Vertical Collusion to Exclude Product Improvement

Online Appendix D: Finite game

By David Gilo[1] and Yaron Yehezkel[2]

October 2023

This Appendix solves for the case where the game is finite and has $N+1$ periods, where $N \geq 1$. We show that vertical collusion to exclude with a reduced fixed fee as a tool mitigating the distortion from persistent predatory pricing, which we identified in Section 3 of the main paper for $\delta > \widetilde{\widetilde{\delta}}$, never emerges in a finite game. This is so even when $N \to \infty$. Instead, for all values of $\delta > \widetilde{\delta}$ and $N \in [1, \infty]$, the manufacturer's only exclusionary tool is persistent predatory pricing. In particular, in order to exclude product 2, the manufacturer sets $w_1 = \widetilde{w}_1(\delta) < c_1$. This is the same predatory price used by the manufacturer for intermediate levels of δ ($\delta \in [\widetilde{\delta}, \widetilde{\widetilde{\delta}}]$) in the infinite-horizon case. Thus, for $\delta > \widetilde{\widetilde{\delta}}$, while exclusion in the infinite-horizon case is achieved via a lower fixed fee, and a less predatory wholesale price ($w_{12} > \widetilde{w}_1(\delta)$), in the finite game it involves only the more predatory wholesale price $\widetilde{w}_1(\delta)$. This result indicates that in the infinite game, the exclusionary equilibrium for $\delta > \widetilde{\widetilde{\delta}}$ involving vertical collusion to exclude relies on trust between the retailer and the manufacturer that cannot exist when the game is expected to end at some point. It further implies that the threshold of δ above which the manufacturer accommodates product 2, δ^{VS}, is higher in the infinite game than in the finite game for $N \to \infty$. That is, it is more profitable for the manufacturer to exclude product 2 in an infinite game than in a finite game in which the number of periods approaches infinity.

We start by solving for the exclusionary contract. When the retailer excludes product 2 throughout the game, in the last period product 2 is no longer improvable, so the manufacturer exploits

[1]Buchmann Faculty of Law, Tel Aviv University (email: gilod@tauex.tau.ac.il)
[2]Coller School of Management, Tel Aviv University (email: yehezkel@tauex.tau.ac.il)

product 2's inferiority by charging $w_1 = c_1$ and $t_1 = \pi_1^{VI} - \pi_2^{VI}$, while the retailer sells product 1 and earns π_2^{VI}. When the retailer sells product 2 in any of the periods before the last period, it earns π_{2H}^{VI} in all future periods. Let $(w_{1,n}, t_{1,n})$ denote the exclusionary contract in period $n < N+1$. If product 2 was excluded in periods 1 to $n-1$, then to motivate the retailer to exclude product 2 in period n, the manufacturer sets $(w_{1,n}, t_{1,n})$ such that:

$$\pi_1^R(w_{1,n}) - t_{1,n} + \sum_{k=1}^{N-n} \delta^k \left(\pi_1^R(w_{1,n+k}) - t_{1,n+k}\right) + \delta^{N-n+1}\pi_2^{VI} \geq \tag{1}$$

$$\max\Big\{ \underbrace{\pi_2^{VI} + \sum_{k=1}^{N-n+1} \delta^k \pi_{2H}^{VI}}_{\text{selling 2}} \;,\; \underbrace{\pi_{12}^R(w_{1,n}) - t_{1,n} + \sum_{k=1}^{N-n+1} \delta^k \pi_{2H}^{VI}}_{\text{selling 1+2}} \Big\}.$$

Notice that in period n the manufacturer and retailer take the contracts in future periods, $(w_{1,n+k}, t_{1,n+k})$, as given. When selling 1+2 binds, the fee in the current period, $t_{1,n}$, cancels out, because it appears in both sides of (1). As shown below, this drives the main difference between the finite and infinite cases. When the current period fee cancels out, the manufacturer cannot use a future promise of a reduced fee as an exclusionary tool. In particular, the parties anticipate a last period, in which the retailer will earn only π_2^{VI} regardless of contractual terms in previous periods.

The following lemma shows that in any exclusionary equilibrium, the manufacturer offers in every period a stationary contract. This contract is identical to the exclusionary contract that we have obtained in the infinite game for $\delta \in [0, \widetilde{\widetilde{\delta}}]$. Yet, when $\delta \in [\widetilde{\widetilde{\delta}}, 1]$, the stationary exclusionary contract in the finite game differs from the infinite one, and continues to involve $w_1^E(\delta) = \widetilde{w}_1(\delta)$ instead of $w_1^E(\delta) = w_{12}(\delta)$.

Lemma 1. ***(Exclusion when vertical relations are of a definite duration)*** *Consider a finite game with $N+1$ periods. In the exclusionary equilibrium, the manufacturer offers in periods $n = 1, ..., N$ the stationery contract:*

$$w_1^E(\delta) = \begin{cases} c_1; & \delta \in [0, \widetilde{\delta}]; \\ \widetilde{w}_1(\delta); & \delta \in [\widetilde{\delta}, 1]; \end{cases} \quad t_1^E(\delta) = \begin{cases} T_2(c_1, \delta); & \delta \in [0, \widetilde{\delta}]; \\ \underline{T}(\widetilde{w}(\delta)); & \delta \in [\widetilde{\delta}, 1]; \end{cases} \tag{2}$$

where $\widetilde{w}_1(\delta)$, $T_2(c_1, \delta)$, $\underline{T}(\widetilde{w}(\delta))$ and $\widetilde{\delta}$ are the same as in the infinite game of Section 3 of the main paper. Then, in the last period ($n = N+1$), the manufacturer offers $w_1 = c_1$ and $t_1 = \pi_1^{VI} - \pi_2^{VI}$.

Proof. Consider a certain period $n < N+1$. We start by showing that, given that in all future periods $n+1, ..., N$, the manufacturer offers the stationary contract defined in Lemma 1, it is optimal for the manufacturer to offer in any period n the same stationary contract. The proof follows for any $n \leq N$. This means that in a two-period game (i.e., $N = 1$), the manufacturer sets in the first period (i.e., in period $n = 1$) the contract defined by Lemma 1. Then, in a three-period game, given that in the second period (when the game becomes a two-period game), the manufacturer offers the same contract, the manufacturer offers in the first period of this three-period game the same stationary contract, and so forth. □

Given that the manufacturer offers in all future periods the contract defined in Lemma 1, $(w_1^E(\delta), t_1^E(\delta))$, the retailer earns in every period (until the last period) $\pi_1^R(w_1^E(\delta)) - t_1^E = \pi_2^{VI} + \delta(\pi_{2H}^{VI} - \pi_2^{VI})$, which holds regardless of whether δ is higher or lower than $\widetilde{\delta}$. To see why, when $\delta < \widetilde{\delta}$, $\pi_1^R(c_1) - T_2(c_1, \delta) = \pi_2^{VI} + \delta(\pi_{2H}^{VI} - \pi_2^{VI})$, and when $\delta \geq \widetilde{\delta}$, $\pi_1^R(\widetilde{w}_1(\delta)) - \underline{T}(\widetilde{w}_1(\delta)) = \pi_2^{VI} + \delta(\pi_{2H}^{VI} - \pi_2^{VI})$, where the equality follows because $\underline{T}(\widetilde{w}(\delta)) = T_2(\widetilde{w}_1(\delta), \delta)$ (from the definition of $\widetilde{w}_1(\delta)$).

Substituting the retailer's profit in future periods $\pi_1^R(w_1^E(\delta)) - t_1^E = \pi_2^{VI} + \delta(\pi_{2H}^{VI} - \pi_2^{VI})$ into (1), we have that to motivate the retailer to exclude product 2, the manufacturer sets in the current period a contract $(w_{1,n}, t_{1,n})$ such that:

$$\pi_1^R(w_{1,n}) - t_{1,n} + \left(\pi_2^{VI} + \delta(\pi_{2H}^{VI} - \pi_2^{VI})\right) \sum_{k=1}^{N-n} \delta^k + \delta^{N-n+1} \pi_2^{VI} \geq \tag{3}$$

$$\max\Bigg\{ \underbrace{\pi_2^{VI} + \sum_{k=1}^{N-n+1} \delta^k \pi_{2H}^{VI}}_{\text{selling 2}} \;,\; \underbrace{\pi_{12}^R(w_1) - t_1 + \sum_{k=1}^{N-n+1} \delta^k \pi_{2H}^{VI}}_{\text{selling 1+2}} \Bigg\}.$$

Comparing the two parts of the second line, "selling 1+2" binds if and only if $t_1 \leq \underline{T}(w_1) = \pi_{12}^R(w_1) - \pi_2^{VI}$, as in the infinite case. When "selling 2" binds ($t_1 > \underline{T}(w_1)$), equating the first line of (3) with the first term in the second line, the highest t_1 that the manufacturer can charge is:

$$t_1 < \pi_1^R(w_1) + \left(\pi_2^{VI} + \delta(\pi_{2H}^{VI} - \pi_2^{VI})\right) \sum_{k=1}^{N-n} \delta^k + \delta^{N-n+1} \pi_2^{VI} - \left(\pi_2^{VI} + \sum_{k=1}^{N-n+1} \delta^k \pi_{2H}^{VI} \right) =$$

$$\pi_1^R(w_1) - \pi_2^{VI} - \delta\left(\pi_{2H}^{VI} - \pi_2^{VI}\right) = T_2(w_1, \delta),$$

where the equality follows because $\sum_{k=1}^{N-n} \delta^k + \delta^{N-n+1} = \sum_{k=1}^{N-n+1} \delta^k$ and $\sum_{k=1}^{N-n+1} \delta^k - \delta \sum_{k=1}^{N-n} \delta^k = \delta$. Notice that $T_2(w_1, \delta)$ is the same as in the infinite case in equation (4) of the main paper. This implies that if $T_2(w_1, \delta) > \underline{T}(w_1)$ (or $w_1 < \widetilde{w}_1(\delta)$), the highest t_1 that the manufacturer sets is

$t_1 = T_2(w_1, \delta)$.

When selling 1+2 binds ($t_1 \leq \underline{T}(w_1)$), t_1 cancels out from the two sides of (3). Condition (3) becomes:

$$\pi_1^R(w_{1,n}) + \left(\pi_2^{VI} + \delta(\pi_{2H}^{VI} - \pi_2^{VI})\right) \textstyle\sum_{k=1}^{N-n} \delta^k + \delta^{N-n+1}\pi_2^{VI} > \pi_{12}^R(w_1) + \sum_{k=1}^{N-n+1} \delta^n \pi_{2H}^{VI}$$
$$\Downarrow$$
$$\pi_1^R(w_1) - \pi_{12}^R(w_1) - \delta\left(\pi_{2H}^{VI} - \pi_2^{VI}\right) \geq 0,$$

which is equivalent to the condition $T_2(w_1, \delta) \geq \underline{T}(w_1)$. This implies that the manufacturer cannot set $w_1 > \widetilde{w}_1(\delta)$, because regardless of t_1, setting $w_1 > \widetilde{w}_1(\delta)$ motivates the retailer to sell products 1+2 instead of selling only product 1. Hence, given that the manufacturer wants to exclude product 2, the manufacturer can either set $w_1 < \widetilde{w}_1(\delta)$ and $t_1 = T_2(w_1, \delta)$ (and selling 2 binds), or set $w_1 = \widetilde{w}_1(\delta)$ and $t_1 = T_2(\widetilde{w}_1(\delta), \delta) = \underline{T}(\widetilde{w}_1(\delta))$ (and both selling 2 and selling 1+2 bind).

Recall that $w_1 = c_1$ maximizes the manufacturer's profit $\pi_1^M(w_1) + T_2(w_1, \delta)$, and $\widetilde{w}_1(\delta) > c_1$ when $\delta < \widetilde{\delta}$. Therefore, we can apply the proof of Proposition 1 of the main paper and find that for $\delta \leq \widetilde{\delta}$, the manufacturer sets $w_1 = c_1$ and $t_1 = T_2(c_1, \delta)$. For $\delta > \widetilde{\delta}$, the manufacturer sets $w_1 = \widetilde{w}_1(\delta)$ and $t_1 = T_2(\widetilde{w}_1(\delta), \delta) = \underline{T}(\widetilde{w}_1(\delta))$.

Notice that Lemma 1 holds for any number of periods, including $N \to \infty$. Recall that by Proposition 1 of the main paper, in the infinite game when $\delta^{VS} \in [\widetilde{\widetilde{\delta}}, 1)$, the manufacturer uses vertical collusion to exclude as an additional exclusionary tool. The higher is δ in this range, the lower is the fixed fee and the less predatory is the wholesale price. By contrast, in the finite game (Lemma 1) the manufacturer's only exclusionary tool is persistent predatory pricing: as δ rises, $\widetilde{w}_1(\delta)$ becomes more predatory, and the fixed fee, $\underline{T}(\widetilde{w}(\delta))$, rises, to extract profits.

This further implies that in the finite game, evaluated at $N \to \infty$, the manufacturer excludes product 2 less than in the infinite game. The manufacturer's inability to implement the more efficient exclusionary tool of vertical collusion to exclude in the finite game shrinks the manufacturer's profitability from exclusion. In particular, the manufacturer's per-period exclusionary profits until the period before last are: $\Pi_1^M(w_1^E(\delta)|\text{finite}) \equiv (w_1^E(\delta) - c_1)q_1(w_1^E(\delta)) + t_1^E(\delta)$, or:

$$\Pi_1^M(w_1^E(\delta)|\text{finite}) = \begin{cases} \pi_1^{VI} - \pi_2^{VI} - \delta\Big(\pi_{2H}^{VI} - \pi_2^{VI}\Big); & \delta < \widetilde{\delta}; \\ \pi_1^{VI}(\widetilde{w}_1(\delta)) - \pi_2^{VI} - \delta\Big(\pi_{2H}^{VI} - \pi_2^{VI}\Big); & \delta \geq \widetilde{\delta}. \end{cases} \tag{4}$$

Then, in the last period, the manufacturer earns $\pi_1^{VI} - \pi_2^{VI}$. The manufacturer's profits from accommodating product 2 are the same as in the infinite game, $\Pi_{12}^{M}(c_1) = \pi_{12}^{VI} - \pi_2^{VI}$, since they are earned only in the current period. Comparing the manufacturer's profits under accommodation and exclusion when $N \to \infty$, we have the following result:[3]

Proposition 1. ***(over-accommodation is intensified in the finite game relative to the infinite game)*** *Consider a finite game with $N+1$ periods and suppose that $N \to \infty$. Then there is a unique threshold, $\delta^{VS}(N \to \infty)$ such that the manufacturer accommodates product 2 iff $\delta > \delta^{VS}(N \to \infty)$. Compared with the threshold in the infinite game, δ^{VS}:*

(i) If $\delta^{VS} < \widetilde{\widetilde{\delta}}$, then $\delta^{VS}(N \to \infty) = \delta^{VS}$, and the exclusionary equilibrium in the finite case is identical to the infinite case;

(ii) If $\delta^{VS} > \widetilde{\widetilde{\delta}}$, then $\widetilde{\widetilde{\delta}} < \delta^{VS}(N \to \infty) < \delta^{VS}$. For $\delta > \widetilde{\widetilde{\delta}}$, the exclusionary equilibrium in the finite case involves lower wholesale and retail prices than in the infinite case. Moreover, the wholesale price in the exclusionary equilibrium becomes more predatory as δ increases.

Proof. The manufacturer's discounted sum of exclusionary profits, given N, is:

$$\left(1 + \delta + ... + \delta^{N-1}\right) \Pi_1^M(w_1^E(\delta)|\text{finite}) + \delta^N \left(\pi_1^{VI} - \pi_2^{VI}\right)$$

$$= \frac{\sum_{n=1}^{N} \delta^n}{\delta} \Pi_1^M(w_1^E(\delta)|\text{finite}) + \delta^N \left(\pi_1^{VI} - \pi_2^{VI}\right),$$

where $\Pi_1^M(w_1^E(\delta)|\text{finite})$ is given by (4) and the equality follows because $\sum_{n=1}^{N} \delta^n = \delta(1 + \delta + ... + \delta^{N-1})$. Evaluated at $N \to \infty$, $\delta^N \to 0$ and $\sum_{n=1}^{N} \delta^n \to \frac{\delta}{1-\delta}$, hence the manufacturer's discounted sum of exclusionary profits approaches $\frac{\Pi_1^M(w_1^E(\delta)|\text{finite})}{1-\delta}$. The manufacturer's profit from accommodating product 2 is the same as in the infinite game: $\Pi_{12}^M(c_1)$ as given by equation (9) of the main paper. The manufacturer excludes product 2 if $\frac{\Pi_1^M(w_1^E(\delta)|\text{finite})}{1-\delta} - \Pi_{12}^M(c_1) \geq 0$.

Suppose first that $\delta < \widetilde{\delta}$. In this case, $\Pi_1^M(w_1^E(\delta)|\text{finite}) = \Pi_1^M(c_1|\text{finite}) = \Pi_1^M(c_1)$, where $\Pi_1^M(c_1)$ is the per-period profit in the infinite case, defined in the first line of equation (6) of the main paper. From the proof of Proposition 2 of the main paper (inequality (17) in the main paper), $\frac{\Pi_1^M(c_1)}{1-\delta} - \Pi_{12}^M(c_1) \geq 0$ for all $\delta < \widetilde{\delta}$. We therefore have that when $\delta < \widetilde{\delta}$, the manufacturer always excludes product 2.

[3] It is possible to show that given any finite number of periods, there is a threshold, $\delta^{VS}(N)$, such that the manufacturer excludes product 2 if $\delta < \delta^{VS}(N)$, where $\delta^{VS}(1) = min\left\{\frac{\pi_1^{VI} - \pi_{12}^{VI}}{\pi_{2H}^{VI} - \pi_1^{VI}}, 1\right\}$, $\delta^{VS}(N)$ is (weakly) decreasing with N, and $\delta^{VS}(N \to \infty) = \frac{\pi_1^{VI}(\widetilde{w}_1(\delta)) - \pi_2^{VI}}{\pi_{2H}^{VI} - \pi_2^{VI}}$. Moreover, for any finite N, the manufacturer accommodates product 2 more than under vertical integration.

The rest of the proof depends on whether δ^{VS}, as defined in the infinite game, is higher or lower than $\tilde{\tilde{\delta}}$. Suppose first that $\delta^{VS} < \tilde{\tilde{\delta}}$. In this case, $\Pi_1^M(w_1^E(\delta)|\text{finite}) = \Pi_1^M(\widetilde{w}_1(\delta)|\text{finite}) = \Pi_1^M(\widetilde{w}_1(\delta))$, where $\Pi_1^M(\widetilde{w}_1(\delta))$ is the first (also equal to the second) line in equation (6) of the main paper, evaluated at $w_1 = \widetilde{w}_1(\delta)$. The condition $\frac{\Pi_1^M(w_1^E(\delta)|\text{finite})}{1-\delta} - \Pi_{12}^M(c_1) \geq 0$ is identical to inequality (18) in the main paper, in the proof of Proposition 2. Therefore, the manufacturer excludes product 2 if $\delta < \delta^{VS}(N \to \infty) = \delta^{VS}$, where δ^{VS} is the same as the δ that solves (18) of the main paper in equality.

Finally, suppose that $\delta^{VS} \geq \tilde{\tilde{\delta}}$. The threshold $\delta^{VS}(N \to \infty)$ in the finite case (evaluated at $N \to \infty$) is still the solution to (21) of the main paper in equality, because in the finite game, the manufacturer continues to charge $\widetilde{w}_1(\delta)$ when $\delta \geq \tilde{\tilde{\delta}}$. Yet, by revealed preference, in the infinite case when $\delta > \tilde{\tilde{\delta}}$, the manufacturer prefers setting $w_1 = w_{12}(\delta)$ over $w_1 = \widetilde{w}_1(\delta)$, and hence $\Pi_1^M(w_{12}(\delta)) > \Pi_1^M(\widetilde{w}_1(\delta)|\text{finite})$. This in turn implies that when $\delta \geq \tilde{\tilde{\delta}}$, $\delta^{VS}(N \to \infty) < \delta^{VS}$. Moreover, $\delta^{VS}(N \to \infty) > \tilde{\tilde{\delta}}$, because from the proof of Proposition 2 of the main paper, when $\delta^{VS} > \tilde{\tilde{\delta}}$, the gap in inequality (21) of the main paper is positive for all $\delta < \tilde{\tilde{\delta}}$. □

Proposition 1 shows that when $\delta^{VS} > \tilde{\tilde{\delta}}$, the infinite game involves more exclusion of product 2, with a less predatory wholesale price, compared to the finite game for $N \to \infty$. When $\delta^{VS} < \tilde{\tilde{\delta}}$, vertical collusion to exclude is not used in the infinite game either, so the finite exclusionary equilibrium converges to the infinite one as $N \to \infty$.[4]

[4] It is possible to show that in the finite game, as $N \to \infty$, a vertically integrated firm would want to exclude product 2 for the same levels of δ as in the infinite game (i.e., for $\delta < \delta^{VI}$). For any finite number of periods, $N+1$, there is a threshold, $\delta^{VI}(N)$, such that a vertically integrated firm excludes product 2 if $\delta < \delta^{VI}(N)$. In a two-period game ($N=1$), $\delta^{VI}(1) = \min\left\{\frac{\pi_1^{VI} - \pi_{12}^{VI}}{\pi_{2H}^{VI} - \pi_1^{VI}}, 1\right\} > \delta^{VI}$. $\delta^{VI}(N)$ is decreasing in N, and as $N \to \infty$, $\delta^{VI}(N) \to \delta^{VI}$.