Group Hug: Platform Competition with User-group

Online Appendix B: Absolute Number of Group and Individual Users

By

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1 Introduction

In our base model we measure the size of the group in proportional terms. Alternatively, one could assume a market with x group members and y individual users. In this case, however, changes in the size of the group are confounded with changes in network effects. In this appendix, we extend our model to the case where the size of the individual users is not affected by a change in the size of the group. We show that, as in our base model, a pivotal group joins the low-quality platform when it is not too large relative to the size of the individual users. The larger the number of individual users, the larger the group needs to be to choose the efficient platform. Moreover, the equilibrium utility of an individual user is independent of the size of the group as their utility is only a function of the network effects they create to each other. The utility of a group user is, in general, increasing in the group size with the exception of the case where the quality difference across the platforms is relatively large. Total consumers surplus, platforms' profits, and thus total welfare, are always increasing in the size of the group as it does not imply a decrease in the size of the individual users.

2 Model

Consider our linear example: $V_A(n_A) = \lambda n_A$ and $V_B(n_B) = Q + \lambda n_B$, where λ represents the network effects and Q the relative quality advantage platform B offers. Further, assume that there is a fixed mass of individual users of size y and a group of size x such

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that the overall size of the market is $x + y$. Under this setting, a change in the size of the group does not affect the size of the individual users. Rather, a change in the size of the group changes the overall size of the market. This setup allows us to disentangle the effect of changes in the size of the group from changes in the size of the individual users.

As in our base model, suppose that network effects are more important to individual users than platform B's quality advantage: $0 < Q < \lambda y$. This is an equivalent assumption to the one in our base model, where we assume that $0 < Q < \lambda$. This assumption implies that without the group, platform B cannot win the individual users. The timing of the game is the same as in our base model: platforms first compete on the group and then compete on individual users.

2.1 Solution to the second-stage: competition on the individual users

As in our base model, we start with solving the second stage–competition over the individual users. Suppose that the group joined platform A. An equilibrium in which platform A wins the individual users satisfies the following two conditions:

$$\lambda(x + y) - p_A \geq Q - p_B, \quad p_B = 0 \quad \Longrightarrow \quad p_A = \lambda(x + y) - Q, \tag{1}$$

$$\pi_A(x, y; A) = y p_A = y\left(\lambda(x + y) - Q\right) > 0. \tag{2}$$

That is, platform A charges the highest price that ensures that individual users prefer joining its focal platform A over joining platform B; and platform B charges the lowest price that ensures non-negative profits. The second condition guarantees that platform A earns positive profit from attracting the individual users. Since $Q < \lambda y$, the inequality in equation (2) holds for all $x > 0$.

To see that given that the group joins platform A there is no equilibrium in which platform B wins the individual users, note that if such equilibrium were to exist, $p_A = 0$ and $\lambda(x + y) - p_A = Q - p_B$, implying that $p_B = Q - \lambda(x + y)$ and platform B earns: $\pi_B(x, y; A) = y\left(Q - \lambda(x + y)\right) < 0$. That is, when platform A wins the group, it always also wins the individual users.

Suppose now that the group joins platform B. An equilibrium in which platform A

wins the individual users satisfies the following condition:

$$\lambda y - p_A \geq Q + \lambda x - p_B, \quad p_B = 0 \quad \Longrightarrow \quad p_A = \lambda(y - x) - Q, \tag{3}$$

Platform A then earns: $\pi_A(x, y; B) = y\left(\lambda(y - x) - Q\right)$. Likewise, in an equilibrium in which platform B wins the individual users, it charges and earns, respectively,

$$p_B = \lambda(x - y) + Q, \quad \pi_B(x, y; B) = y\left(\lambda(x - y) + Q\right). \tag{4}$$

Hence, platform A wins the individual users iff $y\left(\lambda(y - x) - Q\right) > 0$. As in our base model, letting $\widehat{x}$ denote the solution to $\pi_A(x, y; B) = 0$, we get that $\widehat{x} = y - \frac{Q}{\lambda}$. That is, if $x < y - \frac{Q}{\lambda}$, platform A wins the individual users regardless of the group's decision. Once the group becomes larger and $x > y - \frac{Q}{\lambda}$, the group becomes pivotal and the individual users join the platform the group joins. Notice that the threshold $\widehat{x} = y - \frac{Q}{\lambda}$ is equivalent to the threshold in subsection 4.3 in the paper. To see why, we can impose the restriction $x + y = 1$ by substituting $y = 1 - x$ into $x = y - \frac{Q}{\lambda}$ and obtain $x = \frac{1}{2} - \frac{Q}{2\lambda}$, which is the same threshold as in example 4.3. The quality gap Q and the degree of network effects λ affect $\widehat{x}$ in a qualitatively similar way as in our base model. Moreover, $\widehat{x}$ is increasing in y. Intuitively, when the group joins platform B, the larger the number of individual users, the larger the group needs to be in order to balance platform A's focality advantage and enable platform B to attract the individual users. The following Lemma summarizes the results:

Lemma 1. ***(The group may be pivotal)*** *Suppose that there are y individual users and a group of size x. Then, there is a threshold $\widehat{x} = y - \frac{Q}{\lambda}$ such that when $x < \widehat{x}$, platform A always wins the individual users. When $x > \widehat{x}$, the group is pivotal and the platform that wins the group wins the individual users.*

2.2 Solution to the first stage: competition on the group

Moving to the first stage, as in our base model, we start with the case where the group is pivotal, $x \geq \widehat{x}$. In this case, the platform that wins the group also wins the individual users. The group prefers joining platform A over joining platform B if:

$$x\lambda(x + y) - p_A^G > x(Q + \lambda(x + y)) - p_B^G. \tag{5}$$

The lowest price that platform B is willing to set for the group is its profit from winning the individual users; i.e., $p_B^G = -\pi_B(x, y; B) = -y\,(\lambda(x - y) + Q)$. Substituting p_B^G in equation (5), in an equilibrium where platform A wins the group, it charges $p_A^G = \lambda y(y - x) - Q(x + y)$.

Following the same logic, the lowest price that A is willing to set to attract the group is $p_A^G = -y\,(\lambda(x + y) - Q)$. Substituting this p_A^G in equation (5), in an equilibrium where B wins the group, B sets $p_B^G = (x + y)(Q - \lambda y)$. Substituting these prices into $\Pi_A(x, y; A) = \pi_A(x, y; A) + p_A^G$, and $\Pi_B(x, y; B) = \pi_B(x, y; B) + p_A^G$ and rearranging the terms we get that $\Pi_B(x, y; B) = -\Pi_A(x, y; A)$ and platform A wins iff $x < 2y(\frac{\lambda y}{Q} - 1)$ while platform B wins otherwise.[3] The following proposition summarizes the results.

Proposition 1. ***(A pivotal group may join the low-quality platform)*** *Suppose that there are y individual users and a group of size x such that the group is pivotal, $x \geq \widehat{x}$. Then, there is a threshold, $\widetilde{x} = 2y(\frac{\lambda y}{Q} - 1)$, where $\widehat{x} < \widetilde{x}$, such that if $x < \widetilde{x}$ ($x > \widetilde{x}$) platform A (B) wins the group and the individual users.*

Figure 1 illustrates the thresholds $\widehat{x}$ and $\widetilde{x}$ as a function of the quality gap between the two platforms, adjusted by the level of network effects (i.e., Q/λ) for two different individuals size: $y = 0.4$ and 0.5. The figure shows that, just like in our base model, as the quality gap between the platforms increases, the range within which the group is pivotal yet chooses the inefficient platform, $[\widehat{x}, \widetilde{x})$, becomes smaller and the range of group sizes that result in an efficient choice of platform B increases. Moreover, as Q/λ approaches y, $\widetilde{x}$ and $\widehat{x}$ approach 0, in a similar way to which $\widetilde{x}$ and $\widehat{x}$ approach 0 as Q/λ approaches 1 in our base model (recall that this appendix assumes that $Q/\lambda < y$ while our base model assumes that $Q/\lambda < 1$). Turning to changes in the absolute number of individual users, we have that both thresholds $\widehat{x}$ and $\widetilde{x}$ are increasing with the absolute number of individual users. Intuitively, a high number of individual users reduces the proportional size of the group and hence makes it more difficult for platform B to use the group for winning the individual users. As a result, the inefficient range, $[\widehat{x}, \widetilde{x})$, expands with an increase in y.

For completeness, suppose now that the group is not pivotal ($x < \widehat{x}$). In this case, platform A wins the individual users, regardless of the group's decision. In an equilibrium in which platform A wins the group, the group prefers joining A over joining

[3] It is straightforward to show that imposing $x + y = 1$ by substituting $y = 1 - x$ into $x = 2y(\frac{\lambda y}{Q} - 1)$ and solving for x, we have the same $\widetilde{x}$ as in our base model.

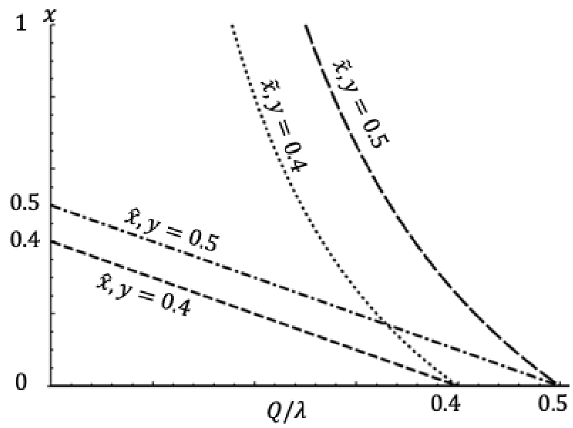

Figure 1: $\widehat{x}$ and $\widetilde{x}$ as a function of Q/λ

B if:

$$\lambda x(x+y) - p_A^G \geq x(Q+\lambda x) - p_B^G \quad, p_B^G = 0 \Longrightarrow \quad p_A^G = x\left(\lambda y - Q\right), \tag{6}$$

and platform A earns higher profit if it wins the group than if it does not attract it:

$$\pi_A(x,y;A) + p_A^G \geq \pi_A(x,y;B)$$
$$\Downarrow$$
$$x(3y\lambda - Q) \geq 0,$$

which is always positive because by assumption $\lambda y > Q$.

Next, we show that there is no equilibrium in which platform B wins the group. In this equilibrium, the lowest price that platform A is willing to charge the group is : $p_A^G = \pi_A(x,y;B) - \pi_A(x,y;A) = -2\lambda xy$. Substituting it into (6), we have that platform B can charge the group at most $p_B^G = -x(3y\lambda - Q) < 0$. As platform B cannot win the individual users when attracting the group, platform B cannot profitably win the group.

We therefore have that, just like in the base model, when the group is not pivotal, there is a unique equilibrium where platform A wins the entire market.

Proposition 2. *Suppose that there are y individual users and a group of size x such that the group is not pivotal, $x < \widehat{x}$. Then, there is a a unique equilibrium in which platform A wins the group and individual users.*

3 Utility of an individual user

The utility of each individual user is $u(x,y) = \lambda(x+y) - p_A$ if A wins, and $u(x,y) = Q + \lambda(x+y) - p_B$ if B wins. Substituting p_A and p_B, given by equations (1) and (4), we get that the utility of an individual user is:

$$u(x,y) = \begin{cases} Q, & \text{if } x < \widehat{x}, \\ 2\lambda y, & \text{otherwise.} \end{cases} \tag{7}$$

As in our base model, as long as platform A wins, the utility of individual users remains fixed at Q and jumps up once the group is pivotal and joins platform B. Unlike our base model, however, when $x > \widetilde{x}$ an increase in the size of the group does not affect the utility of individual users, as their utility only depends on the network effect they create to each other. That is, in the case where the size of the individual users remains unchanged, the size of the group affects the utility of an individual users only through the groups' platform choice. As expected, as the size of individuals users (y) increases, so does the individual utility (see Figure 2).

4 Utility of a single group user

Moving to the utility of a single group user, each group user enjoys $u^G(x,y) = \lambda(x+y) - \frac{p_A^G}{x}$ if A wins, and $u^G(x,y) = Q + \lambda(x+y) - \frac{p_B^G}{x}$ if B wins, where p_A^G is given by (6) when $x \in [0, \widehat{x})$, $p_A^G = \lambda y(y-x) - Q(x+y)$ when $x \in [\widehat{x}, \widetilde{x}]$, and $p_B^G = (x+y)(Q - \lambda y)$ when $x > \widetilde{x}$. Hence,

$$u^G(x,y) = \begin{cases} Q + \lambda x, & \textit{if } x \in [0, \widehat{x}), \\ \lambda(x+2y) + \frac{Q(x+y) - \lambda y^2}{x}, & \textit{if } x \in [\widehat{x}, \widetilde{x}), \\ \frac{\lambda(x+y)^2 - Qy}{x}, & \textit{if } x > \widetilde{x}. \end{cases} \tag{8}$$

It is easy to see from equation (8) that when $x \leq \widetilde{x}$, the utility of a group user increases with the size of the group. This is consistent with our base model and the

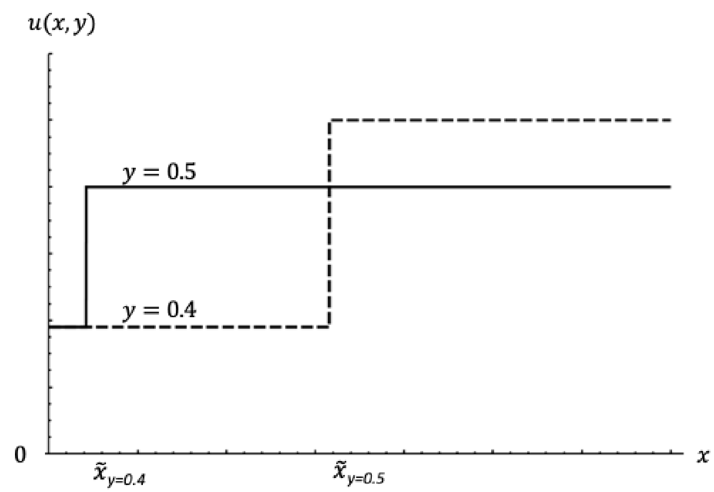

Figure 2: Individual user's utility as a function of x

Individual user's utility when $\frac{Q}{\lambda} = 0.38$ *and* y=*{0.4,0.5}*.

intuition is the same: a larger group has a better outside option of joining platform B. Also, as in our base model there is a discontinuous climb at $\widehat{x}$, when the group becomes pivotal. For $x \in [\widehat{x}, \widetilde{x})$, the utility always increases with x. This differs from the case of a proportional increase in the group. As we explain in the paper, when $x \in [\widehat{x}, \widetilde{x})$ and the proportion of a pivotal group increases, on one hand its alternative option increases but at the same time the decrease in the proportion of the individual users decreases the group's market power over platform A. The second effect vanishes when we consider an absolute increase in the size of the group while keeping the size of the individual users constant. The effect of the size of the group when $x > \widetilde{x}$ is more subtle. Specifically, for group size close to $\widetilde{x}$, a group user's utility either increases with the size of the group or first decreases and then increases in it. Figure 3 presents the utility of a group user for the case where it always increases ($y = 0.5$) as well as for the case where in the area of $\widetilde{x}$ the utility first decreases and then increases ($y = 0.4$). Intuitively, as in our base model, the group needs platform B for its superior quality while platform B needs the group for attracting (and profiting from) the individual users. As x increases, the first effect becomes stronger (again, as in our base model), but now the second effect does not become weaker but instead becomes stronger because the increase in the group size does not come at the expense of the number of individual users. Hence, unlike our base model, here the utility of a group-user may decrease with x if the group is small, but

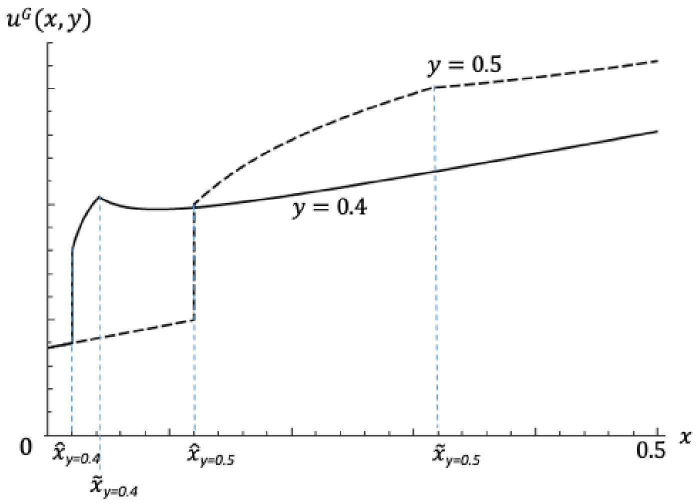

Figure 3: A group user's utility as a function of x

A group user's utility when $\frac{Q}{\lambda} = 0.38$ *and* $y=\{0.4, 0.5\}$.

increases otherwise.

5 Total consumer surplus and profits

Recall that $\Pi_i(x, y; i) \equiv \pi_i(x, y; i) + p_i^G$ denotes platform i's total profit from group and individual users for $i = \{A, B\}$ when the group chooses platform i. The platforms' profits as a function of the size of the group and individual users are then given by:

$$\Pi(x,y) = \begin{cases} (\lambda y - Q)(x+y) + \lambda xy, & \text{if } x \in [0, \widehat{x}), \\ 2y(\lambda y - Q) - Qx, & \text{if } x \in [\widehat{x}, \widetilde{x}), \\ Qx - 2y(\lambda y - Q), & \text{if } x \in [\widetilde{x}, 1]. \end{cases} \tag{9}$$

Total users' surplus is $CS(x, y) = y \times u(x, y) + x \times u^G(x, y)$. Figure 4 illustrates $CS(x, y)$ as a function of x, for two selected y values. Given the above, it is not surprising that we find that consumer surplus always increases in x. In contrast to our base model where an increase in x implies a decrease in y, when the sizes of x and y are independent, an increase in x always implies an increase in the number of users. As long as the decline in per-user utility is small, the increase in the number of overall users outweighs the decrease in per-user utility. Yet, consumer surplus may increase or decrease with

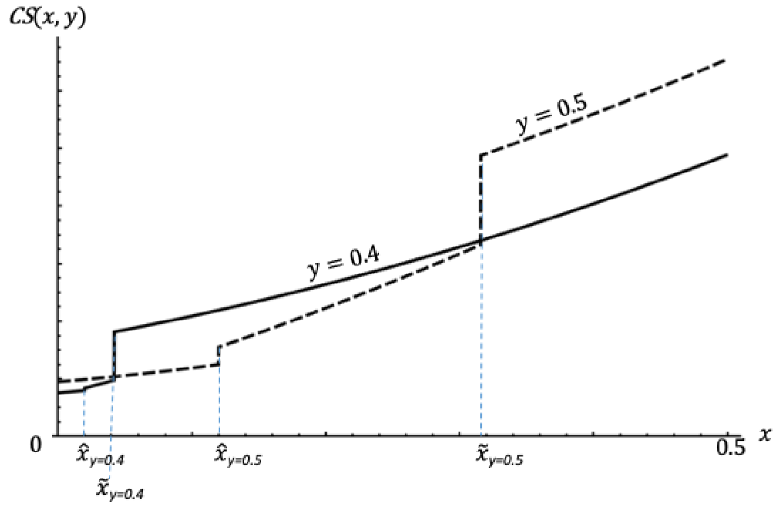

Figure 4: A group user's utility as a function of x

A group user's utility when $\frac{Q}{\lambda} = 0.38$ *and* $y=\{0.4, 0.5\}$.

y. On the one hand, an increase in y increases the threshold value $\widetilde{x}$, which reduces consumer's surplus. On the other hand, when x is sufficiently larger than $\widetilde{x}$, an increase in y increases consumer surplus because both individual and group users benefit from interacting with more individual users.