms. Biancini and Ettinger (2017) analyze downstream collusion and vertical integration under both upstream and downstream oligopolies. They show that vertical integration on one hand increases the integrated firm's collusive profit by eliminating double marginalization, but for the same reason increases the integrated firm's profit from deviating as well as the profit in the punishment stage. Yet, the collusive profit sharing scheme and the optimal punishment can resolve this second effect, such that vertical integration ultimately facilitates collusion.

Repeated vertical relations can also be used to exclude a new competitor. Asker and Bar-Isaac (2014) consider an incumbent supplier that can exclude the entry of a forward-looking entrant by offering forward-looking retailers a share of the incumbent's monopoly profits. Gilo and Yehezkel (2023) study collusion between a manufacturer and a retailer on excluding an entrant selling a new product that is initially inferior but can improve if sold over time.

All the above papers on repeated vertical relations and anti-competitive behavior assume that there is symmetric information concerning the demand. The main contribution of this paper is by introducing asymmetric information to dynamic vertical relations and by showing how retailers' information rents affect the retailers' incentive to facilitate collusion.

The third strand of related literature concerns with relational-contracts. This literature considers a repeated game between a principal and an agent when the agent has some private information and in addition, the agent can choose an uncontractible, though publicly observable action. In the context of this paper, the relationship between each manufacturer and its retailer can be interpreted as such relational contract. Notable contributions are Levin (2003), Halac (2012), Akifumi (2016) Calzolari and Spagnolo (2017) and Martimort, Semenov and Stole (2017). This paper extends this literature to the case where the agents' private information becomes public at the end of each period, and when there are two competing principal-agent supply chains. In a closely related paper, Shamir and Yehezkel (forthcoming) consider a dynamic relational contract between a monopolistic manufacturer and a monopolistic retailer when the retailer has private information concerning the demand. The focus of this paper is different. Shamir and Yehezkel (forthcoming) focus on the question of whether the retailer would like to share information concerning the demand with the manufacturer ex-post. This paper assumes that all information becomes public at the end of each period, and focuses on competition and collusion.

2 The model and the competitive, static benchmark

Consider two supply chains, $M_1 - R_1$ and $M_2 - R_2$, where M_i is an upstream manufacturer that serves the downstream retailer R_i. The four firms interact for an infinite number of periods and discount future profits by δ ($0 \leq \delta \leq 1$).

In every period, there are two states of demand, high (H) and low (L), with probabilities p and $1-p$, respectively. The demand realization is i.i.d between periods. Joint profit of $M_i - R_i$ ($i = 1, 2$) in state $\theta = \{H, L\}$ is $\pi_{\theta i}(q_i, q_j)$, where q_i is the quantity sold by R_i and $\pi_{\theta i}(q_i, q_j)$ is an inverse U-shape function of q_i and decreasing with q_j. Suppose that given the quantity of the competing supply chain, q_j, the joint profit of $M_i - R_i$ is higher in periods of high demand: $\pi_{Hi}(q_i, q_j) > \pi_{Li}(q_i, q_j)$. Furthermore, suppose that given any q_j and q'_j that satisfy $\pi_{Hi}(q_i, q_j) \geq \pi_{Li}(q_i, q'_j)$, $\pi_{Hi}(q_i, q_j) - \pi_{Li}(q_i, q'_j)$ is increasing in q_i. This last assumption, which holds when the gap between q_j and q'_j is not too high, is sufficient for the results and is satisfied under linear demand. Let $\partial \pi_{\theta i}(q_i, q_j)/\partial q_i$ ($\partial \pi_{\theta i}(q_i, q_j)/\partial q_j$) denote the partial derivative of $\pi_{\theta i}(q_i, q_j)$ with respect to the first (second) argument of $\pi_{\theta i}(q_i, q_j)$, respectively, where $\partial \pi_{\theta i}(q_i, q_j)/\partial q_j < 0$. The two supply chains may sell horizontally differentiated products, or homogeneous products. The main results are derived given any profit functions satisfying the above assumptions. For results that depend on the model's parameters (in particular, on p and on the gap between $\pi_{Hi}(q_i, q_j)$ and $\pi_{Li}(q_i, q_j)$), I adopt for simplicity a linear demand example with homogeneous products.

The timing and information structure of each period is the following. At the beginning of the period, the two retailers privately observe whether the demand is H or L in the current period (recall that states are i.i.d.). Each M_i offers R_i a take-it-or-leave-it menu $\{(q_{Hi}, T_{Hi}), (q_{Li}, T_{Li})\}$ from which R_i chooses a contract, where $T_{\theta i}$ is a fixed payment for the quantity $q_{\theta i}$. If R_i rejects the menu, there is no trade between M_i and R_i in the current period and R_j is a monopolist. Otherwise, R_i chooses a contract from the menu. The bilateral contracting stage between M_i and R_i is secret: R_j cannot observe the menu that M_i offers R_i and which contract R_i chooses from the menu, if any. Then, the two retailers sell their receptive quantities.[4] Each R_i earns $\pi_{\theta i}(q_{\theta i}, q_{\theta j}) - T_{\theta i}$ and M_i earns $T_{\theta i}$. At the end of the period, all information becomes public, including the contract offers, which contract

[4]As long as retailers have to sell their entire quantities, the results do not depend on whether retailers compete in prices or quantities.

was chosen and the demand realization.[5]

Let q_θ^M denote the monopoly quantities that maximize total profits of the two supply chains defined by: $\partial\pi_{\theta i}(q_\theta^M, q_\theta^M)/\partial q_{\theta i} + \partial\pi_{\theta j}(q_\theta^M, q_\theta^M)/\partial q_{\theta i} = 0$ (notice that q_θ^M is the quantity that *each* of the two supply chains should set in order to maximize total industry profits).

As a benchmark, consider asymmetric information in a competitive, static game: firms interact for one period. This benchmark case is also an equilibrium in the dynamic game when firms do not believe that their strategies in the current period affect the equilibrium in future periods. This scenario serves as a useful benchmark because I will assume that a deviation from collusion result in playing the competitive static game indefinitely. Under asymmetric information, each M_i offers a menu as to maximize the expected profit, $pT_{Hi} + (1-p)T_{Li}$, subject to:

$$IR_L^S: \qquad \pi_{Li}(q_{Li}, q_{Lj}) - T_{Li} \geq 0, \tag{1}$$

$$IC_H^S: \qquad \pi_{Hi}(q_{Hi}, q_{Hj}) - T_{Hi} \geq \pi_{Hi}(q_{Li}, q_{Hj}) - T_{Li}. \tag{2}$$

Given that M_j offered R_j a menu $\{(q_{Hj}, T_{Hj}), (q_{Lj}, T_{Lj})\}$ and that R_j accepted a contract that corresponds to the true state, M_i offers a menu $\{(q_{Hi}, T_{Hi}), (q_{Li}, T_{Li})\}$ such that R_i agrees to accept the contract (q_{Li}, T_{Li}) in state L (the static Individual Rationality constraint in state L: IR_L^S), and prefers (q_{Hi}, T_{Hi}) over (q_{Li}, T_{Li}) in state H (the static Incentive Compatibility constraint in state H: IC_H^S).[6] Solving IR_L^S and IC_H^S in equality for T_{Hi} and T_{Li} and substituting into $pT_{Hi} + (1-p)T_{Li}$, each M_i earns an expected profit:

$$\begin{aligned}\Pi_{Mi}^S(q_{Hi}, q_{Li}, q_{Hj}, q_{Lj}) &= p\pi_{Hi}(q_{Hi}, q_{Hj}) + (1-p)\pi_{Li}(q_{Li}, q_{Lj}) \\ &\quad - p\big[\pi_{Hi}(q_{Li}, q_{Hj}) - \pi_{Li}(q_{Li}, q_{Lj})\big].\end{aligned} \tag{3}$$

The first two terms are the expected $M_i - R_i$ joint profit. The term in the squared brackets is the "information rents" that each M_i needs to leave R_i in state H, for motivating R_i to reveal the type by choosing (q_{Hi}, T_{Hi}) instead of (q_{Li}, T_{Li}). The information rents are increasing with q_{Li} as long as the gap between q_{Hj} and q_{Lj} is not too high such that $\pi_{Hi}(q_{Li}, q_{Hj}) -$

[5] In Shamir and Yehezkel (forthcoming), we study why a retailer may profit from sharing ex-post information with the manufacturer by providing past-sales information. In this paper, the main focus is on competition or collusion between vertical supply chains and hence I make the simplifying assumption that valuable information becomes observable at the end of the period.

[6] It is possible to show that R_i's participation constraint in state H and incentive compatibility constraint in state L are not binding. See Appendix C.

$\pi_{Li}(q_{Li}, q_{Lj}) > 0$, which holds because the IR_L^S and IC_H^S require that the information rents are positive. Maximizing (3) with respect to q_{Hi} and q_{Li} yields the first-order conditions (respectively):

$$\frac{\partial \pi_{Hi}(q_{Hi}, q_{Hj})}{\partial q_i} = 0, \qquad \frac{\partial \pi_{Li}(q_{Li}, q_{Lj})}{\partial q_i} - p\frac{\partial \pi_{Hi}(q_{Li}, q_{Hj})}{\partial q_i} = 0. \tag{4}$$

Let $q_{Hi}^S(q_{Hj})$ and $q_{Li}^S(q_{Lj}, q_{Hj})$ denote M_i's best response functions in the static, asymmetric information case (the solution to (4)). Notice that $q_{Li}^S(q_{Lj}, q_{Hj})$ is a function of both q_{Lj} and q_{Hj}. The symmetric equilibrium quantities in the static game with asymmetric information, q_L^S and q_H^S, are the solutions to: $q_H^S = q_H^S(q_H^S)$ and $q_L^S = q_L^S(q_L^S, q_H^S)$. I assume that p is not too high to induce $q_L^S = 0$.[7]

As is standard in problems of asymmetric information, M_i sets in state H the full-information best response to q_{Hj}, a feature known as "no distortion at the top". In state L, M_i sets the full-information best response only if $p = 0$. Otherwise, M_i offers a quantity below the full-information best response, in order to reduce the retailer's incentive to mimic L in state H, which enables the manufacturer to reduce the retailer's information rents.

To summarize the competitive, static benchmark, the equilibrium quantities are q_H^S and q_L^S and each manufacturer earns the expected profit $\Pi_M^S \equiv \Pi_M^S(q_H^S, q_L^S, q_H^S, q_L^S)$ while each retailer earns:

$$\Pi_R^S \equiv p\left(\pi_{Li}(q_L^S, q_L^S) - T_L^S\right) + (1-p)\left(\pi_{Hi}(q_H^S, q_H^S) - T_H^S\right) = p[\pi_{Hi}(q_L^S, q_H^S) - \pi_{Li}(q_L^S, q_L^S)].$$

3 Repeated game and collusion

Suppose that the four firms play an infinitely repeated game. The competitive, static equilibrium in the previous section is an equilibrium in the dynamic game as well. It is supported by the firms' beliefs that in every period manufacturers offer the static contract regardless of the past behavior. Yet, a repeated game also supports collusive equilibria that are based on informal understandings between firms. Because collusion in this paper requires the collaboration of both the upstream manufacturers and the downstream retailers, it is possible to distinguish between two special cases of collusive outcomes:

[7] The proof of Proposition 2 below shows that q_L^S is decreasing with p. Hence, I assume that p is not too high as to induce $q_{Li}^S(q_{Lj}, q_{Hj}) = 0$.

1. *Collusion on the monopoly profits* - In every period, each manufacturer offers its retailer a menu with the monopoly quantities, i.e., the quantities that maximize joint industry profits of all four firms. Note that the monopoly quantities are not necessarily the optimal quantities for the two manufacturers. This is because under asymmetric information, each manufacturer has to leave some of the total profit to its retailer as "information rents".

2. *Collusion on maximizing upstream profits* - Manufacturers coordinate on the quantities that maximize their own joint profits. These quantities take into account the manufacturers' incentive to coordinate on reducing the retailers' information rents.

The collusive mechanism is identical in the two collusive possibilities, as they only differ in the collusive quantities. Therefore, in this section I define the general collusive mechanism, and then the next sections study how this mechanism can support each of the collusive cases. Moreover, for each of the collusive possibilities, I compare the case where retailers take part of the collusive scheme and the case where retailers are excluded (on which I explain below).

Consider the following mechanism. In every period, the two manufacturers offer an identical *dynamic* menu, $\{(q_H^D, T_H^D), (q_L^D, T_L^D)\}$ that does not necessarily satisfy the static individual rationality and incentive compatibility constraints from the previous section.[8] Then, retailers accept the menu and reveal the state by choosing a contract that corresponds to the "right" state of demand. All firms expect that as long as they play this dynamic equilibrium, firms will continue playing it in all future periods. Any observable deviation triggers the static equilibrium in all future periods.

Any dynamic menu, $\{(q_H^D, T_H^D), (q_L^D, T_L^D)\}$, has to satisfy the following three constraints. The first constraint, IC_M^D, is the manufacturers' incentive compatibility constraint. As is standard in the literature on tacit collusion, each M_i can offer the dynamic menu at the beginning of each period or deviate to another incentive compatible menu. Since any deviation is observable by $M_j - R_j$ at the end of the period and triggers the competitive, static menu in all future periods, M_i's optimal deviation is to its static, asymmetric-information best-responses given that R_j sells in the current period q_L^D (q_H^D) in state L (H): $q_{Li}^S(q_H^D, q_L^D)$ and $q_{Hi}^S(q_H^D)$ as given by equation (4). In the current period, M_i earns from this deviation the expected profit given by (3), after plugging the best responses. Then, in all future periods,

[8] I focus on stationary mechanisms because it is a repeated game with i.i.d states.

M_i earns the static asymmetric information profit, Π_M^S. Hence:

$$IC_M^D\colon \quad \frac{pT_H^D + (1-p)T_L^D}{1-\delta} \geq \Pi_{Mi}^S(q_{Hi}^S(q_H^D), q_{Li}^S(q_H^D, q_L^D), q_H^D, q_L^D) + \frac{\delta}{1-\delta}\Pi_M^S. \tag{5}$$

The left-hand side (5) is M_i's sum of discounted profits from maintaining collusion and the right-hand side is M_i's one-period profit from deviating, followed by the sum of discounted profits from the competitive equilibrium.

Next, consider the constraints on the retailers. In the context of this model, when the competing manufacturer sell through privately informed and potentially forward-looking retailers, manufacturers can include the retailers in the collusive scheme in which case collusion is also "vertical". To see how, suppose that the retailers, too, can support or stop the collusive scheme if they deviate from the equilibrium in a certain period. In this case, the retailers' individual rationality and incentive compatibility constraints become dynamic. The first constraint, IR_L^D, is R_i's dynamic individual rationality constraint in state L. It ensures that R_i prefers accepting the contract (q_L^D, T_L^D) in state L given that doing so maintains the equilibrium, over rejecting the contract and receiving the static menu (and the static expected information rents) in all future periods:

$$IR_L^D\colon \quad \pi_{Li}(q_L^D, q_L^D) - T_L^D + \frac{\delta}{1-\delta}\big[p(\pi_{Hi}(q_H^D, q_H^D) - T_H^D) + (1-p)(\pi_{Li}(q_L^D, q_L^D) - T_L^D)\big] \geq \tag{6}$$

$$0 + \frac{\delta}{1-\delta}\big[p(\pi_{Hi}(q_L^S, q_H^S) - \pi_{Li}(q_L^S, q_L^S))\big].$$

Notice that this condition is derived for R_i given that M_j offers to R_j the dynamic menu and R_j accepts the contract that corresponds to state L, (q_L^D, T_L^D). Moreover, if R_i deviates by not accepting the contract in state L, $M_j - R_j$ observe it at the end of the period and in all future periods all firms play the competitive, static equilibrium.

The second constraint, IC_H^D, is the retailer's dynamic incentive compatibility constraint in state H. It ensures that R_i prefers accepting the contract (q_H^D, T_H^D) in state H given that doing so maintains the dynamic equilibrium, over accepting the contract (q_L^D, T_L^D). In the latter case both manufacturers and R_j detect the deviation at the end of the period and then play the competitive, static equilibrium in all future periods:

$$IC_H^D\colon \quad \pi_{Hi}(q_H^D, q_H^D) - T_H^D + \frac{\delta}{1-\delta}\big[p(\pi_{Hi}(q_H^D, q_H^D) - T_H^D) + (1-p)(\pi_{Li}(q_L^D, q_L^D) - T_L^D)\big] \geq \tag{7}$$

$$\pi_{Hi}(q_L^D, q_H^D) - T_L + \frac{\delta}{1-\delta}\left[p((\pi_{Hi}(q_L^S, q_H^S) - \pi_{Li}(q_L^S, q_L^S))\right].$$

To summarize, any menu satisfying conditions (5), (6) and (7) can be a dynamic collusive equilibrium that involves all four firms. At the first stage of every period, each M_i prefers offering $\{(q_H^D, T_H^D), (q_L^D, T_L^D)\}$ over deviating to its best responses. At the second stage of every period, each R_i accepts the contract that corresponds to the true state.

Notice that the IR_L^D and IC_H^D constraints are relevant only when retailers are forward-looking and when manufacturers stop colluding following any observable deviation by one of the retailers. In this case, retailers take an active role in maintaining or breaking the collusive scheme. When the two retailers are myopic, or when the two manufacturers ignore any retailer's deviation and continue to offer the dynamic menu following a retailer's deviation, the collusive menu has to satisfy only the manufacturers' dynamic constraint, IC_M^D and the retailer's two static constraints, IR_L^S and IC_H^S, as given by (1) and (2), respectively. An equivalent scenario is when M_i deals with a different retailer in every period. In such cases, retailers do not take an active role in the collusive scheme and behave as if the game is static. This raises the question of what are the market conditions under which including retailers in the collusive scheme (i.e., breaking collusion following a retailer's deviation) facilitates or hinders collusion.

4 Collusion on the monopoly profits

The main question of this section is whether including the two retailers in the collusive scheme (through their dynamic constraints) facilitates or hinders collusion on the monopoly quantities. To this end, I compare the critical value of δ that enables collusion given IR_L^D and IC_H^D with the critical value of δ that enables collusion given IR_L^S and IC_H^S. The main conclusion of this section is that retailers have a positive (negative) contribution to the stability of collusion when the probability of a high demand is above (below) some threshold.

Suppose first that collusion is also "vertical", i.e., involves the two retailers. In order to solve for the highest possible value of δ that supports the collusive equilibrium, IR_L^D, IC_H^D and IC_M^D must hold in equality. I start by deriving M_i's expected one-period profit given arbitrary q_{Li}, q_{Hi}, q_{Lj} and q_{Hj}, when R_i takes part in the collusive scheme. Solving IR_L^D and IC_H^D from (6) and (7) in equality for T_H^D and T_L^D, substituting into M_i's expected one-period profits, $pT_H^D + (1-p)T_L^D$, yields:

$$\Pi_{Mi}^D(q_{Li}, q_{Hi}, q_{Lj}, q_{Hj}) = p\pi_{Hi}(q_{Hi}, q_{Hj}) + (1-p)\pi_{Li}(q_{Li}, q_{Lj}) - p\big[\pi_{Hi}(q_{Li}, q_{Hj}) - \pi_{Li}(q_{Li}, q_{Lj})\big]$$

$$+ \left[\frac{\delta p(1+p)}{1+\delta p}\right]\big(\pi_{Hi}(q_{Li}, q_{Hj}) - \pi_{Li}(q_{Li}, q_{Lj}) - (\pi_{Hi}(q_L^S, q_H^S) - \pi_{Li}(q_L^S, q_L^S))\big)\,. \quad (8)$$

When firms collude on the monopoly quantities, $q_{Li} = q_{Lj} = q_L^M$, $q_{Hi} = q_{Hj} = q_H^M$ and M_i and R_i earn in each period the expected profits:

$$\Pi_{Mi}^{D,M} \equiv \Pi_{Mi}^D(q_L^M, q_H^M, q_L^M, q_H^M),$$

$$\Pi_{Ri}^{D,M} = p\pi_{Hi}(q_H^M, q_H^M) + (1-p)\pi_{Li}(q_L^M, q_L^M) - \Pi_{Mi}^{D,M}.$$

As in the competitive, static, case, M_i's profit as defined by (8) is the expected joint profits of M_i and R_i minus R_i's information rents, evaluated at the monopoly quantities. In the dynamic case, when retailers take part in the collusive scheme, R_i's information rents have two components. The last term of the first line in (8) is the "static" information rents. The second line is the additional profits that M_i can collect from R_i due to the dynamic IR_L^D and IC_H^D. This is the "dynamic" component of the retailer's information rents. If retailers are myopic or if the two manufacturers exclude the retailers from the collusive scheme (by setting T_H and T_L according to the static IR_L^S and IC_H^S), the second line in (8) vanishes. Hence, manufacturers benefit from including retailers in the collusive scheme when $\pi_{Hi}(q_L^M, q_H^M) - \pi_{Li}(q_L^M, q_L^M) > \pi_{Hi}(q_L^S, q_H^S) - \pi_{Li}(q_L^S, q_L^S)$. The following proposition provides the initial intuition for this result (all proofs are in Appendix A):

Proposition 1. ***(retailers may benefit or hurt from collusion on the monopoly outcome)*** *If $\pi_{Hi}(q_L^M, q_H^M) - \pi_{Li}(q_L^M, q_L^M) > (<)\pi_{Hi}(q_L^S, q_H^S) - \pi_{Li}(q_L^S, q_L^S)$:*

(i) retailers gain higher (lower) one-period information rents in the collusive equilibrium on the monopoly outcome than in the competitive, competitive, static equilibrium, for all δ;

(ii) retailers' one-period information rents under collusion on the monopoly outcome are decreasing (increasing) with δ.

Proposition 1 shows that retailers prefer the collusive equilibrium on the monopoly outcome over the competitive, static outcome if the static component of their information rents,

$\pi_{Hi}(q_L, q_H) - \pi_{Li}(q_L, q_L)$, is higher given the monopoly quantities than given the static quantities. Yet, in such a case, including retailers in the collusive scheme by adopting their dynamic constraints reduces their information rents and in turn increases the manufacturer's profits. The following corollary summarizes this result.

Corollary 1. ***(When is it profitable for manufacturers to include their retailers in the collusive scheme on the monopoly outcome?)*** *Including the retailers in the collusive scheme on the monopoly outcome increases the manufacturers' profits and facilitates collusion if and only if* $\pi_{Hi}(q_L^M, q_H^M) - \pi_{Li}(q_L^M, q_L^M) > \pi_{Hi}(q_L^S, q_H^S) - \pi_{Li}(q_L^S, q_L^S)$.

The intuition for this result is the following. When retailers' information rents given the monopoly quantities are higher than given the static quantities, reverting back to the static equilibrium serves as a punishment not only for the two manufacturers – as in standard "horizontal" collusion – but also for the two retailers. In such a case, forward - looking retailers have an incentive to maintain the collusive equilibrium. This in turn makes such collusion more feasible when retailers are included, i.e., when collusion involves IR_L^D and IC_H^D rather than IR_L^S and IC_H^S.

The results above raise the question of whether $\pi_{Hi}(q_L^M, q_H^M) - \pi_{Li}(q_L^M, q_L^M)$ is higher or lower than $\pi_{Hi}(q_L^S, q_H^S) - \pi_{Li}(q_L^S, q_L^S)$. To this end, it is useful to compare between the competitive and the monopoly quantities. While the literature on collusion mainly focused on restricting output, i.e., collusion on quantities below the competitive quantities, in this model collusion may take a different form. Because of asymmetric information between retailers and their manufacturers, collusion on the monopoly quantities may involve collusion on a higher quantity than the competitive one. The following proposition compares between the monopoly and the competitive quantities.

Proposition 2. ***(Comparison between the competitive and monopoly quantities)*** *For low values of* p*, the monopoly quantities in both states are below the competitive quantities. In particular,* $q_H^S > q_H^M$ *for all* $p \in [0, 1]$*. Moreover,* $q_L^S > q_L^M$ *if* p *is close to 0. Yet,* q_L^S *is decreasing in* p*.*

Notice that because $q_L^M > 0$ and because q_L^S is decreasing with p, it is possible that there is a threshold of p above which $0 < q_L^S < q_L^M$. To illustrate, consider the following linear demand example:

Example. ***(Homogeneous products and linear demand)*** *Suppose that the two supply chains sell homogeneous products. There are no product3, 3ion or retail costs and the inverse demand functions are $p_{Hi}(q_i, q_j) = v_H - q_i - q_j$ and $p_{Li}(q_i, q_j) = v_L - q_i - q_j$, where $v_H > v_L > pv_H$.* [9] *Let $\pi_{\theta i}(q_{\theta i}, q_{\theta j}) = p_{\theta i}(q_{\theta i}, q_{\theta i})q_{\theta i} = (v_\theta - q_{\theta i} - q_{\theta i})q_{\theta i}$. Then:*

$$q_H^S = \frac{v_H}{3}, \quad q_L^S = \frac{3v_L - 2pv_H}{9 - 6p} \quad \text{and} \quad q_H^M = \frac{v_H}{4}, \quad q_L^M = \frac{v_L}{4}.$$

Hence, $q_H^S > q_H^M$ for all p, v_H and v_L. Yet, $q_L^S < q_L^M$ if and only if: $v_H > \frac{6}{5}v_L$ and $\frac{3v_L}{8v_H - 6v_L} < p < \frac{v_L}{v_H}$.[10]

In comparison with the monopoly outcome, the static, competitive outcome on one hand increases quantities because of the standard competitive effect. Yet, asymmetric information has the opposite effect of deriving manufacturers to distort the quantity in state L downward, in order to reduce the retailers' information rents. Because this second effect only holds in state L, only the first effect holds in state H and consequently the quantity under competition is higher than the monopoly quantity. Yet, in state L the second effect dominates when state H is likely (p is high) and the demand in state H is sufficiently large (v_H is high enough), because then manufacturers have a strong incentive to reduce the information rents in state H by distorting their quantities in state L downward.

These results are important for explaining how dynamics enable firms to collude on the monopoly outcome. Recall that the retailer's information rents depend on the retailers' incentive to report the true state, which in turn depend on q_L. This is because when a retailer misrepresent state H as state L, the retailer is "punished" by selling q_L instead of q_H. When p is small such that q_L^S is high relative to q_L^M, retailers face a stronger punishment and consequently a lower information rents under the monopoly quantities than under the static quantities. In this case, retailers do not have an incentive to facilitate collusion on the monopoly outcome. Yet, when p is high such that q_L^M is high relative to q_L^S, retailers benefit from higher information rents under collusion on the monopoly quantities than under competition, hence retailers facilitate such collusion.

[9] The condition $v_L > pv_H$ ensures that $q_L^S > 0$.

[10] The information rents given q_i, evaluated at $q_j' = q_H^S$ and $q_j = q_L^S$, are $\pi_{Hi}(q_i, q_H^S) - \pi_{Li}(q_i, q_L^S) = q_i \frac{2(1-p)(v_H - v_L)}{3-2p}$, which are positive and increasing with q_i. Likewise, the information rents given q_i, evaluated at $q_j' = q_H^M$ and $q_j = q_L^M$, are $\pi_{Hi}(q_i, q_H^M) - \pi_{Li}(q_i, q_L^M) = q_i \frac{3(v_H - v_L)}{4}$, which are positive and increasing with q_i.

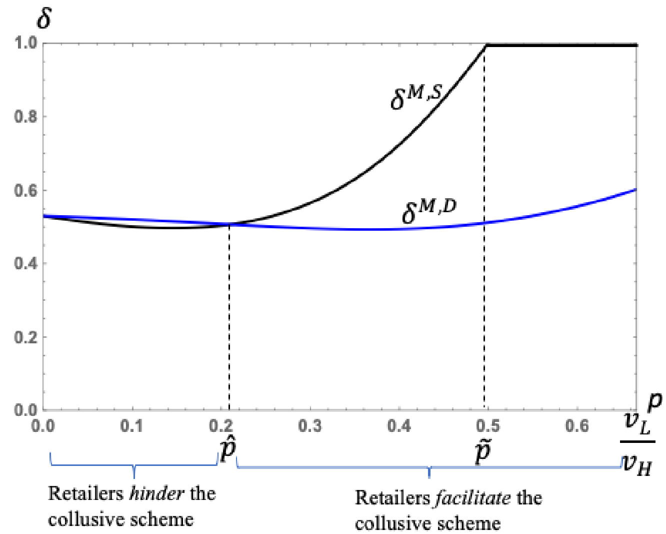


Figure 1: $\delta^{M,D}$ and $\delta^{M,S}$ as a function of p (given the linear demand example, when $v_H = 3$ and $v_L = 2$)

Let $\delta^{M,D}$ denote the threshold of δ above which collusion on the monopoly outcome is an equilibrium, given that retailers are included in the collusive scheme. This threshold solves the constraints IR_L^D, IC_H^D and IC_M^D in equality, evaluated at the monopoly quantities. Likewise, let $\delta^{M,S}$ denote the threshold of δ above which collusion on the monopoly outcome is an equilibrium, given the retailers' static constraints. That is, the solution to IR_L^S, IC_H^S and IC_M^D in equality, evaluated at the monopoly quantities. Including retailers in the collusive scheme facilitates collusion when $\delta^{M,D} < \delta^{M,S}$. The comparison between $\delta^{M,D}$ and $\delta^{M,S}$ depends only on the manufacturer's collusive profits in the two collusive schemes. This is because the quantities are identical in the two schemes, implying that the manufacturer's profit from deviation are also the same. The following corollary is a direct consequence of the analysis so far:

Corollary 2. ***(Forward-looking retailers hinder collusion on the monopoly outcome when a low demand is more likely)*** *If p is sufficiently small, including retailers in the collusive scheme hinders collusion on the monopoly outcome: $\delta^{M,D} > \delta^{M,S}$. Yet if p is sufficiently high such that $\pi_{Hi}(q_L^M, q_H^M) - \pi_{Li}(q_L^M, q_L^M) > \pi_{Hi}(q_L^S, q_H^S) - \pi_{Li}(q_L^S, q_L^S)$, retailers facilitate collusion on the monopoly outcome: $\delta^{M,D} < \delta^{M,S}$.*

The general case cannot identify whether the threshold of p above which retailers facilitate collusion satisfy the assumption that $q_L^S > 0$. To generate further insights and to show when does $\delta^{M,D} < \delta^{M,S}$, consider the linear demand example. Then, there is a threshold:

$$\widehat{p} = \frac{16v_H - 3v_L}{2(16v_H - 9v_L)} - \frac{2}{16v_H - 9v_L}\sqrt{16v_H^2 - 21v_H v_L + 9v_L^2}, \tag{9}$$

such that for $p < \widehat{p}$ ($p > \widehat{p}$) $\pi_{Hi}(q_L^M, q_H^M) - \pi_{Li}(q_L^M, q_L^M) <(>) \pi_{Hi}(q_L^S, q_H^S) - \pi_{Li}(q_L^S, q_L^S)$ and consequently $\delta^{M,D} > (<) \delta^{M,S}$. Moreover, $\widehat{p} < \frac{v_L}{v_H}$ and $q_L^S > 0$ evaluated at $\widehat{p}$. Figure 1 illustrates this result for $0 < p < \frac{v_L}{v_H}$.[11] When $p > \widehat{p}$, it is easier to maintain the collusion on the monopoly outcome with forward-looking retailers than with myopic retailers, because retailers gain higher information rents with the monopoly quantities than with the static quantities. Notice that in this case for high p (above $\widetilde{p}$), it is impossible to maintain the monopoly outcome without forward-looking retailers for all values of δ, because the retailers' incentive to deviate from truthful telling is too high. The opposite case occurs when $p < \widehat{p}$, where it is easier to maintain the monopoly outcome without including the retailers. Notice also that for $p = 0$, $\delta^{M,D} = \delta^{M,S}$, because manufacturers do not pay retailers information rents, hence retailers do not play a positive or negative role in the collusive scheme. In Section 6, I show that this result holds for a collusive equilibrium on any arbitrary collusive quantities, (q_H^*, q_L^*). That is, I show that generally, as p increases, the range of quantities that retailers facilitate collusion on increases.

5 Collusion on upstream profits

The previous section considered collusion on the monopoly quantities: the quantities that maximize the sum of profits of M_1, M_2, R_1 and R_2. Because retailers have private information, manufacturers do not earn all the monopoly profit in the collusive equilibrium because they need to leave retailers with information rents. Hence, regardless of whether collusion on the monopoly quantities is sustainable, manufacturers may prefer to collude on the quantities that maximize only their own join profits: the quantities that maximize the joint profit of M_1 and M_2. By so doing, manufacturers coordinate on decreasing their retailers' information rents. In this section I consider collusion on the quantities that maximize the manufacturer's joint profit, i.e., upstream profits. I ask three questions. First, what are the features of these quantities and how they are affected by the repeated interactions between manufacturers and

[11] The parameter values are chosen as to illustrate the results, which hold for the linear demand example. In Appendix B, I illustrate $\delta^{M,D}$ and $\delta^{M,S}$ as a function of p for $v_H = 3$ and $v_L = \left\{2\frac{1}{3}, 2\frac{1}{2}, 2\frac{2}{3}\right\}$.

their retailers. Second, when is it easier to maintain collusion on upstream profits, in comparison with maintaining collusion on monopoly profits. Third, whether including retailers in the collusive scheme on upstream profits facilitates collusion. The main conclusions of this section are that manufacturers find it optimal to coordinate on a quantity above (below) the monopoly quantity when demand is high (low). In comparison with collusion on the monopoly profits, collusion on upstream profits is easier to maintain when the probability of a high demand is sufficiently high. Finally, forward-looking retailers facilitate upstream collusion when the probability of high demand is high.

Consider a dynamic equilibrium where in each period the two manufacturers offer the quantities that maximize their joint profit, excluding the profit of their retailers, given that IR_L^D and IC_H^D hold in equality and retailers accept. As before, any observable deviation at an end of a period (either a manufacturer deviates from the quantities that maximize upstream profits, or a retailer deviates by rejecting a contract or misrepresenting the state) is followed by a diversion to the competitive, static equilibrium.

I solve for collusion on the manufacturers' optimal unconstrained quantities: the quantities that maximize their joint profit given that the manufacturers' discount factor is sufficiently high to sustain collusion on these quantities. Then, I solve for the critical value of δ that enables manufacturers to collude on these quantities. Notice that even though these unconstrained quantities are unaffected by the manufacturers' discount factor, they are going to be affected by the retailers' discount factor, because retailers internalize that if they deviate, collusion will stop in future periods.

Let q_H^U and q_L^U denote the quantities that maximize the joint upstream profits of M_1 and M_2, excluding R_1's and R_2's profits:

$$\Pi_M^D(q_L, q_H) \equiv \Pi_{Mi}^D(q_L, q_H, q_L, q_H) + \Pi_{Mj}^D(q_L, q_H, q_L, q_H).$$

Recall that each M_i's profit, $\Pi_{Mi}^D(q_{Li}, q_{Hi}, q_{Lj}, q_{Hj})$, can be written as the joint profit of M_i and R_i, minus R_i's information rents. Hence, the quantities that maximize the manufacturers' joint profit are the quantities that maximize the monopoly profit minus the retailers' information rents. This implies that manufacturers collude on reducing their retailers' information rents.

Using the definition of $\Pi_{Mi}^D(q_{Li}, q_{Hi}, q_{Lj}, q_{Hj})$ from (8), the first order conditions of q_H^U and

q_L^U, respectively, are:

$$\frac{\partial \Pi_M^D(q_L, q_H)}{\partial q_H} = \frac{\partial \pi_{Hi}(q_H^U, q_H^U)}{\partial q_i} + \frac{\partial \pi_{Hj}(q_H^U, q_H^U)}{\partial q_i} - \left[\frac{1-\delta}{1+\delta p}\right] \frac{\partial \pi_{Hj}(q_L^U, q_H^U)}{\partial q_i} = 0, \tag{10}$$

$$\frac{\partial \Pi_M^D(q_L, q_H)}{\partial q_L} = \frac{\partial \pi_{Li}(q_L^U, q_L^U)}{\partial q_i} + \frac{\partial \pi_{Lj}(q_L^U, q_L^U)}{\partial q_i} - \left[\frac{(1-\delta)p}{1-\delta p^2}\right] \frac{\partial \pi_{Hi}(q_L^U, q_H^U)}{\partial q_i} = 0. \tag{11}$$

To see the intuition behind these conditions, the first two terms in (10) are the first-order condition of the monopoly profits with respect to q_H (and equal to zero at $q_H^U = q_H^M$). The last term in (10) is the effect of the quantities in reducing the retailers information rents: the effect of the quantity set by M_i in state H on the information rents that M_j has to pay R_j. Hence, under upstream collusion, the two manufacturers' joint interest is not only to coordinate on the monopoly quantities, but also on reducing each other's information rents. Likewise, the first two terms in (11) are the first-order condition of the monopoly profits with respect to q_L (and equal to zero at $q_L^U = q_L^M$), while the last term is the effect of the quantity set by M_i in state L on R_i's information rents. The following proposition characterizes the features of q_H^U and q_L^U:

Proposition 3. ***(Collusion on upstream profits involves upward (downward) distortion in state H (L))*** *The quantities that maximize the manufacturers' joint profits involve a quantity above (below) the monopoly quantity in state H (L): $q_H^U > q_H^M$ and $q_L^U < q_L^M$. Moreover, q_H^U (q_L^U) is decreasing (increasing) in δ and converges to the monopoly quantity as $\delta \to 1$.*

Figure 2 illustrates the results of Proposition 3.[12] The proposition finds two main results with implications to the comparison between collusion on upstream profits and on monopoly profits. The first main result is that in state H (L), manufacturers coordinate on a quantity above (below) the monopoly quantity. This result differs from the standard result of "no distortion at that top" in the literature on asymmetric information. The intuition for this result is that manufacturers have two incentives in setting the quantities that maximize their joint profit. First, the standard incentive to eliminate competition, which motivates manufacturers to set quantities close to the monopoly level. Second, an incentive to reduce

[12]Notice that the results of Proposition 3 hold in the general case and do not depend on the parameter values selected in the figure. In Appendix B, I illustrate the q_H^U and q_L^U as a function of δ for $v_H = 3$ and $v_L = \{2\frac{1}{3}, 2\frac{1}{2}, 2\frac{2}{3}\}$.

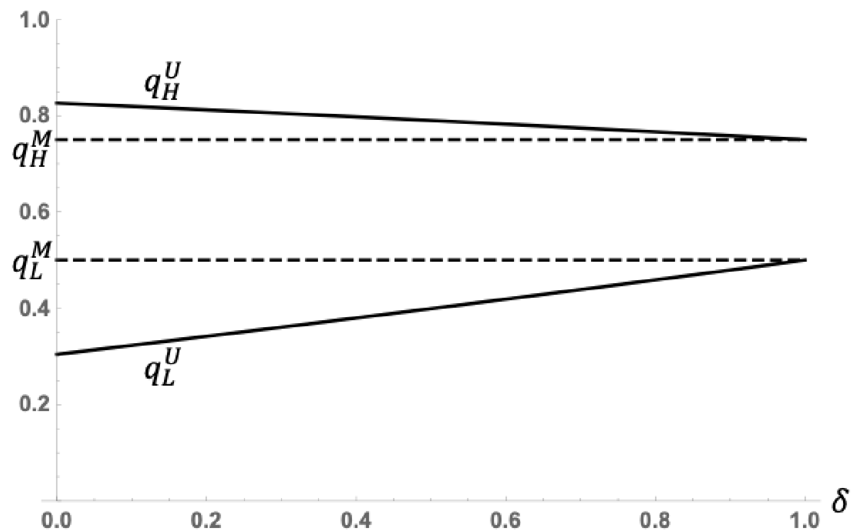


Figure 2: The upstream collusive quantities, q_H^U and q_L^U, as a function of δ (given the linear demand example when $v_H = 3$, $v_L = 2$ and $p = \frac{1}{2}$)

the retailers' information rents. To do so, manufacturers need to increase the gap between the quantities in periods of high a low demand. This increased gap makes it less attractive for a retailer in state H to misrepresent the state as L, because then the retailer sells a low quantity (i.e., below the monopoly quantity in state L), while the competing retailer sells a high quantity (i.e., above the monopoly quantity in state H).

The second main result is that as retailers become forward-looking (δ increases), manufacturers decrease (increase) the quantities in state H (L), and these quantities converge to the monopoly quantities. Intuitively, as retailers become more forward looking, they have a stronger incentive to maintain the collusive equilibrium if they expect to earn higher information rents in future periods. The manufacturers' second incentive, of reducing the retailers' information rents, become weaker, and because of the first effect, quantities becomes closer to the monopoly level.

Remark: I assume that the gap between q_H^U and q_L^U is sufficiently small such that the $\pi_{Hi}(q_L^U, q_H^U) - \pi_{Li}(q_L^U, q_L^U) > 0$. This assumption ensures that if δ is close to 0, the gap between q_H^U and q_L^U is such that R_i's static information rents in state H is positive. Otherwise, The quantities that maximize upstream profits has a corner solution in which both $IR_L^D$and IR_H^D bind. In the linear demand example, it is possible to show that $\pi_{Hi}(q_L^U, q_H^U) - \pi_{Li}(q_L^U, q_L^U) >$ 0 and consequently there is an internal solution to q_H^U and q_L^U for all δ when $0 < p <$

$\min\left\{\frac{12v_H-13v_L}{9(v_H-v_L)}, \frac{v_L}{v_H}\right\}$.

Next, I turn to the question of whether upstream collusion is easier to maintain than collusion on the monopoly outcome. I compare the two collusive schemes given that they both include the retailers (i.e., IR_L^D and IC_H^D are dynamic). Let $\delta^{U,D}$ denote the lowest possible δ that maintains collusion on upstream profits given dynamic retailers. This $\delta^{U,D}$ solves the constraints IR_L^D, IC_H^D and IC_M^D in equality, evaluated at q_L^U and q_H^U. Recall that $\delta^{M,D}$ is the lowest possible δ that maintains collusion on the monopoly outcome, given dynamic retailers. Hence, upstream collusion is easier to maintain than collusion on the monopoly outcome when $\delta^{U,D} < \delta^{M,D}$. To compare $\delta^{U,D}$ with $\delta^{M,D}$, notice first that at $p = 0$, $\delta^{U,D} = \delta^{M,D}$. To see why, recall that evaluated at $p = 0$, $q_L^U = q_L^M$ because manufacturers do not need to pay their retailers information rents. Hence, the manufacturers' profits from both maintaining and deviating from collusion are the same under both upstream collusion and collusion on the monopoly quantities.

When $p > 0$, the comparison between $\delta^{U,D}$ and $\delta^{M,D}$ is inconclusive and depends on the model's parameters. To illustrate the various economic forces that effect the comparison, consider the linear demand example. Figure 3 illustrates $\delta^{U,D}$ and $\delta^{M,D}$ as functions of p when $0 < p < \min\left\{\frac{12v_H-13v_L}{9(v_H-v_L)}, \frac{v_L}{v_H}\right\}$ for $v_H = 3$ and $v_L \in \left\{2\frac{1}{3}, 2\frac{1}{2}, 2\frac{2}{3}\right\}$. Panel (a) illustrates the case where v_L is small. In this case, collusion on monopoly profits is easier to maintain then collusion on upstream profits (i.e., $\delta^{U,D} > \delta^{M,D}$) as long as p is not too high. Panels (b) and (c) illustrate the case where v_L is high. In this case, $\delta^{U,D} > \delta^{M,D}$ for all $0 < p < \min\left\{\frac{12v_H-13v_L}{9(v_H-v_L)}, \frac{v_L}{v_H}\right\}$.

The intuition for these results is that the comparison between $\delta^{U,D}$ and $\delta^{M,D}$ exhibits the following tradeoff. First, when $p > 0$, manufacturers' profits from colluding on upstream profits are higher than colluding on the monopoly profits, as the latter profits do not maximize the manufacturers' joint profits. This effect makes collusion on upstream profits easier to maintain than collusion on monopoly profits. Second, each manufacturer's profit from deviating from collusion might also be higher when manufacturers collude on upstream profits. This second effect is stronger when the probability of state L is high. Recall that the quantity that maximizes upstream profits in state H (L) is higher (lower) than the monopoly quantity. Hence, the incentive to deviate from collusion is higher in state L than in state H, because then the manufacturer would like to deviate to a higher quantity. When v_L is small, the second effect dominates for a low p, while when v_L is high, the second effect dominates

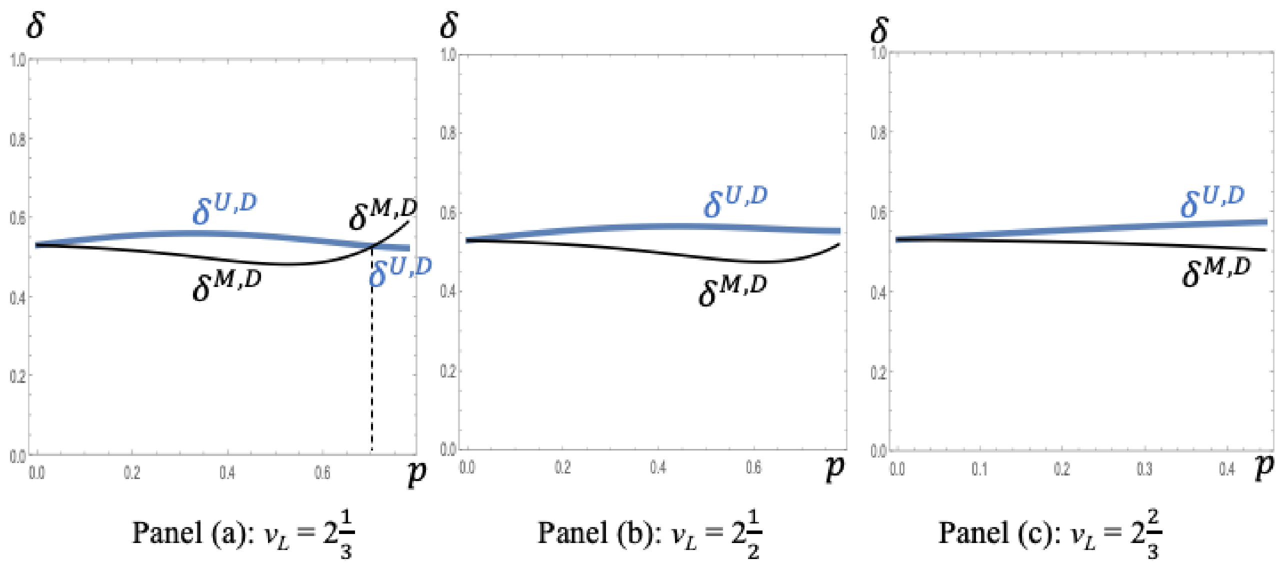


Figure 3: $\delta^{U,D}$ and $\delta^{M,D}$ as a function of p (given the linear demand example, when $v_H = 3$ and $v_L \in \{2\frac{1}{3}, 2\frac{1}{2}, 2\frac{2}{3}\}$)

for all p.

Next, I turn to evaluate the effect of including forward-looking retailers in the collusive scheme on upstream profits. Manufacturers can collude on upstream profits without including the retailers, by dealing with myopic retailers or by ignoring a retailer's deviation from the collusive scheme. This raises the question of whether retailers facilitate of hinder collusion on upstream profit.

Let $q_L^{U,S}$ and $q_H^{U,S}$ denote the quantities that maximize joint manufacturers' profits when retailers are static. The $q_L^{U,S}$ and $q_H^{U,S}$ are the solutions to the first-order-conditions (11) and (10), evaluated at $\delta = 0$. Because q_L^U is increasing with δ while q_H^U is decreasing with δ, $q_L^{U,S}$ and $q_H^{U,S}$ are at their lowest and highest levels of q_L^U and q_H^U, respectively (as shown at Figure 2 when $\delta = 0$). Let $\delta^{U,S}$ denotes the lowest value of δ that sustains collusion on upstream profits when retailers are myopic. This $\delta^{U,S}$ solves IR_L^S, IC_H^S and IC_M^D in equality, evaluated at $q_L^{U,S}$ and $q_H^{U,S}$.

The comparison between $\delta^{U,D}$ and $\delta^{U,S}$ is inconclusive and depends on the model's parameters. To illustrate how, Figure 4 illustrates $\delta^{U,D}$ and $\delta^{U,S}$ as a function of p, for $0 < p < \min\left\{\frac{12v_H - 13v_L}{9(v_H - v_L)}, \frac{v_L}{v_H}\right\}$, $v_H = 3$ and for $v_L \in \{2\frac{1}{3}, 2\frac{1}{2}, 2\frac{2}{3}\}$. Panels (a) and (b) il-

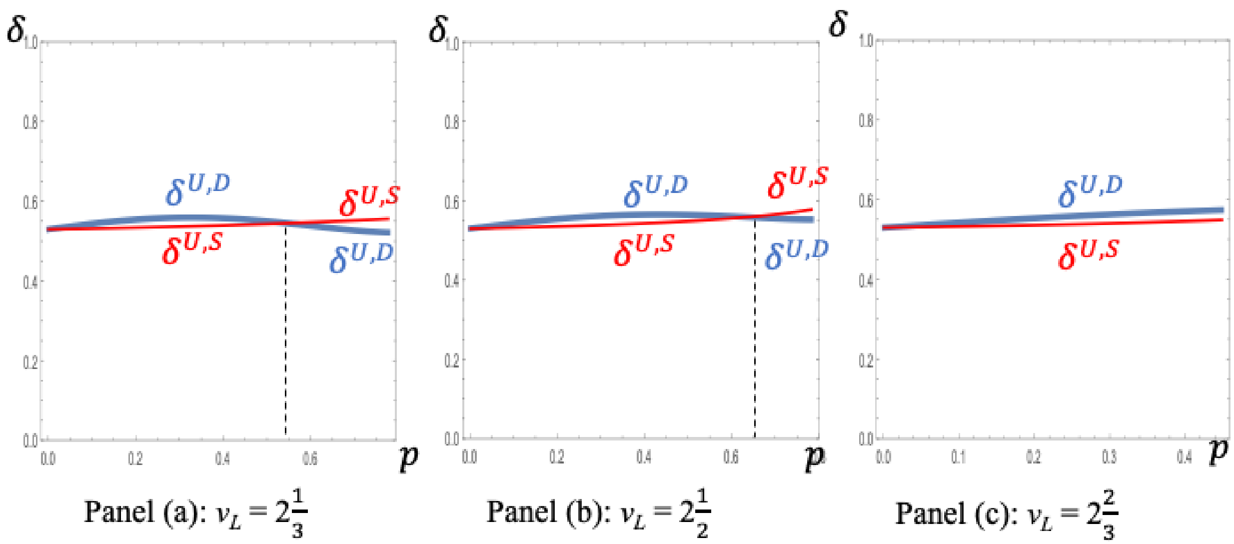


Figure 4: $\delta^{U,D}$ and $\delta^{M,D}$ as a function of p (given the linear demand example, when $v_H = 3$ and for $v_L \in \{2\frac{1}{3}, 2\frac{1}{2}, 2\frac{2}{3}\}$.)

lustrate the case where v_L is small, hence collusion with myopic retailers is easier to sustain then collusion with forward-looking retailers ($\delta^{U,S} < \delta^{U,D}$) when p is below a threshold. In Panel (c), v_L is high and consequently $\delta^{U,S} < \delta^{U,D}$ for all relevant values of p.

Following the same intuition as in Section 4, retailers' information rents in the static equilibrium can be higher or lower than under collusion on upstream profits. When p is small, then $\delta^{U,D} > \delta^{U,S}$ because the gap between $q_H^{U,D}$ and $q_L^{U,D}$ is wide, hence a low level of information rents are needed to motivate retailers to reveal the state under upstream collusion. Yet, this reduces their incentive to facilitate the collusive scheme. The opposite case accuses when p is high. In the next section, I show that this result is general, in that as p increases, retailers facilitate collusion on a wider range of arbitrary quantities that firms may collude on.

6 When retailers facilitate collusion: a general collusive scheme

This section generalizes the results obtained so far in two directions. First, the following subsection formally shows when retailers facilitate collusion given any arbitrary collusive quantities. Then, the next subsection informally comment on a more general punishment phase in which firms adopt a stick-and carrot or other optimal punishment mechanisms

6.1 General collusive quantities

The paper, so far, compared the firms' ability to collude when including or excluding retailers from the collusive scheme given specific quantities: the monopoly quantities (in Section 4) and the quantities that maximize upstream profits (Section 5). In both cases, the paper finds that retailers are harmful to collusion when p is low but may facilitate collusion when p is high. In this subsection, I offer a general assessment of when including retailers in the collusive scheme facilitates collusion. The main conclusion of this subsection is that given a collusive scheme on a certain quantities, it is more likely that retailers facilitate collusion on these quantities when p is high. This result is consistent with the results obtained in the special cases of collusion on monopoly quantities and on the quantities that maximize upstream profit.

To this end, consider a collusive scheme on arbitrary collusive quantities, (q_L^*, q_H^*). Because the main question of this paper is whether retailers facilitate or hinder collusion, I focus on whether retailers can facilitate collusion on these arbitrary (q_L^*, q_H^*), without solving for a specific collusive equilibrium.

Recall that the manufacturers' profit given a collusive equilibrium on the quantities (q_L^*, q_H^*) when retailers are excluded from the collusive scheme (such that constraints (1) and (2) bind, evaluated at (q_L^*, q_H^*)), is given by $\Pi_{Mi}^S(q_L^*, q_H^*, q_L^*, q_H^*)$ in equation (3). Likewise, the manufacturers' profit given a collusive equilibrium on the same quantities (q_L^*, q_H^*) when retailers are included (such that constraints (6) and (7) bind), is given by $\Pi_{Mi}^D(q_L^*, q_H^*, q_L^*, q_H^*)$ in equation (8). Comparing the manufacturers' profit in the two cases, retailers facilitate collusion on (q_L^*, q_H^*) when manufacturers earn higher collusive profit when retailers are included.[13]

[13]To see why, notice that given (q_L^*, q_H^*), each manufacturer's profit from deviating from collusion when

The following proposition characterizes the set of quantities under which retailers facilitate collusion, and, more importantly, how p affects this set:

Proposition 4. *Consider collusion on a certain (q_L^*, q_H^*). Then, the range of (q_L^*, q_H^*) that retailers facilitate collusion on expands as p increases.*

That is, there is a cutoff, $\overline{q}_H(q_L; p)$, such that retailers facilitate collusion on (q_L^*, q_H^*) (i.e.,$\Pi_{Mi}^D(q_L^*, q_H^*, q_L^*, q_H^*) > \Pi_{Mi}^S(q_L^*, q_H^*, q_L^*, q_H^*)$) if $q_H^* < \overline{q}_H(q_L^*; p)$ and are harmful to collusion otherwise. Moreover, $\overline{q}_H(q_L; p)$ is increasing with p and with q_L.

Proposition 4 shows that as p increases, it is more likely that retailers facilitate collusion. Intuitively, recall that as p increases, asymmetric information becomes more of a problem for the manufacturers in the competitive equilibrium, and manufacturers have a stronger incentive to reduce the retailer's information rents. Consequently, retailers know that should collusion breaks, retailers will earn low information rents in the competitive equilibrium and hence they have the incentive to facilitate collusion.

This result generalizes the results of sections 4 and 5 that show that retailers hinder collusion on the monopoly quantities or the quantities that maximize upstream profit when p is small, but may facilitate collusion otherwise. In both cases, if when p is small, the quantities that firms collude on are in the set of (q_L^*, q_H^*) in which retailers hinder collusion, i.e., $q_H^* > \overline{q}_H(q_L^*; p)$. As p increases, $\overline{q}_H(q_L^*; p)$ increases and the set of quantities on which retailers facilitate collusion expands. For a sufficiently high p, the set of quantities on which retailers facilitate collusion may includes the monopoly quantities or the quantities that maximize upstream profits.

6.2 Discussion on alternative punishment stages

The paper assumes that if collusion breaks, firms play the competitive, static equilibrium in all future periods. Yet, because of the dynamic nature of the game and because the market includes both upstream and downstream firms, it is possible to think of other punishment strategies. Such strategies may include stick-and-carrot strategies that impose a more sever punishment on the deviating firm than in the competitive, static equilibrium. A full characterization of such punishment strategies is beyond the scope of this paper, but it is

retailers are included is the same as when retailers are excluded. Moreover, the competitive profit that follows a deviation is also the same. Hence, when manufacturers earn higher collusive profit, their IC_M^D constraint as given by (5) is more likely to satisfy.

possible to discuss how the paper's main result is affected by the possibility of adopting other punishment strategies.

Intuitively, notice that Proposition 1 and consequently Corollary 1 follows to any punishment strategy that inflicts lower information rents on retailers than the collusive information rents. This is because whenever retailers expect that following a deviation, the punishment mechanism result in lower information rents than the collusive information rents, retailers will have an incentive to facilitate collusion. Applying the intuition of Corollary 1, suppose that manufacturers can coordinate on a stick-and-carrot strategy that imposes an even more severe punishment on a deviating retailer than the competitive, static equilibrium. This would make including the retailers in the collusive scheme even more profitable for the manufacturers, because retailers will have a stronger incentive to keep the collusive equilibrium going and earn the collusive information rents.

Moreover, as p increases each manufacturers faces a stronger asymmetric information problem. Consequently, manufacturers are able to coordinate on a more severe punishment on a deviating retailer because each manufacturer has a stronger incentive to reduce its retailer's information rents (which follows from equation (4)). Hence, I expect that the result that retailers facilitate collusion if p is high follows to a collusive scheme that includes a more elaborate punishment mechanism.

7 Conclusion and policy implications

This paper considers an infinitely repeated game between two vertical supply chains. Each retailer has private information concerning the demand, so manufacturers offer a menu of contracts and pay retailers information rents. The repeated interaction between the two supply chains enables firms to collude, and the repeated interaction within each supply chain enables the manufacturers to include their retailers in the collusive scheme. Studying such markets has implications for competition policy. While from a legal perspective, it is rather difficult to prosecute firms for engaging in tacit collusion, detecting such collusion enables competition authorities to evaluate the market power and concentration in such markets. Moreover, in real-life, such collusion may involve informal communication which can be illegal.

The paper finds that retailers facilitate collusion when their information rents in the collusive equilibrium are higher than in the competitive outcome. When firms collude on the monopoly outcome, retailers facilitate collusion when the probability of a high demand is high. Otherwise, manufacturers are better off without including their retailers in the collusive scheme, hence manufacturers ignore a retailer's deviation from the collusive path. This result can explain why, in some recent legal cases, collusion involved both upstream and downstream firms, while other legal cases involved only the upstream firms. For policy, this result implies that when competition authorities detect collusion among upstream firms, it is also important to investigate the role of their retailers in facilitating such collusion. In particular, when the market demand fluctuates over time, such that information concerning the demand is important for the distribution contract, retailers could potentially participate in such collusion. Then, manufacturers will repeatedly deal with the same retailers along time and establish long-term relationships based on trust. Counterintuitively, the paper also finds that in some market conditions, short-term relationship between manufacturers and retailers may actually facilitate collusion. In such a case, manufacturers prefer to exclude retailers from the collusive scheme by dealing with short-sighted retailers, alternating between retailers in each period, or ignoring a retailer's deviation from collusion.

Because asymmetric information forces manufacturers to leave their retailers a share of the collusive profits, the paper finds that manufacturers can benefit from colluding on the quantities that maximize upstream profits, rather than total profits. In this case, collusion involves an upward (downward) deviation in periods of high (low) demand, in comparison with the monopoly quantities. The policy implication of this result is that in order to detect collusion, competition authorities should not look at the quantity of the current period alone, but on the stream of quantities along time, and try to identify cases in which low demand result in an ultra low quantity. Wide variations in quantities between periods can be indicative of collusion on the quantities that maximize upstream profits.

Finally, the paper finds that when the probability of a low demand is high enough, collusion on monopoly profits is easier to maintain than collusion on upstream profits. Although the two manufacturers earn higher collusive profits when colluding on maximizing upstream profits, each manufacturer's incentive to deviate is also higher, which makes collusion more difficult to maintain than collusion on monopoly profits.

Appendix A

Below are the proofs of Propositions 1 - 4.

Proof of Proposition 1:

Part (i): R_i's information rents in the collusive equilibrium on the monopoly outcome are:

$$\Pi_{Ri}^{D,M} = p\big[\pi_{Hi}(q_L^M, q_H^M) - \pi_{Li}(q_L^M, q_L^M)\big] \tag{12}$$

$$- \left[\frac{\delta p(1+p)}{1+\delta p}\right] \left(\pi_{Hi}(q_L^M, q_H^M) - \pi_{Li}(q_L^M, q_L^M) - (\pi_{Hi}(q_L^S, q_H^S) - \pi_{Li}(q_L^S, q_L^S))\right).$$

Hence:

$$\Pi_{Ri}^{D,M} - \Pi_{Ri}^S = \frac{p(1-\delta)}{1+\delta p}\left[\pi_{Hi}(q_L^M, q_H^M) - \pi_{Li}(q_L^M, q_L^M) - (\pi_{Hi}(q_L^S, q_H^S) - \pi_{Li}(q_L^S, q_L^S))\right],$$

which is positive (negative) when the sign of the squared brackets is positive (negative).
Part (ii): The term $\left[\frac{\delta p(1+p)}{1+\delta p}\right]$ is increasing in δ, implying that the retailers' information rents are decreasing in δ when the term in the squared brackets is positive.

Proof of Proposition 2:

I start by showing that $q_{Hi}^S(q_H^M) > q_H^M$, which implies that $q_H^S > q_H^M$. Recall that q_H^M is the solution to $\partial\pi_{Hi}(q_H^M, q_H^M)/\partial q_i + \partial\pi_{Hj}(q_H^M, q_H^M)/\partial q_i = 0$, where the second term is negative because $\partial\pi_{\theta j}(q_{\theta j}, q_{\theta i})/\partial q_i < 0$, implying that $\partial\pi_{Hi}(q_H^M, q_H^M)/\partial q_i > 0$. Recall further that $q_{Hi}^S(q_H^M)$ is the solution to $\partial\pi_{Hi}(q_{Hi}, q_H^M)/\partial q_i = 0$. Because evaluated at $q_{Hi}^S(q_H^M) = q_H^M$, $\partial\pi_{Hi}(q_H^M, q_H^M)/\partial q_i > 0$, I have that $q_{Hi}^S(q_H^M) > q_H^M$.

Next, consider the comparison between q_L^S and q_L^M. To show that $q_{Li}^S(q_L^M, q_H^M) > q_L^M$ when $p = 0$, recall that $q_{Li}^S(q_L^M, q_H^M)$ is the solution to:

$$\frac{\partial\pi_{Li}(q_L^M, q_L^M)}{\partial q_i} - p\frac{\partial\pi_{Hi}(q_L^M, q_H^M)}{\partial q_i} = 0. \tag{13}$$

When $p = 0$, the second term in (13) vanishes and $q_{Li}^S(q_L^M, q_H^M) > q_L^M$ by applying the same argument as the first part of this proof for the case where $\theta = L$. When $p > 0$, $\partial\pi_{Hi}(q_L^M, q_H^M)/\partial q_i > \partial\pi_{Hi}(q_H^M, q_H^M)/\partial q_i > 0$, where the first inequality follows because by assumption $\pi_{Hi}(q_i, q_j)$ is concave in q_i (hence $\partial\pi_{Hi}(q_i, q_j)/\partial q_i$ is decreasing in q_i) and because $q_H^M > q_L^M$. The second inequality follows by the first pat of this proof. I therefore have that (13) is decreasing in p, implying that $q_{Li}^S(q_L^M, q_H^M)$ and consequently q_L^S is decreasing in p.

Proof of Proposition 3:

Consider first q_H^U. I show that the first-order-condition of q_H^U, (10), is positive when evaluated at $q_H^U = q_H^M$ and $q_L^U = q_L^M$. The first two terms in (10) are the first-order-condition for the monopoly quantity hence equals 0 at $q_H^U = q_H^M$ and $q_L^U = q_L^M$. The last term is positive for all quantities because by assumption $\partial\pi_{Hj}(q_L^U, q_H^U)/\partial q_i < 0$. Because the term $\left[\frac{1-\delta}{1+\delta p}\right]$ is positive at $\delta = 0$, positive and decreasing with δ for $\delta > 0$ and equals 0 at $\delta = 1$, it follows that $q_H^U > q_H^M$ for $\delta = 0$, q_H^U is decreasing in δ and converges to q_H^M as $\delta \to 1$.

Next consider q_L^U. I show that the first-order-condition of q_L^U, (11), is negative when evaluated at $q_L^U = q_L^M$ and $q_H^U = q_H^M$. The first two terms in (11) are the first-order-condition for the monopoly quantity hence equals 0 at $q_H^U = q_H^M$ and $q_L^U = q_L^M$. The last term, evaluated at $q_H^U = q_H^M$ and $q_L^U = q_L^M$ is negative when $\partial\pi_{Hi}(q_L^M, q_H^M)/\partial q_i > 0$. To see why $\partial\pi_{Hi}(q_L^M, q_H^M)/\partial q_i > 0$, recall that $\partial\pi_{Hi}(q_H^M, q_H^M)/\partial q_i > 0$ and $\partial^2\pi_{Hi}(q_i, q_H^M)/\partial q_i^2 < 0$. Because $q_H^M > q_L^M$, $\partial\pi_{Hi}(q_L^M, q_H^M)/\partial q_i > \partial\pi_{Hi}(q_H^M, q_H^M)/\partial q_i > 0$. Because the term $-\left[\frac{(1-\delta)p}{1-\delta p^2}\right]$ is negative at $\delta = 0$, negative and increasing in δ for $\delta > 0$ and equals 0 at $\delta = 1$, it follows that that $q_L^U < q_L^M$ for $\delta < 1$, q_L^U is increasing in δ and converges to q_L^M as $\delta \to 1$.

Proof of Proposition 4:

I first establish that exists a threshed value of q_H^* such that retailers facilitate collusion if $q_H^* < \overline{q}_H(q_L^*; p)$. To this end, notice that the results of Proposition 1 and Corollary 1 hold given any arbitrary quantities. Hence, retailers facilitate collusion (i.e., $\Pi_{Mi}^D(q_L^*, q_H^*, q_L^*, q_H^*) > \Pi_{Mi}^S(q_L^*, q_H^*, q_L^*, q_H^*)$) if and only if $\Delta(q_L^*, q_H^*) \equiv \pi_{Hi}(q_L^*, q_H^*) - \pi_{Li}(q_L^*, q_L^*) - \left(\pi_{Hi}(q_L^S, q_H^S) - \pi_{Li}(q_L^S, q_L^S)\right) > 0$. By definition, $\Delta(q_L^S, q_H^S) = 0$. Moreover, $\pi_{Hi}(q_L^*, q_H^*) - \pi_{Li}(q_L^*, q_L^*)$ is decreasing with q_H^*, because $\pi_{Hi}(q_i, q_j)$ is decreasing with q_j. Hence, there is a unique threshold of q_H^*, $\overline{q}_H(q_L^*; p)$, such that $\Delta(q_L^*, q_H^*) > 0$ if and only if $q_H^* < \overline{q}_H(q_L^*; p)$, where $q_H^S = \overline{q}_H(q_L^S; p)$. Notice that it is not always the case that $\overline{q}_H(q_L^*; p) > 0$, though $\overline{q}_H(q_L^*; p) > 0$ at least when q_L^* is sufficiently close to q_L^S because $q_H^S = \overline{q}_H(q_L^S; p) > 0$.

Next, I turn to the features of $\overline{q}_H(q_L^*; p)$. I first show that $\overline{q}_H(q_L^*; p)$ is increasing with p. To this end, recall that q_L^S is decreasing with p, which follows from (4). Moreover, $\pi_{Hi}(q_L^S, q_H^S) - \pi_{Li}(q_L^S, q_L^S)$ is increasing with q_L^S. To see why, I have:

$$\frac{\partial}{\partial q_L^S}\left(\pi_{Hi}(q_L^S, q_H^S) - \pi_{Li}(q_L^S, q_L^S)\right) =$$

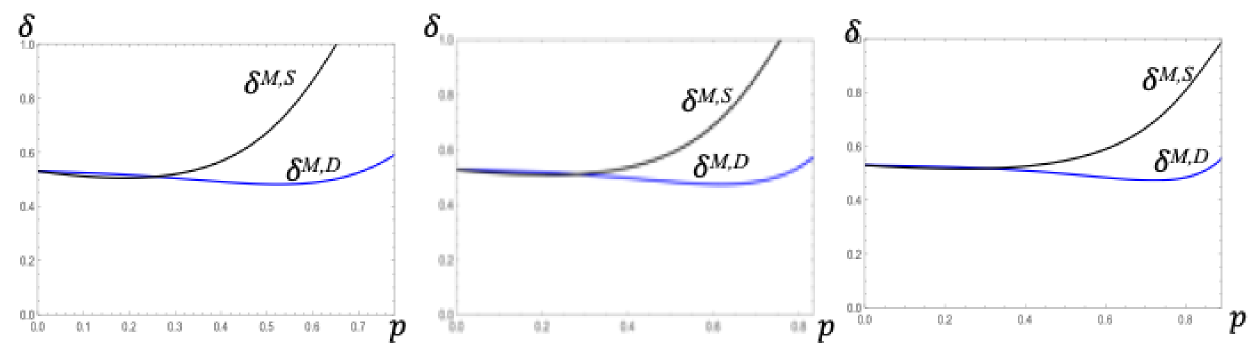
$$\frac{\partial \pi_{Hi}(q_L^S, q_H^S)}{\partial q_i} - \frac{\partial \pi_{Li}(q_L^S, q_L^S)}{\partial q_i} - \frac{\partial \pi_{Li}(q_L^S, q_L^S)}{\partial q_j} > 0,$$

where the sum of the first two terms is positive because by assumption, $\pi_{Hi}(q_i, q_H^S) - \pi_{Li}(q_i, q_L^S)$ is increasing in q_i (recall that $\pi_{Hi}(q_i, q_H^S) - \pi_{Li}(q_i, q_L^S) > 0$ from the IR_L^S constraint), and the last term is positive because by assumption, $\pi_{Li}(q_L^S, q_j)$ is decreasing in q_j. Hence, $\Delta(q_L^*, q_H^*)$ is increasing with p. Because $\Delta(q_L^*, q_H^*)$ is decreasing with q_H^*, it follows that $\overline{q}_H(q_L^*; p)$ is increasing with p.

Finally, I turn to show that $\overline{q}_H(q_L^*; p)$ is increasing with q_L^*. To this end, using the proof above yields that $\pi_{Hi}(q_L^*, q_H^*) - \pi_{Li}(q_L^*, q_L^*)$ is increasing in q_L^*. Consequently, $\Delta(q_L^*, q_H^*)$ is increasing with q_L^*. Because $\Delta(q_L^*, q_H^*)$ is decreasing with q_H^*, it follows that $\overline{q}_H(q_L^*; p)$ is increasing with q_L^*.

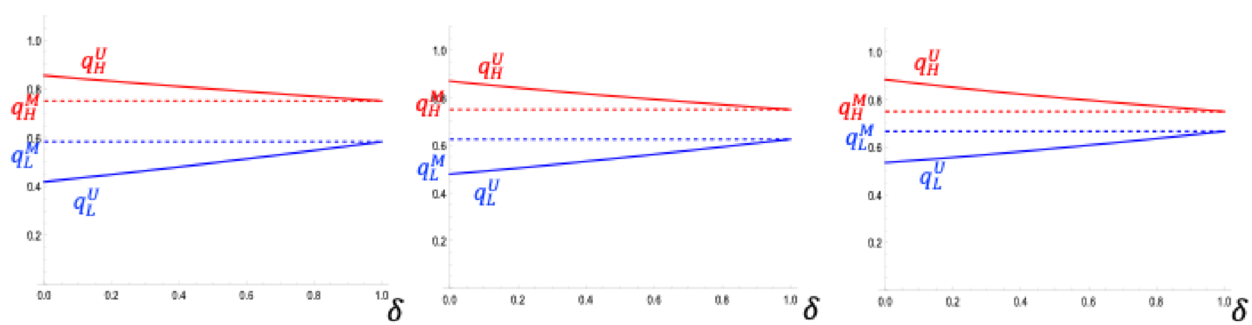
Appendix B

Below are other numerical examples for figures 1 and 2, respectively:



Panel (a): $v_L = 2\frac{1}{3}$ Panel (b): $v_L = 2\frac{1}{2}$ Panel (c): $v_L = 2\frac{2}{3}$

Figure 5: $\delta^{M,S}$ and $\delta^{M,D}$ as a function of p, for $v_H = 3$ and v_L={$2\frac{1}{3}$, $2\frac{1}{2}$, $2\frac{2}{3}$}



Panel (a): $v_L = 2\frac{1}{3}$ Panel (b): $v_L = 2\frac{1}{2}$ Panel (c): $v_L = 2\frac{2}{3}$

Figure 6: The quantities that maximize upstream profits as a function of δ, for p = ½, $v_H = 3$ and v_L ={$2\frac{1}{3}$, $2\frac{1}{2}$, $2\frac{2}{3}$}

Appendix C: Proof that IR_H and IC_L are not binding

The competitive, static equilibrium

I show below that in the general case, given that IR_L^S and IC_H^S hold in equality, the IR_H^S and IC_L^S hold.

Consider a static menu that satisfies IR_L^S and IC_H^S in equality. Suppose that $q_H^S > q_L^S$ and $\pi_{Hi}(q_i, q_H^S) - \pi_{Li}(q_i, q_L^S)$ is positive and increasing with q_i (all of these assumptions are satisfied in the linear demand example). I show that IR_H^S and IC_L^S hold.

I start with IR_H^S. This constraint holds when $\pi_{Hi}(q_H^S, q_H^S) - T_{Hi} > 0$, where:

$$\pi_{Hi}(q_H^S, q_H^S) - T_{Hi} = \pi_{Hi}(q_L^S, q_H^S) - \pi_{Li}(q_L^S, q_L^S) > 0, \tag{14}$$

where the first equality follows by substituting the T_{Hi} that solves IR_L^S and IC_H^S and the second inequality follows by assumption.

Turning to IC_L^S, this condition holds if this term is positive:

$$\pi_{Li}(q_L^S, q_L^S) - T_{Li} - \big(\pi_{Li}(q_H^S, q_H^S) - T_{Hi}\big) =$$

$$\pi_{Hi}(q_H^S, q_H^S) - \pi_{Li}(q_H^S, q_L^S) - \big(\pi_{Hi}(q_L^S, q_H^S) - \pi_{Li}(q_L^S, q_L^S)\big) > 0,$$

where the first equality follows by substituting the T_{Hi} and T_{Li} that solve IR_L^S and IC_H^S and the second inequality follows because by assumption, $\pi_{Hi}(q_i, q_H^S) - \pi_{Li}(q_i, q_L^S)$ is increasing with q_i and because $q_H^S > q_L^S$.

The dynamic menu

The IR_H^D and the IC_L^D hold for all the numerical examples in the paper. For the general case, I show below that the IR_H^D holds. The IC_L^D holds whenever $\pi_{Hi}(q_L^D, q_H^D) - \pi_{Li}(q_L^D, q_L^D) > (\pi_{Hi}(q_L^S, q_H^S) - \pi_{Li}(q_L^S, q_L^S))$, i.e., when retailers indeed facilitate collusion and manufacturers have the incentive to adopt the dynamic menu. Moreover, the IC_L^D holds even when retailers are harmful to collusion, i.e., whenever $\pi_{Hi}(q_L^D, q_H^D) - \pi_{Li}(q_L^D, q_L^D) - (\pi_{Hi}(q_L^S, q_H^S) - \pi_{Li}(q_L^S, q_L^S)) < 0$, as long as the gap is not too high.

Consider any arbitrary dynamic menu that satisfies IR_L^D and IC_H^D in equality. Again, suppose that $q_H^D > q_L^D$ and $\pi_{Hi}(q_i, q_H^D) - \pi_{Li}(q_i, q_L^D)$ is positive and increasing with q_i. All of these assumptions are satisfied in the linear demand example for the monopoly quantities and the quantities that maximize upstream profit.

I start with IR_H^D. The IR_H^D holds when:

$$\pi_{Hi}(q_H^D, q_H^D) - T_H^D + \frac{\delta}{1-\delta}\left[p(\pi_{Hi}(q_H^D, q_H^D) - T_H^D) + (1-p)(\pi_{Li}(q_L^D, q_L^D) - T_L^D)\right]$$

$$- \left(0 + \frac{\delta}{1-\delta}\left[p(\pi_{Hi}(q_L^S, q_H^S) - \pi_{Li}(q_L^S, q_L^S))\right]\right)$$

$$= \frac{\pi_{Hi}(q_L^D, q_H^D) - \pi_{Li}(q_L^D, q_L^D) + \delta p\left(\pi_{Hi}(q_L^S, q_H^S) - \pi_{Li}(q_L^S, q_L^S)\right)}{1+\delta p} > 0,$$

where the first equality follows by substituting the T_{Hi} and T_{Li} that solve IR_L^D and IC_H^D and the second inequality follows by assumption.

Turning to IC_L^S, this condition holds if this term is positive:

$$\pi_{Li}(q_L^D, q_L^D) - T_L^D + \frac{\delta}{1-\delta}\left[p(\pi_{Hi}(q_H^D, q_H^D) - T_H^D) + (1-p)(\pi_{Li}(q_L^D, q_L^D) - T_L^D)\right]$$

$$- \left(\pi_{Li}(q_H^D, q_H^D) - T_H^D + \frac{\delta}{1-\delta}\left[p(\pi_{Hi}(q_L^S, q_H^S) - \pi_{Li}(q_L^S, q_L^S))\right]\right)$$

$$= \frac{\pi_{Hi}(q_H^D, q_H^D) - \pi_{Li}(q_H^D, q_L^D) - \left(\pi_{Hi}(q_L^D, q_H^D) - \pi_{Li}(q_L^D, q_L^D)\right)}{1+\delta p}$$

$$+ \frac{\delta p\left[\pi_{Hi}(q_H^D, q_H^D) - \pi_{Li}(q_H^D, q_L^D) + \pi_{Hi}(q_L^D, q_H^D) - \pi_{Li}(q_L^D, q_L^D) - 2(\pi_{Hi}(q_L^S, q_H^S) - \pi_{Li}(q_L^S, q_L^S))\right]}{1+\delta p}.$$

The first equality follows from substituting the T_{Hi} and T_{Li} that solve IR_L^D and IC_H^D. The third line is positive because $q_H^D > q_L^D$ and $\pi_{Hi}(q_i, q_H^D) - \pi_{Li}(q_i, q_L^D)$ is increasing with q_i. The term in the squared brackets of the forth line satisfies:

$$\pi_{Hi}(q_H^D, q_H^D) - \pi_{Li}(q_H^D, q_L^D) + (\pi_{Hi}(q_L^D, q_H^D) - \pi_{Li}(q_L^D, q_L^D)) - 2(\pi_{Hi}(q_L^S, q_H^S) - \pi_{Li}(q_L^S, q_L^S))$$

$$> \pi_{Hi}(q_L^D, q_H^D) - \pi_{Li}(q_L^D, q_L^D) + (\pi_{Hi}(q_L^D, q_H^D) - \pi_{Li}(q_L^D, q_L^D)) - 2(\pi_{Hi}(q_L^S, q_H^S) - \pi_{Li}(q_L^S, q_L^S))$$

$$= 2\left((\pi_{Hi}(q_L^D, q_H^D) - \pi_{Li}(q_L^D, q_L^D)) - (\pi_{Hi}(q_L^S, q_H^S) - \pi_{Li}(q_L^S, q_L^S))\right),$$

where the inequality follows because $q_H^D > q_L^D$ and $\pi_{Hi}(q_i, q_H^D) - \pi_{Li}(q_i, q_L^D)$ is increasing with q_i. The last term is positive when $\pi_{Hi}(q_L^D, q_H^D) - \pi_{Li}(q_L^D, q_L^D) > (\pi_{Hi}(q_L^S, q_H^S) - \pi_{Li}(q_L^S, q_L^S))$. Because the previous inequalities are strict, it follows that IC_L^D is satisfied when $\pi_{Hi}(q_L^D, q_H^D) - \pi_{Li}(q_L^D, q_L^D) > (\pi_{Hi}(q_L^S, q_H^S) - \pi_{Li}(q_L^S, q_L^S))$ or when $\pi_{Hi}(q_L^D, q_H^D) - \pi_{Li}(q_L^D, q_L^D) - (\pi_{Hi}(q_L^S, q_H^S) -$

$\pi_{Li}(q_L^S, q_L^S)) < 0$ but the gap is not too large.

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