

# Vertical Collusion to Exclude Product Improvement

## Online Appendix B: How the discount factor affects the welfare comparison among the three antitrust regimes

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This Appendix extends Section 5 in the paper on how the discount factor affects the welfare comparison among the three antitrust regimes. Let us first compare regime (ii) (exclusive dealing is banned and predatory pricing is allowed) with regime (i) (exclusive dealing is allowed). One clear observation is that, when  $\tilde{\delta} < \delta < \delta^{VS}$ , regime (ii) is unambiguously better for social welfare than regime (i). While in both regimes product 2 is excluded, in regime (ii) the manufacturer engages in (welfare enhancing) predatory pricing, while in regime (i) the manufacturer charges  $w_1 = c_1$ . Intuitively, when the manufacturer cannot explicitly restrain the retailer with an exclusive dealing prohibition, the manufacturer needs to exclude product 2 via price reduction. This reduction in prices, although predatory, alleviates the market's monopoly distortion and expands output, to the benefit of end consumers.

Turning to the range  $\delta^{VS} < \delta < \delta^{VI}$ , in both regimes (i) and (ii)  $w_1 = c_1$ , but in regime (ii), without exclusive dealing, product 2 is accommodated, while in regime (i), with exclusive dealing, it is excluded. Thus, in this range, the comparison between the two regimes hinges only on whether the accommodation of product 2 increases or decreases social welfare. To this end, let  $SW_1(w_1)$  denote per-period social welfare when the retailer sells only product 1 (the quantity is  $q_1(w_1)$ ). Similarly,  $SW_{12}(w_1)$  denotes social welfare in a period in which the retailer sells both products. In

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the latter case, the retailer sells the same total quantity  $q_1(w_1)$  (recall that  $\hat{q}_1(w_1) = q_1(w_1) - \underline{q}$ ) hence:

$$SW_1(w_1) = \int_0^{q_1(w_1)} (p(q) - c_1) dq, \quad SW_{12}(w_1) = \int_0^{q_1(w_1)} p(q) dq - c_2 \underline{q} - c_1(q_1(w_1) - \underline{q}).$$

Since  $c_1 < c_2$ , we have that  $SW_1(w_1) > SW_{12}(w_1)$ . This stems from the short-run downside, in terms of social welfare, to selling the inferior product in the current period. Turning to the up-side, of improving product 2 in the future, let  $SW_{2H}$  denote per-period social welfare when the retailer sells the improved product 2. We assume that the improved product 2 offers not only higher vertically integrated profits (i.e.,  $\pi_{2H}^{VI} > \pi_1^{VI}$ ), but also (weakly) higher consumer surplus:  $SW_{2H} - \pi_{2H}^{VI} \geq SW_1(c_1) - \pi_1^{VI}$ . This would be the case when the improved product 2 is of higher quality than product 1, or when its marginal costs are lower than those of product 1, and hence its quantity is higher.

Under exclusive dealing (regime (i)), recall that product 2 is excluded in the range we are considering,  $\delta \in [\delta^{VS}, \delta^{VI}]$ , and furthermore  $w_1 = c_1$ . Conversely, under regime (ii), product 2 is accommodated in this range, while the wholesale price is the same as in regime (i) ( $w_1 = c_1$ ). Accordingly, we assess the socially optimal level of accommodation while taking the pricing in these two regimes as given:  $w_1 = c_1$ . In other words, since in this range of  $\delta$  both regimes involve similar (monopoly) quantities and prices, we derive the socially optimal level of accommodation given that monopoly pricing in the industry persists. Accommodation of product 2 improves welfare iff  $\delta > \delta^{SW}$ , where  $\delta^{SW}$  is the solution to:

$$SW_{12}(c_1) + \frac{\delta}{1 - \delta} SW_{2H} \geq \frac{SW_1(c_1)}{1 - \delta}. \quad (1)$$

Because  $SW_{2H} - \pi_{2H}^{VI} \geq SW_1(c_1) - \pi_1^{VI}$ , exclusive dealing (regime (i)) always involves over-exclusion of product 2 compared to what is socially optimal. Recall from Section 4 of the main paper that under exclusive dealing, the industry replicates the vertically integrated outcome. This further implies that a vertically integrated firm over-excludes product 2 compared to what is socially desirable:  $\delta^{SW} < \delta^{VI}$ . Intuitively, the vertically integrated monopoly does not internalize the benefits that improving product 2 has on consumers, and hence “under-improves” product 2.

While exclusive dealing (regime (i)) always involves over-exclusion of product 2, regime (ii) may involve over accommodation of product 2 compared to the social optimum. In particular,  $\delta^{SW}$  can

be higher or lower than  $\delta^{VS}$ , depending on the model's parameters. Intuitively,  $\delta^{SW} < \delta^{VS}$  if there are welfare-enhancing advantages to accommodating product 2 that the manufacturer and retailer do not fully internalize. This occurs when the short-term sacrifice to social welfare of selling the inferior product 2 in a certain period is small enough. Conversely, when  $\delta^{SW} > \delta^{VS}$ , the vertically separated industry accommodates product 2 more than what is socially optimal. Recall that the manufacturer's losses from its exclusionary strategies become prohibitively costly to it for  $\delta > \delta^{VS}$ . The social planner is indifferent to these losses, so the parties might over-accommodate product 2.

Suppose first that  $\delta^{SW} < \delta^{VS}$ , so that even regime (ii), when exclusive dealing is banned, involves over-exclusion of product 2 compared to the social optimum. The corollary below follows directly from Table 1 in our paper:

**Corollary 1.** *Suppose that  $\delta^{SW} < \delta^{VS}$ . Then regime (ii) (banning exclusive dealing and allowing predatory pricing), is better for social welfare than regime (i) (allowing exclusive dealing).*

The intuition for this result is that for  $\delta^{SW} < \delta^{VS}$ , regime (ii) dominates regime (i) along both dimensions, of quantities supplied and accommodation of product 2. With respect to quantities supplied, regime (ii) is superior to regime (i), because it involves persistent predatory pricing by the manufacturer (for  $\tilde{\delta} < \delta < \delta^{VS}$ ). This persistent predatory pricing helps alleviate the industry's monopoly distortion.<sup>3</sup> As for the dimension of the exclusion of product 2, even regime (ii) involves over-exclusion of product 2 compared to the social optimum. Hence regime (i), which allows exclusive dealing, must have even more severe over-exclusion ( $\delta^{SW} < \delta^{VS} < \delta^{VI}$ ).

Next, consider the case where  $\delta^{VS} < \delta^{SW}$  (hence,  $\tilde{\delta} < \delta^{VS} < \delta^{SW} < \delta^{VI}$ ). We have:

**Corollary 2.** *Suppose that  $\delta^{SW} > \delta^{VS}$ . Then, regime (i) (allowing exclusive dealing) is socially superior to regime (ii) (banning exclusive dealing and allowing predatory pricing) if  $\delta \in [\delta^{VS}, \delta^{SW}]$ , and the converse is true for  $\delta \notin [\delta^{VS}, \delta^{SW}]$ .*

The intuition for this result is that when  $\delta \in [\delta^{VS}, \delta^{SW}]$ , from a social perspective it is optimal to exclude product 2 ( $\delta < \delta^{SW}$ ), yet the industry under regime (ii) accommodates product 2 ( $\delta > \delta^{VS}$ ). This is while in regime (i) the industry excludes product 2 ( $\delta < \delta^{VI}$ ), conforming to what is socially optimal. With respect to the other dimension, of pricing, in both cases the manufacturer sets  $w_1 = c_1$  for  $\delta \in [\delta^{VS}, \delta^{SW}]$ . Hence the welfare-enhancing effect of regime (ii)'s predatory pricing is

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<sup>3</sup>Notice that a predatory wholesale price,  $w_1 < c_1$ , can never result in selling too much of product 1 in comparison with the quantity that maximizes social welfare. This is because the manufacturer would never set  $w_1 < c_1$  so low such that  $p(q_1(w_1)) < c_1$ , as doing so involves negative joint profits. For any  $w_1 < c_1$  for which  $p(q_1(w_1)) > c_1$ , social welfare is decreasing in  $p$ , and hence in  $w_1$ .

not relevant. This results in higher social welfare in this range under regime (i), of allowing exclusive dealing. We illustrate this using our linear demand example in the main paper.

The converse is true for  $\delta \notin [\delta^{VS}, \delta^{SW}]$ . In particular, if  $\delta \in [\tilde{\delta}, \delta^{VS}]$  both regimes exclude product 2, but regime (ii) involves a lower price. If  $\delta \in [\delta^{SW}, \delta^{VI}]$ , both regimes involve monopoly pricing, but it is socially optimal to accommodate product 2, as regime (ii) achieves in this range. Regime (i) excludes product 2 for such discount factors, thereby over-excluding from a welfare perspective.

Next, we turn to evaluate the effects of regime (iii): both predatory pricing and exclusive dealing are banned. As noted in Table 1 in the paper, this regime triggers accommodation when  $\delta > \tilde{\delta}^{VS}$  – more than in regime (ii) ( $\delta > \delta^{VS}$ ) and regime (i) ( $\delta > \delta^{VI}$ ). At the same time, the ban on predatory pricing of the wholesale price increases the retail price to monopoly levels. Consider first the comparison between regimes (ii) and (iii). When  $\delta \in [\delta^{VS}, \delta^{VI}]$ , both regimes are identical. However, when  $\delta \in [\tilde{\delta}^{VS}, \delta^{VS}]$  the comparison between regimes (ii) and (iii) is inconclusive. Regime (ii) in this range involves predatory pricing with exclusion of product 2, while regime (iii) involves monopoly pricing and accommodation of product 2. Hence, along the dimension of pricing and quantity, regime (ii) dominates regime (iii), because regime (ii) involves welfare-enhancing predatory pricing, while regime (iii) involves monopoly pricing.

The other dimension, of accommodating product 2, may point to the opposite direction, depending on market circumstances. We have seen above that  $\delta^{SW}$ , the socially optimal cutoff for accommodation given monopoly pricing, may be either below or above  $\delta^{VS}$ , the corresponding cutoff under regime (ii). But market conditions could also be such that  $\delta^{SW}$  is either above or below  $\tilde{\delta}^{VS}$ , the cutoff for accommodation under regime (iii). This implies that the welfare comparison between regimes (ii) and (iii) for  $\delta \in [\tilde{\delta}^{VS}, \delta^{VS}]$  is affected by both the industry quantity dimension and the over-accommodation or over-exclusion dimension in a way that critically depends on market parameters. Conversely, for  $\delta \in [\tilde{\delta}, \tilde{\delta}^{VS}]$  regime (ii), of allowing predatory pricing and banning exclusive dealing, is unambiguously better than regime (iii), of banning both practices, because in this range product 2 is excluded in both regimes, but regime (ii) involves a lower price.

Next consider the comparison between regimes (i) and (iii). For  $\delta \in [\tilde{\delta}^{VS}, \delta^{VI}]$ , both regimes include a wholesale price equal to marginal cost, though regime (iii) accommodates product 2 and regime (i) excludes it. Hence, the comparison between the two regimes depends only on whether accommodating product 2 is socially superior to exclusion. As noted, regime (i) always involves over-exclusion of product 2, while regime (iii) may over-exclude or over-accommodate product 2,

depending on whether  $\delta^{SW}$  lies above or below  $\tilde{\delta}^{VS}$ . The following corollaries summarize the results:

**Corollary 3.** *Suppose that  $\delta^{SW} < \tilde{\delta}^{VS}$ . Then regime (iii) (banning both exclusive dealing and predatory pricing), is better for social welfare than regime (i) (allowing exclusive dealing).*

The intuition is that if  $\delta^{SW} < \tilde{\delta}^{VS}$ , even regime (iii) over-excludes product 2 compared to what is socially optimal, so regime (i) involves even more harmful exclusion.

**Corollary 4.** *Suppose that  $\delta^{SW} > \tilde{\delta}^{VS}$ . Then, regime (i) (allowing exclusive dealing) is socially superior to regime (iii) (banning both exclusive dealing and predatory pricing) if  $\delta \in [\tilde{\delta}^{VS}, \delta^{SW}]$ , and the converse is true for  $\delta \notin [\tilde{\delta}^{VS}, \delta^{SW}]$ .*

This implies another scenario where allowing exclusive dealing could be socially beneficial. Note, though, that for  $\delta \notin [\tilde{\delta}^{VS}, \delta^{SW}]$ , regime (iii) dominates regime (i): it is socially better to ban both exclusive dealing and predatory pricing than to allow exclusive dealing.