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TARGET ZONES AND EXCHANGE RATE DYNAMICS

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This paper develops a simple model of exchange rate behavior under a target zone regime. It shows that the expectation that monetary policy will be adjusted to limit exchange rate variation affects exchange rate behavior even when the exchange rate lies inside the zone and is thus not being defended actively. Somewhat surprisingly, the analysis of target zones turns out to have a strong formal similarity to problems in option pricing and investment under uncertainty.

Wide attention has been given to proposals to establish "target zones" for exchange rates. Indeed, we are already arguably living under a weak target zone regime. The reference zones established under the Louvre Accord and in subsequent consultations among the major industrial nations are not publicly announced, nor is it clear how strongly they will be defended, but the principle of setting limits on the range of exchange rate variation has been established.

A target zone differs from a fixed rate regime in allowing a fairly wide range of variation for the exchange rate around some reference rate. Williamson [1985], for example, called for a range of 10 percent on either side of the central rate. The appeal of this idea, as opposed to a more strict pegging, is that a target zone should not need as much maintenance. Some exchange rate flexibility would be allowed, and thus the defense of the exchange rate would become only an occasional problem rather than a continuous preoccupation.

Given the widespread attention devoted to target zones and the apparent drift of actual exchange rate policy toward something that looks increasingly like such zones, one might have supposed that the theory of how a target zone system would work would be fully worked out. In particular, since the whole idea of target zones is that exchange rates will normally be inside the zone, one would expect considerable focus on how the exchange rate will actually behave inside the band. In fact, however, there is essentially no literature on this question. Even advocates of target zones, such as Miller and Williamson [1987], have not actually modeled the behavior of rates inside the zone; instead they attempt to approximate a target zone by a continuous monetary policy of leaning against the wind.

The principal issue in modeling exchange rate dynamics under a target zone regime is the formation of expectations. A naive view would suppose that the exchange rate behaves as if the regime were one of free floating until the rate hits the edge of the band, whereupon the regime switches to a fixed rate. However, this cannot be right. The existence of a band constrains possible future paths of the exchange rate; exchange markets, knowing this, should behave differently than they would if there were no target zone. In other words, the existence of a band should affect exchange rate behavior even when the exchange rate is inside the band and the zone is not actively being defended.

This paper offers a new approach to the modeling of exchange rate dynamics under a target zone regime. The approach is related to earlier work by Flood and Garber [1983], but the methods employed are different and, as it turns out, much simpler. More surprisingly, the analysis of target zones turns out to have a strong formal similarity to that of problems in option pricing and irreversible investment. This unexpected linkage with other areas may give the paper some interest even to those who are not concerned with target zones per se.

The model used to exposit the logic of target zones is a minimalist monetary model. Clearly, the next step is to move to more sophisticated and realistic underlying models. In work based on earlier versions of this paper, Miller and Weller [1989] have already shown how the same general approach may be applied to a Dornbusch-type model with intrinsic dynamics as well as the "extrinsic" dynamics that, as we shall see, arise from a target zone.

The paper is in five parts. The first part lays out the basic model and presents some intuition on the results. The second part derives an explicit solution for exchange rate behavior under a target zone, except for the question of "tying down the ends of the S" (a concept whose meaning will become clear in context). The third part then tackles this issue, uncovering a fundamental similarity between target zones and option analysis. The fourth part considers the money supply behavior necessary to enforce the target zone. Finally, the fifth part examines the behavior of an imperfectly credible target zone.

I. THE BASIC MODEL

We consider a minimalist log-linear monetary model of the exchange rate. Expressing all variables in natural logarithms, the exchange rate at any moment in time is assumed equal to

$$s = m + v + \gamma E[ds]/dt, \tag{1}$$

where s is the (log of the) spot price of foreign exchange, m the domestic money supply, v a shift term representing velocity shocks, and the last term is the expected rate of depreciation.

There are two "fundamentals" in (1); the money supply and the velocity shift term. I shall assume that monetary policy is passive; m is shifted only in order to maintain a target zone. Specifically, the monetary authority is prepared to reduce m in order to prevent s from exceeding some maximum value $\bar{s}$, and to increase m to prevent s from falling below some minimum value $\underline{s}$. As long as s lies within the band between $\bar{s}$ and $\underline{s}$, the money supply remains unchanged. The actual money supply dynamics implied by this policy are best described in the context of a full solution of the model.

With no loss of generality, and some saving in notation, we can choose units to center the target zone around zero, so that $\underline{s} = -\bar{s}$.

The velocity term v will be the only exogenous source of exchange rate dynamics. It will be assumed to follow a continuous-time random walk:

$$dv = \sigma dz. \tag{2}$$

There is no good economic reason for assuming a random walk on v. The assumption is made here for two reasons. First, the random walk assumption allows us to focus entirely on the dynamics caused by the presence of a target zone, as opposed to the effects of predictable future changes in v. Second, the random walk assumption gives rise to a simple analytic solution; more realistic target zone models with (say) some inherent autoregression require the use of numerical methods (see Miller and Weller [1989]).

This is the complete model. One might not at first suppose that there could be any interesting exchange rate dynamics arising from so simple a structure, yet it will yield some surprising insights about the functioning of a target zone.

Before proceeding to algebraic analysis, it is useful to start with an intuitive approach to the effects of a target zone on exchange rate behavior. Figure I plots the exchange rate against v; the target zone is indicated by the broken lines that define a band that bounds the exchange rate between $-\bar{s}$ and $\bar{s}$. We consider the behavior of the exchange rate when starting with some initial money supply, say $m = 0$.

Now a naive view would run as follows: since m is locally held constant and since v follows a random walk, there should be no predictable change in the exchange rate— $E[ds/dt] = 0$. Thus, the exchange rate might simply be expected to equal $m + v$ inside the band, i.e., to behave like a freely floating rate inside the target zone. If successive shocks to v push the exchange rate to the edge of the band, then the money supply will be adjusted to prevent s from drifting any further; thus, this naive view would suppose a relationship between v and s that looks like the heavy line in Figure I.

Why is this not right? Suppose that it were the correct description of exchange rate behavior, and consider the situation at an exchange rate that is just inside the band, say at point 2. Starting at point 2, if v falls a little, the exchange rate would retreat

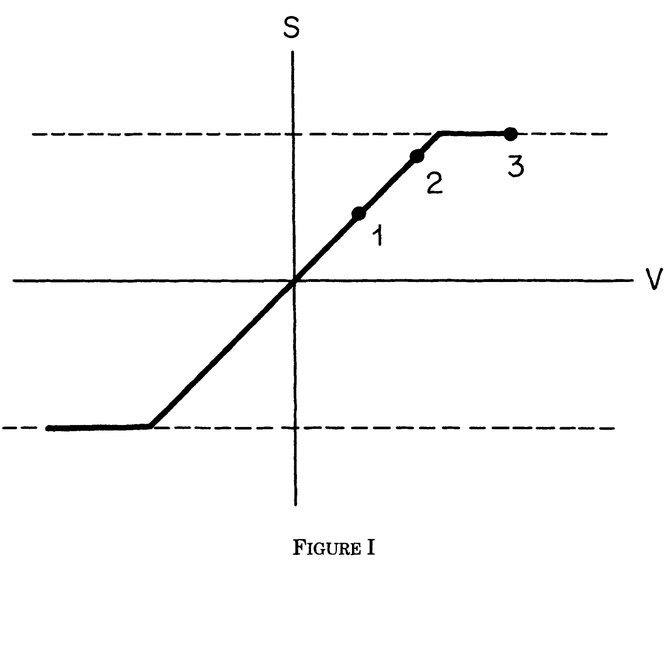

FIGURE I

down the 45-degree line, to a point like 1. If v rises a little, however, the exchange rate will not rise by an equal amount, because the monetary authority will act to defend the target zone. So the exchange rate will move to a point like 3.

But this says that when we are near the top of the band, a fall in v will reduce s more than a rise in v will increase s. Since v is assumed to follow a random walk, *the expected rate of change of s is negative.* Because expected depreciation enters the basic exchange rate equation (1), this will affect the exchange rate itself: the exchange rate would be "dragged" down from 2 to a somewhat lower point. The same must be true at the bottom of the band. In effect, the relationship between v and s must be bent as it approaches the edges of the target zone.

We cannot stop here, however; once exchange rate behavior near the edges of the zone lies off the 45-degree line, this will affect exchange rate expectations further inside the zone as well. It seems intuitively obvious that repeated revisions of exchange rate expectations will lead to a relationship between v and s that looks like the S-shaped curve in Figure II, below the 45-degree line in the upper half of the target zone and above it in the lower half.

There are two important points to make about the S-curve in Figure II. First is the relationship between geometry and behavior.

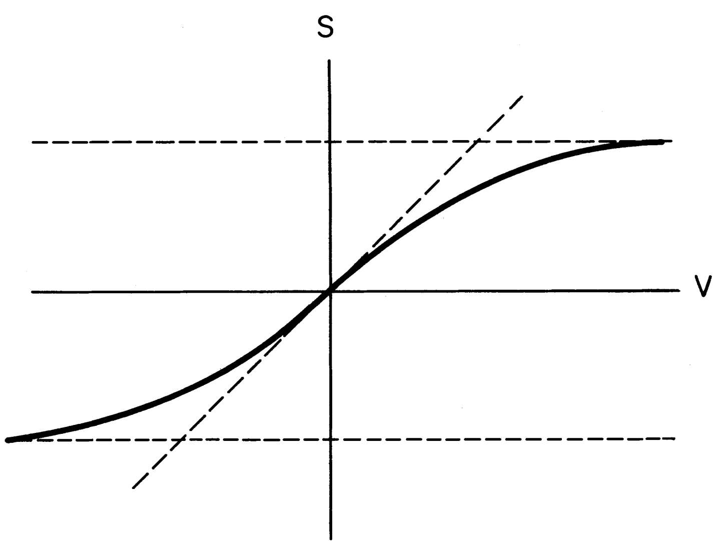

FIGURE II

We note that the exchange rate lies below the 45-degree line in the upper half of the figure, below it in the lower half; by (1) this must mean that the expected rate of change of s is negative in the upper portions, positive below. Yet how is this possible, with m constant and v following a random walk? The answer lies in the *curvature* of the S. Because the S is concave in the upper half, even though the expected rate of change in v is zero, the expected rate of change of s is negative; and conversely in the lower half. It is thus the concavity or convexity of the relationship between v and s that itself "drags" the relationship off the 45-degree line.

Second, we may note that the effect of the target zone on exchange rates is stabilizing. With a constant money supply and no target zone, the exchange rate would simply move up and down the 45-degree line. The S-curve in Figure II, however, is flatter than the 45-degree line; that is, shocks to velocity have a smaller effect on exchange rates, and thus, the exchange rate itself has less variation than under a free float. Notice that this reduction in variation occurs even while the exchange rate is inside the band, and thus no current effort is being made to stabilize it.

This is about as far as we can go on the basis of rough intuition and geometry. The next step must be to develop an explicit analysis of the S-curve.

II. Algebraic Analysis

We have implicitly defined equilibrium as a relationship between the "fundamentals" m and v and the exchange rate. More formally, we want to determine a relationship,

$$s = g(m,v,\bar{s},\underline{s}), \tag{3}$$

that is consistent with (1) and the assumed monetary behavior. The S-curve in Figure II is a partial relationship between v and s, for a given m; we shall actually define (3) initially by finding a family of such curves.

Suppose that we hold m constant; i.e., we consider a situation where s lies inside the band. Then the only source of expected changes in s lies in the random movement of v. By the usual rules of stochastic calculus, we have

$$E[ds/dt] = (\sigma^2/2)g_{vv}(m,v,\underline{s},\bar{s}). \tag{4}$$

Substituting (4) into (1), we have

$$g(m,v,\bar{s},\underline{s}) = m + v + (\gamma\sigma^2/2)g_{vv}(m,v,\underline{s},\bar{s}). \tag{5}$$

The general solution of (5) has the form,

$$g(m,v,\bar{s},\underline{s}) = m + v + Ae^{\rho v} + Be^{-\rho v}, \tag{6}$$

where

$$\rho = (2/\gamma\sigma^2)^{1/2} \tag{7}$$

and A and B are constants still to be determined.

We can further simplify the problem by appealing to symmetry. Suppose that $m = 0$; then surely we would expect the relationship to go through the middle of Figure II, i.e., to have $s = 0$ when $v = 0$. This can only be true if $B = -A$; so we can simplify (6) to

$$g(m,v,\bar{s},\underline{s}) = m + v + A[e^{\rho v} - e^{-\rho v}]. \tag{8}$$

To get the S-curve illustrated in Figure II, we clearly need to have $A < 0$; this will yield a value of s that falls increasingly below $m + v$ for positive v, increasingly above for negative v. However, we need something else to determine the precise value of A. Equivalently, we may note that the value of A determines where (8) intersects the edges of the band. The problem of determining A is thus equivalently viewed as a problem of "tying down the ends of the S."

III. Tying Down the Ends of the S

The choice of A, which determines the S-curve for any given m, must be such as to produce the following result: *the curve defined by (8) must be tangent to the edges of the band.* This condition ties down the ends of the S.

To see why this condition must obtain, we consider what would happen if it did not. Figure III illustrates part of a hypothetical S-curve that crosses the band rather than forming a tangency. For this to be the right S-curve, a point like 2, very close to the edge of the band, must in fact represent an equilibrium value of s given v.

Now the construction of the S-curve (8) is such that any curve (i.e., a curve corresponding to any value of A) is self-validating. If the values of s corresponding to future realizations of v are expected to lie on a curve passing through the current v,s, then the current s is in fact an equilibrium. Specifically, if a small fall in v will bring us to point 1, while a small increase will bring us to point 3, then point 2 will be an equilibrium.

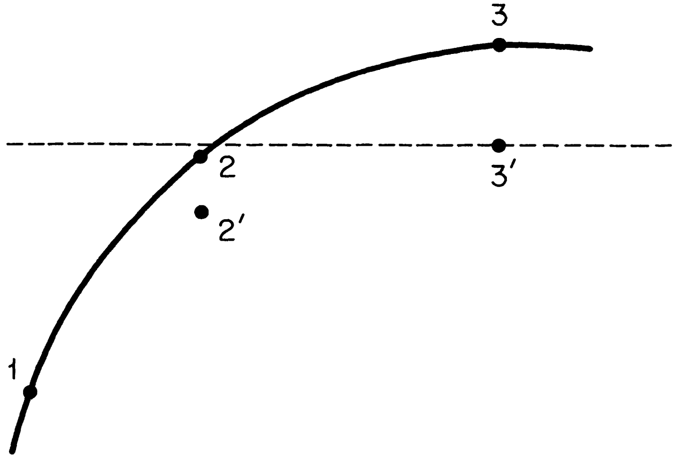

FIGURE III

But if point 2 is right at the edge of the band, then an increase in v will not lead to 3, because s will not be allowed to rise. Instead, it will lead to 3′, a point with a lower value of s. This in turn means that the expected rate of appreciation of s will be *larger* than that consistent with equilibrium at point 2; the s corresponding to that v would instead be lower, say at 2′. But this means that 2 was not an equilibrium after all. So it is not possible to approach the edge of the band on an S-curve that actually crosses that edge. The only possible curves are those that are just tangent to the edge.

Evidently, what we have here is a result that is closely related to the "high-order contact" or "smooth pasting" conditions that occur in option theory and in the analysis of irreversible investment. The analogy with option pricing comes as a surprise. However, we can show that there is indeed an option-pricing interpretation of a target zone.

To see this, note that the basic exchange rate equation (1) can be viewed as arising from a more underlying equation,

$$s_t = \left(\frac{1}{\gamma}\right) \int_t^{\infty} (m + v) e^{-(1/\gamma)(\tau - t)}\, d\tau. \tag{9}$$

Differentiating (9) with respect to t yields (1). Thus, the current exchange rate may be viewed as a sort of present discounted value of future realizations of $(m + v)$.

Now suppose that we were to consider an imaginary asset whose price is the present discounted value of $m + v$ holding m

constant at its current level, say m_0. The value of this asset would be

$$\tilde{s}_t = \left(\frac{1}{\gamma}\right) \int_t^{\infty} (m_0 + v) e^{-(1/\gamma)(\tau - t)} \, d\tau. \tag{10}$$

Now the actual exchange rate may be viewed as the price of a compound asset. This asset consists of the imaginary asset whose price is determined by (10), plus the right to sell the asset at a price $\underline{s}$, plus the obligation to sell at the price $\bar{s}$ on demand.

The deviation of the S-curve from the 45-degree line may now be viewed as the combined price of the two options. Not surprisingly, the requirement to sell on demand at $\bar{s}$ becomes more important the higher is $\tilde{s}$, so that the price of the compound asset falls below $\tilde{s}$ at high v; conversely, the right to sell at $\underline{s}$ supports the value of the asset at low v.

The options pricing analogy could be followed up further, but the basic point is that at a formal level the target zone is essentially the same as a variety of problems involving choice under uncertainty. S-shaped curves tangent to a band at top and bottom have in fact appeared recently in some seemingly unrelated papers on entry and exit under exchange rate fluctuations by Dixit [1989] and analysis of irreversible investment problems by Dumas [1988].

We may determine A, then, by the requirement that the curve (8) be tangent to the band at top and bottom. Let v be the value of v at which s reaches the top of the band; then we have

$$\bar{s} = \bar{v} + A[e^{\rho\bar{v}} - e^{\rho\bar{v}}] \tag{11}$$

and

$$0 = 1 + \rho A[e^{\rho\bar{v}} + e^{\rho\bar{v}}]. \tag{12}$$

These equations implicitly define A and v.

IV. Money Supply Behavior

So far, the paper has not been explicit about the monetary behavior implied by the defense of a target zone. However, this is easily derived. Let $\bar{v}$ be the value of v at which a particular S-curve touches the top of the band. This is of course dependent on the money supply: $\bar{v} = \bar{v}(\overline{m},\bar{s},\bar{s})$. If v goes beyond $\bar{v}$, the money supply must be reduced. This will shift the market to a new S-curve displaced to the right; the money supply reduction must always be such as to imply that at the current v the market is at the top of the new S-curve. What we shall see, then, is a family of curves, as illustrated in Figure IV.

The dynamics of the money supply can perhaps best be illustrated by considering a possible cycle. Suppose that initially the market is at point 1 and that for a while there is a series of positive shocks to v. Initially this will move the market along the original curve, until point 2 is reached. Any further increase in v will, however, be offset by reductions in m, so that the exchange rate would remain constant as we move from 2 to 3. Next suppose that v starts to have negative shocks. Then the market will not retrace its steps, since the monetary authority will not react to shocks that push s into the band. Thus, the market will move back down a new S-curve, to a point like 4.

What we have, then, is a family of S-curves; the market stays on any one curve as long as v remains within the range where s lies inside the band. The money supply shifts whenever the edge of the band is reached, placing the market on a new curve. The monetary behavior is identical to what one would observe under a gold standard with costly shipment of gold: specie flows out whenever the upper gold point is reached and does not return unless the lower gold point is hit.

We can actually write a simple expression for the whole family of curves. Let A be determined so that the curve is tangent for some

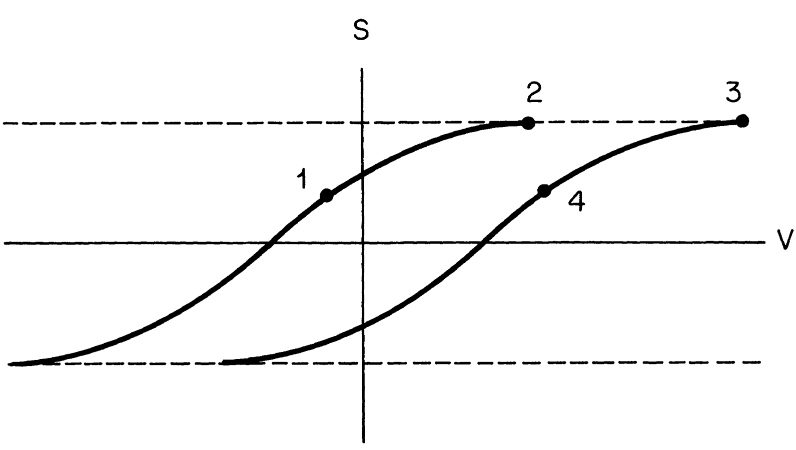

FIGURE IV

particular m. Then the whole family of curves is defined by

$$g(m,v,\bar{s},\underline{s}) = m + v + A[e^{\rho(m+v)} - e^{-\rho(m+v)}] \tag{13}$$

with the same A. Whenever positive shocks to v push (13) to the edge of the band, m will be reduced to keep $m + v$ constant; clearly this will keep s at the edge of the band and also preserve the tangency.

It follows that we can draw the whole family of S-curves as a single curve in $(m + v)$, s space, as shown in Figure V. The edges of the band now in effect represent reflecting barriers. Whenever s hits the edge of the band, m adjusts to keep $m + v$ from going any farther; if shocks move s back into the band, m remains unchanged, and thus $m + v$ is allowed to change.

V. Imperfect Credibility

Up to this point it has been assumed throughout that the commitment to defend the target zone is completely credible. One may wonder whether the results are fragile, in that some lack of credibility will completely undermine the description of exchange rate behavior. Indeed, some economists have been concerned that an imperfectly credible target zone could actually lead to instabil-

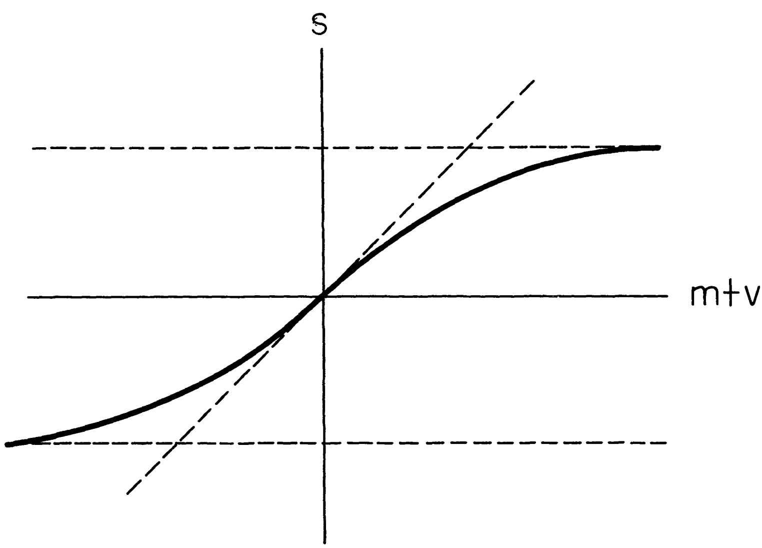

Figure V

ity, with markets driving the exchange rate to the edge of the band in order to test the authorities' resolve. What we can show is that this is not the case in this model.

Suppose that the market does not know whether the monetary authority is actually prepared to alter policy to defend the zone. A probability ϕ is assigned to the probability that the zone will in fact be defended; $1 - \phi$ to the probability that it will not. The only way to resolve the issue is to see what happens when the edge of the band is actually reached.

When the exchange rate reaches the end of the band, one of two things will happen. Either the monetary authorities will reveal their willingness to do what is necessary, making the zone fully credible thereafter, or they will not, and the market will discover that it is living under a free float after all. If the zone proves credible, the exchange rate will jump to the full credibility locus derived earlier in this paper. If it does not, the exchange rate will jump to its free-float value.

What ties this down is the requirement that there not be an infinite rate of expected capital gain when the target zone is challenged; so the *expected* jump of the exchange rate must be zero. Let $\tilde{v}$ be the value of v at which the target zone must be defended. Then if the target zone is proved credible, s jumps to the full credibility value $g(m,\tilde{v},\bar{s},\underline{s})$. If the zone proves to be a sham, s jumps to its free-floating value $m + \tilde{v}$. The condition of an expected zero jump may therefore be written as

$$\bar{s} = \phi g(m,\tilde{v},\bar{s},\underline{s}) + (1 - \phi)(m + \tilde{v}), \tag{14}$$

which implicitly defines $\tilde{v}$.

Once $\tilde{v}$ is known, the behavior of the exchange rate inside the band may be determined. Within the band, s must still lie on an S-curve of the form (8), with A chosen so that

$$\bar{s} = m + \tilde{v} + A[e^{\rho\tilde{v}} - e^{-\rho\tilde{v}}]. \tag{15}$$

The situation is illustrated in Figure VI. The full credibility locus is shown along with the tangency point $\bar{v}$. If there is to be a zero expected jump in s, then $\tilde{v}$ must lie between $\bar{v}$ and the 45-degree line. The imperfect-credibility exchange rate behavior is then seen to be a curve that is steeper than the full-credibility locus but still flatter than the 45-degree line. That is, the target zone is less stabilizing if it is imperfectly credible, but it is still stabilizing. Also, it is apparent that the extent of stabilization depends on the degree of credibility. As ϕ approaches one, the imperfect-credibility

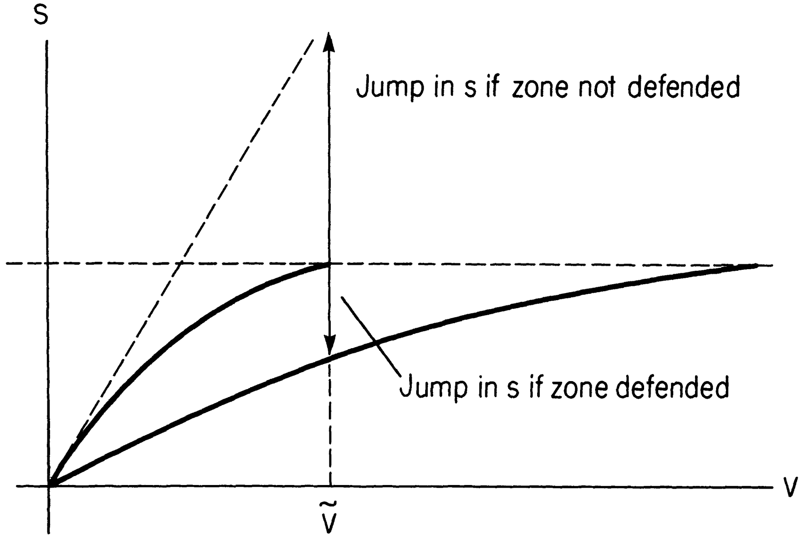

FIGURE VI

locus approaches the full-credibility one; as ϕ goes to zero, it approaches the 45-degree line.

VI. POSTSCRIPT

An early version of this paper was presented in November 1987 [Krugman, 1987]; a literature of several dozen papers using related methodology has already emerged. Several of the important extensions to the results should be mentioned.

First, Froot and Obstfeld [1989] have recast the formalization in a somewhat different way, by positing directly that the central bank places limits on the range of variation of an otherwise stochastic fundamental. This approach allows the use of known results from Harrison [1985], placing the analysis on a more solid formal footing.

Flood and Garber [1989] have examined the case in which the central bank makes discrete interventions to defend the band. This extension is of interest in its own right and provides a rather neat way of deriving the "smooth pasting" result for infinitesimal interventions.

Miller and Weller [1989] have developed a geometric technique for analyzing cases in which the stochastic process assumed does not lead to a closed-form analytical solution. Their work shows that the intuition derived from the simpler models carries over when more realistic assumptions, such as autoregressive velocity, are made.

Bertola and Caballero [1990] have extended the analysis to the case in which drift in the fundamentals may sometimes provoke realignments, i.e., shifting of the band itself.

Finally, in Krugman [1989] a bridge is built between the target zone model and the older literature on speculative attack; smooth pasting is shown to be a limiting case that emerges when reserves are sufficiently large.

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