The Search and matching Model
The Great Recession and other Business Cycles

April 2018
The DMP search and matching model

- An equilibrium model of unemployment

- Firms and workers have to spend time and resources before a match is formed. (A model of frictional unemployment)

- A match involves rents and a positive surplus

- Search externalities

- Based on Diamond (1982), Mortensen (1982), Pissarides (1985). The model we’ll use is closer to Pissarides (1985). (Also in Pissarides (2000), and LS ch. 26.3 26.4)
ASSUMPTIONS

- Risk neutral firms and workers
- Discrete time; future values discounted by a factor $\beta = \frac{1}{1+r}$
- No labor force participation decision. All unemployed workers search for jobs
- Firms decision is how many vacancies to maintain
- Unemployed workers and open vacancies “meet” according to a matching technology
  - Timing: match at $t$; begin production at $t + 1$
- Surplus is divided according to bargaining; bargaining occurs every period
- A constant exogenous separation rate $\lambda$
Firms

- Produce a final homogeneous good

- CRS production technology - produce \( p \) units with one unit of labor (all workers are identical and equally productive)

- Maintain \( v \) open vacancies at a cost \( c \) per vacancy, per period

- “Free entry” - many potential firms/vacancies willing to enter if there are positive profits
Matching Technology

\[ M(v, u) = A v^\alpha u^{1-\alpha} \]

- \( M = \) number of new hires
- \( v = \) number of open vacancies
- \( u = \) number of unemployed/searchers
Matching Technology

Properties

- Constant returns to scale

- Constant elasticities

\[
\frac{\partial M}{\partial v} \frac{v}{M} = \alpha A v^{\alpha-1} u^{1-\alpha} \frac{v}{M} = \alpha \frac{A v^{\alpha} u^{1-\alpha}}{M} = \alpha
\]

- Define the tightness ratio \( \theta \equiv \frac{v}{u} \); with CRS:

  - Job filling prob: \( q(\theta) = \frac{M}{v} = \frac{A v^{\alpha} u^{1-\alpha}}{v} = A \left( \frac{v}{u} \right)^{\alpha-1} = A \theta^{\alpha-1} \)

  - Job finding prob:

\[
\mu(\theta) = \frac{M}{u} = \frac{A v^{\alpha} u^{1-\alpha}}{u} = A \left( \frac{v}{u} \right)^{\alpha} = A \theta^{\alpha} = \theta q(\theta)
\]

- Firms and workers take these probabilities parametrically. This is a source for search externality.
**Bellman Values**

The rest of the model can be described with a set of 4 Bellman equations:

- $V =$ the value of an unfilled vacancy
- $J =$ the value of a filled/active job to the firm
- $U =$ the value of an unemployed worker
- $W =$ the value of an employed worker

We use $'$ to denote next period’s variables
**The Value of an Open Vacancy**

\[ V = \max \{0, -c + \beta E [q(\theta)J' + (1 - q(\theta))V']\} \]

- Can choose note to open a vacancy - value is zero
- If a vacancy is “open”:
  - Pay a flow cost \(c\)
  - With prob \(q\) the job is filled; prob \((1 - q)\) the job is not filled
  - Because of timing assumption, discount by a factor \(\beta\).
- Can think about \(J - V\) as a “capital gain”; this is the surplus the firm derives from a match
Value of a Filled Job

\[ J = p - \omega + \beta E \left[ \lambda V' + (1 - \lambda) J' \right] \]

The value of an active job to the employer consists of

- productivity \( p \)
- the wage that is paid to the worker \( \omega \)
- Continuation
  - With prob \( \lambda \) the match is separated and the firm receives a value \( V \)
  - With prob \( (1 - \lambda) \) the match continues and the firm receives a value \( J' \) next period
THE ZERO PROFIT CONDITION

► The central condition of the model...with free entry and exit

► Assume $V > 0 \rightarrow$ for the given costs and probabilities, more vacancies created $\rightarrow \theta$ increases $\rightarrow$ job filling probability $q(\theta) = A\theta^{\alpha-1}$ declines $\rightarrow V$ declines

► Assume $V < 0 \rightarrow$ for the given costs and probabilities, some existing vacancies close (generate zero value) $\rightarrow \theta$ decreases $\rightarrow$ job filling probability $q(\theta) = A\theta^{\alpha-1}$ increases $\rightarrow V$ increases

► $V=0$ in equilibrium

► $V = 0 \iff c = \beta q(\theta) E[J']$

► $J = p - \omega + \beta E[\lambda V' + (1 - \lambda)J'] = p - \omega + \beta(1 - \lambda)E[J']$
**Workers**

\[ U = z + \beta E \left[ \mu(\theta)W' + (1 - \mu(\theta))U' \right] \]

\[ W = \omega + \beta E \left[ \lambda U' + (1 - \lambda)W' \right] \]

- All workers are either working or searching
- All workers are equally productive
- An employed worker receives a wage \( \omega \); unemployed workers receive constant unemployment benefits \( z \) per period
- An unemployed worker finds a job with prob \( \mu(\theta) \); an employed worker separates from job with prob \( \lambda \)
Wage Bargaining

- Once there is a match, there exists a (total) surplus $W - U + J - V = W - U + J$

- This surplus needs to be divided between the worker and the employer.

- Both the worker and the employer have “outside options” in the event of disagreement. (Worker can get the value $U$; Firm receives the value $V = 0$). This implies that reservation wages exist, and the actual wage is between them.

- Bargain every period (the outside options may change as shocks hit the economy).
Nash Bargaining

With Nash bargaining:

Denote by $\tau$ the worker’s share of the match surplus

The wage level is maximizing the Nash product

\[
\omega = \argmax (W - U)\tau (J - V)^{1-\tau}
\]

\[
= \argmax (W - U)\tau J^{1-\tau}
\]
**Deriving the Wage Function (1)**

Take the first order condition of the Nash product (and set equal to zero)

Note that \( \frac{\partial(W-U)}{\partial \omega} = 1 \) and \( \frac{\partial J}{\partial \omega} = -1 \)

\[
\tau (W - U)^{\tau-1} \frac{\partial(W - U)}{\partial \omega} J^{1-\tau} + (W - U)^\tau (1 - \tau) J^{-\tau} \frac{\partial J}{\partial \omega} = 0
\]

\[\Rightarrow\]

\[
\tau (W - U)^{\tau-1} J^{1-\tau} - (W - U)^\tau (1 - \tau) J^{-\tau} = 0
\]

\[\Rightarrow\]

\[
\tau (W - U)^{\tau-1} J^{1-\tau} = (W - U)^\tau (1 - \tau) J^{-\tau}
\]

\[\Rightarrow\]

\[
\tau J = (1 - \tau) (W - U)
\]

Surpluses are proportional; Bilateral efficiency: when the firm’s surplus is positive, so is the worker’s surplus
Deriving the Wage Function (2)

Rearrange the last equation

\[ \tau J = (1 - \tau)(W - U) = (W - U) - \tau(W - U) \]

\[ W - U = \tau(J + W - U) = \tau \times \text{Total Surplus} \]

\[ J = TS - (W - U) = TS - \tau TS = (1 - \tau)TS \]

The Nash rule implies that total surplus is divided according to the bargaining weights. The bargaining weights reflect the worker/firm share of the surplus.
Deriving the Wage Function (3)

To actually derive the wage function, let’s substitute the values $U, W, J$ into the first order condition

$$
\tau J = (1 - \tau)(W - U)
$$

$$
\tau \left[ p - \omega + (1 - \lambda)\beta E \left[ J' \right] \right] = \\
(1 - \tau)\left\{ \omega + \beta E \left[ (1 - \lambda)W' + \lambda U' \right] \\
- z - \beta E \left[ \mu(\theta)W' + (1 - \mu(\theta))U' \right] \right\} = \\
(1 - \tau)\left\{ \omega - z + \beta E \left\{ W' [1 - \lambda - \mu(\theta)] \\
U' [1 - \mu(\theta) - \lambda] \right\} \right\} = \\
(1 - \tau)\left\{ \omega - z + (1 - \lambda - \mu(\theta))\beta E \left[ W' - U' \right] \right\}
$$
\[
\tau [p - \omega] + \tau (1 - \lambda) \beta E [J'] = \\
(1 - \tau)(\omega - z) - (1 - \tau) \mu(\theta) \beta E [W' - U'] \\
+ (1 - \tau)(1 - \lambda) \beta E [W' - U']
\]

With the assumptions on bargaining, we know that
\( \tau J = (1 - \tau)(W - U) \) every period. Therefore the last term on the LHS and the last term on the RHS cancel, and we have
\[
\tau (p - \omega) = \omega - z - \tau \omega + \tau z - (1 - \tau) \mu(\theta) \beta E [W' - U']
\]

\[
\omega = \tau p + (1 - \tau) z + (1 - \tau) \mu(\theta) \beta E [W' - U']
\]
The Wage Equation

Interpretation

\[ \omega = \tau p + (1 - \tau)z + (1 - \tau)\mu(\theta)\beta E [W' - U'] \]

The wage is a function of

- productivity
- the value of leisure (unemployment benefits)
- an opportunity cost – when the worker takes a job, he forgoes the opportunity to find a different job (with probability \( \mu \))
The Wage Equation

As a function of current period variables

In most cases we want to express the wage as a function of current period variables. This usually involve expressing \( \omega \) as a function of \( \theta \) (the tightness ratio). This, in turn, reduces the solution of equilibrium to one equation with one unknown – \( \theta \).

Use three assumptions/results:

First, note that \( \tau J = (1 - \tau)(W - U) \). Therefore we can write the last equation as

\[
\omega = \tau p + (1 - \tau)z + \tau \mu(\theta) \beta E \left[ J' \right]
\]

Second, recall that \( \mu(\theta) = \theta q(\theta) \) (using the prob defined by the matching technology)

\[
\omega = \tau p + (1 - \tau)z + \tau \theta q(\theta) \beta E \left[ J' \right]
\]
The Wage Equation
As a Function of Current Period Variables

Finally, note that in any equilibrium the zero profit condition 
\[ c = \beta q(\theta) E[J'] \] must hold:

\[ \omega = \tau p + (1 - \tau)z + \tau \theta c \]

The interpretation is the same as before. The first two terms depend on productivity and flow value of leisure. The last term still reflects to the opportunity cost: When \( \theta \) is higher, the job finding rate \( \mu(\theta) \) is higher \( \rightarrow \) easier to find a job \( \rightarrow \) the outside option of remaining unemployed is “better”, and therefore the wage must be higher.

Alternative interpretation – cost saving
**Unemployment**

This is the final piece needed before defining the steady state of the model. Based on transitions – each period:

- a fraction $\lambda$ of employed workers transition for emp to unemp
- a fraction $\mu(\theta)$ of unemployed transition from unemp to emp
- recall that we assume that everyone is in the labor force $n + u = 1$

$$u' = (1 - \mu(\theta))u + \lambda(1 - u)$$
A steady state equilibrium is a triple \((u, \theta, \omega)\) that satisfies

- the zero profit condition (job creation condition):
  \[ c = \beta q(\theta^{ss}) J^{ss} \]

- the wage equation:
  \[ \omega^{ss} = \tau p + (1 - \tau)z + \tau \theta^{ss} c \]

- the unemployment equation
  \[ u^{ss} = (1 - \mu(\theta^{ss})) u^{ss} + \lambda (1 - u^{ss}) \]

\[
\begin{align*}
\lambda &= u^{ss} (\lambda + \mu(\theta^{ss})) \\
u^{ss} &= \frac{\lambda}{\lambda + \mu(\theta^{ss})}
\end{align*}
\]
**Steady State Equilibrium**

Use the definition of the value $J$ in steady state:

$$J^{ss} = p - \omega^{ss} + \beta(1 - \lambda)J^{ss} \iff J^{ss} = \frac{p - \omega^{ss}}{1 - \beta(1 - \lambda)}$$

Substitute back in the job creation condition

$$c = \beta q(\theta^{ss})J^{ss} = \beta q(\theta^{ss})\frac{p - \omega^{ss}}{1 - \beta(1 - \lambda)}$$

or

$$\frac{c}{\beta q(\theta^{ss})} = \frac{p - \omega^{ss}}{1 - \beta(1 - \lambda)}$$

The job creation equation and the wage equation: two equations with two unknowns $\theta^{ss}, \omega^{ss} \rightarrow$ can solve for $\theta^{ss}, \omega^{ss}$. Once we have a solution for $\theta$ we can solve for $\mu$, and solve for $u^{ss}$. 
Graphical Representation I

Similar to the “usual” labor demand and labor supply curves we can look at the job creation condition and the wage equations graphically. The price is still the real wage $\omega$. The “quantity” is the tightness ratio $\theta$.

The wage equation implies that when $\theta$ increases, $\omega$ should increase $\rightarrow$ an upward sloping curve (acts like a supply curve)

Looking at the job creation condition: when $\theta$ increases, $q(\theta)$ decreases, and so the left hand side of the equation increases. For the equation to hold, the wage should adjust so that the RHS increased as well. It is clear that the wage should decrease. This establishes an inverse relation between $\theta$ and $\omega$ $\rightarrow$ a downward sloping curve (acts like a demand curve)
Graphical Representation I

ω

wage equation

Job creation (JC)

θ
Graphical Representation II

Alternatively, we can describe the steady state equilibrium on the $v, u$ space.

First, substitute the wage equation into the job creation condition. This results in one equation with one unknown $\theta$. The solution itself is independent of $u$. Therefore, when a solution exists, we can describe the relation between $u$ and $v$ as a linear line with a slope $\theta$. (Recall that $\theta \equiv \frac{v}{u}$.)

We can also use the steady state unemployment equation $u = \frac{\lambda}{\lambda+\mu}$ to plot the Beveridge Curve: a negative relation between unemployment and vacancies.
Graphical Representation II

$v$

Job creation (JC)

Beveridge Curve

$u$
Comparing steady states

To develop some intuition, let's look at a few simple examples.

Higher unemployment benefits $z$:

- $\omega = \tau p + (1 - \tau)z + \tau \theta c \rightarrow$ clearly the wage curve should shift up
- A new equilibrium with a lower $\theta$ and a higher wage.
- The lower $\theta$ implies a “flatter” JC line in the $(v, u)$ space $\rightarrow$ moving down along the Beveridge curve to an equilibrium with higher $u$ and lower $v$.
- Makes sense... higher unemployment benefits imply that the outside option for the worker is better, increasing the worker’s reservation wage and the equilibrium wage. This implies that matches are less profitable to firms, leading to creation of fewer vacancies.
- What does this model miss? insurance aspects of unemployment benefits.
Comparing steady states

Higher separation rate $\lambda$

- The JC condition: $\frac{c}{\beta q(\theta^{ss})} = \frac{p-\omega^{ss}}{1-\beta(1-\lambda)}$
- $\lambda$ does not enter the wage equation directly, so no shift of the wage curve
- A higher $\lambda$ reduces the value of the RHS of the JC equation
- Since the RHS ↓, the LHS should ↓ as well $\Rightarrow q(\theta) ↑$, and $\theta ↓$ for any level of wages
- The JC curve shifts down. New steady state at lower $\theta$ and a lower $\omega$
- Moving down along the Beveridge Curve
Comparing steady state

Higher labor productivity $p$

- A direct effect on wages through the wage equation. Wage curve shifts up.
- A direct effect on job creation:
  \[ p \uparrow \Rightarrow RHS \uparrow \Rightarrow LHS \uparrow \Rightarrow q(\theta) \downarrow \Rightarrow \theta \uparrow \]
- JC curve shifts up; wage curve shifts up; Based on the diagram alone, ambiguous prediction with respect to $\theta$, and the wage increases

- If we use the JC equation, and substituting for the wage:
  \[ \frac{c}{\beta q(\theta^{ss})} = \frac{(p-z)(1-\tau)-\tau\theta^{ss}c}{1-\beta(1-\lambda)} \]
  we can see that as long as $\tau < 1$, the effect on $\theta$ will be positive.

- Higher $\theta$ implies a move up along the Beveridge curve to a new steady state with higher $v$ and lower $u$
**Business Cycles Application**

The examples can be interpreted as the long run response to permanent shocks.

In business cycles analysis, we are often interested in the short and medium run responses to transitory (yet persistent) shocks.

For that, we have to look at the dynamic problem, rather than focusing on steady state.

In many cases the starting point is calibrating the model and calculating a non-stochastic steady state. Then the model is “hit” by stochastic shocks, and we use that to evaluate the response of the model variables to the shocks.

Popular way to evaluate a model: second moment statistics
Unemployment

Year

Unemployment

Year
UNEMPLOYMENT - TREND AND DEVIATIONS

Trend

Year

Deviaions

Year
UNEMPLOYMENT AND VACANCIES

![Graph showing the relationship between unemployment and vacancies over the years from 1982 to 2012.](image)

The graph compares the deviations of unemployment and vacancies over the years. The y-axis represents the deviations, ranging from -0.4 to 0.3, and the x-axis represents the years from 1982 to 2012. The blue line represents unemployment, while the green line represents vacancies.
**Shimer’s puzzle**

- Can the model explain business cycles \((v, u, \theta)\)?
- The exercise goes as follows:
  - Given shocks to the model and parameters...
  - What’s the volatility of \(u\) and \(v\) in the model?
  - How does this volatility compares with the data?
- Shocks to:
  - labor productivity (BLS)
  - Separations (CPS, time aggregation)
- Value of leisure \(= 0.4\)
- Matches elasticity \(= 0.72\)
- Bargaining power \(=\) Matches elasticity (Hosios)
Shimer’s puzzle

Results

Quarterly Summary Statistics from
U.S. Data, 1951:1 to 2003:4

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<th>$u$</th>
<th>$v$</th>
<th>$v/u$</th>
<th>$p$</th>
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<td>Correlation $p$</td>
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**Shimer’s puzzle**

**Results**

**Quarterly Summary Statistics from Model Simulations**

<table>
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<tr>
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<tr>
<td>Correlation $p$</td>
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<td>1.000</td>
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UNEMPLOYMENT AND PRODUCTIVITY

![Graph showing the relationship between unemployment and productivity over time. The graph displays two lines, one representing unemployment and the other productivity, with deviations over the years from 1982 to 2012.]
Mechanisms

- If productivity falls or separations increase then:
  - Value of a filled job goes down
  - Vacancies go down
- BUT!
  - Wage goes down
  - Unemployment goes up
- These feedback effects increase vacancies back
What if productivity was very volatile?
Three types of solutions

- Robert Hall (2005) - Fine tune the model

- Marcus Hagedorn and Iourii Manoskii (2008) - Change the calibration

THREE TYPES OF SOLUTIONS

- Hall (2005) introduces *sticky wages*
  - Strenghtens shock b/c firms need to pay the previous wage

- Hagedorn and Manovskii (2008) change the *calibration* as follows:
  - a very high leisure value (0.955)
  - a low bargaining power for workers (0.05)
  - Wages do not change very much (i.e., sticky)

  - Interest rate: cost of capital and cost of vacancy fluctuate
  - Financial spread: implies a high probability of default and separation
  - Model’s volatility of both $v$ and $u$ is same magnitude of data
ECKSTEIN-ET AL

RESULTS

QUARTERLY SUMMARY STATISTICS
FROM THE CALIBRATED MODEL

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<tr>
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<tr>
<td>Correlation ( r )</td>
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<td>-</td>
<td>1.000</td>
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UNEMPLOYMENT AND INTEREST RATE

![Graph showing deviations in unemployment and interest rate over years from 1982 to 2012. The x-axis represents years from 1982 to 2012, and the y-axis represents deviations. The graph compares unemployment (blue line) and interest rate (green line) over time.]
Financial Risk and Unemployment

Zvi Eckstein, Ofer Setty, David Weiss

Tel Aviv U. and IDC, Tel Aviv U., Tel Aviv U.

February 2015
Introduction

- Volatility in unemployment $u$, vacancies $v$, tightness $\theta = \frac{v}{u}$

- Firms experience a large volatility in financial risk:
  - Interest rate fluctuations (BAA)
  - Spread ($\Rightarrow$ default) fluctuations (BAA-Treasury)

- Relationship? $\rightarrow$
Unemployment, Interest rate and Spread

Figure: US time-series data 1982-2012
Figure: US time-series data 1982-2012
**Unemployment, Interest rate and Spread**

*Figure:* US time-series data 1982-2012

Spread & interest rate Granger cause $u$ with lag 2.
Research Question & Methodology

How does financial risk (interest rate and credit spread) affect unemployment, vacancies, and market tightness?

- What are the mechanisms?
- What is the quantitative power?

Methodology:
- Use a search-and-matching (DMP) model with capital
- Exogenous financial intermediary cost and default shocks together determine interest rate and default
- Outline mechanisms for how shocks affect
- Calibrate model to US economy (w/o targeting volatility)
- Ask how much volatility is generated
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Mechanisms

Productivity shocks:

- $p \downarrow \rightarrow \text{profits} \downarrow \rightarrow v \downarrow \rightarrow u \uparrow \quad \Rightarrow \quad \theta = \frac{v}{u} \downarrow$
Mechanisms

Productivity shocks:

- $p \downarrow \rightarrow \text{profits} \downarrow \rightarrow v \downarrow \rightarrow u \uparrow \Rightarrow \theta = \frac{v}{u} \downarrow$

Interest rate rises:

- higher capital costs lead to a lower profits (Flow Profits)
- more expensive vacancies (Vacancy Cost)
Mechanisms

Productivity shocks:

▶ \( p \downarrow \rightarrow \) profits \( \downarrow \rightarrow \) \( v \downarrow \rightarrow \) \( u \uparrow \) \( \Rightarrow \theta = \frac{v}{u} \downarrow \)

Interest rate rises:

▶ higher capital costs lead to a lower profits (Flow Profits)

▶ more expensive vacancies (Vacancy Cost)

Spread (default) rises:

▶ increase in chances of losing claim to profits (Ownership)
DMP with productivity shocks:

- Puzzle: Shimer (2005)
- Solutions: Hall (2005), Hagedorn & Manovskii (2008)....
- Fundamental surplus: Ljungqvist and Sargent (2014)
LITERATURE

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- DMP with financial shocks:
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  - Boeri, Garibaldi and Moen (2014)
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- Financial shocks:
  - Christiano, Eichenbaum and Trabandt (2014)
  - Jermann and Quadrini (2012)
  - ....
Model

Key Features

- Risk-neutral workers, $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t i_t$
  - Employed: $i_t = w_s + r_f k_s$
  - Unemployed: $i_t = b + r_f k_s$
  - Make consumption/savings choice wrt risk free $r_f$

- Banks:
  - Competitive banks borrow from workers, lend to firms
  - Perceive financial intermediation costs and default risk

- Firms:
  - Matched: produce, pay labor & capital costs: $w_s$, $(r_s + \delta)k$
  - $\delta$ is the depreciation rate, $r_s$ is state dependent
  - Unmatched: post vacancies $v$ at a cost $c_s(r_s)$
  - Face state-dependent default

- Workers and firms match in a frictional labor market

- Wages - Nash Bargaining
**Banks**

- Banks borrow from workers at \( r_f = \frac{1-\beta}{\beta} \)
- Lend to firms at rate \( r \)
- Default rate \( d \) (with recovery rate \( \zeta \)), intermediation costs \( x \)
- Maximize profits given by:

\[
\pi_b = (1 - d)(1 + r - x) + d\zeta(1 + r - x) - (1 + r_f)
\]

- Free entry
Matching

- A C.R.S. matching function $M(v, u)$: new matches

- Define market tightness as: $\theta = \frac{v}{u}$
  - Job finding rate for worker: $\frac{M(u,v)}{u} = \lambda^w(\theta)$
  - Job filling rate for firm: $\frac{M(u,v)}{v} = \lambda^f(\theta)$

- Use: $M(u, v) = \frac{uv}{(u^l+v^l)^{\frac{1}{t}}}$ (Ramey, den Haan, and Watson)
Firms and Production

- Matched firms: output $p$ using capital $K$ and labor $L$

$$Q(L, K) = \min \left( pL, \frac{K}{\phi} \right)$$

- Capital per worker is $k = \frac{K}{\phi p}$
- Allows constant productivity
- Look at business cycle frequencies

- Flow profits per match: $\pi = p - w_s - (r_s + \delta)k$

Hiring and investment
Firms and workers face state-independent separations $\bar{\sigma}$

In addition firms separate at $d$, due to default

Separation rate for firms: $\sigma_s^f = \bar{\sigma} + (1 - \bar{\sigma})d$

Separation rate for workers: $\sigma^w = \bar{\sigma}$
**Value Functions - Workers**

Employed worker:

\[ W_s = w_s + r_f k + \beta((1 - \sigma^w)E_s W_{s'} + \sigma^w E_s U_{s'}) \]

Unemployed worker:

\[ U_s = b + r_f k + \beta(\lambda^w(\theta)E_s W_{s'} + (1 - \lambda^w(\theta)) E_s U_{s'}) \]
Value Functions - Firms

The value of a matched firm is:

\[ J_s = p - w_s - (r_s + \delta)k + \beta \left( \left(1 - \sigma^f_s\right) E_s J_s' + \sigma^f_s E_s V_s' \right) \]

Vacancy posting firm:

\[ V_s = -c_s(r_s) + \beta \left( \lambda^f(\theta) E_s J_s' + \left(1 - \lambda^f(\theta)\right) E_s V_s' \right), \]

with vacancy cost: \( c_s(r_s) = c_r r_s + c_\delta + c_l \)
Wages - Nash Bargaining

- Wages solve: \( \max_{W_s} (W_s - U_s) \gamma (J_s - V_s)^{1-\gamma} \)
  
  - where \( \gamma \) is the worker’s bargaining weight

- The solution is: \( W_s - U_s = \gamma S_s; \quad J_s = (1 - \gamma) S_s \)
  
  - where \( S_s = (W_s - U_s) + (J_s - V_s) \)
**EQUILIBRIUM**

Given free entry for banks:

\[ r = f(x, d|r_f, \zeta) \]

Solve for \( S_s, \theta_s \) using:

- Free entry condition \((V = 0)\):

\[
\frac{c_s}{f(\theta)} = \beta(1 - \gamma)E_s S_s' (= \beta E_s J_s')
\]

- Evolution of surplus:

\[
S_s = \frac{p - b - (r_s + \delta)k + \beta \left\{ \left(1 - \sigma_s^f \right) E_s S_s' - \frac{(\theta q(\theta) - (1 - \bar{\sigma})d)}{(1 - \gamma) q(\theta)} \gamma \frac{c_s}{\beta} \right\}}{1 - \sigma_s^f}
\]
Abstracting from Default

- Default is a shock to ownership (continuation value)
- How big is it?
  - Separation rate is on average 2% \textit{a month} (Shimer, 2005)
  - Default rate is on average 1\% \textit{a year} (Elton, 2001)
- Formalize that this is small using Ljungqvist and Sargent (2014) \textit{Fundamental Surplus} approach
Calibration strategy

- Normalize $p - (\bar{r} + \delta)k = 1$
  - Flow surplus is $1 - b - \Delta rk$, where $\Delta r$ is deviation from mean
  - compared with $p - b$ in the productivity shocks literature

- Some parameters set a priori
- Exogenous shocks to $x (r)$
- Set some parameters to match data moments
## A-Priori Parameter Values

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Value</th>
<th>Identification</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>Job separation</td>
<td>0.0081</td>
<td>Literature</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount rate</td>
<td>0.99$^{\frac{1}{12}}$</td>
<td>Literature</td>
</tr>
<tr>
<td>$c$</td>
<td>Vacancy Costs</td>
<td>0.584</td>
<td>Hagedorn &amp; Manovskii (2008)</td>
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<tr>
<td>$\gamma$</td>
<td>Worker Bargaining Weight</td>
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<td>Literature</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation Rate</td>
<td>0.06$^{\frac{1}{52}}$</td>
<td>Annual rate of 6%</td>
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</table>
**Calibration - Matching Moments**

Parameter values and identification:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning</th>
<th>Value</th>
<th>Jointly Identified</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>Flow utility when $u$</td>
<td>0.60</td>
<td>Job finding rate</td>
</tr>
<tr>
<td>$l$</td>
<td>Matching elasticity</td>
<td>0.41</td>
<td>Market Tightness</td>
</tr>
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</table>

Model fit:

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Job Finding Rate</td>
<td>0.139</td>
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</tr>
<tr>
<td>Market Tightness</td>
<td>0.634</td>
<td>0.634</td>
</tr>
</tbody>
</table>
INTEREST RATE SHOCKS

- Without default the free entry condition for banks becomes:
  \[ r_s = r_f + x_s \]

- The estimated process is given by
  \[ r = \rho_r r_{-1} + \epsilon_r \]
  \[ \epsilon_r \sim N(0, \sigma_r) \]

- Estimating the process and converting to weekly frequency:

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>Quarterly</td>
<td>Quarterly</td>
<td>Weekly</td>
</tr>
<tr>
<td>( \rho_r )</td>
<td>0.799</td>
<td>0.798</td>
<td>0.996</td>
</tr>
<tr>
<td>( \sigma_r )</td>
<td>0.075</td>
<td>0.074</td>
<td>0.028</td>
</tr>
</tbody>
</table>
## Results - Data

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>$u$</th>
<th>$v$</th>
<th>$\theta$</th>
<th>$r$</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>St Dev</em></td>
<td>Data</td>
<td>0.11</td>
<td>0.12</td>
<td>0.22</td>
<td>0.14</td>
</tr>
<tr>
<td><em>Pers</em></td>
<td>Data</td>
<td>0.94</td>
<td>0.91</td>
<td>0.93</td>
<td>0.79</td>
</tr>
<tr>
<td><em>Corr $U$</em></td>
<td>Data</td>
<td>1</td>
<td>-0.89</td>
<td>-0.97</td>
<td>0.26</td>
</tr>
<tr>
<td><em>Corr $V$</em></td>
<td>Data</td>
<td>-</td>
<td>1</td>
<td>0.97</td>
<td>-0.23</td>
</tr>
<tr>
<td><em>Corr $\theta$</em></td>
<td>Data</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-0.47</td>
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</tbody>
</table>

**Table:** Quarterly moments: data: 1982-2012 versus Model In ($var, r$) correlations, var is 2 quarters lagged
# Results - Data versus Model

<table>
<thead>
<tr>
<th></th>
<th>$u$</th>
<th>$v$</th>
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<th>$r$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>St Dev</strong></td>
<td>Data</td>
<td>0.11</td>
<td>0.12</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>Model</td>
<td>0.09</td>
<td>0.11</td>
<td>0.19</td>
</tr>
<tr>
<td><strong>Pers</strong></td>
<td>Data</td>
<td>0.94</td>
<td>0.91</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>Model</td>
<td>0.86</td>
<td>0.61</td>
<td>0.78</td>
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<tr>
<td><strong>Corr $U$</strong></td>
<td>Data</td>
<td>1</td>
<td>-0.89</td>
<td>-0.97</td>
</tr>
<tr>
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<td>Model</td>
<td>1</td>
<td>-0.71</td>
<td>-0.91</td>
</tr>
<tr>
<td><strong>Corr $V$</strong></td>
<td>Data</td>
<td>-</td>
<td>1</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>Model</td>
<td>-</td>
<td>1</td>
<td>0.94</td>
</tr>
<tr>
<td><strong>Corr $\theta$</strong></td>
<td>Data</td>
<td>-</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Model</td>
<td>-</td>
<td>-</td>
<td>1</td>
</tr>
</tbody>
</table>

**Table:** Quarterly moments: data: 1982-2012 versus Model

In ($var$, $r$) correlations, var is 2 quarters lagged
# Break Down of Mechanisms

<table>
<thead>
<tr>
<th>Mechanisms</th>
<th>$u$</th>
<th>$v$</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>0.11</td>
<td>0.12</td>
<td>0.22</td>
</tr>
<tr>
<td>Both mechanisms</td>
<td>0.09</td>
<td>0.11</td>
<td>0.19</td>
</tr>
<tr>
<td>Profit</td>
<td>0.06</td>
<td>0.07</td>
<td>0.13</td>
</tr>
<tr>
<td>Vacancy cost</td>
<td>0.03</td>
<td>0.04</td>
<td>0.06</td>
</tr>
</tbody>
</table>

**Table:** Breakdown - Just Standard Deviation
# Robustness

<table>
<thead>
<tr>
<th>Robustness</th>
<th>u</th>
<th>v</th>
<th>v/u</th>
<th>r</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>0.11</td>
<td>0.12</td>
<td>0.22</td>
<td>0.14</td>
</tr>
<tr>
<td>Benchmark</td>
<td>0.09</td>
<td>0.11</td>
<td>0.19</td>
<td>0.14</td>
</tr>
<tr>
<td>b=0.4</td>
<td>0.06</td>
<td>0.07</td>
<td>0.13</td>
<td>0.14</td>
</tr>
<tr>
<td>γ=0.30</td>
<td>0.08</td>
<td>0.10</td>
<td>0.18</td>
<td>0.14</td>
</tr>
<tr>
<td>γ=0.72</td>
<td>0.10</td>
<td>0.13</td>
<td>0.20</td>
<td>0.14</td>
</tr>
<tr>
<td>δ=0.08</td>
<td>0.08</td>
<td>0.09</td>
<td>0.15</td>
<td>0.14</td>
</tr>
</tbody>
</table>
Elasticity of tightness w.r.t. the shock

Example: profits channel

- A continuous time model w/ only profits mechanism \((r_s k)\)

\[
\frac{\partial \log \theta}{\partial \log p} = \frac{p}{p - b} \quad \ast \Upsilon \quad \text{productivity shocks}
\]

fundamental surplus

\[
\frac{\partial \log \theta}{\partial \log r_k} = \frac{\bar{r}_k}{p - \bar{r}_k - \delta_k - b} \ast \Upsilon \quad \text{interest rate shocks}
\]

\[
\Upsilon = \left( r + \sigma \right) + \gamma \theta q \left( \theta \right) \alpha \left( r + \sigma \right) + \gamma \theta q \left( \theta \right) \text{where } \alpha \text{ is the elasticity of matching w.r.t. } u
\]

In Shimer-based calibration:

\[
p - z = 1 \]

\[
\bar{r}_k - \bar{r}_k - \delta_k - z = 0
\]

Conclusion: elasticity is about 2 times smaller in our model, But:

\[
\frac{\partial \log \theta}{\partial \log r_k} = 0
\]
Elasticity of tightness w.r.t. the shock

Example: profits channel

- A continuous time model w/ only profits mechanism \((r_s k)\)

\[
\frac{\partial \log \theta}{\partial \log p} = \frac{p}{p - b} \frac{\gamma}{\text{fundamental surplus}} \quad \text{productivity shocks}
\]

\[
\frac{\partial \log \theta}{\partial \log rk} = \frac{-\bar{r}k}{p - \bar{r}k - \delta k - b} \frac{\gamma}{\text{fundamental surplus}} \quad \text{interest – rate shocks}
\]
Elasticity of tightness w.r.t. the shock

Example: profits channel

- A continuous time model w/ only profits mechanism \((r_s k)\)

\[
\frac{\partial \log \theta}{\partial \log p} = \frac{p}{p - b} \star \Upsilon \quad \text{productivity shocks}
\]

\[
\text{fundamental surplus}
\]

\[
\frac{\partial \log \theta}{\partial \log rk} = \frac{-\bar{r}k}{p - \bar{r}k - \delta k - b} \star \Upsilon \quad \text{interest – rate shocks}
\]

\[
\text{fundamental surplus}
\]

- \(\Upsilon = \frac{(r+\sigma)+\gamma q(\theta)}{\alpha(r+\sigma)+\gamma q(\theta)}\) where \(\alpha\) is the elasticity of matching w.r.t. \(u\)
Elasticity of tightness w.r.t. the shock

Example: profits channel

- A continuous time model w/ only profits mechanism \((r_s k)\)

\[
\frac{\partial \log \theta}{\partial \log p} = \frac{p}{p - b} \quad \text{fundamental surplus} \\
\frac{\partial \log \theta}{\partial \log rk} = \frac{-\bar{r}k}{p - \bar{r}k - \delta k - b} \quad \text{fundamental surplus}
\]

- \(\Upsilon = \frac{(r+\sigma)+\gamma q(\theta)}{\alpha(r+\sigma)+\gamma q(\theta)}\) where \(\alpha\) is the elasticity of matching w.r.t. \(u\)

- In Shimer-based calibration: \(\frac{p}{p-z} = 1.67, \frac{\bar{r}k}{p-\bar{r}k-\delta k-z} = 0.83\)

- Conclusion: elasticity is about 2 times smaller in our model, But:

- \((r,\text{spread})\) are \(\sim 14\) times more volatile than labor productivity
Elasticity of tightness w.r.t. the shock

Example: profits channel

- A continuous time model w/ only profits mechanism \((r_s k)\)

\[
\frac{\partial \log \theta}{\partial \log p} = \frac{p}{p - b} \cdot Y \quad \text{productivity shocks}
\]

fundamental surplus

\[
\frac{\partial \log \theta}{\partial \log rk} = \frac{-\bar{r}k}{p - \bar{r}k - \delta k - b} \cdot Y \quad \text{interest-rate shocks}
\]

fundamental surplus

- \(Y = \frac{(r+\sigma)+\gamma\theta q(\theta)}{\alpha(r+\sigma)+\gamma\theta q(\theta)}\) where \(\alpha\) is the elasticity of matching w.r.t. \(u\)

- In Shimer-based calibration: \(\frac{p}{p - z} = 1.67, \frac{\bar{r}k}{p - \bar{r}k - \delta k - z} = 0.83\)

- Conclusion: elasticity is about 2 times smaller in our model, But:

- \((r,\text{spread})\) are \(\sim 14\) times more volatile than labor productivity

- \(\frac{\partial \log \theta}{\partial \log rk} \cdot \sigma_r = 0.12\)
**Interest Rate**

Comparison by looking at only data:

<table>
<thead>
<tr>
<th></th>
<th>$u$</th>
<th>$v$</th>
<th>$\theta$</th>
<th>$r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>St Dev</td>
<td>0.11</td>
<td>0.12</td>
<td>0.22</td>
<td>0.14</td>
</tr>
<tr>
<td>Pers</td>
<td>0.94</td>
<td>0.91</td>
<td>0.93</td>
<td>0.79</td>
</tr>
<tr>
<td>Corr $U$</td>
<td>1</td>
<td>-0.89</td>
<td>-0.97</td>
<td>0.26</td>
</tr>
<tr>
<td>Corr $V$</td>
<td>-</td>
<td>1</td>
<td>0.97</td>
<td>-0.23</td>
</tr>
<tr>
<td>Corr $\theta$</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-0.25</td>
</tr>
</tbody>
</table>

**Table:** Quarterly moments: data: 1982-2012
In $(\text{var}, r)$ correlations, var is 2 quarters lagged

Note: exact value for $\sigma_P$ is 0.0095. Go to comparison without lag.
Interest Rate vs. Productivity Shocks

Comparison by looking at only data:

<table>
<thead>
<tr>
<th></th>
<th>$u$</th>
<th>$v$</th>
<th>$\theta$</th>
<th>$r$</th>
<th>$p$</th>
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<td>0.77</td>
</tr>
<tr>
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<td>-0.97</td>
<td>0.26</td>
<td>-0.32</td>
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<tr>
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<td>-0.25</td>
<td>0.41</td>
</tr>
</tbody>
</table>

Table: Quarterly moments: data: 1982-2012
In $(var, r)$ correlations, var is 2 quarters lagged

Note: exact value for $\sigma_p$ is 0.0095.
What About the Great Recession?

- Simulate the model for 2008Q2-2012Q4
What About the Great Recession?

- Simulate the model for 2008Q2-2012Q4
- Simulate if the Fed had not intervened:

\[ r_c = r_t + (f_{2008Q2} - f_t) \]
What About the Great Recession?

- Simulate the model for 2008Q2-2012Q4
- Simulate if the Fed had not intervened:

\[ r_c = r_t + \left( f_{2008Q2} - f_t \right) \]
CONCLUSION

We studied:

- Mechanisms for *financial risk* affecting unemployment
- The quantitative effect of those shocks using DMP literature
Conclusion

We studied:

▶ Mechanisms for *financial risk* affecting unemployment

▶ The quantitative effect of those shocks using DMP literature

We found:

▶ Financial conditions matter a lot

▶ The main driving force is the interest rate
Calibration of vacancy cost

- Vacancy cost is \( c_s(r_s) = c_r r_s + c_\delta + c_l \)

- Capital component: \( c_r r_s + c_\delta \)
  - Assume capital required one period in advance
  - Capital share = \( \frac{1}{3} \)
  - Labor productivity is 1 → capital cost \( \sim 0.5 \)
  - Correct for capital in vacancies: \( c_r r_s + c_\delta = 0.474 \)

- Labor component: \( c_l \)
  - 11% of average labor productivity based on micro evidence

- Total vacancy cost = 0.474 + 0.11 = 0.584
Hires from JOLTS, Inv. is real gross private domestic
Correlation = 0.73
St dev of log is 0.11 for investment, 0.10 for hires
**UNEMPLOYMENT, PRODUCTIVITY, SPREAD**

**Figure:** US time-series data 1982-2012
UNEMPLOYMENT, PRODUCTIVITY AND INTEREST RATE

Figure: US time-series data 1982-2012
Compare to productivity?

- Standard Dev of U: 0.11 V: 0.12 Tightness ($\frac{v}{u}$): 0.22
**Compare to productivity?**

- Standard Dev of U: 0.11 V: 0.12 Tightness ($\frac{v}{u}$): 0.22
- Productivity: 0.01 (Shimer Puzzle)
Compare to productivity?

- Standard Dev of U: 0.11 V: 0.12 Tightness ($\frac{\nu}{\mu}$): 0.22
- Productivity: 0.01 (Shimer Puzzle)
- Interest rate: 0.17 Spread: 0.35
Compare to productivity?

- Standard Dev of U: 0.11 V: 0.12 Tightness \( \frac{v}{u} \): 0.22
- Productivity: 0.01 (Shimer Puzzle)
- Interest rate: 0.17 Spread: 0.35
- Correlation with u? P: -0.32 R: 0.53 Spread: 0.71

HP Filtered time series
## Interest Rate vs. Productivity Shocks

Comparison by looking at **only** data:

<table>
<thead>
<tr>
<th></th>
<th>$u$</th>
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<tr>
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<td>-0.89</td>
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<td>Corr $V$</td>
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<td>0.17</td>
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<tr>
<td>Corr $\theta$</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-0.30</td>
<td>0.06</td>
</tr>
</tbody>
</table>

**Table**: Quarterly moments: data: 1982-2012
($var, r$) correlations are contemporaneous

Note: exact value for $\sigma_p$ is 0.0095.