The Basic New Keynesian Model

by

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Motivation and Outline

Evidence on Money, Output, and Prices:

• Short Run Effects of Monetary Policy Shocks
  (i) persistent effects on real variables
  (ii) slow adjustment of aggregate price level
  (iii) liquidity effect

• Micro Evidence on Price-setting Behavior: significant price and wage rigidities

Failure of Classical Monetary Models

A Baseline Model with Nominal Rigidity

• monopolistic competition
• sticky prices (staggered price setting)
• competitive labor markets, closed economy, no capital accumulation
Figure 1. Estimated Dynamic Response to a Monetary Policy Shock

Source: Christiano, Eichenbaum and Evans (1999)
Figure 1 - Examples of individual price trajectories (French and Italian CPI data)

Note: Actual examples of trajectories, extracted from the French and Italian CPI databases. The databases are described in Baudry et al. (2004) and Veronese et al. (2005). Prices are in levels, denominated in French Francs and Italian Lira respectively. The dotted lines indicate events of price changes.

Source: Dhyne et al. WP 05
Households

Representative household solves

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U (C_t, N_t)$$

where

$$C_t \equiv \left[ \int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} \, di \right]^\frac{\epsilon}{\epsilon-1}$$

subject to

$$\int_0^1 P_t(i) \, C_t(i) \, di + Q_t \, B_t \leq B_{t-1} + W_t \, N_t - T_t$$

for \( t = 0, 1, 2, \ldots \) plus solvency constraint.
Optimality conditions

1. Optimal allocation of expenditures

\[ C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t \]

implying

\[ \int_0^1 P_t(i) C_t(i) \, di = P_t C_t \]

where

\[ P_t \equiv \left[ \int_0^1 P_t(i)^{1-\epsilon} \, di \right]^{\frac{1}{1-\epsilon}} \]

2. Other optimality conditions

\[
\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}
\]

\[
Q_t = \beta \, E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\}
\]
Specification of utility:

\[ U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \]

implied log-linear optimality conditions (aggregate variables)

\[
\begin{align*}
w_t - p_t &= \sigma \ c_t + \varphi \ n_t \\
c_t &= E_t\{c_{t+1}\} - \frac{1}{\sigma} \ (i_t - E_t\{\pi_{t+1}\} - \rho)
\end{align*}
\]

where \( i_t \equiv -\log Q_t \) is the nominal interest rate and \( \rho \equiv -\log \beta \) is the discount rate.

Ad-hoc money demand

\[
m_t - p_t = y_t - \eta \ i_t
\]
Firms

- Continuum of firms, indexed by \( i \in [0, 1] \).
- Each firm produces a differentiated good.
- Identical technology
  \[ Y_t(i) = A_t \, N_t(i)^{1-\alpha} \]
- Probability of resetting price in any given period: \( 1 - \theta \), independent across firms (Calvo (1983)).
- \( \theta \in [0, 1] \) : index of price stickiness.
- Implied average price duration \( \frac{1}{1-\theta} \).
Aggregate Price Dynamics

\[ P_t = \left[ \theta \left( P_{t-1}\right)^{1-\epsilon} + (1 - \theta) \left( P^*_t \right)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \]

Dividing by \( P_{t-1} \):

\[ \Pi_t^{1-\epsilon} = \theta + (1 - \theta) \left( \frac{P^*_t}{P_{t-1}} \right)^{1-\epsilon} \]

Log-linearization around zero inflation steady state

\[ \pi_t = (1 - \theta) \left( p^*_t - p_{t-1} \right) \quad (1) \]

or, equivalently

\[ p_t = \theta p_{t-1} + (1 - \theta) p^*_t \]
Optimal Price Setting

\[
\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} \left( P_{t}^{*} Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t}) \right) \right\}
\]

subject to

\[
Y_{t+k|t} = \left( \frac{P_{t}^{*}}{P_{t+k}} \right)^{-\epsilon} C_{t+k}
\]

for \( k = 0, 1, 2, \ldots \) where

\[
Q_{t,t+k} \equiv \beta^k \left( \frac{C_{t+k}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+k}} \right)
\]

Optimality condition:

\[
\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} Y_{t+k|t} \left( P_{t}^{*} - M \psi_{t+k|t} \right) \right\} = 0
\]

where \( \psi_{t+k|t} \equiv \Psi_{t+k}'(Y_{t+k|t}) \) and \( M \equiv \frac{\epsilon}{\epsilon-1} \)
Equivalently,

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} \ Y_{t+k|t} \left( \frac{P^*_t}{P_{t-1}} - \mathcal{M} \ MC_{t+k|t} \ \Pi_{t-1,t+k} \right) \right\} = 0$$

where $MC_{t+k|t} \equiv \psi_{t+k|t}/P_{t+k}$ and $\Pi_{t-1,t+k} \equiv P_{t+k}/P_{t-1}$

**Perfect Foresight, Zero Inflation Steady State:**

$$\frac{P^*_t}{P_{t-1}} = 1 \ ; \ \Pi_{t-1,t+k} = 1 \ ; \ Y_{t+k|t} = Y \ ; \ Q_{t,t+k} = \beta^k \ ; \ MC = \frac{1}{\mathcal{M}}$$
Log-linearization around zero inflation steady state:

\[ p_t^* - p_{t-1} = (1 - \beta \theta) \sum_{k=0}^{\infty} (\beta \theta)^k E_t\{\hat{mc}_{t+k|t} + p_{t+k} - p_{t-1}\} \]

where \(\hat{mc}_{t+k|t} \equiv mc_{t+k|t} - mc\).

Equivalently,

\[ p_t^* = \mu + (1 - \beta \theta) \sum_{k=0}^{\infty} (\beta \theta)^k E_t\{mc_{t+k|t} + p_{t+k}\} \]

where \(\mu \equiv \log \frac{e}{\epsilon - 1}\).

Flexible prices (\(\theta = 0\)):

\[ p_t^* = \mu + mc_t + p_t \]

\(\implies\) \(mc_t = -\mu\) (symmetric equilibrium)
Particular Case: $\alpha = 0$ (constant returns)

$$\implies MC_{t+k|t} = MC_{t+k}$$

Rewriting the optimal price setting rule in recursive form:

$$p_t^* = \beta \theta E_t\{p_{t+1}^*\} + (1 - \beta \theta) \widehat{mc}_t + (1 - \beta \theta)p_t$$  \hspace{1cm} (2)

Combining (1) and (2):

$$\pi_t = \beta \ E_t\{\pi_{t+1}\} + \lambda \widehat{mc}_t$$

where

$$\lambda \equiv \frac{(1 - \theta)(1 - \beta \theta)}{\theta}$$
**Generalization to** $\alpha \in (0, 1)$ (decreasing returns)

Define

$$mc_t \equiv (w_t - p_t) - mpn_t$$

$$\equiv (w_t - p_t) - \frac{1}{1 - \alpha} (a_t - \alpha y_t) - \log(1 - \alpha)$$

Using $mc_{t+k|t} = (w_{t+k} - p_{t+k}) - \frac{1}{1-\alpha} (a_{t+k} - \alpha y_{t+k|t}) - \log(1 - \alpha)$,

$$mc_{t+k|t} = mc_{t+k} + \frac{\alpha}{1-\alpha} (y_{t+k|t} - y_{t+k})$$

$$= mc_{t+k} - \frac{\alpha \epsilon}{1-\alpha} (p^*_t - p_{t+k}) \quad (3)$$

**Implied inflation dynamics**

$$\pi_t = \beta \ E_t \{ \pi_{t+1} \} + \lambda \ \widehat{mc}_t \quad (4)$$

where

$$\lambda \equiv \frac{(1 - \theta)(1 - \beta \theta)}{\theta} \frac{1 - \alpha}{1 - \alpha + \alpha \epsilon}$$
Equilibrium

Goods markets clearing

\[ Y_t(i) = C_t(i) \]

for all \( i \in [0, 1] \) and all \( t \).

Letting \( Y_t \equiv \left( \int_0^1 Y_t(i)^{1-\frac{1}{\epsilon}} \, di \right)^{\frac{\epsilon}{\epsilon-1}} \),

\[ Y_t = C_t \]

for all \( t \). Combined with the consumer’s Euler equation:

\[ y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma} (i_t - E_t\{\pi_{t+1}\} - \rho) \quad (5) \]
\textit{Labor market clearing}

\[ N_t = \int_0^1 N_t(i) \, di \]
\[ = \int_0^1 \left( \frac{Y_t(i)}{A_t} \right)^{\frac{1}{1-\alpha}} \, di \]
\[ = \left( \frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\frac{\alpha}{1-\alpha}} \, di \]

Taking logs,

\[ (1 - \alpha) \, n_t = y_t - a_t + d_t \]

where \( d_t \equiv (1 - \alpha) \log \int_0^1 (P_t(i)/P_t)^{-\frac{\alpha}{1-\alpha}} \, di \) (second order).

Up to a first order approximation:

\[ y_t = a_t + (1 - \alpha) \, n_t \]
Marginal Cost and Output

\[ mc_t = (w_t - p_t) - mpn_t \]
\[ = (\sigma y_t + \varphi n_t) - (y_t - n_t) - \log(1 - \alpha) \]
\[ = \left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) y_t - \frac{1 + \varphi}{1 - \alpha} a_t - \log(1 - \alpha) \] (6)

Under flexible prices

\[ mc = \left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) y_t^n - \frac{1 + \varphi}{1 - \alpha} a_t - \log(1 - \alpha) \] (7)

\[ \implies y_t^n = -\delta_y + \psi_{ya} a_t \]

where \( \delta_y = \frac{(\mu - \log(1 - \alpha))(1 - \alpha)}{\sigma + \varphi + \alpha(1 - \sigma)} > 0 \) and \( \psi_{ya} = \frac{1 + \varphi}{\sigma + \varphi + \alpha(1 - \sigma)} \).

\[ \implies \hat{mc}_t = \left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) (y_t - y_t^n) \] (8)

where \( y_t - y_t^n \equiv \bar{y}_t \) is the output gap
New Keynesian Phillips Curve

\[ \pi_t = \beta \, E_t\{\pi_{t+1}\} + \kappa \, \tilde{y}_t \]  \hspace{1cm} (9)

where \( \kappa \equiv \lambda \left( \sigma + \frac{\varphi + \alpha}{1-\alpha} \right) \).

Dynamic IS equation

\[ \tilde{y}_t = E_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma} \left( i_t - E_t\{\pi_{t+1}\} - r^n_t \right) \]  \hspace{1cm} (10)

where \( r^n_t \) is the natural rate of interest, given by

\[ r^n_t \equiv \rho + \sigma \, E_t\{\Delta y^n_{t+1}\} = \rho + \sigma \psi_y a \, E_t\{\Delta a_{t+1}\} \]

Missing block: description of monetary policy (determination of \( i_t \)).
Equilibrium under a Simple Interest Rate Rule

\[ i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t \]  \hspace{1cm} (11)

where \( v_t \) is exogenous (possibly stochastic) with zero mean.

Equilibrium Dynamics: combining (9), (10), and (11)

\[
\begin{bmatrix}
\tilde{y}_t \\
\pi_t
\end{bmatrix} = A_T \begin{bmatrix}
E_t\{\tilde{y}_{t+1}\} \\
E_t\{\pi_{t+1}\}
\end{bmatrix} + B_T (\tilde{r}_t - v_t) \hspace{1cm} (12)
\]

where

\[
A_T \equiv \Omega \begin{bmatrix}
\sigma & 1 - \beta\phi_\pi \\
\sigma\kappa & \kappa + \beta(\sigma + \phi_y)
\end{bmatrix} ; \hspace{1cm} B_T \equiv \Omega \begin{bmatrix}
1 \\
\kappa
\end{bmatrix}
\]

and \( \Omega \equiv \frac{1}{\sigma + \phi_y + \kappa\phi_\pi} \)
**Uniqueness** \(\iff\) \(A_T\) has both eigenvalues within the unit circle

Given \(\phi_\pi \geq 0\) and \(\phi_y \geq 0\), (Bullard and Mitra (2002)):

\[
\kappa (\phi_\pi - 1) + (1 - \beta) \phi_y > 0
\]

is necessary and sufficient.
Effects of a Monetary Policy Shock

Set $\tilde{r}_t^n = 0$ (no real shocks).

Let $v_t$ follow an AR(1) process

\[ v_t = \rho_v \ v_{t-1} + \varepsilon_t^v \]

Calibration:

\[ \rho_v = 0.5, \ \phi_\pi = 1.5, \ \phi_y = 0.5/4, \ \beta = 0.99, \ \sigma = \varphi = 1, \ \theta = 2/3, \ \eta = 4. \]

Dynamic effects of an exogenous increase in the nominal rate (Figure 1).

Exercise: analytical solution
Figure 3.1: Effects of a Monetary Policy Shock (Interest Rate Rule)
Effects of a Technology Shock

Set $v_t = 0$ (no monetary shocks).

Technology process:

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a.$$

Implied natural rate:

$$\hat{r}_t^n = -\sigma \psi y_a (1 - \rho_a) a_t$$

Dynamic effects of a technology shock ($\rho_a = 0.9$) (Figure 2)

Exercise: AR(1) process for $\Delta a_t$
Figure 3.2: Effects of a Technology Shock (Interest Rate Rule)
Equilibrium under an Exogenous Money Growth Process

\[ \Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t^m \]  \hspace{1cm} (13)

Money market clearing

\[ \hat{l}_t = \hat{y}_t - \eta \hat{i}_t \]  \hspace{1cm} (14)

\[ = \tilde{y}_t + \tilde{y}_t^n - \eta \hat{i}_t \]  \hspace{1cm} (15)

where \( l_t \equiv m_t - p_t \) denotes (log) real money balances.

Substituting (14) into (10):

\[ (1 + \sigma \eta) \tilde{y}_t = \sigma \eta E_t\{\tilde{y}_{t+1}\} + \hat{l}_t + \eta E_t\{\pi_{t+1}\} + \eta \tilde{r}_t - \tilde{y}_t \]  \hspace{1cm} (16)

Furthermore, we have

\[ \hat{l}_{t-1} = \hat{l}_t + \pi_t - \Delta m_t \]  \hspace{1cm} (17)
Equilibrium dynamics

\[
\mathbf{A}_{M,0} \begin{bmatrix} \tilde{y}_t \\ \pi_t \\ \hat{l}_{t-1} \end{bmatrix} = \mathbf{A}_{M,1} \begin{bmatrix} E_t \{ \tilde{y}_{t+1} \} \\ E_t \{ \pi_{t+1} \} \\ \hat{l}_{t-1} \end{bmatrix} + \mathbf{B}_M \begin{bmatrix} \hat{r}_t^n \\ \hat{y}_t^n \\ \Delta m_t \end{bmatrix} \quad (18)
\]

where

\[
\mathbf{A}_{M,0} \equiv \begin{bmatrix} 1 + \sigma \eta & 0 & 0 \\ -\kappa & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}; \quad \mathbf{A}_{M,1} \equiv \begin{bmatrix} \sigma \eta & \eta & 1 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad \mathbf{B}_M \equiv \begin{bmatrix} \eta & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}
\]

Uniqueness $\iff \mathbf{A}_M \equiv \mathbf{A}_{M,0}^{-1} \mathbf{A}_{M,1}$ has two eigenvalues inside and one outside the unit circle.
Effects of a Monetary Policy Shock
Set \( \hat{r}_t^n = y_t^n = 0 \) (no real shocks).

Money growth process

\[
\Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t^m
\] (19)

where \( \rho_m \in [0,1) \)

Figure 3 (based on \( \rho_m = 0.5 \))

Effects of a Technology Shock
Set \( \Delta m_t = 0 \) (no monetary shocks).

Technology process:

\[
a_t = \rho_a a_{t-1} + \varepsilon_t^a.
\]

Figure 4 (based on \( \rho_a = 0.9 \)).

Empirical Evidence
Figure 3.3: Effects of a Monetary Policy Shock (Money Growth Rule)
Figure 3.4: Effects of a Technology Shock (Money Growth Rule)
Figure 3.5: Estimated Effects of a Permanent Technology Shock

Figure 4: Estimated Impulse Responses from a Five-Variable Model: U.S. Data, First-Differenced Hours (Point Estimates and ±2 Standard Error Confidence Intervals)
Technical Appendix

Optimal Allocation of Consumption Expenditures

Maximization of $C_t$ for any given expenditure level $\int_0^1 P_t(i) \, C_t(i) \, di \equiv Z_t$ can be formalized by means of the Lagrangean

$$\mathcal{L} = \left[ \int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} \, di \right]^{\frac{1}{1-\frac{1}{\epsilon}}} - \lambda \left( \int_0^1 P_t(i) \, C_t(i) \, di - Z_t \right)$$

The associated first order conditions are:

$$C_t(i)^{-\frac{1}{\epsilon}} \frac{\partial C_t}{\partial C_t} = \lambda P_t(i)$$

for all $i \in [0,1]$. Thus, for any two goods $(i,j)$ we have:

$$C_t(i) = C_t(j) \left( \frac{P_t(i)}{P_t(j)} \right)^{-\epsilon}$$

which can be plugged into the expression for consumption expenditures to yield

$$C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} Z_t \frac{P_t}{P_t}$$

for all $i \in [0,1]$. The latter condition can then be substituted into the definition of $C_t$, yielding

$$\int_0^1 P_t(i) \, C_t(i) \, di = P_t \, C_t$$

Combining the two previous equations we obtain the demand schedule:

$$C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t$$

Log-Linearized Euler Equation
We can rewrite the Euler equation as
\[ 1 = E_t\{\exp(i_t - \sigma \Delta c_{t+1} - \pi_{t+1} - \rho)\} \] (20)

In a perfect foresight steady state with constant inflation $\pi$ and constant growth $\gamma$ we must have:
\[ i = \rho + \sigma \gamma + \pi \]

with the steady state real rate being given by
\[ r \equiv i - \pi = \rho + \sigma \gamma \]

A first order Taylor expansion of $\exp(i_t - \sigma \Delta c_{t+1} - \pi_{t+1} - \rho)$ around that steady state yields:
\[ \exp(i_t - \sigma \Delta c_{t+1} - \pi_{t+1} - \rho) \approx 1 + (i_t - i) - \sigma (\Delta c_{t+1} - \gamma) - (\pi_{t+1} - \pi) = 1 + i_t - \sigma \Delta c_{t+1} - \pi_{t+1} - \rho \]

which can be used in (20) to obtain, after some rearrangement of terms, the log-linearized Euler equation
\[ c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma} (i_t - E_t\{\pi_{t+1}\} - \rho) \]

**Aggregate Price Level Dynamics**

Let $S(t) \subset [0, 1]$ denote the set of firms which do not re-optimize their posted price in period $t$. The aggregate price level evolves according to
\[
P_t = \left[ \int_{S(t)} P_{t-1}(i)^{1-\epsilon} \, di + (1 - \theta) (P_t^*)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \\
= \left[ \theta (P_{t-1})^{1-\epsilon} + (1 - \theta) (P_t^*)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}
\]
where the second equality follows from the fact that the distribution of prices among firms not adjusting in period $t$ corresponds to the distribution of effective prices in period $t-1$, with total mass reduced to $\theta$.

Equivalently, dividing both sides by $P_{t-1}^*$:

$$\Pi_t^{1-\epsilon} = \theta + (1-\theta) \left( \frac{P_t^*}{P_{t-1}} \right)^{1-\epsilon}$$

(21)

where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$. Notice that in a steady state with zero inflation $P_t^* = P_{t-1}$.

Log-linearization around a zero inflation ($\Pi = 1$) steady state implies:

$$\pi_t = (1-\theta) (p_t^* - p_{t-1})$$

(22)

**Price Dispersion**

From the definition of the price index:

$$1 = \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{1-\epsilon} di$$

$$= \int_0^1 \exp\{ (1-\epsilon)(p_t(i) - p_t) \} di$$

$$\simeq 1 + (1-\epsilon) \int_0^1 (p_t(i) - p_t) di + \frac{(1-\epsilon)^2}{2} \int_0^1 (p_t(i) - p_t)^2 di$$

thus implying the second order approximation

$$p_t \simeq E_i\{p_t(i)\} + \frac{(1-\epsilon)}{2} \int_0^1 (p_t(i) - p_t)^2 di$$
where $E_i\{p_t(i)\} \equiv \int_0^1 p_t(i) \, di$ is the cross-sectional mean of (log) prices.

In addition,

$$
\int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\frac{\epsilon}{1-\alpha}} \, di = \int_0^1 \exp \left\{ -\frac{\epsilon}{1-\alpha} \left( p_t(i) - p_t \right) \right\} \, di
$$

$$
\simeq 1 - \frac{\epsilon}{1-\alpha} \int_0^1 \left( p_t(i) - p_t \right) \, di + \frac{1}{2} \left( \frac{\epsilon}{1-\alpha} \right)^2 \int_0^1 \left( p_t(i) - p_t \right)^2 \, di
$$

$$
\simeq 1 + \frac{1}{2} \frac{\epsilon(1-\epsilon)}{1-\alpha} \int_0^1 \left( p_t(i) - p_t \right)^2 \, di + \frac{1}{2} \left( \frac{\epsilon}{1-\alpha} \right)^2 \int_0^1 \left( p_t(i) - p_t \right)^2 \, di
$$

$$
= 1 + \frac{1}{2} \left( \frac{\epsilon}{1-\alpha} \right) \frac{1}{\Theta} \int_0^1 \left( p_t(i) - p_t \right)^2 \, di
$$

$$
\simeq 1 + \frac{1}{2} \left( \frac{\epsilon}{1-\alpha} \right) \frac{1}{\Theta} \text{var}_i\{p_t(i)\} > 1
$$

where $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha^2}$, and where the last equality follows from the observation that, up to second order,

$$
\int_0^1 \left( p_t(i) - p_t \right)^2 \, di \simeq \int_0^1 \left( p_t(i) - E_i\{p_t(i)\} \right)^2 \, di
$$

$$
\equiv \text{var}_i\{p_t(i)\}
$$

Finally, using the definition of $d_t$ we obtain

$$
d_t \equiv (1-\alpha) \log \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\frac{\epsilon}{1-\alpha}} \, di \simeq \frac{1}{2} \epsilon \Theta \text{var}_i\{p_t(i)\}
$$