OPTIMAL INFLATION AND THE GOVERNMENT REVENUE MIX

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This paper examines the question of optimal inflation and taxation as a government optimal control problem. By inclusion of bond finance and portfolio decisions by private agents in a perfect foresight framework it is shown that: (i) optimal steady state inflation is independent of government revenue requirements, (ii) inflation and tax rates do not necessarily move together in optimum.

1. Introduction

There has been a recent revival of interest in the question of the optimal collection of seigniorage and taxes. The emerging literature has been partly motivated by current policy issues, such as the large U.S. budget deficits this decade and the way of achieving monetary integration in Western Europe in the next decade. One strand in this literature has derived optimal values for inflation and taxes by formulating a dynamic optimal control problem to be solved by the government. This approach was taken by Barro (1979) in deriving the principle that tax rates should be smoothed over time. More recent papers by Mankiw (1987) and Grilli (1988), employing a similar framework, replicated the tax smoothing result. Moreover, as they explicitly modeled seigniorage revenues, an important implication of their results was that an increase in government revenue requirements increases the use of both taxes and inflation and hence tax and inflation rates move together.

The present paper re-examines this question by introducing bond issues (as well as money printing) to finance primary deficits. Thus three instruments, i.e. taxes, bonds and money, are used to raise revenue. This is obviously a more general, and in many cases a more realistic, description of government revenue sources. In addition we allow for private agents determining their financial portfolio composition. Perfect foresight is assumed to simplify the discussion.

The results obtained are markedly different from the ones cited above. In particular we show that optimal inflation in the steady state is independent of government expenditures, being a function only of its discount rate and the real rate of interest. Whenever these two rates coincide we obtain Friedman’s (1969) optimum quantity of money rule which was replicated by Kimbrough (1986) in a general equilibrium welfare-maximization framework.

The paper is organized as follows: Section 2 presents the model. Section 3 discusses the implications of the first-order conditions of the government’s optimization problem. Section 4 provides some concluding remarks.

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2. The model

We consider a closed economy with constant output \( \bar{Y} \) and a given real rate of interest \( r \). The government finances real expenditures \( g \) by taxes on output \( t \), money creation \( \dot{M}/P \) in real terms) and bonds issue \( \dot{b} \). The government budget (flow) constraint is therefore

\[
g + rb = t + \mu m + \dot{b},
\]

where \( rb \) are interest payments on government debt and

\[
\mu = \dot{M}/M, \quad m = M/P.
\]

In order to focus on the optimal revenue mix, government expenditures shall be taken as exogenous.

The private sector's financial wealth consists of (outside) money and government bonds:

\[
w = m + b. \tag{2}
\]

The public chooses the composition of this portfolio as a function of inflation and the real rate of interest:

\[
b = P(\pi, r)w, \quad \partial P/\partial \pi, \partial P/\partial r > 0, \quad 0 \leq P \leq 1,
\]

\[
m = (1 - P)w. \tag{3}
\]

Differentiating (2) with respect to time and inserting (1) and (3) we get

\[
\dot{w} = g - t + rPw - \pi(1 - P)w. \tag{4}
\]

This is basically the constraint faced by the government, generated both by the budget equation and by private portfolio choice. The 'benevolent' government seeks to minimize the social losses incurred by inflation and taxes. The former losses consist of increased menu costs, disruption of the efficient functioning of markets and the deadweight losses associated with inflation as a tax on real balances. The latter losses are the traditional deadweight social losses of taxes on output (such as an income or sales tax).

Thus the following loss function may be considered:

\[
L = \int_0^\infty e^{-\rho s} \left[ \frac{\alpha}{2} \pi^2 + \frac{\beta}{2} t^2 \right] ds, \tag{5}
\]

\( \alpha, \beta > 0, \rho \) the government discount rate.

The government's problem is to minimize (5) with respect to \( \pi \) and \( t \) subject to constraint (4).

3. Implications of the optimal solution

The appropriate current value Hamiltonian for the government optimal control problem is

\[
H = (\alpha/2)\pi^2 + (\beta/2)t^2 + \lambda \left[ g - t + rPw - \pi(1 - P)w \right], \tag{6}
\]

where \( \lambda \) is the co-state variable.

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1 Note that the nominal yield of bonds is \( i = \pi + r \) and that expected values equal actual values.
2 The nature of these costs is discussed in Frenkel (1976), Fischer and Modigliani (1978) and Jovanovic (1982).
First-order conditions are:

\[ \alpha \pi + \lambda \omega \left[ (r + \pi) \frac{\partial P}{\partial \pi} (1 - P) \right] = 0, \]
\[ \beta t - \lambda = 0, \]
\[ \dot{\lambda} = \rho \lambda - \lambda \left[ rP - \pi (1 - P) \right], \]
\[ \dot{\omega} = g - t + rPw - \pi (1 - P)w, \]
\[ \lim_{s \to \infty} \lambda e^{-\rho s} \omega = 0. \]

From (7a)–(7d) the control variables \( \pi \) and \( t \), the state variable \( w \) and the co-state variable \( \lambda \) are solved as a function of the exogenous variables \( g, r, \rho, \alpha \) and \( \beta \).

3.1. Co-movement of optimal inflation and tax rates

Combining (7a) and (7b),

\[ \frac{\pi}{t} = \frac{\beta}{\alpha} \omega \left[ (r + \pi) \frac{\partial P}{\partial \pi} (1 - P) \right]. \]

The expression in square brackets may be either positive or negative; the sign is determined by the relative strength of two contradictory consequences of an increase in inflation on government revenues. The first consequence is an increase in interest payments and a decline in the inflation tax base due to a portfolio shift from money to bonds. The second is an increase in the inflation tax itself due to increased inflation. Thus, if the former effect is dominant, government net expenditures increase and tax rates must be raised; if the latter effect is dominant, government net revenues increase and tax rates should be lowered. The basic idea is that optimal inflation and tax rates do not necessarily move together.

3.2. Optimal inflation in the steady state

From (7c) the following relation is obtained in the steady state:

\[ \rho = rP \{ \pi, r \} - \pi \left[ 1 - P \{ \pi, r \} \right]. \]

This condition implies that optimal inflation in the steady state depends only on the discount rate and the real rate of interest. This rate must be chosen such that the rate used to discount future social losses equals the weighted average yield on financial wealth [the weights being \( P \) and \( 1 - P \)]. Note that when \( \rho = r \) the optimal steady-state inflation is given by \( \pi = -r \), which is Friedman’s optimal inflation rule.

The optimal solution may be understood as follows: The discount rates, which are exogenous, determine steady-state inflation; given this rate the tax rate and the asset stock are determined by eqs. (7d) and (8). The latter represents preferences derived from the loss function, while the former represents the budget constraint binding those two variables. Note that this structure generates a distinction between the determination of the inflation rate (\( \pi \)) and the determination of the inflation tax \( \pi (1 - P)w \).

3 Note that output being given an optimal value of \( t \), which is the amount of taxes, implies also an optimal tax rate.
4. Conclusions

Introduction of bond finance and private portfolio choice into the government optimal control problem causes a significant change in the results obtained when using money printing and taxes alone. In particular the optimal steady-state inflation is seen not to depend either on fiscal policy or on social losses, but rather on intertemporal prices – the government’s time preferences and the real rate of interest. Given a change in these prices, which generates a change in optimal steady-state inflation, the ensuing tax rate change depends on the tradeoff between interest payments on bonds and the inflation tax on money balances.

References

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