On the robustness of the full-information separating equilibrium in multi-sender signaling games

by

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***Abstract*:** This paper considers a signaling game between $N \geq 2$ competing senders that have common private information and receivers that can observe the senders' individual signals. I define the minimal restriction on out-of-equilibrium beliefs that eliminates all separating equilibria but the full-information. This restriction is based on ε-continuous beliefs and is supported by experimental evidence. I show that for $N > 2$, the full-information separating equilibrium that survives the ε-continuous requirement exists regardless of senders' preferences concerning receivers' beliefs.

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1. Introduction

A common feature of signaling games is that senders typically need to distort their actions away from their full information levels in order to credibly signal their private information to receivers. However, most previous literature on signaling games has focused on cases where there is only one privately informed sender. In their seminal paper, Bagwell and Ramey (1991) consider a signaling game with two senders, in the context of an Industrial Organization model with two competing incumbents that use prices to signal their costs to an uniformed potential entrant. Bagwell and Ramey focus on separating equilibria that satisfy an "unprejudiced belief" refinement according to which if one of the incumbent sets an out-of-equilibrium price, then as long as the price of the second incumbent corresponds to the separating price in one of the states, the entrant should ignore the unilateral deviation of the first incumbent and believe that the state is as signaled by the second incumbent. They show that the only separating equilibrium that survives the unprejudiced beliefs refinement is the full information equilibrium. Moreover, the full information equilibrium in their model is always separating.

This result raises two interesting questions that I address in this paper. First, the unprejudiced belief refinement has the deterministic and somewhat unintuitive feature that receivers completely ignore a unilateral deviation from the equilibrium path regardless of the extent of the deviation, and blindly believe the signal of the sender that played the equilibrium strategy. In an experimental examination of Bagwell and Ramey's paper, Müller et al. (2006) do not find empirical support for this conjecture. In practice, subjects in their experiments have by and large adopted a bipolar beliefs according to which they divided the space of potential signals into two "zones", each zone corresponded to a different state, and attached a high probability for the corresponding state in each zone. A small and insignificant unilateral deviation from the separating equilibrium had negligible effect (if at all) on the beliefs because the new pair of signals remained in the same zone as the equilibrium signal. However, a large unilateral deviation shifted the pair of strategies to the other zone, resulting in a dramatic change in beliefs.[1] Nonetheless, subjects played by and large the full information equilibrium even under asymmetric information. This raises the question of what happens if beliefs are unprejudiced only for a small unilateral deviation, and what are the exact condition that eliminates all separating equilibria but the full information.

Second, even if the full information equilibrium is the only plausible separating equilibrium, the full information equilibrium may not always be separating. In particular, previous literature have shown that if senders differ in their preferences concerning receivers' beliefs, then under some model specification it is impossible to find beliefs that support the full information equilibrium as separating. For example, Schultz (1999) extends Bagwell and Ramey's paper to the case where one incumbent wants to deter entry while the other incumbent wants to accommodate entry. In this case, the two senders (incumbents) have conflicting interests concerning the receiver's beliefs: one incumbent wants to signal that costs are low (such that entry is profitable) while the other incumbent wants to signal the opposite thing. Schultz show that in this case the full information equilibrium may not always be separating. In the context of Political Economy, Schultz (1996) show that if two political parties are privately informed about the costs of producing a pubic good, then the full information equilibrium may not be separating if the parties defer in their ideology concerning the importance of this good. In the context of advertising, Hertzendorf and Overgaard (2001) and Fluet and Garella (2002) show that the full information equilibrium may not be separating when two firms signal their qualities using prices and uninformative advertising. Finally, Kim (2003) show that the full information equilibrium may not be separating in a general framework of two competing senders. These results are puzzling because they show that in a multi-sender setting, asymmetric information either has no

[1] In Bagwell and Ramey's model, incumbents have the same costs that can be either high or low, and the entrant wants to enter if costs are low. Müller et al. find that under asymmetric information, subjects divided the space of the potential price combinations of the two incumbents to "low" price pairs that were associated with low costs and thereby with high entry rates by entrants, and "high" price pairs that were associated with high costs and thereby with low entry rates.

effect in that the full information equilibrium is separating, or has a significant effect in that there is no "reasonable" separating equilibrium at all.[2] This raises the question of what are the general conditions under which the full information equilibrium is indeed separating.

In this paper I consider a signaling game between $N \geq 2$ competing senders that have common private information. I establish two main results. First, I propose an equilibrium refinement which is based on a "small" deviation from the equilibrium path. This refinement is a generalization of both Bagwell and Ramey's unprejudiced beliefs and the bipolar beliefs found in Müller et al. (2006), and eliminates all possible separating equilibria but the full information. Second, I show that the full information equilibrium satisfy this refinement if $N > 2$. This last result stands in stark contrast with previous literature that showed that in some cases it is impossible to find beliefs that support the full information equilibrium as separating. The difference in results emerges because previous literature have focused on the case of $N = 2$, which turns out to be a crucial assumption.

The rest of the paper is organized as follows. In Section 2 I describe the model. Section 3 presents an ε-continuous beliefs refinement and show that it eliminates separating equilibria but the full information. Section 4 shows that for $N > 2$, such equilibrium exists. In Section 5 I discuss the implication of the results to the theoretical research on signaling games.

2. The model

Consider a signaling game between $N \geq 2$ senders, $(z_1,\ldots,z_N) = Z$, that observe a common state, $\theta \in \{A, B\}$, and receivers that do not observe θ. In the first stage, senders observe θ and choose their strategies, $s_1,\ldots,s_N$, simultaneously and non-cooperatively, where $s_i \in \Re$. In the second stage,

[2] Hertzendorf and Overgaard (2001) solve this puzzle by introducing a restricted version of unprejudiced beliefs according to which a unilateral deviation does not change beliefs only if the deviation is to a strategy belonging to another separating equilibrium. They show that there is a unique separating equilibrium that survives this refinement in which prices, advertising, or both are distorted. Bontems and Meunier (forthcoming) consider this restricted version of unprejudiced beliefs in the context of a spatial competition model and show that separating equilibrium will result in a distortion in the location choice of the firms.

receivers observe the vector of strategies $S = (s_1,...,s_N)$ but not θ, update their prior beliefs concerning θ and choose their strategies, $v_1,\ldots v_M$, simultaneously and non-cooperatively. Note that I assume that receivers can observe the individual strategy of each sender, and that θ is identical and known to all senders. These two assumptions are crucial for the results, as I will discuss in the Conclusion. The payoffs of z_i and each receiver are $\tilde{\pi}_i(s_1,...,s_N, v_1,\ldots v_M, \theta)$ and $U_i(s_1,...,s_N, v_1,\ldots v_M, \theta)$ respectively.

This setting is applicable to numerous cases. In the context of Industrial Organization, senders can represent incumbents that are privately informed about a common industry parameter such as production costs or demand, and the receivers can represent potential entrants. In this case s_i can represent the price of incumbent i, and v_i can represent the entry decision of each receiver. In the context of Political Economy, senders can represent political candidates that are privately informed about the social costs of investing in a certain public good, while receivers are voters. In this example s_i can represent senders' declared policy, or any other action the depends on the true costs of the project, and v_i can represent voters' choice between candidates. Note that s_i can also represent a vector of strategies. For example, in the context of uninformative advertising, two competing firms may use both prices and advertising as a signal of quality. In this case we can think of $s_i = \{p_i, A_i\}$, where p_i and A_i are the price and advertising expenditure of firm i.

Let $\alpha(S) \in [0,1]$ denote the receivers' posterior probability that $\theta = A$ given S, where the posterior probability that the state is B is $1 - \alpha(S)$. Thus in the second stage each receiver solves:

$$v_i = \arg\max_{v_i} \alpha(S) U_i(S, v_1, ..., v_M, A) + (1 - \alpha(S)) U_i(S, v_1, ..., v_M, B). \tag{1}$$

Let $V(S,\alpha(S)) = (v_1(S,\alpha(S)),\ldots,v_M(S,\alpha(S)))$ denote the vector of second – stage strategies that solve (1). Note that S affects $V(S,\alpha(S))$ both directly because receivers' strategies may depend on the senders' strategies, and indirectly through $\alpha(S)$, because receivers' strategies also depend on their posterior beliefs concerning θ, which in turn depend on the observed strategies S. Turning to the first stage, each sender earns $\tilde{\pi}_i\,(S,V(S,\ \alpha(S),\theta)$. To facilitate exposition, I can rewrite each senders' payoff as $\pi_i(S,\alpha(S),\theta) \equiv \tilde{\pi}_i\,(S,V(S,\alpha(S),\theta)$. Hence the first entry in $\pi_i(S,\alpha(S),\theta)$ indicates the direct effect of S on z_i's payoff through both senders and receivers actions. The second entry indicates the net effect of S on z_i's payoff through the effect of S on receivers' beliefs. Let s_{-i} denote the vector of senders' strategies excluding s_i. I make three mild assumptions concerning senders' payoff:

Assumption 1: $\partial^2\pi_i(s_i, s_{-i}, \alpha(S),\theta)/\partial^2 s_i < 0,\ \forall z_i \in Z$.

Assumption 2: $\pi_i(S, \alpha(S),\theta)$ is monotonic in $\alpha(S)$, $\forall z_i \in Z$.

Assumption 3: $\partial\pi_i(s_i, s_{-i}, 1,A)/\partial s_i \neq \partial\pi_i(s_i, s_{-i}, 0,B)/\partial s_i,\ \forall z_i \in Z$.

The first assumption requires that $\pi_i(s_i, s_{-i}, \alpha(S),\theta)$ is concave in s_i, taking $\alpha(S)$ as given. The second assumption requires that $\pi_i(S,\alpha(S),\theta)$ is either weakly increasing in $\alpha(S)$ for all $\alpha(S) \in [0,1]$, or weakly decreasing. This assumption allows for the possibility that all senders have the same preferences over receivers' beliefs. For example, in the case of limit pricing, potential entrants may choose to stay out if they believe that industry costs are high, and thereby the payoff of all incumbents are increasing in the posterior probability that entrants attach to the possibility that costs are high. Alternatively, the second assumption also allows for the possibility that different senders have different preferences over receivers' beliefs, such that $\pi_i(S, \alpha(S),\theta)$ is increasing with $\alpha(S)$ for some $z_i \in Z$ while decreasing with $\alpha(S)$ for another $z_j \in Z$, $j \neq i$. For example, in the context of Industrial Organization, some firms produce goods that relay on a certain technology, while other firms produce substitute goods that do not relay on this technology. If all firms have better information concerning this technology than consumers, then the first group of firms has an incentive to signal that the technology is of high quality, while the second group has the incentive to signal the opposite. In the context of Political Economy, the ideology of a certain candidate may be in favorer of producing a public good and thereby this candidate may wish to signal that the cost of this good are low, while another candidate may have the opposite ideology and may thereby wish to signal the opposite. The third assumption requires that senders' full information best response differs between states. For example, incumbents' full information best response prices typically depend of their costs and thereby differ between states. Likewise, firms' full information best response prices depends on their qualities.

Under full information, each sender takes receivers' beliefs as given and beliefs are correct: $\alpha(S) = 1$ for $\forall S$ if $\theta = A$ and $\alpha(S) = 0$ for $\forall S$ otherwise. Thereby, let $s_i(s_{-i},\theta)$ denote the full information best response of z_i, where

$$s_i(s_{-i},\theta) = \arg\max_{s_i} \pi_i(s_i, s_{-i},\alpha(\theta),\theta), \quad \alpha(\theta) = \begin{cases} 1, & if\ \theta = A, \\ 0, & if\ \theta = B. \end{cases}$$

From Assumption 1, $s_i(s_{-i},\theta)$ is uniquely defined for any s_{-i} and θ. From assumption 3, $s_i(s_{-i},A) \neq s_i(s_{-i},B)$. Let $S^*(\theta) = (s_1^*(\theta),\ldots,s_N^*(\theta))$ denotes the of full – information equilibrium, satisfying $s_i^*(\theta) = s_i(s_{-i}^*(\theta),\theta)$, $\forall z_i \in Z$. From Assumption 3, $S^*(A) \neq S^*(B)$ because $s_i^*(A) \neq s_i^*(B)$ for at least one sender.

Turning back to asymmetric information, I focus throughout this paper on separating equilibria. I use standard definition for separating equilibrium in the context of multi-senders games:[3]

Definition 1: A separating equilibrium is a pair of vectors $\{S^{**}(A), S^{**}(B)\}$ such that:

1) $s_i^{**}(A) \neq s_i^{**}(B)$ for at least one $z_i \in Z$ (but not necessarily all).

2) $\alpha(S^{**}(A)) = 1$ and $\alpha(S^{**}(B)) = 0$.

3) $s_i^{**} = \arg\max_{s_i} \pi_i(s_i, s_{-i},\alpha(s_i, s_{-i}),\theta)$, $\forall z_i \in Z$.

[3] See for example Bagwell and Ramey (1991).

Since $N \geq 2$, Definition 1 does not require that all senders should play different strategies in different states: it is sufficient that one sender play $s_i^{**}(A) \neq s_i^{**}(B)$ for receivers to infer the true type from this sender. I thereby distinguish between different "types" of separating equilibria as follows:

Definition 2: A separating equilibrium is "*n-sided*" if $1 \leq n < N$ senders play $s_i^{**}(A) \neq s_i^{**}(B)$, while $N - n$ senders play $s_i^{**}(A) = s_i^{**}(B)$. An equilibrium is *fully* separating if $n = N$.

Note that any n – sided separating equilibrium satisfies Definition 1. Fully separating equilibrium is also a private case of Definition 1 in which all senders play different strategies in different states.

3. Equilibrium refinement

Out-of-equilibrium beliefs are any $\alpha(S)$ for $S \neq S^{**}(\theta)$. Definition 1 does not restrict the choice of $\alpha(\mathrm{S})$ for $S \neq S^{**}(\theta)$. Clearly, different assumptions concerning out-of-equilibrium beliefs may give raise to different equilibria. To refine the set of potential equilibria, I propose the following generalization of Bagwell and Ramey's unprejudiced beliefs refinement:

Definition 3: $\alpha(S)$ is "ε - *continuous*" if there is a sufficiently small ε such that for all $z_i \in Z$, if $s_{-i}^{**}(A) \neq s_{-i}^{**}(B)$ for at least one z_{-i}:

1) $\alpha(s_i^{**}(A) \pm \varepsilon, s_{-i}^{**}(A)) = 1$ if $s_i^{**}(A) \pm \varepsilon \neq s_i^{**}(B)$,

2) $\alpha(s_i^{**}(B) \pm \varepsilon, s_{-i}^{**}(B)) = 0$ if $s_i^{**}(B) \pm \varepsilon \neq s_i^{**}(A)$.

That is, suppose that in a separating equilibrium the vector $S^{**}(A)$ signals state A, but suppose that receivers observe $(s_1, s_{-1}^{**}(A))$, where $s_1 \neq \{s_1^{**}(A), s_1^{**}(B)\}$ and $s_{-1}^{**}(A) \neq s_{-1}^{**}(B)$. In this case the strategies of all senders but z_1 correspond to state is A while z_1's signal seems uninformative: it does not match any state. What should receivers believe in this case? Note that since $s_{-1}**(A) \neq s_{-1}**(B)$, the equilibrium signals of the other senders are informative: they signal that the state is A. According to Bagwell and Ramey's unprejudiced beliefs refinement, receivers should ignore the unilateral deviation by z_1 and believe that the state is A regardless of the size of the deviation. However, it is reasonable to expect that if s_1 is significantly different from $s_1**(A)$, then receivers may not completely ignore this significant deviation from the equilibrium path and place some probability on state B. Indeed, in an experimental investigation in the context of limit pricing, Müller et al. (2006) find that beliefs are bipolar and thereby can change dramatically if a unilateral deviation from separating strategy is significant enough. Nonetheless, since receivers believe that the state is A if they observe $s_1**(A)$ given that all other senders play $s_{-1}**(A)$, then by argument of continuity it is natural to expect that receivers will still place a high probability on A if they observe $(s_1, s_{-1}**(A))$, as long $s_1**(A) - \varepsilon < s_1 < s_1**(A) + \varepsilon$, if ε is not too high. Moreover, if s_1 is very close to $s_1**(A)$, in that ε is very small, then $\alpha(s_i**(A) \pm \varepsilon, s_{-i}**(A)) = 1$. The same argument applies of course for state B.

I start by applying the ε- continuous beliefs requirement on potential fully separating equilibria:

Proposition 1: *If $\alpha(S)$ is ε-continuous, then for any given ε, the only possible fully separating equilibrium, if exists, is the full information.*

Proof: Suppose by contradiction that there is a fully separating equilibrium such that $S**(\theta) \neq S*(\theta)$ and that beliefs are ε-continuous. In this case there is at least one sender satisfying $s_i**(\theta) \neq s_i(s**_{-i}(\theta),\theta)$. Without loss of generality, suppose that $s_1**(A) > s_1(s**_{-1}(A),A)$. Since by definition $s_1**(A) \neq s_1**(B)$ and $s**_{-1}(A) \neq s**_{-1}(B)$ (the equilibrium is fully separating), there is an $\varepsilon > 0$ such that $s_1**(A) - \varepsilon \neq s_1**(B)$ and $s_1**(A) -\varepsilon > s_1*(s**_{-1}(A),A)$. However,

$$\begin{aligned}
&\pi_1(s_1**(A) - \varepsilon, s_{-1}**(A), \alpha(s_1**(A) - \varepsilon, s_{-1}**(A)),A)\\
&= \pi_1(s_1**(A) - \varepsilon, s_{-1}**(A), 1,A)\\
&> \pi_1(s_1**(A), s_{-1}**(A), 1,A),
\end{aligned}$$

where the (first) equality follows since beliefs are ε-continuous and the (second) inequality follows because $s_1^{**}(A) > s_1(s^{**}_{-1}(A),A)$ and thereby $\pi_1(s_1, s_{-1}^{**}(A), 1,A)$ is monotonically decreasing with s_1 for any $s_1 > s_1(s^{**}_{-1}(A),A)$. Thereby, z_1 will deviate from $s_1^{**}(A)$ and the equilibrium fails. □

Proposition 1 shows that out-of-equilibrium beliefs that eliminate all distorted fully separating equilibria only require that a small deviation from the equilibrium vector by one sender does not, or have a negligible effect on the beliefs. The unprejudiced beliefs refinement is a special (and somewhat extreme) case of ε-continuous beliefs, in which a deviation from $s_i^{**}(\theta)$ to $s_i^{**}(\theta) \pm \varepsilon$ does not change beliefs for all ε. Bipolar beliefs proposed by Müller et al. (2006) are also a special case of ε-continuous beliefs, because a small deviation does not change the beliefs because the signal still remains within the same "zone", although a large deviation may switch the signal to another "zone" in which beliefs change dramatically. Moreover, note that the result that ε-continuous beliefs eliminate all fully separating equilibria but the full information is a general result that holds for a large set of signaling games with common private information and perfect observability of senders' individual strategies. In particular, Proposition 1 does not depend on whether senders' strategies are complements or substitutes, or whether senders have similar or conflicting preferences concerning receivers' beliefs.

Next consider n – sided separating equilibria. If $n = 1$, such that only one sender separates between states, then clearly we cannot apply ε-continuous beliefs for deviations by this sender, implying that the equilibrium strategies of this sender can be distorted away from the full information best response. However, if $n > 1$, then for any z_i, regardless of whether this z_i pools or separates, there is at least one z_{-i} such that $s_{-i}^{**}(A) \neq s_{-i}^{**}(B)$, from which receivers can infer the true state even if z_i slightly deviates form its equilibrium strategy. Consequently:

Proposition 2: *If* $\alpha(S)$ *is* ε*-continuous, then for any given* ε*, the only possible n-sided separating equilibrium with* $n > 1$*, if exist, is the full information.*

Proof: Suppose by contradiction that there is an n – sided separating equilibrium such that $S^{**}(\theta) \neq S^*(\theta)$, beliefs are ε-continuous, and $n > 1$. Without loss of generality, suppose that for z_1, $s_1^{**}(A) > s_1(s^{**}_{-1}(A),A)$. If $s_1^{**}(A) \neq s_1^{**}(B)$, it follows directly from the proof of Proposition 1 that z_1 will play its full information best response strategy. If $s_1^{**}(A) = s_1^{**}(B)$, then z_1 does not change beliefs by deviating to $s_1^{**}(A) - \varepsilon$ because $s_1^{**}(A) - \varepsilon \neq s_1^{**}(B)$ and $s_{-1}^{**}(A) \neq s_{-1}^{**}(B)$, and I can apply the rest of the proof of Proposition 1.

4. Existence

Next I turn to the question of existence. Since ε-continuous beliefs only allow for the full information equilibrium, a separating equilibrium which is ε-continuous may potentially not exist. To prove existence, note first that from Assumption 3, $S^*(A) \neq S^*(B)$ for at least one sender, implying that the first condition in Definition 1 is satisfied by any full information equilibrium. This leaves us with the task of finding beliefs that ensure that both the third condition in Definition 1 and Definition 3 are satisfied for the full information strategies.

To this end, suppose first that the full information equilibrium is potentially fully separating in that $s_i^*(A) \neq s_i^*(B)$, $\forall i \in S$. Notice that if a certain z_i prefers, say, state A (in that $\pi_i(S,\alpha(S),\theta)$ is increasing with ,$\alpha(S)$), then in state A, z_i will never deviate from $s_i^*(A)$. Intuitively, in this case along the equilibrium path z_i both plays the best response to $s_{-i}^*(A)$ and beliefs are in z_i's favor. However, if the state is B, then this z_i may wish to deviate from $s_i^*(B)$ if doing so mislead receivers into increasing their posterior probability that the state is A. The binding constraints are therefore that such deviations are unprofitable for all senders. Indeed, previous literature has shown that for some parameters, it is impossible to find belief functions that prevent such deviation from the full information strategies. More precisely, define two subsets of senders,

Z^A,Z^B∈Z, such that $\pi_i(S,\alpha(S),\theta)$ is increasing with $\alpha(S)$ for $\forall z_i \in Z^A$ and decreasing with $\alpha(S)$ for $\forall z_i \in Z^B$. Let $s^{A}*(\theta)$,$s^{B}*(\theta) \in S*(\theta)$ denote the vectors of full information strategies in state θ for senders belonging to Z^A and Z^B respectively. Clearly, any $z_i \in Z^A$ will never deviate from $s_i*(A)$ and any $z_i \in Z^B$ will never deviate from $s_i*(B)$ no matter what the beliefs are, so we only need to limit beliefs such that $z_i \in Z^\theta$ will not deviate from the equilibrium in state θ'≠ θ. To this end, let

$$\alpha(s_i,s_{-i}^{A}*(B),s^{B}*(B)) = \varepsilon, \qquad \forall z_i \in Z^A, \tag{2}$$

$$\alpha(s_i,s_{-i}^{B}*(A),s^{A}*(A)) = 1 - \varepsilon, \quad \forall z_i \in Z^B. \tag{3}$$

It is possible to find a sufficiently low ε (such as ε = 0) such that in state θ' sender $z_i \in Z^\theta$ has no incentive to deviate from $s_i*(\theta')$, because doing so deflects z_i from the full information best response and does not increase the posterior probability on z_i's preferred state, θ. Thereby the constraints (2) and (3) ensure the third condition of Definition 1. However, we still need to ensure that constraints (2) and (3) never contradict each other. If $N = 2$ and, say, $z_1 \in Z^A$ while $z_2 \in Z^B$, then (2) and (3) contradict because substituting $s_i = s_1*(A)$ into (2) requires that $\alpha(s_1*(\mathrm{A}), s_2*(B))$ is close to 0 while substituting $s_i = s_2*(B)$ into (3) requires that $\alpha(s_1*(A), s_2*(B))$ is close to 1. Kim (2003) refers to this problem as "signal jamming": both senders have conflicting preferences concerning receivers' beliefs and each sender signals its own preferred state. For some parameters of the signaling model, it may be impossible to find $\alpha(s_1*(A), s_2*(B)) \in [0,1]$ that both prevents z_1 from deviating to $s_1*(A)$ when the state is B, and prevents z_2 from deviating to $s_2*(B)$ when the state is A. However, from (2) and (3) it is evident that this problem emerged because for $N = 2$, both $s_{-i}^{A}*(B)$ and $s_{-i}^{B}*(A)$ are empty. However, for $N > 2$, the number of senders in Z^θ is higher than 1 for at least one state. Thereby, either $s_{-i}^{A}*(B)$ or $s_{-i}^{B}*(A)$ (or both) are nonempty implying that signal jamming is impossible: $z_i \in Z^A$ cannot "jam" the signal of $z_j \in Z^B$ by choosing the strategy $s_i*(A)$ in state B, because there is at least one other $z_k \in Z^A$ ($k \neq i,j$) that sets in equilibrium

$s_k^*(B)$. For example, suppose that $N = 3$ and $z_1 \in Z^A$ while $z_2, z_3 \in Z^B$. Substituting $s_i = s_1^*(A)$ into (2) now requires that $\alpha(s_1^*(A), s_2^*(B), s_3^*(B))$ is close to 0 while substituting $s_i = s_2^*(B)$ into (3) requires that $\alpha(s_1^*(A), s_2^*(B), s_3^*(A))$ is close to 1, which do not contradict each other. Clearly, the same holds for any given $N > 2$. Finally, it is straightforward to see that (2) and (3) satisfy Definition 3. I summarize this result as follows:

Proposition 3: *If $N > 2$, and $s_i^*(A) \neq s_i^*(B)$, $\forall z_i \in Z$, then the full information equilibrium is fully separating and ε-continuous.*

Proposition 2 shows that the result from previous papers that for some parameters it is impossible to find beliefs that support the full information equilibrium as separating, depends heavily on the number of senders. In particular, for $N > 2$, the full information equilibrium is separating if all senders play under full information different strategies indifferent states. The existence of the separating equilibrium for $N > 2$ does not depend on senders' preferences over receivers' beliefs, or on whether senders' strategies are substitutes or complements.

Next consider the case where the full information equilibrium is potentially n – sided separating, in that $s_i^*(A) = s_i^*(B)$ for $N - n > 0$ senders. Suppose first that $n = 1$ and that z_1 plays $s_1^*(A) \neq s_1^*(B)$ and $z_1 \in Z^A$, while all other senders play s_{-1}^* in both states. In this case we only need to prevent z_1 from deviating from $s_1^*(B)$ in state B (recall that $z_1 \in Z^A$ will never deviate from $s_1^*(A)$ in state A no matter what beliefs are). Applying (2) to this case, $\alpha(s_1, s_{-1}^*) = \varepsilon$ for any $s_1 \neq s_1^*(A)$ will prevent z_1 from deviating to any other $s_1 \neq s_1^*(A)$ in state B if ε is sufficiently low, but condition (2) in Definition 1 still requires that $\alpha(s_1^*(A), s_{-1}^*) = 1$, thereby z_1 may deviate to $s_1^*(A)$ in state B if the benefit from misleading receivers that the state is A outweighs the loss from deviating from z_1's full information best response. Thus, the full information equilibrium may not be 1 – sided separating, depending on z_1's payoff function.

A similar problem emerges if $n = 2$ senders separate under full information, say, z_1 and z_2 where $z_1 \in Z^A$ and $z_2 \in Z^B$. In this case, we need to define some $\alpha(s_1^*(A), s_2(B), s_j^*) \in [0, 1]$ that on one hand will be small enough to ensure that z_1 will not deviate to $s_1^*(A)$ in state B but at the same time be high enough to ensure that z_2 will not deviate to $s_2^*(B)$ in state A. Beliefs $\alpha(s_1^*(A), s_2(B), s_j^*) \in [0, 1]$ that prevent this signal jamming possibility may or may not exist depending on senders' payoff functions.

However, following the same argument behind Proposition 2, this problem disappears if $n > 2$. In this case we can apply the out-of-equilibrium beliefs defined in (2) and (3). Although not all senders separate in this case, still z_i cannot jam the full information equilibrium signal of other senders because there is at least one other sender that separates and share the same preferences as z_i concerning receivers beliefs. More precisely, let s^{P*} denote the vector of full information strategies for senders that play under full information $s_i^*(A) = s_i^*(B)$. Then, for any z_i that plays $s_i^*(A) \neq s_i^*(B)$, let:

$$\alpha(s_i, s_{-i}^{A*}(B), s^{B*}(B), s^{P*}) = \varepsilon, \qquad \forall z_i \in Z^A, \tag{4}$$

$$\alpha(s_i, s_{-i}^{A*}(A), s^{B*}(A), s^{P*}) = 1 - \varepsilon, \qquad \forall z_i \in Z^B, \tag{5}$$

and for any z_i that plays $s_i^{P*} = s_i^*(A) = s_i^*(B)$,let

$$\alpha(s_i, s_{-i}^*(\mathrm{B})) = \varepsilon, \qquad \forall z_i \in Z^A, \tag{6}$$

$$\alpha(s_i, s_{-i}^*(A)) = 1 - \varepsilon, \qquad \forall z_i \in Z^B. \tag{7}$$

For a sufficiently small ε, no sender can change beliefs in its favor by deviating from its full information best response and since $n > 2$, (4) - (7) never contradict each other. Finally, these beliefs satisfy Definition 3.

Proposition 4: *Suppose that under full information $n < N$ senders play $s_i*(A) \neq s_i*(B)$ and $N - n$ senders play s_i* in both states. Then the full information equilibrium is n – sided separating and* ε *- continuous if* $n > 2$.

5. Discussion

This paper shows that in multi-senders signaling games, it is possible to find beliefs that support the full information equilibrium as separating if the number of senders is high enough. Moreover, the full information equilibrium is the only separating equilibrium that survives a simple and intuitive ε-continuous refinement, that coincides with the experimental results reported by Müller et al. (2006). From first glance, these results undermine the importance of theoretical research concerning signaling games with multiple senders, because they show that compared with full information, asymmetric information actually has no effect on market performance. However, recall that the results of this paper relay on three important features. First, the number of senders has to be higher than 2 (and in the case of n – sided separating equilibria, higher than 3). In contrast, in many real-life situations the number of senders is indeed low. For example, in many countries there are only two main political candidates or parties. Likewise, entry deterrence is mainly relevant in markets in their premature stage, i. e. when then number of incumbents is low. This makes the analysis of two-sender games important, although it should be taken into account that the results obtained from these games may not hold as the number of senders increase.

Second, the results depend on the assumption of perfect observability of the individual signal of each sender. In some cases, receivers may not be able to distinguish between individual signals and instead observe only an aggregate composition of these signals. With imperfect observability of individual signals, each sender cannot free-ride on the signal set by competing senders because receivers cannot detect a unilateral deviation from the equilibrium path. Consequently ε-continuous beliefs has no force, and other separating equilibria apart from the full information are possible. For example, Harrington (1987) considers a Cournot oligopoly limit pricing model with

N incumbents that are privately informed about costs and compete by setting quantities, and a potential entrant that can only observe the market price and not individual quantities. Harrington shows that in the separating equilibrium, incumbents' strategies are distorted away from the full information strategies even if $N > 2$.

Third, the results clearly depend on the assumption of common private information, such that asymmetric information is only between senders and receivers, and not between senders and themselves. If each sender is privately informed concerning an individual characteristic, each sender cannot free-ride on the signal of other senders because these signals are uninformative concerning this sender's type. For example, in the context of price competition between firms that each is privately informed about it own quality, Daughety and Reinganum (2006) find upwards price distortion in the context of two firms, and Daughety and Reinganum (forthcoming) find upwards price distortion for $N \geq 2$ firms.

References

Bagwell K. and G. Ramey (1991), "Oligopoly Limit Pricing," *Rand Journal of Economics*, 22, 155-172.

Bontems, P. and Meunier V. "Advertising and Price Signaling of Quality in A Duopoly with Endogenous Locations," forthcoming, *Journal of Economics and Management Strategy*.

Daughety, Andrew F. and Jennifer F. Reinganum, "Competition and Confidentiality: Signaling Quality in a Duopoly When There is Universal Private Information," forthcoming, *Games and Economic Behavior*.

Daughety, Andrew F. and Jennifer F. Reinganum, (2006), "Imperfect Competition and Quality Signaling," working paper.

Fluet C. and P. Garella (2002), "Advertising and Prices as Signals of Quality in a Regime of Price Rivalry," *International Journal of Industrial Organization*, 20, 907-930.

Harrington J. (1987), "Oligopolistic Entry Deterrence under Incomplete Information," *Rand Journal of Economics,* 18, 211-231.

Hertzendorf M. and P. Overgaard (2001), "Price Competition and Advertising Signals: Signaling by Competing Senders," *Journal of Economics and Management Strategy*, 10, 621-662.

Kim J-Y. (2003), "Signal Jamming in Games with Multiple Senders," *Contribution to Theoretical Economics*, 2, 1 – 15.

Martin S. (1995), "Oligopoly Limit pricing: Strategic Substitutes, Strategic Complements," *International Journal of Industrial Organization*, 13, 41-65.

Müller W., Spiegel Y. and Yehezkel Y. (2006), "Oligopoly limit-pricing in the lab," working paper.

Schultz C. (1996), "Polarization and Inefficient Policies," *Review of Economic Studies*, 63, 331-344.

Schultz C. (1999), "Limit Pricing when Incumbents have Conflicting Interests," *International Journal of Industrial Organization*, 17, 801-825.