Vertical Collusion to Exclude Product Improvement

Online Appendix A: A strategic firm is selling the new product

By David Gilo[1] and Yaron Yehezkel[2]

October 2023

1 Introduction

In our base model we assumed that the new product is supplied to the retailer by a perfectly competitive fringe and is available at marginal costs. In this appendix, we extend our model to the case where there are two competing manufacturers, one that sells the established product and a second manufacturer that sells the new product. The two manufacturers compete in setting two-part-tariffs and after the two manufacturers make their offers, the retailer chooses whether to carry one of the products, both or none.

We show that the results of our base model hold when the new product is sold by an independent manufacturer who is a strategic player. In the exclusionary equilibrium, the manufacturer of the established product offers the same contract as in our base model, while the competing manufacturer offers a contract that replicates the retailer's profit when the new product is available to the retailer by a perfectly competitive market. The exclusionary equilibrium holds for the same discount factors as in our base model, and holds either under simultaneous competition between the two manufacturers or when the manufacturer of the established product makes the contract offer before the manufacturer of the new product. In the accommodation equilibrium, again the manufacturer of the established product sets the same contract as in our base model. The joint profit of the retailer and the manufacturer of the new product is also the same as in our base model, but

[1]Buchmann Faculty of Law, Tel Aviv University (email: gilod@tauex.tau.ac.il)
[2]Coller School of Management, Tel Aviv University (email: yehezkel@tauex.tau.ac.il)

now the manufacturer of the new product earns positive profits at the expense of the retailer. This implies that the accommodation equilibrium in a simultaneous game does not have pure strategies. However, the main model's accommodation equilibrium always holds in a sequential game in which the manufacturer of the established product makes the contract offer before the manufacturer of the new product.

We then extend the main model's analysis of the vertically integrated case to the case where the competing manufacturer is a strategic player and show that the main model's results extend to this case.

2 Model

Consider our base model, and suppose that there are two firms, $M1$, which sells product 1, and $M2$, which sells product 2. Each firm offers the retailer a two-part tariff contract, (w_i, t_i) $(i = 1, 2)$, and then the retailer decides whether to accept $M1$'s offer, or $M2$'s offer, or both.

When the retailer sells product 2 in a certain period, in each of the future periods, product 2 improves, creating a joint profit of $\pi_{2H}^{VI} > \pi_1^{VI}$ for $M2$ and the retailer. It is clear that in such a case, the retailer sells only the product granting it the highest profit, product 2. Hence, there is a unique equilibrium in which the two manufacturers charge wholesale prices equal to their marginal costs and fixed fees of $t_1 = 0$ and $t_2 = \pi_{2H}^{VI} - \pi_1^{VI}$. The retailer accepts only $M2$'s offer and earns $\pi_{2H}^{VI} - t_2 = \pi_1^{VI}$, $M2$ earns $t_2 = \pi_{2H}^{VI} - \pi_1^{VI}$ and $M1$ earns 0. Notice that these are the equilibrium strategies under both simultaneous contract offers and when $M1$ offers the contract before $M2$.

We proceed as follows. In Section 3 we show that it is a dominant strategy for $M2$ to set $w_2 = c_2$. That is, $M2$ will not engage in below-cost pricing nor will it want to charge an above-cost wholesale price. Recall that $M1$ engaged in persistent predatory pricing when the retailer considered selling both products, so as to inflate the retailer's short-term sacrifice from improving product 2. The intuition for this difference between $M1$ and $M2$'s incentives is that unlike $M1$, $M2$ has no reason to induce the retailer to hold only product 2 in the first period. $M2$'s best strategy is to induce the retailer to hold product 2 along-side product 1, in order to improve product 2. Therefore, it is always more profitable for $M2$ to use t_2 as the only tool for motivating the retailer to sell product 2. Using the result that $w_2 = c_2$, we can then show, in Section 4, that in any exclusionary equilibrium, $M1$ sets the same contract as in our base model. The intuition is that t_2 does not affect the retailer's decision on whether to sell product 2 or products 1+2. Hence, "selling 2" and "selling 1+2" are binding in a similar way as in our base model. Finally, Section 5 solves for the accommodation equilibrium.

3 $M2$'s strategy

In this section we show that it is optimal for $M2$ to set $w_2 = c_2$ as a response to (w_1, t_1), regardless of whether "selling 2" or "selling 1+2" bind. Suppose first that "selling 2" binds. Then, $M2$ sets t_2 such that:

$$(p(q_2(w_2)) - w_2)q_2(w_2) - t_2 + \frac{\delta}{1-\delta}\pi_1^{VI} \geq \frac{\pi_1^R(w_1) - t_1}{1-\delta}, \tag{1}$$

where the left-hand-side is the retailer's profit from selling only the inferior product 2 in the first period and the profits $M2$ leaves the retailer with in the following periods, in which product 2 will have been improved if the retailer sells only product 2 in the first period. The right-hand-side is the retailer's profit from selling product 1 in all periods. Extracting t_2 and substituting into $M2$'s profit when "selling 2" binds, $\Pi_2^{M2}(w_2) \equiv (w_2 - c_2)q_2(w_2) + t_2 + \frac{\delta}{1-\delta}(\pi_{2H}^{VI} - \pi_1^{VI})$, we have:

$$\Pi_2^{M2}(w_2) = (p(q_2(w_2)) - c_2)q_2(w_2) + \frac{\delta}{1-\delta}\pi_{2H}^{VI} - \frac{\pi_1^R(w_1) - t_1}{1-\delta}. \tag{2}$$

The w_2 that maximizes (2) is $w_2 = c_2$. Hence, if at $w_2 = c_2$ selling 2 binds, $M2$ will not deviate to any other w_2 in which selling 2 continues to bind.

When "selling 1+2" binds, $M2$ sets t_2 such that:

$$p(\widehat{q}_1(w_1) + \underline{q})(\widehat{q}_1(w_1) + \underline{q}) - w_1\widehat{q}_1(w_1) - w_2\underline{q} - t_1 - t_2 + \frac{\delta}{1-\delta}\pi_1^{VI} \geq \frac{\pi_1^R(w_1) - t_1}{1-\delta}, \tag{3}$$

where the left-hand-side is the profit left to the retailer when selling 1+2 in the current period and then the retailer's continuation payoff from selling the improved product 2 in all future periods. Notice that $\widehat{q}_1(w_1)$ is the same as in our base model, because $\widehat{q}_1(w_1)$ is not affected by the retailer's marginal cost of selling product 2, so it is not a function of w_2. Extracting t_2 and substituting into $M2$'s profit when 1+2 binds, $\Pi_{12}^{M2}(w_2) \equiv (w_2 - c_2)\underline{q} + t_2 + \frac{\delta}{1-\delta}(\pi_{2H}^{VI} - \pi_1^{VI})$, we have:

$$\Pi_{12}^{M2}(w_2) = p(\widehat{q}_1(w_1) + \underline{q})(\widehat{q}_1(w_1) + \underline{q}) - w_1\widehat{q}_1(w_1) - c_2\underline{q} + \frac{\delta}{1-\delta}\pi_{2H}^{VI} \tag{4}$$

$$-\frac{\pi_1^R(w_1) - \delta t_1}{1-\delta}.$$

We have that w_2 does not affect (4). Hence if at $w_2 = c_2$ selling 1+2 binds, $M2$ will not deviate to any other w_2 in which selling 1+2 continues to bind.

It is left to verify that when selling 1+2 binds (evaluated at $w_2 = c_2$), $M2$ will not deviate to another w_2, which makes selling 2 bind. Likewise, when selling 2 binds (evaluated at $w_2 = c_2$), we need to verify that $M2$ will not deviate to any other w_2, which makes selling 1+2 bind. Comparing the two left-hand sides of (1) and (3), the retailer prefers selling product 2 over selling 1+2 if and only if:

$$t_1 > \underline{T}(w_1, w_2) \equiv p(\widehat{q}_1(w_1) + \underline{q})(\widehat{q}_1(w_1) + \underline{q}) - w_1\widehat{q}_1(w_1) - w_2\underline{q} - (p(q_2(w_2)) - w_2)q_2(w_2). \tag{5}$$

The fixed fee t_2 does not affect the retailer's decision on whether to sell 2 or 1+2, because in both cases, the retailer needs to pay it. Yet given w_1, a low w_2 may motivate the retailer to prefer selling only 2 over selling 1+2, and vice versa: a high w_2 may motivate the retailer to prefer selling 1+2 over selling only 2. To see why, we have (using the envelope theorem) that $\partial \underline{T}(w_1, w_2)/\partial w_2 = q_2(w_2) - \underline{q} \geq 0$, where the inequality follows because the retailer has to sell at least $\underline{q}$ to improve product 2.[3] This implies that, given w_1, $M2$ can cause the retailer's binding constraint to be selling 2 by reducing w_2, thereby causing (5) to hold in equality. To pin down this level of w_2, let $\widetilde{w}_2$ denote the solution to $t_1 = \underline{T}(w_1, w_2)$. We have that if $t_1 < \underline{T}(w_1, c_2)$ ($t_1 > \underline{T}(w_1, c_2)$), then $\widetilde{w}_2 < c_2$ ($\widetilde{w}_2 > c_2$). The gap in $M2$'s profits when the retailer sells 1+2 and when it sells 2, evaluated at $t_1 = \underline{T}(w_1, \widetilde{w}_2)$, is:

$$\Pi_{12}^{M2}(\widetilde{w}_2) - \Pi_2^{M2}(\widetilde{w}_2) = (c_2 - \widetilde{w}_2)(q_2(\widetilde{w}_2) - \underline{q}).$$

The term $q_2(\widetilde{w}_2) - \underline{q}$ is positive (see note 3) so the sign of $\Pi_{12}^{M2}(\widetilde{w}_2) - \Pi_2^{M2}(\widetilde{w}_2)$ depends on whether $\widetilde{w}_2$ is higher or lower than c_2. If $\widetilde{w}_2$ is higher than c_2, $\Pi_{12}^{M2}(\widetilde{w}_2) < \Pi_2^{M2}(\widetilde{w}_2)$. Conversely, if $\widetilde{w}_2$ is lower than c_2, $\Pi_{12}^{M2}(\widetilde{w}_2) > \Pi_2^{M2}(\widetilde{w}_2)$. Accordingly, using our result that $\partial \underline{T}(w_1, w_2)/\partial w_2 \geq 0$, if at $w_2 = c_2$ 1+2 binds (i.e., $t_1 < \underline{T}(w_1, w_2)$) and $M2$ attempts to shift w_2 to make "selling 2" bind, $M2$ needs to decrease w_2 (thereby lowering $\underline{T}(w_1, w_2)$) to $w_2 = \widetilde{w}_2 < c_2$. Such a deviation is not profitable to $M2$, because $\Pi_{12}^{M2}(c_2) = \Pi_{12}^{M2}(\widetilde{w}_2) > \Pi_2^{M2}(\widetilde{w}_2)$ (where the first equality follows because $\widetilde{w}_2$ does not affect $\Pi_{12}^{M2}(c_2)$ and the second inequality follows because $\widetilde{w}_2 < c_2$). Likewise, if at $w_2 = c_2$ selling 2 binds (i.e., $t_1 > \underline{T}(w_1, w_2)$), and $M2$ attempts to shift w_2 to make "selling 1+2"

[3] Our assumption that $q_2(c_2) > \underline{q}$ implies that $q_2(w_2) > \underline{q}$ for all $w_2 < c_2$. If, for $w_2 > c_2$, $q_2(w_2) < \underline{q}$, then we assume the retailer's future profits are large enough so that the retailer does not avoid improving product 2 by setting $q_2 < \underline{q}$, so it instead sets $q_2 = \underline{q}$. In such a case, $\partial \underline{T}(w_1, w_2)/\partial w_2 = 0$.

bind, $M2$ needs to increase w_2 (thereby increasing $\underline{T}(w_1, w_2)$) to $w_2 = \widetilde{w}_2 > c_2$. Such a deviation is not profitable to $M2$ either, because $\Pi_2^{M2}(c_2) > \Pi_2^{M2}(\widetilde{w}_2) > \Pi_{12}^{M2}(\widetilde{w}_2)$ where the first inequality follows because $w_2 = c_2$ maximizes $\Pi_2^{M2}(w_2)$ and the second inequality follows because $\widetilde{w}_2 > c_2$. We therefore have that $M2$ always sets $w_2 = c_2$.

4 Exclusionary equilibrium

In an exclusionary equilibrium, $M2$ sets the lowest t_2 that ensures $M2$ overall profits of zero. Recall that $M2$ always sets $w_2 = c_2$, and, assuming the retailer accepts $M2$'s offer in a certain period, $M2$ earns in all subsequent periods $\pi_{2H}^{VI} - \pi_1^{VI}$. Accordingly, in an exclusionary equilibrium $M2$ sets $t_2 = -\frac{\delta}{1-\delta}(\pi_{2H}^{VI} - \pi_1^{VI})$. Therefore, the retailer's exclusionary contract with $M2$ mimics vertical integration between the retailer and $M2$. The retailer's profit from accepting $M2$' offer exclusively is:

$$\pi_2^R(w_1) - t_2 + \frac{\delta}{1-\delta}\pi_1^{VI} = \pi_2^{VI} + \frac{\delta}{1-\delta}\pi_{2H}^{VI}, \tag{6}$$

(after substituting $t_2 = -\frac{\delta}{1-\delta}(\pi_{2H}^{VI} - \pi_1^{VI})$). Notice that this term is identical to the retailer's profit in equation (2) in our base model, when "selling 2" binds. Likewise, the retailer's profit from accepting both offers is:

$$\pi_{12}^R(w_1) - t_1 - t_2 + \frac{\delta}{1-\delta}\pi_1^{VI} = \pi_{12}^R(w_1) - t_1 + \frac{\delta}{1-\delta}\pi_{2H}^{VI}, \tag{7}$$

(after substituting $t_2 = -\frac{\delta}{1-\delta}(\pi_{2H}^{VI} - \pi_1^{VI})$). This term is identical to the retailer's profit in equation (2) in our base model, when "selling 1+2" binds. Hence, applying the analysis in our base model, the optimal exclusionary contract for $M1$ given $M2$'s contract is the exclusionary contract defined in our base model. If $M1$ deviates from this exclusionary equilibrium to an accommodating contract, given $M2$'s strategies of $w_2 = c_2$ and $t_2 = -\frac{\delta}{1-\delta}(\pi_{2H}^{VI} - \pi_1^{VI})$, $M1$'s strategies and profit are also the same as in our base model. Hence, $M1$ prefers to exclude product 2 if $\delta < \delta^{VS}$, where δ^{VS} is the same as in our base model. The following corollary summarizes these results:

Corollary 1. *Suppose that a strategic firm, $M2$, sells product 2. Then, for $\delta \leq \delta^{VS}$, there is an exclusionary equilibrium in which $M2$ sets $w_2 = c_2$ and $t_2 = -\frac{\delta}{1-\delta}(\pi_{2H}^{VI} - \pi_1^{VI})$ and $M1$ sets the same exclusionary contract defined in our base model. This equilibrium holds under both a simultaneous game ($M1$ and $M2$ make simultaneous contract offers) and a sequential game ($M1$ makes the offer* *before $M2$).*

The exclusionary equilibrium holds under both the simultaneous and sequential games because, as shown in section 3, $M2$'s wholesale price is never affected by $M1$'s strategies. Furthermore, in the sequential game too $M2$'s fee in an exclusionary equilibrium will guaranty it zero overall profits.[4]

5 Accommodation equilibrium

If $M1$ chooses to accommodate product 2, $M1$ sets t_1 that makes the retailer indifferent between selling 1+2 and selling 2. From our analysis above, $M2$ sets $w_2 = c_2$. Moreover, the retailer's decision on whether to sell 1+2 or just 2 is independent of t_2 (as the retailer pays t_2 in both options). Hence, $M1$'s accommodation contract is the same as in our base model. That is, $M1$ sets t_1 as the solution to equation (8) in our base model (such that the retailer is indifferent between selling 2 and selling 1+2, given $w_2 = c_2$). Then, $M1$ sets $w_1 = c_1$ (maximizing equation (9) in our base model), $t_1 = \pi_{12}^{VI} - \pi_2^{VI}$, and earns the same accommodation profit as in our base model.

Turning to $M2$, given $w_1 = c_1$ and $t_1 = \pi_{12}^{VI} - \pi_2^{VI}$, in the current period $M2$ sets t_2 such that the retailer prefers to accept $M2$'s contract over the out-of equilibrium option of accepting only $M1$'s accommodation contract in all periods:[5] [6]

$$\pi_2^{VI} - t_2 + \frac{\delta}{1-\delta}\pi_1^{VI} \geq \frac{\pi_1^{VI} - \pi_{12}^{VI} + \pi_2^{VI}}{1-\delta}. \tag{8}$$

Hence, $t_2 = \frac{\pi_{12}^{VI} - \delta\pi_2^{VI}}{1-\delta} - \pi_1^{VI}$, and $M2$ earns:

$$\Pi_{12}^{M2}(c_2) = \frac{\delta(\pi_{2H}^{VI} - \pi_2^{VI}) - (\pi_1^{VI} - \pi_{12}^{VI})}{1-\delta}, \tag{9}$$

where we obtain (9) by adding t_2 to $M2$'s continuation payoff, derived in Section 2, of $\frac{\delta(\pi_{2H}^{VI} - \pi_1^{VI})}{1-\delta}$. Notice that $\delta(\pi_{2H}^{VI} - \pi_2^{VI}) - (\pi_1^{VI} - \pi_{12}^{VI}) > 0$ when $\delta > \widetilde{\delta}$ (see equation (13) and Lemma 1 in the main paper). Thus, $M2$ makes positive profits in the accommodation equilibrium.

[4] Note that, as in our base model, the exclusionary equilibrium is stationary because product 2 remains inferior in all periods. In an accommodation equilibrium, the fixed fee $M2$ offers the retailer in the first period (to be explored in the next section) is different than the fixed fee $M2$ offers in any sub-game following the improvement of product 2, as shown in Section 2.

[5] The retailer earns from selling 1+2 in the current period $\pi_2^{VI} - t_2$ because given $w_1 = c_1$ and $t_1 = \pi_{12}^{VI} - \pi_2^{VI}$, $\pi_{12}^{VI} - t_1 - t_2 = \pi_2^{VI} - t_2$.

[6] We assume that if the retailer rejects $M2$'s offer, the retailer expects $M1$ to continue offering the same accommodation contract in all future periods. The retailer's profit from accepting only $M1$'s contract in the current period is $\pi_1^{VI} - t_1 = \pi_1^{VI} - \pi_{12}^{VI} + \pi_2^{VI} = \pi_2^{VI} + (c_2 - c_1)\underline{q} > 0$.

While the joint profit of $M2$ and the retailer is the same as the retailer's profit in our base model ($\pi_2^{VI} + \frac{\delta}{1-\delta}\pi_{2H}^{VI}$), now $M2$ exploits $M1$'s accommodation contract to collect some of this joint profit, and the retailer earns $\frac{\pi_1^{VI}-\pi_{12}^{VI}+\pi_2^{VI}}{1-\delta}$, less than in our base model. This implies that in a game in which $M1$ and $M2$ make simultaneous offers and then the retailer decides whether to accept both offers or one of them, the accommodation strategies cannot yield a pure-strategy equilibrium. If $M2$ expects $M1$ to accommodate product 2, $M2$ exploits this by offering a higher t_2 than in the exclusionary equilibrium. But given this fee, $M1$ will deviate to its exclusionary offer.

Accordingly, we consider a sequential game, in which $M1$ plays slightly before $M2$. This is a reasonable assumption, given that $M1$ is the incumbent supplier, which is likely to have a first-mover advantage in offering the retailer a contract before the supplier of the new product. When making its offer, $M1$ has two options. First, to set the exclusionary contract that we identified in our base model, which will induce $M2$ to set the fixed fee appropriate to the exclusionary equilibrium (recall that $M2$ sets $w_2 = c_2$ regardless of $M1$'s strategy). $M1$'s second option is to set the accommodation contract, which will be answered by $M2$'s accommodation fixed fee. In both cases, $M1$ earns the exclusion and accommodation profits that we identified in our base model, respectively. Hence $M1$ accommodates product 2 iff $\delta > \delta^{VS}$. This is summarized in the following corollary:

Corollary 2. *Suppose that a strategic firm, $M2$, sells product 2 and $M1$ makes the contract offer before $M2$. Then $M1$ accommodates product 2 iff $\delta > \delta^{VS}$. For $\delta > \delta^{VS}$ there is an accommodation equilibrium in which $M2$ sets $w_2 = c_2$ and $t_2 = \frac{\pi_{12}^{VI}-\delta\pi_2^{VI}}{1-\delta} - \pi_1^{VI}$ and $M1$ sets the same accommodation contract defined in our base model. $M1$ earns the same profits as in our base model. The joint profits of $M2$ and the retailer are also the same as in our base model, but now the retailer earns a lower share of these joint profits and $M2$ earns positive profits.*[7]

Had the game between $M1$ and $M2$ been simultaneous, then in any mixed strategy equilibrium the retailer would exclude the new product with some probability and accommodate it with the complementary probability. Note that any mixed strategy equilibrium is short-lived, in the sense that once the probability of accommodating product 2 is realized in a certain period, in all following periods, all players play a pure strategy. In this stationary pure-strategy equilibrium, the retailer sells only the improved product 2, $M1$ offers the retailer the contract $t_1 = 0$ and $w_1 = c_1$ and makes no sales, and $M2$ offers $t_2 = \pi_{2H}^{VI} - \pi_1^{VI}$ and $w_2 = c_2$.

[7] It is straightforward that $M2$ would not want to induce the retailer to hold only (inferior) product 2 (e.g., by offering a different fixed fee depending on whether the retailer also holds product 1). Such an exclusive dealing contract would harm $M2$ and the retailer's joint profits, by missing the opportunity to sell only a quantity $\underline{q}$ of inferior product 2 that suffices to improve it.

6 Vertical integration between the manufacturer of the established product and the retailer

We now extend the main model's analysis of vertical integration to the case where the manufacturer of the competing product is a strategic player. Suppose that $M1$ is vertically integrated with the retailer and $M2$ sells product 2. We show that in equilibrium, the market outcome is identical to the vertically integrated outcome and, in particular, product 2 is accommodated if and only if $\delta > \delta^{VI}$.

To solve for the exclusion and accommodation equilibria, we start by solving the game following accommodation in the previous period. Consider the case where product 2 has improved. In any subgame following such improvement, $M2$ offers the vertically integrated firm a contract (w_2, t_2) that makes it indifferent between selling only product 1 and selling only the improved product 2. If the vertically integrated firm sells only product 1 it earns π_1^{VI} per-period, and if it sells only the improved product 2 it earns $\pi_{2H}^{RVI}(w_2) - t_2$ per period, where $\pi_{2H}^{RVI}(w_2)$ denotes the vertically integrated firm's profits from selling the improved product 2 excluding the fixed fee, when the wholesale price it pays $M2$ is w_2. Hence $t_2 = \pi_{2H}^{RVI}(w_2) - \pi_1^{VI}$. $M2$'s profit is $\pi_{2H}^{M2VI}(w_2) + t_2$, where $\pi_{2H}^{M2VI}(w_2)$ denotes $M2$'s profits from selling its improved product, excluding the fixed fee. Substituting $t_2 = \pi_{2H}^{RVI}(w_2) - \pi_1^{VI}$, we have that $\pi_{2H}^{M2VI}(w_2) + t_2 = \pi_{2H}^{M2VI}(w_2) + \pi_{2H}^{RVI}(w_2) - \pi_1^{VI} = \pi_{2H}^{VI}(w_2) - \pi_1^{VI}$, where $\pi_{2H}^{VI}(w_2) \equiv \pi_{2H}^{M2VI}(w_2) + \pi_{2H}^{RVI}(w_2)$ is the vertically integrated profit from selling the improved product 2 given w_2, which is maximized at $M2$'s marginal cost of supplying the improved product 2. The vertically integrated firm's per-period profit in this case is π_1^{VI} and $M2$'s profit is $\pi_{2H}^{VI} - \pi_1^{VI}$.

Suppose now that $M2$ offers the vertically integrated firm its inferior product. According to the results of Section 3 in this appendix, $M2$ sets $w_2 = c_2$. This is because Section 3 establishes that $M2$ sets $w_2 = c_2$ regardless of $M1$'s contract, so this result also holds for $w_1 = c_1$ and $t_1 = 0$, which is equivalent to the case where $M1$ and the retailer are vertically integrated.

Consider first the equilibrium in which the vertically integrated firm excludes product 2. In this equilibrium, $M2$ sets $w_2 = c_2$ and $t_2 = -\frac{\delta}{1-\delta}(\pi_{2H}^{VI} - \pi_1^{VI})$, because $M2$ expects to earn $\pi_{2H}^{VI} - \pi_1^{VI}$ in each of the future periods, should product 2 improve and in an exclusionary equilibrium $M2$'s overall profits must be zero. The vertically integrated firm expects to earn π_1^{VI} in each period, if it excludes product 2, or $\pi_{12}^{VI} - t_2$, if it accommodates product 2 in the current period, and then earns

π_1^{VI} in all future periods. Hence, the vertically integrated firm prefers to exclude product 2 if:

$$\frac{\pi_1^{VI}}{1-\delta} \geq \pi_{12}^{VI} - t_2 + \frac{\delta}{1-\delta}\pi_1^{VI}. \tag{10}$$

Substituting $t_2 = -\frac{\delta}{1-\delta}(\pi_{2H}^{VI} - \pi_1^{VI})$, the above condition holds if and only if $\delta \leq \delta^{VI}$.

In the accommodation equilibrium, $M2$ sets $w_2 = c_2$ and t_2 that solves (10) in equality, hence: $t_2 = \pi_{12}^{VI} - \pi_1^{VI}$. Substituting $t_2 = \pi_{12}^{VI} - \pi_1^{VI}$ into $M2$'s profit from accommodation, $t_2 + \frac{\delta}{1-\delta}(\pi_{2H}^{VI} - \pi_1^{VI})$, yields a positive profit if and only if $\delta > \delta^{VI}$.[8]

We summarize these results in the following corollary:

Corollary 3. *Suppose that a strategic firm, $M2$, sells product 2 and $M1$ and the retailer are vertically integrated. Then, in equilibrium, the integrated firm accommodates product 2 if and only if $\delta > \delta^{VI}$ and the market implements the vertically integrated outcome.*

[8]It is straightforward that $M2$ would not induce the vertically integrated firm to hold only product 2. $M2$ cannot use the fixed fee for this purpose, since it applies whether the vertically integrated firm sells only product 2 or both products. $M2$ cannot use the wholesale price for this purpose either, since, as shown in this appendix, $M2$ sets $w_2 = c_2$, and to make selling only product 2 preferable it must be that $w_2 \leq c_1 < c_2$.