U.S. LABOR MARKET DYNAMICS
AND THE BUSINESS CYCLE

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Abstract

The picture of U.S. labor market dynamics is disturbingly opaque. Different empirical studies have yielded contradictory findings, debates have emerged regarding their business cycle implications, and there is also disagreement as to how much the search and matching model—a key model in this context—can explain the data.

This paper tries to determine what facts can be established, what are their implications for the business cycle, and what remains to be further investigated. It then re-examines the model to see whether it fits the data, what generates the fit, and where it fails.

On the data facts issue, some key moments of the flows between the employment and unemployment pools were found to be similar across studies; a set of clear business cycle facts emerges, including countercyclical and volatile hiring and separation rates, pro-cyclical job finding rates, with considerable volatility of both accessions and separations. However, there is no agreement on the computation of flows between the out of the labor force and employment pools. In terms of the business cycle implications, it turns out that both job finding and separation are key to the understanding of the cycle. On the fit of the search and matching model, there is a mixed answer. The model captures the persistence, volatility, and some of the co-movement in the data. It is shown that convex hiring costs and the appropriate stochastic process for separation shocks are needed for this fit. But, at the same time, wage behavior is not captured.

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1 Introduction

The picture of U.S. labor market dynamics and its implications for the study of business cycles are disturbingly opaque. There are three, related issues of concern:

First, different empirical studies of U.S. gross worker flows and labor market dynamics over the past two decades have yielded contradictory findings. Reading these different studies, it is not easy to get a sense of what the key data moments are and how they compare with each other.

Second, debates have emerged regarding the implications of these worker flows for the understanding of the business cycle. The ‘conventional wisdom,’ based on the reading of Blanchard and Diamond (1989, 1990), Davis and Haltiwanger (1999), and Bleakley, Ferris, and Fuhrer (1999), was that worker separations from jobs are the more dominant cyclical phenomenon than hirings of workers, and that therefore it is important to analyze the causes for separations or job destruction. In particular, it was believed that in order to study the business cycle it is crucial to understand the spikes and volatility of employment destruction. This view was challenged by Hall (2005) and Shimer (2005a,b), who claimed that separations are roughly constant over the cycle, and that the key to the understanding of the business cycle is in the cyclical behavior of the job finding rate.

Third, there is also disagreement as to how much the search and matching model – a key model in this context – can explain the data. Thus, for example, Mortensen and Pissarides (1994) extended the basic Pissarides (1985) model to cater for endogenous separations in order to capture the stylized facts on the importance of job destruction. But a number of subsequent papers claimed that the model does not fit the data well and that the key patterns of the data (to be fitted) are different from what Mortensen and Pissarides had in mind.

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This paper aims at clarifying the picture. It tries to determine what facts can be established, what are their implications for the business cycle, and what remains to be further investigated. The paper examines CPS data used by five key studies, as well as JOLTS data, and establishes the key facts. It then re-examines the model – in partial equilibrium form and in general equilibrium form – to see whether it fits the data, what generates the fit, and where it fails.

The paper proceeds as follows: Section 2 discusses the data issues, establishing a list of facts and a list of issues in need of further research. Section 3 examines the partial equilibrium version of the search and matching model. Section 4 examines the general equilibrium version of the model. In each of these last two sections the model is presented, the empirical methodology used is explained, calibration values are delineated, and results are reported and discussed. Section 5 concludes, summarizing the findings.

2 U.S. Data

In this section I build on the findings in Yashiv (2007b) regarding the properties of U.S. labor market dynamics data.

2.1 Labor Market Dynamics: Basic Equations

The dynamic equations of the labor market recognize the fact that in addition to the official pool of unemployed workers, to be denoted $U$, there is another relevant pool of non-employed workers – the ‘out of the labor force’ category, to be denoted $N$, and that there are substantial flows between the latter and the employment pool $E$.

The evolution of employment proceeds according to the following equation

$$E_{t+1} = E_t + M_{t}^{UE+NE} - S_{t}^{EU+EN}$$

where $E$ is the employment stock, $M_{t}^{UE+NE}$ are gross hiring flows from both unemployment and out of the labor force and $S_{t}^{EU+EN}$ are separation flows to these pools. In terms of rates this equation may be re-written as:
\[
\frac{E_{t+1}}{E_t} - 1 = \frac{M_t^{UE+NE}}{E_t} - \delta_t^{EU+EN}
\]  
(2)

where \( \delta = \frac{S}{E} \) is the separation rate from employment.

A similar equation holds true for unemployment dynamics:

\[
U_{t+1} = U_t(1 - p_t^{UE}) + \delta_t^{EU} E_t + F_t^{NU} - F_t^{UN}
\]  
(3)

where \( U \) is the unemployment stock, \( p^{UE} \) is the job finding rate (moving from unemployment to employment), and \( F_t^{NU} - F_t^{UN} \) is the net inflow of workers from out of the labor force, joining the unemployment pool (computed by deducting the gross flow out of unemployment from the gross flow into it).

This can be re-written:

\[
\frac{U_{t+1}}{U_t} - 1 = -p_t^{UE} + \delta_t^{EU} \frac{E_t}{L_t} \frac{L_t}{U_t} + \frac{F_t^{NU} - F_t^{UN}}{L_t} \frac{L_t}{U_t}
\]  
(4)

In steady state there is a constant growth rate of unemployment at the rate of labor force growth to be denoted \( g^L \):

\[
\frac{U_{t+1}}{U_t} - 1 = g^L
\]  
(5)

Thus the unemployment rate is constant at \( \pi_t \):

\[
\pi_t = \frac{U_t}{L_t}
\]  
(6)

The dynamic equation (4) becomes:

\[
g^L = -p_t^{UE} + \delta_t^{EU} (1 - \frac{\pi_t}{\bar{\pi}}) \frac{1}{\bar{\pi}} + \frac{F_t^{NU} - F_t^{UN}}{L_t} \frac{1}{\bar{\pi}}
\]  
(7)

Hence steady state unemployment is given by

\[
\bar{\pi} = \frac{F_t^{NU} - F_t^{UN}}{p_t^{UE} + g^L + \delta_t^{EU}} + \delta_t^{EU}
\]  
(8)

In case there is no labor force growth or workers joining from out of the labor force, i.e., \( \frac{F_t^{NU} - F_t^{UN}}{L_t} = g^L = 0 \), this becomes:
\[ \bar{\pi} = \frac{\delta^{EU}}{\delta^{EU} + p^{UE}} \]  

Noting that \( M_t = p_t U_t \) and \( \delta_t = \frac{S_t}{E_t} \), the empirical researcher needs data on the stocks \( U_t \) and \( E_t \) and on the flows \( M_t \) and \( S_t \), to investigate the determinants of \( \bar{\pi} \).

Note some implications of these equations:

(i) Taking the whole employment stock, \( E \), as one pool to be explained, it is flows to and from this pool that need to be accounted for. Flows within \( E \) (job to job) do not change \( E \) itself. In what follows, the term ‘separations’ will refer to separations from \( E \) and ‘hires’ will refer to hiring into \( E \), and not to separations or hires within \( E \). This is an important distinction, as some studies focused on separation from employment \( \delta^{EU+EN} \) while others focused on total separations \( \delta^{EU+EN+EE} \).

(ii) Another important distinction is between hiring rates \( \frac{M^{UE}}{E} \) and job finding rates \( p^{UE} = \frac{M^{UE}}{U} \); some studies compared the separation rate from employment \( \delta^{EU} \) to the former, while others emphasized the comparison to the latter.

(iii) The key variables for understanding the rate of unemployment at the steady state are \( p^{UE}, \delta^{EU}, \frac{E^{NU} - E^{UN}}{L} \) and \( g^L \).

2.2 Interpretation of the Data

I briefly summarize the interpretation given in the literature to the gross worker flows data – the variables \( M^{UE}, M^{NE}, S^{EU}, S^{EN} \) – in accounting for U.S. labor market dynamics.

**Trend.** A number of studies recognized trends in the data: Ritter (1993) discussed a downward trend in gross job finding and separation rates that starts around 1984. Bleakley, Ferris, and Fuhrer (1999) too noted a trend decline in flows in and out of employment since the early 1980s.

**Volatility.** Blanchard and Diamond (1989, 1990) found that the amplitude of fluctuations in the flow out of employment is larger than that of the flow into employment, implying that changes in employment are dominated by movements in job destruction rather than in job creation. Bleakley, Ferris, and Fuhrer (1999) found that once the trend is removed, the flows out of employment have more than twice the variance of the flows into employment. These studies places the emphasis on comparing hiring rates \( \frac{M^{UE}}{E} \) to the separation rate from employment \( \delta^{EU} \). But recently Shimer
(2005b) and Hall (2005) claimed that separation rates are not as volatile as job finding rates \( p \) (not hiring rates) and that they can be taken roughly as constant (in detrended terms). These studies typically refer to the total separation rate \( \delta^{EU+EN+EE} \), which includes job to job flows.

**Cyclicality.** Blanchard and Diamond (1989, 1990) found sharp differences between the cyclical behavior of the various flows. In particular, the EU flow increases in a recession while the EN flow decreases; the UE flow increases in a recession, while the NE flow decreases. Ritter (1993) reported that the net drop in employment during recessions is clearly dominated by job separations. Bleakley, Ferris, and Fuhrer (1999) found that the flow into voluntary quits declines fairly sharply during recessions, consistent with the notion that quits are largely motivated by prospects for finding another job. “Involuntary” separations – both layoffs and terminations – rise sharply during recessions and gradually taper off during the expansions that follow. Fallick and Fleischmann (2004) noted that the cyclicality of the flow into employment is unclear, as it combines a countercyclical flow from unemployment to employment with a procyclical flow from not in the labor force to employment. They concluded that the total flow out of employment is probably weakly countercyclical in the United States.

Recently, some authors have presented a new picture of worker flows cyclicality. Hall (2005) developed estimates of separation rates and job-finding rates for the past 50 years, using historical data informed by the detailed recent data from JOLTS. He found that the separation rate is nearly constant while the job-finding rate shows high volatility at business-cycle and lower frequencies. He concluded that this necessitates a revised view of the labor market: during a recession unemployment rises entirely because jobs become harder to find. Recessions involve no increases in the flow of workers out of jobs. Another important finding from the new data is that a large fraction of workers departing jobs move to new jobs without intervening unemployment.

Shimer (2005b) reported that the job finding probability is strongly procyclical while the separation probability is nearly acyclical, particularly during the last two decades. He showed that these results are not due to compositional changes in the pool of searching workers, nor are they due to movements of workers in and out of the labor force. He concluded that the results contradict the conventional wisdom of the last fifteen years. If one wants to understand fluctuations in unemployment, one must understand fluctuations in the transition rate from unemployment to
employment, not fluctuations in the separation rate. Note, that Hall (2005) and Shimer (2005b) focus on comparing \( p \) and \( \delta \), rather than \( \frac{M}{\delta} \) and \( \delta \), and that as noted above – they often refer to the total separation rate, including job to job flows.

In what follows I look at these characterizations and reconcile some of the differences.

2.3 Data Sources

There are two main sources for U.S. aggregate worker flow data: the CPS and JOLTS, both of the BLS. The CPS, available at http://www.bls.gov/cps/, is a household survey and offers a worker perspective. JOLTS data, available at http://www.bls.gov/jlt/home.htm, are based on a survey of employers. This data set includes monthly figures for hires, separations, quits, layoffs, and vacancies.

The CPS is the main basis for the data sets to be analyzed below. These data were computed and analyzed by Blanchard and Diamond (1989, 1990), Ritter (1993), Bleakley, Ferris and Fuhrer (1999), Fallick and Fleischmann (2004), and Shimer (2005b). Note that what is done below is not the analysis of the raw CPS data but rather the analysis of the computed data, i.e., the computed gross flows, based on CPS, as undertaken by the cited authors.

JOLTS data were reported and discussed by Hall (2005). I take the JOLTS data from the BLS website.

2.4 Measurement Issues

The CPS is a rotating panel, with each household in the survey participating for four consecutive months, rotated out for eight months, then included again for four months. With this structure of the survey, not more than three-quarters of survey respondents can be matched, and typically the fraction is lower because of survey dropouts and non-responses. Using these matched records, the gross flows can be constructed. However, there are various problems that need to be addressed when doing so. Thus, while such flows have been tabulated monthly from the CPS since 1949, the BLS has not published them because of their “poor quality.” More specifically, missing observations and

\footnote{Updated further till 2003:12.}
\footnote{A summary of data sources and a discussion of them is to be found in Farber (1999), Davis and Haltiwanger (1998,1999), Fallick and Fleischmann (2004) and Davis, Faberman, and Haltiwanger (2006).}
classification error were noted. These issues are discussed in detail in Abowd and Zellner (1985) and in Poterba and Summers (1986), who offer corrective measures. Additional issues involve methods of matching individuals across months, weighting individuals, aggregation across sectors and over time, survey methodology changes (in particular the 1994 CPS redesign), and seasonal adjustment. The above two studies, as well as the five studies which data are examined here, offer extensive discussion.

2.5 Why data series may differ

In the next sub-section I present an analysis of five data sets, all computed by the different authors on the basis of raw CPS data. They turn out not to be the same. Why so? The preceding discussion makes it clear that there are various measurement issues that need to be treated. It is evident that if treatment methods vary then the resulting series will differ. The discussion in Bleakley et al (1999, pages 72-76) gives important details about these adjustments. As key examples, consider the following points which emerge from this discussion:

Adjustments are substantial. The Abowd-Zellner adjustments for misclassification substantially reduce the transitions between labor market states. The N - E flows have the largest reduction, almost 50 percent. Likewise, Shimer’s (2005b) framework caters for time aggregation and leads to the capturing of more transitions relative to the other data sets that do not deal with this issue.

Application of adjustment methods may vary. The different authors have not used the same corrections of the data. One revelatory example is the following passage from Bleakley et al (1999, page 75):

“In order to apply Abowd and Zellner’s adjustments to the gross flows, we obtained adjusted gross flow data for January 1968 to May 1986 from Olivier Blanchard (Blanchard and Diamond 1990). The data have been Abowd-Zellner adjusted, using the reinterview surveys, and are not seasonally adjusted. By dividing these adjusted data by the raw gross flows, we obtained the multiplicative adjustment factors for each month from January 1976 to May 1986... Adjusting the data after May 1986 proves to be a difficult issue because Abowd and Zellner have not updated their series and we do not have the reinterview survey information to extend their findings. Based on the ad-
justment information we do have, the adjustment factors do change over time. We have estimates of misclassification for 1994 and 1995 from the BLS, which indicate that the 1994 misclassification rates differ dramatically from those for the 1976–86 period. Most error rates dropped substantially, with the exception of those between N and U. Therefore, to accurately adjust the gross flows using reinterview data, we plan on obtaining reinterview survey data from 1986 to the present. For this paper, we have chosen to use the mean adjustment for the period February 1976 to May 1986 for each seasonally adjusted transition (flow).”

This passage makes it clear that Abowd-Zellner adjustments depend on time-varying factors, with the possible result that they will be applied differently by different authors. Moreover, Bleakley et al (1999) use additional adjustments, dealing with the 1994 CPS redesign.

Seasonal adjustment may vary. The gross flows data exhibit very high seasonal variation (see for example the discussion of Tables 1 and 2 in Bleakley et al (1999)). The methodology of seasonally adjusting the series differs across studies: Blanchard and Diamond (1990) use the Census Bureau X11 program. Ritter (1993) also seasonally adjusts using the X-11 procedure but further smooths using a five-month centered moving average. Bleakley et al (1999) note the use of regressions on monthly dummies as well as the X11 methodology. Fallick and Fleischmann (2004) use the newer Census Bureau X12 seasonal adjustment program. Shimer (2005b) uses a ratio-to-moving average technique.

Hence, even though the data source may be the same, the resulting series may differ depending upon the differential application of adjustments.

2.6 Key Moments of the Gross Flows Data

The key findings are reported in Table 1 and Figures 1-6.

Table 1 and Figures 1-6

The following are the emerging points:

Flows from Unemployment.

a. The monthly hiring rate $\left(\frac{M^{UE}}{E}\right)$ is around 1.5%-1.7%, with a standard deviation of 0.1%-0.3%. Four series give a very similar picture. The series from Shimer (2005b), with a 2% mean,
is somewhat higher than the four others. This is probably due to the fact that he captures more transitions by correcting for time aggregation.

b. The monthly job finding rate \( \left( p_{t}^{UE} = \frac{M_{t}^{UE}}{l} \right) \) is around 25%-32% on average. The series from Shimer (2005b), with a 32% mean, is again somewhat higher than the four others. These numbers imply quarterly rates of around 60% - 70%. The average monthly volatility of this rate is around 3%-6%.

**Flows from Out of the Labor Force.** There seem to be two data sets here: Blanchard and Diamond (1989) and Bleakley et al. (1999), report mean hiring rates of 1.3%-1.5% and standard deviation of 0.1%-0.3%. The other two data sets span different sample periods but indicate mean hiring rates of 2.5%-2.9% and standard deviation of 0.2% or 0.4%.

**Total Hires.** The total hires reflect the differences between the data sets as discussed above. There is one addition, though, and that is JOLTS. While it has a mean rate of 3.2% and standard deviation of 0.2%, similar to Bleakley, Ferris and Fuhrer (1999), it has a negative correlation of -0.22 with the latter series.\(^4\)

**Flows from employment to unemployment.** The monthly separation rate into unemployment is around 1.3%-1.5% on average for all studies, except Shimer who puts it at 2%, again because of the treatment of time aggregation. The former imply quarterly separation rates of around 4%, while the latter implies 5.9%. Its volatility is around 0.1%-0.3% in monthly terms according to all studies.

**Flows from Employment to Out of the Labor Force.** The different data sets again seem to suggest different moments: a monthly mean ranging from 1.5% to 3.2% and a standard deviation ranging from 0.2% to 0.5%.

**Total Separations.** As in the case of total hires, the total separations flows reflect the differences between the data sets discussed above; and, again, there is the addition of the JOLTS data set. The picture that emerges is the following: the mean separation rate ranges from around 3% a month according to three sources to as high as 5% according to Shimer. The standard deviation ranges from a low of 0.15% according to the JOLTS data to as high as 0.47% according to Shimer.

\(^4\)It should be remarked, though, that there are only 49 overlapping monthly observations for these two series.
2.7 The Cyclical Behavior of Flows

Table 2 reports correlations and relative standard deviations of hiring rates (U to E, N to E, and both U and N to E), job finding rates, and separation rates (E to U, E to N, and E to both U and N) with real GDP. I use four alternative detrending methods (all on the logged series): first differences, the HP filter with the standard smoothing parameter \( \lambda = 1600 \), with a low frequency filter \( \lambda = 10^5 \), and the Baxter-King band-pass filter.

Table 2

The main findings are as follows.

Filtering effects. As to the cyclical series, the table shows that the filtering method matters. The filtered series are substantially less volatile than the original series, first differencing yields different patterns than the other methods, and the Baxter-King filtered series is less volatile than the HP filtered series. The Baxter-King band pass filter indicates that there is much high frequency movement in both \( p \) and \( \delta \) (over and beyond seasonality). Note, too, that the key comparison – the one between \( p \) and \( \delta \) – depends on the filtering method.

Co-movement. Generally across studies the following holds true:

(i) Hiring rates from unemployment to employment \( \left( \frac{M_{UE}}{E} \right) \) are counter-cyclical, while hiring rates from out of the labor force to employment \( \left( \frac{M_{NE}}{E} \right) \) are pro-cyclical. Summing up the two \( \left( \frac{M_{UE} + M_{NE}}{E} \right) \) yields a flow that is moderately counter-cyclical. This can be seen graphically in Figures 1a, 2 and 3 for the NBER-dated recessions.

(ii) Job finding rates from unemployment to employment \( (p^{UE}) \) are pro-cyclical. This can be seen graphically in Figure 1b for the NBER-dated recessions.

(iii) Separation rates from employment to unemployment \( \left( \frac{\delta_{EU}}{E} \right) \) are counter-cyclical, while those from employment to out of the labor force \( \left( \frac{\delta_{EN}}{E} \right) \) are pro-cyclical. Summing up the two \( \left( \frac{\delta_{EU} + \delta_{EN}}{E} \right) \) yields a flow that is moderately counter-cyclical. This can be seen in graphically Figures 4, 5, and 6 for the NBER-dated recessions.

Volatility. Across studies the following holds true:

(i) Hiring rates \( \frac{M}{E} \), job finding rates \( p \), and separation rates \( \delta \) are highly volatile, roughly 2 to 4 times the volatility of real GDP.
(ii) Hiring rates from unemployment to employment (\(\frac{M_{UE}}{E}\)) are less volatile than the corresponding separation flows (\(\delta^{EU}\)).

(iii) The reverse is true for flows between out of the labor force and employment (i.e., \(\frac{M_{NE}}{E}\) is more volatile than \(\delta^{EN}\)).

(iv) The sum of the hiring flows (\(\frac{M_{UE}+M_{NE}}{E}\)) is less volatile than the sum of the separation flows (\(\delta^{EU+EN}\)).

(v) There is no agreement across studies about the relationship between the volatility of the job finding rate \(p^{UE}\) and the volatility of the separation rate \(\delta^{EU}\). In the Blanchard and Diamond (1989,1990) and Ritter (1993) data the latter is more volatile than the former across all filtering methods; in the Bleakley, Ferris and Fuhrer (1999) data this is generally so too, but using the 10^5 HP filter they have almost the same volatility; in Fallick and Fleischmann (2004) separations are more volatile than hirings, but under the low frequency HP filter this relation is reversed; the Shimer (2005b) data indicate that for most filtering methods the opposite holds true, i.e. \(p^{UE}\) is more volatile than \(\delta^{EU}\). However, even for the latter, it is important to note that the volatility of aggregate job finding \(p^{UE+NE}\) is very similar to that of aggregate separations \(\delta^{EU+EN}\).

These facts are the ones to be explained. I turn now to two versions of the search and matching model to see whether it can explain the facts.

3 The Search and Matching Model: Partial Equilibrium

I first look at a partial equilibrium version of the search and matching model, based on Pissarides (1985)^5. In the next section I look at the general equilibrium version.

3.1 The Model

3.1.1 Basic Set-Up

There are two types of agents: unemployed workers (\(U\)) searching for jobs and firms recruiting workers through vacancy creation (\(V\)). Firms maximize their intertemporal profit functions with the choice variable being the number of vacancies to open. Each firm produces a flow of output (\(F\)),

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^5For a recent survey of the model see Yashiv (2007a).
paying workers wages ($W$) and incurring hiring costs ($\Gamma$). Workers and firms are faced with different frictions such as different locations leading to regional mismatch or lags and asymmetries in the transmission of information. These frictions are embedded in the concept of a matching function which produces hires ($M$) out of vacancies and unemployment, leaving certain jobs unfilled and certain workers unemployed. Workers are assumed to be separated from jobs at a stochastic, exogenous rate, to be denoted by $\delta$. The labor force ($L$) is growing with new workers flowing into the unemployment pool. The set-up, whereby search is costly and matching is time-consuming, essentially describes the market as one with trade frictions. Supply and demand are not equilibrated instantaneously, so at each date $t$ there are stocks of unemployed workers and vacant jobs. The model assumes a market populated by many identical workers and firms. Hence I shall continue the discussion in terms of “representative agents.” Each agent is small enough so that the behavior of other agents is taken as given.

3.1.2 Matching

A matching function captures the frictions in the matching process; it satisfies the following properties:

$$M_t = \bar{M}(U_t, V_t)$$ (10)

$$\frac{\partial \bar{M}}{\partial U} > 0, \quad \frac{\partial \bar{M}}{\partial V} > 0$$

Empirical work [see the survey by Petrongolo and Pissarides (2001)] has shown that a Cobb-Douglas function is useful for parameterizing it:

$$M_t = \mu U_t^\sigma V_t^{1-\sigma}$$ (11)

where $\mu$ stands for matching technology. The parameter $\sigma$ reflects the relative contribution of unemployment to the matching process and determines the elasticity of the hazard rates with respect to market tightness $\frac{V_i}{U_t}$.

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$^6$The hazard rates – $P$, the worker probability of finding a job, and $Q$, the firm’s probability of filling the vacancy – are derived as follows:
3.1.3 Firms

Firms maximize the expected, present value of profits (where all other factors of production have been “maximized out”):

$$\max_{\{V\}} \epsilon_0 \left[ \sum_{t=0}^{\infty} (\prod_{j=0}^{t} \beta_j) [F_t - W_tE_t - \Gamma_t] \right]$$

(12)

where $\epsilon$ is the expectations operator and $\beta_j = \frac{1}{1 + \tau_{t-1,t}}$.

This maximization is done subject to the employment dynamics equation given by:

$$E_{t+1} = (1 - \delta_t)E_t + Q_tV_t$$

(13)

The main F.O.C are:

$$\frac{\partial \Gamma_t}{\partial V_t} = Q_t \epsilon_t \Lambda_{t+1}$$

(14)

$$\Lambda_{t+1} = \epsilon_t \left[ \beta_{t+1} \left[ \frac{\partial F_{t+1}}{\partial E_{t+1}} - \frac{\partial \Gamma_{t+1}}{\partial E_{t+1}} - \frac{\partial (W_{t+1}E_{t+1})}{\partial E_{t+1}} \right] \right] + \epsilon_t \left[ \beta_{t+2}(1 - \delta_{t+2})\Lambda_{t+2} \right]$$

(15)

The first, intratemporal condition (equation 14) sets the marginal cost of hiring $\frac{\partial \Gamma_t}{\partial V_t}$ equal to the expected value of the multiplier times the probability of filling the vacancy. The second, intertemporal condition (equation 15) sets the multiplier equal to the sum of the expected, discounted marginal profit in the next period $\epsilon_t \left[ \beta_{t+1} \left[ \frac{\partial F_{t+1}}{\partial E_{t+1}} - \frac{\partial \Gamma_{t+1}}{\partial E_{t+1}} - \frac{\partial (W_{t+1}E_{t+1})}{\partial E_{t+1}} \right] \right]$ and the expected, discounted (using also $\delta$) value of the multiplier in the next period $\epsilon_t \left[ \beta_{t+2}(1 - \delta_{t+2})\Lambda_{t+2} \right]$.\(^7\)

\[^7\] Other F.O.C are the flow constraint (1) and a transversality condition.

\[^8\] Note that because I postulate that $\Gamma$ depends on $E$ (see below), the net marginal product for the firm depends on $E$. This marginal product is part of the match surplus bargained over, and therefore part of the wage solution discussed below. Hence the term $\frac{\partial W_{t+1}}{\partial E_{t+1}}$, usually absent, is not zero in this formulation.
For production I assume a standard Cobb-Douglas function:

\[ F_t = A_t K_t^{1-\alpha} E_t^\alpha \quad (16) \]

where \( A \) is technology and \( K \) is capital.

Hiring costs refer to the costs incurred in all stages of recruiting: the cost of posting, advertising and screening – pertaining to all vacancies \((V)\), and the cost of training and disrupting production – pertaining to actual hires \((QV)\). For the functional form I use a power function formulation. This modelling relates to the same rationale being used in the capital adjustment costs/Tobin’s Q literature. It emerged as the preferred one – for example as performing better than polynomials of various degrees – in structural estimation of this model reported in Yashiv (2000a,b) and in Merz and Yashiv (2007). The former studies used an Israeli data-set that is uniquely suited for such estimation with a directly measured vacancy series that fits well the model’s definitions. The latter study used U.S. data. Formally this function is given by:

\[ \Gamma_t = \Theta (\phi V_t + (1-\phi)QV_t)^{\gamma+1} F_t \quad (17) \]

Hiring costs are a function of the weighted average of the number of vacancies and the number of hires. They are internal to production and hence are proportional to output. Note that \( \Theta \) is a scale parameter, \( \phi \) is the weight given to vacancies as distinct from actual hires, and \( \gamma \) expresses the degree of convexity.

The function is linearly homogenous in \( V, E \) and \( F \). It encompasses the cases of a fixed cost per vacancy (i.e. linear costs, \( \gamma = 0 \)) and increasing costs (\( \gamma > 0 \)). Note, in particular, two special cases: when \( \gamma = 0 \) and \( \phi = 1 \), I get \( \Gamma_t = \Theta V_t F_t \), which is the standard specification in much of the literature. When \( \gamma = 1 \), I get the quadratic formulation \( \Gamma_t = \Theta (\phi V_t + (1-\phi)QV_t)^2 F_t \), which is analogous to the standard formulation in “Tobin’s q” models of investment where costs are quadratic in \( \frac{I}{K} \).

### 3.1.4 Wages

In this model, the matching of a worker and a vacancy against the backdrop of search costs, creates a joint surplus relative to the alternatives of continued search. The prototypical search and matching
model derives the wage \( W \) as the Nash solution of the bargaining problem of dividing this surplus between the firm and the worker [see the discussion in Pissarides (2000, Chapters 1 and 3)]. Note that I follow the prototypical model in the way it models wage setting in general, and in the specific form of wage bargaining in particular. While alternative mechanisms have been suggested, it is beyond the scope of this paper to examine them.

Formally this wage is:

\[
W_t = \text{arg max}(J_t^N - J_t^U)\xi(J_t^F - J_t^V)^{1-\xi}
\]

where \( J_t^N \) and \( J_t^U \) are the present value for the worker of employment and unemployment respectively; \( J_t^F \) and \( J_t^V \) are the firm’s present value of profits from a filled job and from a vacancy respectively; and \( 0 < \xi < 1 \) reflects the degree of asymmetry in bargaining.

Using the approach of Cahuc, Marque and Wasmer (2004) to solve (18) taking into account the fact that \( \frac{\partial W_{t+1}}{\partial E_{t+1}} \neq 0 \), the wage is given by:

\[
W_t = \xi \left( \alpha A \left( \frac{K_t}{E_t} \right)^{1-\alpha} \left[ \frac{1}{1-(1-\alpha)b_t} + \Theta \left( \frac{\phi V_t + (1-\phi)Q_t V_t}{E_t} \right)^{\gamma+1} \left( \frac{1-\alpha+\gamma}{(1+(1-\xi(1-\eta+\xi)))} \right) \right] + (1-\xi)b_t \right)
\]

where \( b_t \) is the income of the unemployed, such as unemployment benefits. I will posit \( b_t = \tau W_t \), where \( \tau \) is the replacement ratio.

I will also use the concept of the labor share in income:

\[
s_t = \frac{W_t}{E_t}
\]

---

9 The solution entails postulating the asset values of a filled job and of a vacant job for the firm and the asset values of employment and unemployment for the worker in (18). See the technical appendix at http://www.tau.ac.il/~yashiv/research.html for details of the derivation.

10 Below I shall use \( \eta = \frac{\xi}{(\xi+1-\xi\eta)} \).
3.1.5 Equilibrium

The stocks of unemployment and employment and the flow of hiring emerge as equilibrium solutions. Solving the firms’ maximization problem yields a dynamic path for vacancies; these and the stock of unemployment serve as inputs to the matching function; matches together with separation rates and labor force growth change the stocks of employment and unemployment.

This dynamic system may be solved for the five endogenous variables $V, U, M, E$ and $W$ given initial values $U_0, E_0$ and given the path of the exogenous variables. As noted above, this is a partial equilibrium model. The exogenous variables include the worker’s marginal product, the discount factor, the labor force, and the separation rate. If the production function is CRS and if the capital market is perfect – as I shall assume – the capital-labor ratio will be determined in equilibrium at the point where the marginal product of capital equals the interest rate plus the rate of depreciation. This in turn will determine production and the marginal product of labor.

In the following sub-sections I solve explicitly for a stochastic, dynamic equilibrium using a stochastic structure for the exogenous variables.

3.2 Methodology

I use a log-linear approach, transforming the non-linear problem into a first-order, linear, difference equations system through approximation and then solving the system using standard methods. To abstract from population growth, in what follows I cast all labor market variables in terms of rates out of the labor force $L_t$, denoting them by lower case letters. Productivity growth is captured by the evolution of $A$, which enters the model through the dynamics of $F_t$, so I divide all variables by the latter. This leaves a system that is stationary and is affected by shocks to labor force growth, to productivity growth, as well as to the interest rate and to the job separation rate, to be formalized below.

The model has four exogenous variables. These are productivity growth ($G^X = \frac{F_{t+1}}{F_t}$), labor force growth ($g^L = \frac{L_{t+1}}{L_t}$), the discount factor ($\beta$) and the separation rate ($\delta$). These affect dynamics as follows: productivity growth $G^X$ raises the match surplus; the discount rate $\beta$ (related to the rate of interest) and the separation rate $\delta$ are components of the relevant discount rate used in computing the present value of the match. Empirical testing reveals that $g^L$ can be modelled as white noise.
around a constant value. When I tried to add it as a stochastic variable to the framework below the results were not affected, so I treat it as a constant. It is the other three variables that inject shocks into this system. I do not formulate the underlying shocks structurally. Instead, I postulate that they follow a first-order VAR (for each variable $Y$, I use the notation $\hat{Y}_t = \frac{Y_t - Y}{Y_t - Y}$ where $Y$ is the steady state value, so all variables are log deviations from steady state):

$$
\begin{bmatrix}
\hat{G}^X_{t+1} \\
\hat{\beta}_{t+1} \\
\hat{\delta}_{t+1}
\end{bmatrix}
= \Pi
\begin{bmatrix}
\hat{G}^X_t \\
\hat{\beta}_t \\
\hat{\delta}_t
\end{bmatrix} + \Sigma
$$

(21)

I use reduced-form VAR estimates of the data to quantify the coefficient matrix $\Pi$ and the variance-covariance matrix of the disturbances $\Sigma$. Thus the current model is consistent with both RBC-style models that emphasize technology shocks as well as with models that emphasize other shocks. Note also that the shocks may interact through the off-diagonal elements in $\Pi$ and $\Sigma$.

In the non-stochastic steady state the rate of vacancy creation is given by:

$$
\Theta \left( \phi + (1 - \phi)Q \right) \left( \frac{\phi V}{E} + (1 - \phi)QV \right) = Q \frac{G^X \beta}{\left( 1 - (1 - \delta)G^X \beta \right)^\pi}
$$

(22)

The LHS are marginal costs; the RHS is the match asset value. It is the probability of filling a vacancy ($Q$) times the marginal profits accrued in the steady state. The latter are the product of per-period marginal profits $\pi$ and a discount factor $\frac{G^X \beta}{1 - G^X \beta (1 - \delta)}$ that takes into account the real rate of interest, the rate of separation and productivity growth.

As discussed above, steady state unemployment is given by

$$
u = \frac{F^N U - F^U N}{L} + \delta EU
$$

(23)

I log-linearly approximate the deterministic version of the F.O.C in the neighborhood of this steady state. The resulting system is a first-order, linear, difference equation system in the state variable $\hat{e}$ and the co-state $\hat{\lambda}$ with three exogenous variables, $\hat{G}^X$, $\hat{\beta}$ and $\hat{\delta}$. The matrices of coefficients are defined by the parameters $\alpha, \Theta, \gamma, \phi, \mu, \sigma$ and $\eta$ and by the steady state values of various variables. The solution of this system enables me to solve for the control variable —
vacancies, and for other variables of interest, such as unemployment, hires, the matching rate, and the labor share of income.\footnote{I thank Craig Burnside for generously sharing his Gauss simulation program and for very useful advice.}

### 3.3 Calibration

There are three structural parameters that are at the focal point of the model and that reflect the operation of frictions. These are the matching function parameter $\sigma$ (elasticity of unemployment), the wage parameter $\xi$, and the hiring function convexity parameter $\gamma$.

For $\sigma$ I use Blanchard and Diamond's (1989) estimate of 0.4. Structural estimation of the model using U.S. corporate sector data in Merz and Yashiv (2007) indicates a value of $\gamma$, the convexity parameter of the hiring cost function, around 2, i.e. a cubic function ($\gamma + 1 = 3$) for hiring costs. These costs fall on vacancies and on actual hires, with $\phi$ being the weight on the former. I follow the estimates in Yashiv (2000a) and set it at 0.3.

The wage depends on the asymmetry of the bargaining solution ($\xi$). Rather than imposing it, I solve it out of the steady state relations.

For the production function labor parameter $\alpha$ I use a fairly traditional value of 0.68, which is also the structural estimate of this parameter in Merz and Yashiv (2007).

For the values of the exogenous variables I use sample average values. Calibration of $Q$, the matching rate for vacancies, is problematic as there are no wide or accurate measures of vacancy durations for the U.S. economy. Using a 1982 survey, Burdett and Cunningham (1998) estimated hazard functions for vacancies both parametrically and semi-parametrically finding that the general form of the hazard function within the quarter is non-monotonic; based on their estimates the quarterly hazard rate should be in the range of $0.7 - 1$. I thus take $Q = 0.9$ which is also the value used by Merz (1995) and Andolfatto (1996). This implies a particular steady state value for the vacancy rate ($v$). I use the average of the labor share in income $s$ which is 0.58.

With the above values, I solve the steady state relations for the steady state vacancy rate $v$, the hiring cost scale parameter $\Theta$, the matching function scale parameter $\mu$, and the wage parameter $\xi$. I can then solve for the steady state values of market tightness $v_u$, the worker hazard rate $P$, per period profits $\pi$ and the match asset value $\lambda$. 

The following table summarizes the calibrated values.

Table 3

The data appendix specifies definitions and sources.

3.4 Model-Data Fit

I now turn to examine the performance of the model. Table 4 shows the moments implied by the model and those of the data.

Table 4

The following conclusions can be drawn:

Persistence. The model captures the fact that $u, m$ and $s$ are highly persistent. The model tends to somewhat overstate this persistence.

Volatility. The model captures very well the volatility of employment and unemployment. Hiring volatility is understated by the model. As to the labor share, the model substantially overstates its volatility.

Co-Movement. The counter-cyclical behavior of hiring and the pro-cyclical behavior of the worker job finding rate are well captured. But the behavior of the labor share is not captured: while in the data it is about a-cyclical and co-varies moderately with the hiring rate, it is strongly pro-cyclical in the model and has a strong negative relationship with hiring.

Overall fit. The model captures the persistence, volatility, and some of the co-movement in the data. The major problem concerns the labor share in income which is not well captured.

I turn now to look at the mechanism driving this model-data fit.

3.5 The Underlying Mechanism

The natural question to ask now is what underlies the fit and how it differs from studies like Shimer (2005a), which had reached different conclusions. In order to understand the essential mechanism in operation, I analyze the non-stochastic steady state and the stochastic dynamics.
3.5.1 The Non-Stochastic Steady State

Consider the following steady-state equations, the first of which is a re-writing of (22), and the second — equation (23) above — being the equality of unemployment inflows and outflows, usually referred to as the Beveridge curve:

\[ \Theta \tilde{Q}(\frac{\tilde{Q}}{e})^\gamma = Q\Phi \pi \]  

(24)

where \( \tilde{Q} = \phi + (1 - \phi)Q \) and \( \Phi = \frac{G^X \beta}{\left[1 - (1 - \delta)(G^X)^\beta\right]} \).

\[ u = \frac{F^W - F^W}{\mu} + \delta EU \]

(25)

Equation (24) shows the vacancy creation decision as an optimality condition equating the marginal costs of hiring (the LHS) with the asset value of the match (the RHS). It is clear that the responsiveness of vacancies \( \nu \) depends on the elasticity parameter \( \gamma \) of the hiring cost function. The RHS is the asset value of the match. This value can vary because per period profits \( \pi \) vary, because the discount factor \( \Phi \) varies, or because the matching hazard \( Q \) varies. Profits may vary because of changes in the match surplus or changes in the sharing of the surplus, with a key parameter being \( \xi \). Changes in the discount factor \( \Phi \) can happen because of changes in productivity growth \( (G^X) \), changes in the discount factor \( (\beta) \), or changes in match dissolution rate \( (\delta) \). Changes in the matching rate \( Q \) are predicated on changes in market tightness \( \frac{V}{U} \).

The following ingredients are therefore essential:

(i) The shape of the hiring costs function determining the LHS of equation (24) i.e., the marginal cost function, where \( \gamma \) is a key parameter.

(ii) The formulation of the match surplus — this depends both on the data used and on all key parameters of the model.

(iii) the surplus sharing rule, where \( \xi \) is the key parameter.

(iv) the discounting of the match surplus — here the data used (for \( G^X, \beta \) and \( \delta \)) and their stochastic properties are key.

Consider now three cases for the value of \( \gamma \). The ‘classic’ formulation in the literature posits \( \gamma = 0 \) (and \( \phi = 1 \)) so the LHS of equation (24) is given by:
\( \Theta \tilde{Q}(\frac{\tilde{Q}_v}{\tilde{e}}) = \Theta \frac{f_t}{e_t} \)

Hence marginal vacancy costs in terms of average productivity are a constant \( \Theta \).

Based on empirical work (Merz and Yashiv (2007)), the current formulation is \( \gamma = 2 \) and \( 0 < \phi < 1 \), so the LHS of equation (24) is given by:

\[
\Theta \tilde{Q}(\frac{\tilde{Q}_v}{\tilde{e}})^\gamma = \Theta [\phi + (1 - \phi)Q_t] \left[ \frac{v_t}{e_t} \right]^2 \frac{f_t}{e_t}
\]

Marginal costs are a function of both vacancies and hires and are quadratic in the vacancy rate.

There is an intermediate case \( \gamma = 1 \) and \( 0 < \phi < 1 \), so the LHS of equation (24) is given by:

\[
\Theta \tilde{Q}(\frac{\tilde{Q}_v}{\tilde{e}})^\gamma = \Theta [\phi + (1 - \phi)Q_t] \left[ \frac{v_t}{e_t} \right] \frac{f_t}{e_t}
\]

Figure 7 plots equation (24) in \( u - v \) space for these three cases, namely \( \gamma = 0, 1 \) and 2.

The intuition for these shapes is as follows: for the linear \( \gamma = 0 \) case, the RHS of equation (24) depends on market tightness \( \tilde{e} \), but the LHS does not. Hence when \( u \) rises (for whatever reason), ceteris paribus, \( v \) rises too and the curve is upward sloping. The rise in \( u \) causes a decline in the job finding rate \( p \) and so wages decline, profits rise and vacancy creation rises. When \( \gamma > 0 \) while the afore-going consideration holds true, there is an additional one which is quantitatively stronger. As \( u \) rises, \( e \) falls. On the LHS this means that \( v \) falls, ceteris paribus. The reason is that as the employment stock falls, the firm will want to open fewer vacancies for a given present value of profits (costs depend on the vacancy rate). Because the latter effect on costs is stronger than the former effect on profits, the curve in the \( \gamma = 1 \) or 2 cases is downward sloping.

Now consider a rise in the separation rate, which is part of the discount factor \( \Phi \). Figure 8 shows the change in steady state using both equation (24) and equation (25). Panel a shows the \( \gamma = 0 \) case and panel b the \( \gamma = 2 \) case.

Figure 8
In both panels, the curve of equation (24) moves relatively little, in line with the arguments posited by Shimer (2005a). But the Beveridge curve equation (25) moves substantially. The results of panel (a) show that \( v \) rises with \( u \) and market tightness \( \frac{v}{u} \) changes little. The results of panel (b) show that \( v \) falls while \( u \) rises and that market tightness \( \frac{v}{u} \) declines. These results help explain why the standard models (with \( \gamma = 0 \)) have obtained a counter-factual positive relationship between \( u \) and \( v \) and little movement in market tightness \( \frac{v}{u} \). Note that the latter determines the job finding rate \( p \) and the matching rate \( Q \). Likewise, this analysis explains why the current results fit the data better.

I turn now to the stochastic dynamics.

### 3.5.2 The Stochastic Dynamics

In order to see how the same elements affect the dynamics, I undertake some counterfactual simulations, reported in Table 5.12

**Table 5**

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>Persistence and Volatility of Vacancy Creation</th>
<th>Volatility of Matching</th>
<th>Volatility of Employment</th>
<th>Volatility of Wages</th>
<th>Volatility of Hiring and Separation Rates</th>
<th>Volatility of Job Finding Rates</th>
<th>Relationship Between ( u ) and ( v ), the Beveridge Curve, Turns Positive, and Market Tightness ( \frac{v}{u} ) Volatility Falls by a Half</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.021</td>
<td>0.015</td>
<td>0.02</td>
<td>0.01</td>
<td>More Volatile</td>
<td>Less Pro-cyclical</td>
<td>Better</td>
</tr>
</tbody>
</table>

Looking at the convexity of hiring costs (parameterized by \( \gamma \)), the table implies that values of \( \gamma \) determine the persistence and volatility of vacancy creation. The latter then influences the second moments of matching, and consequently the moments of unemployment and employment. To see this, panel (a) of the table presents the outcomes when \( \gamma \) is set to zero, i.e., postulating linear hiring costs. The table shows that when moving from convex (\( \gamma = 2 \)) to linear costs (\( \gamma = 0 \)), all the persistence statistics decline, getting further away from the data; employment volatility falls from the data-consistent 0.021 (in log terms) to 0.015, (and likewise for unemployment); wages become counterfactually more volatile; hiring and separation rates become more disconnected and job finding rates become less pro-cyclical. The relationship between \( u \) and \( v \), the Beveridge curve, turns positive, and market tightness \( \frac{v}{u} \) volatility falls by a half. The only good point is that hiring volatility comes closer to the data.

The other key element discussed above is the role played by the match separation rate \( \delta \). As it is a variable with a relatively high mean, it is the main determinant of the relevant discount.

---

12 Note that the simulations pertain to the full stochastic dynamic system.
factor $\Phi$; as it has relatively high volatility and persistence, it makes the present value of the match volatile and persistent. This in turn engenders the volatility and persistence of vacancies, hiring, and unemployment. To see this point consider panel (b) of Table 5, where, in the column labeled counterfactual 1, the AR coefficient of $\delta$ is reduced to 0.1, counterfactually. The change in the moments is substantial: the persistence of all variables falls from 0.98 to 0.55–0.76 and the volatility of all variables is reduced so that standard deviations are 2%-5% of their benchmark, actual values. The co-movement statistics weaken: most of the co-movement relations weaken moderately, and the negative co-movement relations of the labor share with the hiring rate and of unemployment with vacancies weaken substantially. I reset the persistence to its actual value, and, in the column labeled counterfactual 2, I set the variance and co-variance of $\delta$ to zero. This dramatically lowers the persistence, the volatility, and the co-movement statistics of all the variables.

Next, I examine whether the interest rate has a similar effect via $\beta$. The column labeled counter-factual 3 sets its variance and co-variance to zero but this change hardly has any effect.

Finally, in panel c, I combine both a linear hiring cost function ($\gamma = 0$) and zero variance and co-variance for $\beta$ and $\delta$. The results are negative auto-correlation statistics, very low volatility, weaker co-movement, and a positive, rather than negative, correlation between $u$ and $v$. Comparing panel (c) to panels (a) and (b), one can see that allowing no variance for $\delta$ is the dominant effect on all moments.

Thus the performance of the model hinges to a large extent on the formulation of $\gamma$ and on the stochastic properties of $\delta$. The dynamic analysis has shown that the lack of fit in part of the literature is due to the use of a linear hiring cost function ($\gamma = 0$) instead of a convex one ($\gamma = 2$ here), and due to different stochastic properties assigned to the separation rate $\delta$. Other elements of the model play a smaller role. The wage parameter $\xi$ basically has a scale effect on per period profits and hence on the scale of match asset values. It therefore affects the value of the variables at the steady state but does not affect the dynamics, as it does not affect the response of vacancy creation to asset values. The matching function parameter $\sigma$, that does have this ‘elasticity’ type of effect, has a range of possible variation that is much smaller than the variation in values of $\gamma$. For example, a reasonable change in $\sigma$ would be 0.1 or 0.2 relative to the benchmark value (which is 0.4), but a move from linear ($\gamma = 0$) to cubic ($\gamma = 2$) costs is a change of 2 in the value of
The interest rate and the rate of productivity growth in their turn play a much smaller role than the separation rate in discounting future values. While \( \delta \) has a sample mean of 8.6\% and a standard deviation of 0.8\%, the rate of productivity growth \( (G_X - 1) \) has a sample mean of 0.4\% and standard deviation of 0.6\%. The rate of interest \( (\frac{1}{\beta} - 1) \) has a sample mean of 1.4\% and a relatively high standard deviation, 5.5\%, but as shown in panel (b) of Table 5 it does not play a significant role.

4 The Search and Matching Model: General Equilibrium

(Preliminary and incomplete)

4.1 Model

Following the implementation by Den Haan, Ramey, and Watson (2000) and Krause and Lubik (2007) of the Mortensen and Pissarides (1994) model, the following elements and equations are added to the partial equilibrium model presented above.

4.1.1 Environment

Households maximize an intertemporal utility function via choice of consumption \( (C) \). There is a continuum of jobs within the firm. Productivity has an aggregate component, evolving according to an AR1 process, and a job-specific component. The latter is drawn each period from a time-invariant distribution (with density \( g(a) \) and cdf \( G(a) \)). Workers are assumed to be separated from jobs at a stochastic rate; the latter has an exogenous part, to be denoted by \( \delta^x \), and an endogenous part \( \delta^n \). \( \delta^n \) is the result of the existence of an optimal threshold \( a_t \) for job specific productivity, below which the job and the worker separate. Firms open vacancies \( (V) \) and determine the productivity threshold \( a_t \) optimally.

4.1.2 Households

Households maximize utility:
\[
\max_C \sum_{t=0}^{\infty} \beta^t U(C_t)
\]  

subject to the budget constraint:

\[
C_t + I_t = W_t E_t + b_t U_t + (R_t - 1) K_t
\]

where \( \epsilon \) is the expectations operator, \( C \) is consumption, \( \beta \) is a discount factor, \( I \) is investment in capital \( K \) which depreciates at rate \( \psi \), and \( R \) is the gross rate of interest.

The F.O.C is:

\[
U_C = \beta \epsilon_t [(R_{t+1} - \psi)U_{C_{t+1}}]
\]

In what follows I shall use the notation:

\[
\lambda_t \equiv U_{C_t}
\]

### 4.1.3 Firms

Firms maximize profits:

\[
\max_{\{V,K\}} \Pi_t = \epsilon_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} [F_t - W_t E_t - \Gamma_t - (R_t - 1) K_t]
\]

subject to

\[
E_{t+1} = (1 - \delta_t) E_t + Q_t V_t
\]

\[
K_{t+1} = (1 - \psi) K_t + I_t
\]

The F.O.C are:

\[
\frac{\partial \Gamma_t}{\partial V_t} = Q_t \beta \epsilon_t \lambda_{t+1} \frac{\lambda_t}{\lambda_{t+1}} \left[ \frac{\partial F_{t+1}}{\partial E_{t+1}} - \frac{\partial W_{t+1}}{\partial E_{t+1}} - \frac{\partial (W_{t+1} E_{t+1})}{\partial E_{t+1}} \right] + (1 - \delta_{t+2}) \frac{\partial \Gamma_{t+1}}{\partial Q_{t+1}}
\]

26
\[ \frac{\partial F_t}{\partial a_t} - \frac{\partial \Gamma_t}{\partial a_t} - \frac{\partial (W_t E_t)}{\partial a_t} = \epsilon_t \Lambda_{t+1} \left[ E_t \frac{\partial \delta_t}{\partial a_t} \right] \] (33)

\[ \frac{\partial F_t}{\partial K_t} = (R_t - 1) \] (34)

### 4.1.4 Wage Determination

The Nash wage solution is given by:

\[ W(a_t) = \arg \max (J_t^E - J_t^U) \xi (J_t^F - J_t^V)^{1-\xi} \] (35)

which works out to be:

\[ W(a_t) = \chi_t \left[ \frac{\partial F_t}{\partial E_t} - \frac{\partial \Gamma_t}{\partial E_t} - E_t \frac{\partial W_t}{\partial E_t} \right] + \chi_t P_t \Lambda_t \\
+ (1 - \chi_t) U_t b_t \\
\chi_t = \frac{\xi}{(\xi + (1 - \xi) U_t \tau)} \] (36)

with \( b_t = \tau W_t \).

### 4.1.5 Functional Forms

For functional forms the following will be used:

CRRA utility on consumption.

\[ U(C_t) = \frac{C_t^{1-\omega}}{1-\omega} \] (37)

A log-normal distribution for idiosyncratic productivity.

\[ a \sim LN(g) \] (38)

The separation rate is now given by:
\[ \delta_t = \delta_t^x + (1 - \delta_t^x)\delta_t^n \]  
\[ \delta_t^n = G(a_t) \]  

Production now embeds both aggregate productivity and idiosyncratic productivity.

\[ F_t = A_t E_t^\alpha K_t^{1-\alpha} \]  
\[ \tilde{E}_t = E_t \int_a^\infty a g(a_t) \frac{g(a_t)}{1 - G(a_t)} da_t \]  

The wage bill is also affected by idiosyncratic productivity.

\[ W_t E_t = E_t \int_a^\infty W(a_t) \frac{g(a_t)}{1 - G(a_t)} da_t \]  

4.1.6 Shocks

Aggregate productivity is modeled as follows:

\[ \ln A_{t+1} = \rho_A \ln A_t + \sigma_A \]  

Idiosyncratic productivity shocks are drawn from an i.i.d log normal distribution \( g \) with CDF \( G \).

4.1.7 Equilibrium

The endogenous variables now are \( C, K, R, V, a, U \), and \( w \). Knowing these the variables \( E, Q, P, \) and \( \delta^n \) can be determined. The exogenous variables are \( \tau \) and \( \delta^x \), the parameters of the aggregate productivity process \( A \) and the functional form and moments of \( g(a) \).

4.2 Methodology

The simulation methodology consists of the following steps:

(i) Calibration of the parameters:
• discounting $\beta$
• matching $\mu, \sigma$
• CRRA utility parameter $\omega$
• bargaining $\xi$
• hiring $\Theta, \gamma, \phi$
• aggregate productivity $\rho_A, \sigma_A$
• idiosyncratic productivity – moments of $g$

(ii) Calibration of the exogenous separation rate $\delta^x$ and steady state values.
(iii) Log-linearization of the solution around steady state.
(iv) Simulation of the second moments.\textsuperscript{13}

I follow the calibration values used in Section 3 above. For the additional parameters I use the following: for the CRRA utility parameter, I use the fairly standard coefficient $\omega = 2$; for aggregate productivity I choose the parameters $\rho_A = 0.95$ and $\sigma_A = 0.0049$ so as to match the moments of U.S. GDP time series; in order to calibrate the two moments of the lognormal distribution assumed for idiosyncratic productivity, I normalize the mean to zero and choose the second moment so as to replicate the observed volatility of the job destruction rate. The standard deviation is therefore 0.12.

4.3 Results

To be expanded in the next version of the paper.

Preliminary results indicate the following:

(i) The basic properties of the data that are well captured include counter-cyclical $u, \rho$ and $m$, and pro-cyclical $p$.

\textsuperscript{13} I thank Michal Krause and Thomas Lubik for generously sharing with me their calibration-simulation MATLAB code.
(ii) With conventional linear vacancy costs, vacancy behavior is not captured as it is counter-factually counter-cyclical, not volatile, and has positive correlation with unemployment. But when hiring costs are made convex, vacancies become counter-cyclical and volatile.

(iii) In the basic specification wages are – as in the PE version – very pro-cyclical and too volatile. With convex hiring costs, wages become less volatile but are still too pro-cyclical.

(iv) There is great sensitivity to the calibration of $\delta^x$ and the second moment of $g(a)$. For example, increasing each of these makes $u, v, p, m, \delta$ more volatile and makes $m$ and $\rho$ less cyclical.

4.4 Mechanisms

To be expanded in the next version of the paper.

The same mechanism described above applies. Here in addition there is also the job destruction condition:

$$\frac{\partial F_t}{\partial a_t} - \frac{\partial \Gamma_t}{\partial a_t} - \frac{\partial (W_t E_t)}{\partial a_t} = \varepsilon_t \Lambda_{t+1} \left[ E_t \frac{\partial \delta_t}{\partial a_t} \right]$$

This shows that separations depend—beyond the exogenous factor $\delta^x$—on the present value of the match at the threshold idiosyncratic productivity level. The latter then implies that the factors discussed above, namely productivity and discounting, play a role again.

5 Conclusions

The paper began with the statement that the picture of U.S. labor market dynamics is opaque and has posed three main problems.

On the data facts issue, the following are the key findings: key moments of the flows between the employment and unemployment pools were found to be similar across studies; a set of clear business cycle facts emerges, including countercyclical and volatile hiring and separation rates, procyclical job finding rates, with considerable volatility of both accessions and separations. However, on the computation of flows between the out of the labor force and employment pools there is no agreement. The different computations – probably due to differential adjustments of the raw data
– affect the implied series of job finding and separation rates, and the reconciliation of gross and net flows.

On the business cycle implications, it turns out that both job finding and separation are key to the understanding of the cycle.

On the fit of the search and matching model, there is a mixed answer. On the one hand, the model captures the persistence, volatility, and some of the co-movement in the data. It was shown that convex hiring costs and the appropriate stochastic process for separation shocks are needed for the fit. On the other hand, wage behavior is not captured. Moreover, the nature of the shocks leading to separations remains unclear. In terms of the model, the process generating $\delta_t$ merits further attention, possibly based on micro studies of productivity behavior.
Data for Sections 3 and 4: Sources and Definitions


<table>
<thead>
<tr>
<th>variable</th>
<th>symbol</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployment – official pool</td>
<td>$U^0$</td>
<td>CPS, BLS series id: LNS13000000</td>
</tr>
<tr>
<td>Unemployment – additional pools (see Table 1 below)</td>
<td>$U$</td>
<td>CPS, BLS</td>
</tr>
<tr>
<td>Employment (total), household survey</td>
<td>$E$</td>
<td>CPS, BLS series id: LNS12000000</td>
</tr>
<tr>
<td>Vacancies – Index of Help Wanted ads</td>
<td>$V$</td>
<td>Conference Board$^1$</td>
</tr>
<tr>
<td>Hires</td>
<td>$QV$</td>
<td>CPS, Boston Fed computations$^2$</td>
</tr>
<tr>
<td>Separations</td>
<td>$\delta E$</td>
<td>CPS, Boston Fed computations$^2$</td>
</tr>
<tr>
<td>Working age population$^4$</td>
<td>$POP$</td>
<td>CPS, BLS series id: LNU00000000</td>
</tr>
<tr>
<td>Labor share$^5$</td>
<td>$s = \frac{WE}{F}$</td>
<td>Table 1.12. NIPA, BEA$^3$</td>
</tr>
<tr>
<td>Productivity</td>
<td>$\frac{F}{E}$</td>
<td>BLS</td>
</tr>
<tr>
<td>Cost of finance (equity and debt$^6$)</td>
<td>$r$</td>
<td>Tables 1.1.5; 1.1.6 NIPA, BEA</td>
</tr>
</tbody>
</table>

Notes:

1. BLS series are taken from http://www.bls.gov/cps/home.htm
2. Data were downloaded from Federal Reserve Bank of St. Louis http://research.stlouisfed.org/fred2/series
3. See Bleakley et al (1999) for construction methodology. I thank Jeffrey Fuhrer and Elizabeth Walat for their work on this series.
4. Total civilian noninstitutional population 16 years and older.
5. Total compensation of employees divided by GDP.
6. This is a weighted average of the return.
References


Table 1

Moments of the Gross Flows

a. Hiring Flows to Employment

<table>
<thead>
<tr>
<th>study</th>
<th>sample</th>
<th>$\frac{M^U_E}{E}$</th>
<th>$p^U_E = \frac{M^U_E}{U}$</th>
<th>$M^{NE}_E$</th>
<th>$M^{U+NE}_E$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>std.</td>
<td>mean</td>
<td>std.</td>
<td>mean</td>
</tr>
<tr>
<td>BD</td>
<td>0.017</td>
<td>0.002</td>
<td>0.257</td>
<td>0.053</td>
<td>0.015</td>
</tr>
<tr>
<td>R</td>
<td>0.017</td>
<td>0.002</td>
<td>0.263</td>
<td>0.046</td>
<td>0.029</td>
</tr>
<tr>
<td>BFF</td>
<td>0.016</td>
<td>0.002</td>
<td>0.247</td>
<td>0.030</td>
<td>0.013</td>
</tr>
<tr>
<td>FF</td>
<td>0.015</td>
<td>0.001</td>
<td>0.288</td>
<td>0.029</td>
<td>0.025</td>
</tr>
<tr>
<td>S</td>
<td>0.020</td>
<td>0.003</td>
<td>0.321</td>
<td>0.050</td>
<td>–</td>
</tr>
<tr>
<td>J</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

b. Separation Flows from Employment

<table>
<thead>
<tr>
<th>study</th>
<th>sample</th>
<th>$\delta^{EU}$</th>
<th>$\delta^{EN}$</th>
<th>$\delta^{EN+EU}$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>std.</td>
<td>mean</td>
<td>std.</td>
</tr>
<tr>
<td>BD</td>
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<td>0.003</td>
<td>0.017</td>
<td>0.002</td>
</tr>
<tr>
<td>R</td>
<td>0.015</td>
<td>0.003</td>
<td>0.032</td>
<td>0.004</td>
</tr>
<tr>
<td>BFF</td>
<td>0.013</td>
<td>0.002</td>
<td>0.015</td>
<td>0.001</td>
</tr>
<tr>
<td>FF</td>
<td>0.013</td>
<td>0.001</td>
<td>0.027</td>
<td>0.002</td>
</tr>
<tr>
<td>S</td>
<td>0.020</td>
<td>0.003</td>
<td>0.030</td>
<td>0.004</td>
</tr>
<tr>
<td>S II</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>J</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>
Notes:


2. All numbers are the relevant flows as adjusted by the authors and are divided by seasonally-adjusted employment.

3. All data are monthly except for Shimer (2005b) data, which are quarterly averages of monthly data. The latter were computed by converting the computed transition rate $f$ to the probability rate $F$ using the relation $F_t \equiv 1 - e^{-f_t}$. 
### Table 2

**Business Cycle Properties**

**a. Full Analysis**

<table>
<thead>
<tr>
<th>BFF (1999)</th>
<th>1st diff.</th>
<th>HP (1600)</th>
<th>HP (10^5)</th>
<th>BK</th>
</tr>
</thead>
<tbody>
<tr>
<td>1976 : I − 2003 : IV</td>
<td>ρ</td>
<td>σ_ρ/σ_y</td>
<td>ρ</td>
<td>σ_ρ/σ_y</td>
</tr>
<tr>
<td></td>
<td>M^{UE}_E , y</td>
<td>-0.23</td>
<td>6.9</td>
<td>-0.68</td>
</tr>
<tr>
<td></td>
<td>M^{NE}_E , y</td>
<td>0.06</td>
<td>7.1</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td>M^{UE} + M^{NE}_E , y</td>
<td>-0.12</td>
<td>5.9</td>
<td>-0.43</td>
</tr>
<tr>
<td></td>
<td>p^{UE}_E , y</td>
<td>0.31</td>
<td>7.3</td>
<td>0.76</td>
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<tr>
<td></td>
<td>δ^{EU} , y</td>
<td>-0.41</td>
<td>8.4</td>
<td>-0.77</td>
</tr>
<tr>
<td></td>
<td>δ^{EN} , y</td>
<td>-0.01</td>
<td>6.3</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>δ^{EU} + δ^{EN} , y</td>
<td>-0.28</td>
<td>6.1</td>
<td>-0.53</td>
</tr>
</tbody>
</table>

**b. Abridged Analysis (HP filter 1600)**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ρ</td>
<td>σ_ρ/σ_y</td>
<td>ρ</td>
</tr>
<tr>
<td></td>
<td>M^{UE}_E , y</td>
<td>-0.75</td>
<td>4.4</td>
</tr>
<tr>
<td></td>
<td>M^{NE}_E , y</td>
<td>0.56</td>
<td>4.9</td>
</tr>
<tr>
<td></td>
<td>M^{UE} + M^{NE}_E , y</td>
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<td></td>
<td>p^{UE}_E , y</td>
<td>0.80</td>
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<td></td>
<td>p^{UE} + p^{NE}_E , y</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>JF , y</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>δ^{EU} , y</td>
<td>-0.81</td>
<td>7.2</td>
</tr>
<tr>
<td></td>
<td>δ^{EN} , y</td>
<td>0.54</td>
<td>4.6</td>
</tr>
<tr>
<td></td>
<td>δ^{EU} + δ^{EN} , y</td>
<td>-0.41</td>
<td>3.0</td>
</tr>
</tbody>
</table>

**sample**

Notes:

a. $y$ is real GDP.

b. All variables are logged; then they are either first differenced or are filtered using the Hodrick-Prescott filter (with smoothing parameter 1600 or $10^5$) or with the Baxter King filter. Panel b reports only results with the Hodrick-Prescott filter, using smoothing parameter 1600.

c. $\frac{\sigma_y}{\sigma_y}$ is the relative standard deviation, where the standard deviation of filtered GDP is in the denominator.

d. For the Shimer (2005b) data the following computations were used: (i) Define $\lambda_t^{XY}$ as the Poisson arrival rate of a shock that moves a worker from state $X \in \{U, E, N\}$ to another state during period $t$. $\Lambda^{XY} = 1 - e^{\lambda^{XY}}$ is the associated full-period transition probability. The series $\lambda_t^{NE}$ and $\lambda_t^{UE}$ are available from Shimer’s website (see http://home.uchicago.edu/~shimer/data/flows/).

(ii) b. To obtain $p^{UE+NE}$, the following formula was used: $p^{UE+NE} = (1 - e^{\lambda^{UE}}) \frac{CPS_U}{CPS_U + CPS_N} + (1 - e^{\lambda^{NE}}) \frac{CPS_N}{CPS_U + CPS_N}$ where $CPS_U$ is quarterly average of monthly SA CPS data on the number of unemployed; $CPS_N$ is quarterly average of monthly SA CPS data on the number of persons ‘not in the labor force.’ (iii) c. For Shimer II the $JF$ probability was calculated from the job finding rate $f_t$, given in the above web page using $F_t = 1 - e^{-f_t}$. In Shimer (2007) $F$ is given by $F_t = 1 - \frac{u_{t+1} - u_t^s}{u_t}$ where $u_t = \text{number of unemployed in period } t$, $u_{t+1} = \text{number of unemployed in period } t+1$, and $u_t^s = \text{short term unemployed workers, who are unemployed at date } t+1$ but held a job at some point during period $t$. An explanation of how short term unemployment was calculated is to be found in Shimer’s (2005b), Appendix A.
### Table 3

**Calibration Values**

a. Parameters, Exogenous Variables, and Steady State Values

<table>
<thead>
<tr>
<th>Quarterly Parameter/Variable</th>
<th>symbol</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production</td>
<td>$\alpha$</td>
<td>0.68</td>
</tr>
<tr>
<td>Matching</td>
<td>$\sigma$</td>
<td>0.4</td>
</tr>
<tr>
<td>Hiring (convexity)</td>
<td>$\gamma$</td>
<td>2</td>
</tr>
<tr>
<td>Hiring (vacancy weight)</td>
<td>$\phi$</td>
<td>0.3</td>
</tr>
<tr>
<td>Productivity growth</td>
<td>$\frac{A_{t+1}}{A_t}$</td>
<td>0.003536</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>0.9929</td>
</tr>
<tr>
<td>Separation rate</td>
<td>$\delta$</td>
<td>0.0854</td>
</tr>
<tr>
<td>Unemployment</td>
<td>$u$</td>
<td>0.104</td>
</tr>
<tr>
<td>Labor share</td>
<td>$s = \frac{W}{F/E}$</td>
<td>0.58</td>
</tr>
<tr>
<td>Vacancy matching rate</td>
<td>$Q$</td>
<td>0.9</td>
</tr>
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</table>

b. Implied Values

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Matching scale parameter</td>
<td>$\mu$</td>
</tr>
<tr>
<td>Hiring scale parameter</td>
<td>$\Theta$</td>
</tr>
<tr>
<td>Wage bargaining parameter</td>
<td>$\xi$</td>
</tr>
<tr>
<td>Vacancy rate</td>
<td>$v$</td>
</tr>
<tr>
<td>Market tightness</td>
<td>$\frac{v}{u}$</td>
</tr>
<tr>
<td>Workers’ hazard</td>
<td>$P$</td>
</tr>
</tbody>
</table>

**Note:**

The implied values of $v, \mu, \Theta$ and $\xi$ are solved for using the steady state relationships.
### Table 4
Model and Data Moments

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho(\hat{u}<em>t, \hat{u}</em>{t-1})$</td>
<td>0.97</td>
<td>0.98</td>
</tr>
<tr>
<td>$\rho(\hat{m}<em>t, \hat{m}</em>{t-1})$</td>
<td>0.85</td>
<td>0.99</td>
</tr>
<tr>
<td>$\rho(\hat{s}<em>t, \hat{s}</em>{t-1})$</td>
<td>0.88</td>
<td>0.98</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{std}(\hat{e}_t)$</td>
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<td>0.021</td>
</tr>
<tr>
<td>$\text{std}(\hat{u}_t)$</td>
<td>0.188</td>
<td>0.182</td>
</tr>
<tr>
<td>$\text{std}(\hat{m}_t)$</td>
<td>0.085</td>
<td>0.051</td>
</tr>
<tr>
<td>$\text{std}(\hat{s}_t)$</td>
<td>0.016</td>
<td>0.056</td>
</tr>
</tbody>
</table>
co-movement

<table>
<thead>
<tr>
<th>( \rho(\hat{u}_t, \hat{m}_t) )</th>
<th>Data</th>
<th>0.81</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Model</td>
<td>0.997</td>
</tr>
<tr>
<td>( \rho(\hat{u}_t, \hat{P}_t) )</td>
<td>Data</td>
<td>-0.933</td>
</tr>
<tr>
<td></td>
<td>Model</td>
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</tr>
<tr>
<td>( \rho(\hat{n}_t, \hat{s}_t) )</td>
<td>Data</td>
<td>-0.16</td>
</tr>
<tr>
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<td>Model</td>
<td>0.997</td>
</tr>
<tr>
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<td>Data</td>
<td>0.45</td>
</tr>
<tr>
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<td>Model</td>
<td>-0.99</td>
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</table>

**Note:**

All data are quarterly for the period 1970:I - 2003:IV, except for hires and separations which begin in 1976:I and end in 2003:III. Data sources and definitions are elaborated in the appendix.
Table 5
Counterfactuals

a. Convexity of Hiring Costs

<table>
<thead>
<tr>
<th>hiring costs</th>
<th>Data benchmark</th>
<th>counter-factual convex $\gamma = 2$</th>
<th>linear $\gamma = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho(\tilde{u}<em>t, \tilde{u}</em>{t-1})$</td>
<td>0.971</td>
<td>0.983</td>
<td>0.881</td>
</tr>
<tr>
<td>$\rho(\tilde{m}<em>t, \tilde{m}</em>{t-1})$</td>
<td>0.853</td>
<td>0.986</td>
<td>0.467</td>
</tr>
<tr>
<td>$\rho(\tilde{s}<em>t, \tilde{s}</em>{t-1})$</td>
<td>0.884</td>
<td>0.976</td>
<td>0.592</td>
</tr>
<tr>
<td>$std(\tilde{c}_t)$</td>
<td>0.022</td>
<td>0.021</td>
<td>0.015</td>
</tr>
<tr>
<td>$std(\tilde{u}_t)$</td>
<td>0.188</td>
<td>0.183</td>
<td>0.126</td>
</tr>
<tr>
<td>$std(\tilde{m}_t)$</td>
<td>0.085</td>
<td>0.052</td>
<td>0.090</td>
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<tr>
<td>$std(\tilde{m}_t)$</td>
<td>–</td>
<td>0.219</td>
<td>0.103</td>
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<tr>
<td>$std(\tilde{s}_t)$</td>
<td>0.016</td>
<td>0.056</td>
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</tr>
<tr>
<td>$\rho(\tilde{u}_t, \tilde{m}_t)$</td>
<td>0.810</td>
<td>0.997</td>
<td>0.888</td>
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<tr>
<td>$\rho(\tilde{u}_t, \tilde{P}_t)$</td>
<td>-0.933</td>
<td>-1.00</td>
<td>-0.743</td>
</tr>
<tr>
<td>$\rho(\tilde{m}_t, \tilde{P}_t)$</td>
<td>-0.160</td>
<td>0.997</td>
<td>0.936</td>
</tr>
<tr>
<td>$\rho(\tilde{m}_t, \tilde{s}_t)$</td>
<td>0.451</td>
<td>-0.989</td>
<td>-0.993</td>
</tr>
<tr>
<td>$\rho(\tilde{m}_t, \tilde{b}_t)$</td>
<td>0.908</td>
<td>0.862</td>
<td>0.599</td>
</tr>
<tr>
<td>$\rho(\tilde{u}_t, \tilde{b}_t)$</td>
<td>–</td>
<td>-0.985</td>
<td>0.580</td>
</tr>
</tbody>
</table>
### b. The Interest Rate and the Separation Rate

<table>
<thead>
<tr>
<th></th>
<th>Data benchmark</th>
<th>counterfactual 1</th>
<th>counterfactual 2</th>
<th>counterfactual 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho(\hat{u}<em>t, \hat{u}</em>{t-1}) )</td>
<td>0.971</td>
<td>0.983</td>
<td>0.763</td>
<td>0.704</td>
</tr>
<tr>
<td>( \rho(\hat{m}<em>t, \hat{m}</em>{t-1}) )</td>
<td>0.853</td>
<td>0.986</td>
<td>0.696</td>
<td>-0.152</td>
</tr>
<tr>
<td>( \rho(\hat{s}<em>t, \hat{s}</em>{t-1}) )</td>
<td>0.884</td>
<td>0.976</td>
<td>0.553</td>
<td>0.262</td>
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<tr>
<td>( \text{std}(\hat{e}_t) )</td>
<td>0.022</td>
<td>0.021</td>
<td>0.001</td>
<td>0.0006</td>
</tr>
<tr>
<td>( \text{std}(\hat{u}_t) )</td>
<td>0.188</td>
<td>0.183</td>
<td>0.004</td>
<td>0.0006</td>
</tr>
<tr>
<td>( \text{std}(\hat{m}_t) )</td>
<td>0.085</td>
<td>0.052</td>
<td>0.001</td>
<td>0.0005</td>
</tr>
<tr>
<td>( \text{std}(\hat{s}_t) )</td>
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<td>0.056</td>
<td>0.002</td>
<td>0.0006</td>
</tr>
<tr>
<td>( \rho(\hat{u}_t, \hat{m}_t) )</td>
<td>0.810</td>
<td>0.997</td>
<td>0.886</td>
<td>0.279</td>
</tr>
<tr>
<td>( \rho(\hat{u}_t, \hat{P}_t) )</td>
<td>-0.933</td>
<td>-1.00</td>
<td>-0.984</td>
<td>-0.620</td>
</tr>
<tr>
<td>( \rho(\hat{u}_t, \hat{s}_t) )</td>
<td>-0.160</td>
<td>0.997</td>
<td>0.917</td>
<td>0.304</td>
</tr>
<tr>
<td>( \rho(\hat{m}_t, \hat{s}_t) )</td>
<td>0.451</td>
<td>-0.989</td>
<td>-0.627</td>
<td>0.830</td>
</tr>
<tr>
<td>( \rho(\hat{m}_t, \hat{\delta}_t) )</td>
<td>0.908</td>
<td>0.862</td>
<td>0.304</td>
<td>–</td>
</tr>
<tr>
<td>( \rho(\hat{u}_t, \hat{v}_t) )</td>
<td>–</td>
<td>-0.985</td>
<td>-0.733</td>
<td>-0.140</td>
</tr>
</tbody>
</table>
c. Convexity and the Separation Rate

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>benchmark</th>
<th>counter-factual</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\gamma = 0; \sigma_\delta = \sigma_\beta = 0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho(\hat{u}<em>t, \hat{u}</em>{t-1})$</td>
<td>0.971</td>
<td>0.983</td>
<td>-0.118</td>
</tr>
<tr>
<td>$\rho(\hat{m}<em>t, \hat{m}</em>{t-1})$</td>
<td>0.853</td>
<td>0.986</td>
<td>-0.556</td>
</tr>
<tr>
<td>$\rho(\hat{s}<em>t, \hat{s}</em>{t-1})$</td>
<td>0.884</td>
<td>0.976</td>
<td>-0.556</td>
</tr>
<tr>
<td>$\text{std}(\hat{e}_t)$</td>
<td>0.022</td>
<td>0.021</td>
<td>0.00004</td>
</tr>
<tr>
<td>$\text{std}(\hat{u}_t)$</td>
<td>0.188</td>
<td>0.183</td>
<td>0.00004</td>
</tr>
<tr>
<td>$\text{std}(\hat{m}_t)$</td>
<td>0.085</td>
<td>0.052</td>
<td>0.0007</td>
</tr>
<tr>
<td>$\text{std}(\hat{v}_t)$</td>
<td>–</td>
<td>0.219</td>
<td>0.0008</td>
</tr>
<tr>
<td>$\text{std}(\hat{s}_t)$</td>
<td>0.016</td>
<td>0.056</td>
<td>0.0004</td>
</tr>
<tr>
<td>$\rho(\hat{u}_t, \hat{m}_t)$</td>
<td>0.810</td>
<td>0.997</td>
<td>0.720</td>
</tr>
<tr>
<td>$\rho(\hat{u}_t, \hat{P}_t)$</td>
<td>-0.933</td>
<td>-1.00</td>
<td>0.255</td>
</tr>
<tr>
<td>$\rho(\hat{m}_t, \hat{s}_t)$</td>
<td>-0.160</td>
<td>0.997</td>
<td>0.773</td>
</tr>
<tr>
<td>$\rho(\hat{m}_t, \hat{v}_t)$</td>
<td>0.451</td>
<td>-0.989</td>
<td>-0.997</td>
</tr>
<tr>
<td>$\rho(\hat{m}_t, \hat{s}_t)$</td>
<td>0.908</td>
<td>0.862</td>
<td>-</td>
</tr>
<tr>
<td>$\rho(\hat{u}_t, \hat{v}_t)$</td>
<td>–</td>
<td>0.985</td>
<td>0.588</td>
</tr>
</tbody>
</table>

**Note:**

All data are quarterly for the period 1970:I - 2003:IV, except for hires and separations which begin in 1976:I and end in 2003:III.
Figure 1
Flows from Unemployment to Employment

a. Hiring Rates $\frac{M^{UE}}{E}$
b. Job Finding Rates $\frac{MUE}{r}$
Figure 2
Flows from Out of the Labor Force to Employment

Matching Rates $\frac{\Delta^{NE}}{E}$
Figure 3
Total Hirings

Hiring Rates $\frac{MUE+NE}{E}$
Figure 4

Flows from Employment to Unemployment

Separation rates $\delta^{EU}$
Figure 5
Flows from Employment to Out of the Labor Force

Separation Rates $\delta^{EN}$
Figure 6
Total Separation Rates
Figure 7
FOC for Vacancy Creation
Alternative Values of $\gamma$
Figure 8
Increase in the Separation Rate $\delta$

a. $\gamma = 0$
b. $\gamma = 2$