Capital Values, Job Values, and the Joint Behavior of Hiring and Investment

Eran Yashiv

Tel Aviv University and CEPR

May 19, 2014

Abstract

This paper explores how the joint behavior of hiring and investment is governed by the expected discounted values of capital and of jobs. It uses a model of frictions, akin to search models of the labor market and Q-type models of the capital market, emphasizing the interaction of capital and labor frictions. Relying on structural estimation of private sector U.S. data, it studies cyclical behavior, future determinants, and the implications for U.S. labor market developments.

Key findings include (i) a substantial effect of the expected value of capital on hiring; (ii) the cyclical behavior of hiring and of investment are markedly different; (iii) future returns series are shown to play a dominant role in determining capital and job values; and (iv) U.S. labor market developments, including the inward and subsequent outward shift of the Beveridge curve, can be accounted for by changes in job values, as well as in labor force growth rates.

Capital Values, Job Values, and the Joint Behavior of Hiring and Investment[1]

1 Introduction

This paper explores how the joint behavior of hiring and investment is governed by the expected discounted value of capital and labor. It uses a model of frictions, a combination of a search model of the labor market and a Q-type model of the capital market, emphasizing the interaction of capital and labor frictions. Hiring and investment are modelled as the outcomes of a dynamic, intertemporal optimization problem of the representative firm. The paper uses structural estimation of private sector U.S. data to answer the following four specific questions: (i) how do capital and labor expected present values behave over the business cycle and what cyclical hiring and investment patterns do they generate? (ii) how big are these values, i.e., how big are the relevant frictions? (iii) what determinants drive expected present values? (iv) how can recent U.S. labor market developments – including the Great Recession period – be understood in terms of capital and labor (job) values?

The answers to these questions are important for a number of key issues: the evolution of employment and of the capital stock are essential for the understanding of macroeconomic fluctuations. It has been shown that gross hiring is a major factor for understanding employment and unemployment dynamics.[2] Hiring frictions were shown to play a key role in determining the business cycle properties of labor productivity (including its declining

[1] *I thank Jordi Gali, Robert Hall, Giuseppe Moscarini, Richard Rogerson, and Gianluca Violante; conference participants at the NBER- RSW group, ESSIM- CEPR, Aarhus University- Sandbjerg and SaM-Edinburgh; seminar audiences at Yale, LSE, Tel Aviv University, CREI, EUI, Bristol, Queen Mary, the Bank of England, Birbeck College and Keio for helpful comments on previous versions; and Noa Pasternak and Avihai Lifschitz for excellent research assistance. Any errors are my own.*

[2] See, for example, Hall (2007) and Rogerson and Shimer (2011).

pro-cyclicality) and of the job finding rate (including its high volatility).[3] Investment is key for the understanding of the evolution of the capital stock and consequently of firm market value.[4]

Key business cycle facts are explained by the results of this inquiry, including the outward shift of the Beveridge curve and the big rise in unemployment in the Great Recession, the counter-cyclicality of the hiring rate and of the value of jobs, the negative co-movement of gross investment and gross hiring rates, and the role of future returns. These findings have implications for business cycle modelling, such as the importance of incorporating joint investment and hiring costs, complete with the cited interaction, into DSGE models.

A major implication of the findings is that hiring and investment can be treated as forward-looking variables, reflecting the expectations of future discounted profits from employing labor and capital. Using the results of estimation, I employ a restricted VAR analysis, such as the one used in the asset pricing literature, to study this forward-looking aspect. The analysis shows how investment and hiring are related to their expected, future determinants, with future returns turning out to play the dominant role.

This approach naturally links up with stock prices that are also forward-looking and relate to the same expected discounted future profits. Indeed, in previous work, joint with Monika Merz (Merz and Yashiv (2007)), we have shown that this set-up allows one to define asset values for hiring and for investment and that these values can be used to explain the time variation of equity values of firms in the U.S. economy. The current paper retains the

[3] Gali and van Rens (2014) show that a lower degree of hiring frictions may lower the cyclicality of labor productivity in ways which are consistent with actual U.S. aggregate data dynamics. Coles and Mortensen (2013a,b), building on Merz and Yashiv (2007), study the role of hiring costs in dynamic environments which generate a result whereby there is no Shimer "puzzle" and the job finding rate volatility matches the data.

[4] See, for example, Cochrane (2011).

focus on forward-looking behavior but does not make use of stock market data or tries to explain them. It updates the previous estimates, using a longer sample period, one that includes the Great Recession and its aftermath, but then proceeds to examine a totally different set of empirical implications.

The paper is structured as follows: Section 2 briefly discusses the relevant strands of literature. Section 3 presents the firm's optimization problem and the resulting optimality conditions. Section 4 discusses estimation issues and presents the results. It uses the results to look at the implied magnitude of frictions and to gauge the plausibility of the estimates. Section 5 discusses hiring and investment as driven by their present values and examines cyclical behavior. It compares the results to those obtained in a standard search and matching model. Section 6 undertakes the restricted VAR analysis and decomposes the present value relationships embodied in the model. Section 7 looks at the ability of the results to provide a stylized account of U.S. labor market developments, including the shift of the Beveridge curve and high unemployment of the Great Recession. Section 8 concludes. Technical matters and data issues are treated in appendices.

2 Background Literature

The literature on hiring and on investment is very large. In what follows I allude to those papers that relate directly to the focus of this paper.

First is the literature on search and matching models, which feature dynamic, optimal hiring decisions by firms in the face of frictions; see Pissarides (2000), Yashiv (2007) and Rogerson and Shimer (2011) for overviews and surveys. Hiring costs and time lags are the expression of frictions in these models. The first order condition for optimal hiring is a key ingredient and this is one of the two estimating equations examined here. The finding in this literature, as indicated above, is that gross hiring, subject to these frictions, is key in accounting for employment and unemployment dynamics.

The model here features a generalization of the hiring problem and a wider concept of costs relative to what has been considered by these models. The second strand of relevant literature includes investment models, mostly following the seminal contributions of Lucas and Prescott (1971) and of Tobin (1969) and Hayashi (1982). These models have been studied extensively for over four decades. The idea in these models is that costs are key to the understanding of investment behavior. These models have encountered a lot of empirical difficulties and have engendered much debate (see, for example, the discussion in Chirinko (1993) and Smith (2008)). Like search and matching models, much of this literature does not feature the other factor of production, namely labor. In the current paper I present results both from the "traditional" formulation of the investment costs model and from a formulation which allows for the interaction of investment costs and hiring costs. It should also be noted that models of the business cycle (evidently) feature optimal hiring and investment decisions. Many of them do not feature frictions, though a large part of the RBC literature assumes lags in the installation of capital. The latest vintage of business cycle models, surveyed by Christiano, Trabandt and Walentin (2010), posits costs for investment but no frictions in hiring. Note, too, that in business cycle models there is no explicit interaction between hiring costs and investment costs.

A key issue in the current paper is the mutual dependence of hiring and investment and the interaction of their costs. This is not a new issue. Mortensen (1973) has examined the interrelation of costs in a theoretical model and over the years some empirical work was attempted; prominent examples include Nadiri and Rosen (1969), Shapiro (1986), and Hall (2004). These studies point to the potential importance of including costs on both capital and labor. However key differences with the current study are that these papers do not model at least one of three elements, which the empirical work below finds to be of crucial importance: (i) an interaction term between the two costs; (ii) gross, as opposed to net, hiring flows; and (iii) aggregate, as opposed to micro-level, hiring and investment. It should also be emphasized, that the current paper stays within the representative firm framework of the cited literature and does not at all attempt to go into a firm-level or sector-level analysis. Hence most of the findings of the latter type of studies are different from what is reported here.

This paper stresses the forward-looking aspect of hiring and investment. Consequently an important issue is the future determinants of current behavior. This issue is studied, for the case of stock prices, by a sizeable strand of literature in Finance, launched by the work of Campbell and Shiller (1988). A key concern in this literature has been the question of what is the relative importance of dividend growth and of future returns for stock price volatility. I make use of the methodology developed in this literature, examined by Cochrane (2011), to determine the relative importance of the future determinants of current hiring and current investment. Recently, Hall (2014) has taken up this issue, albeit using a different empirical methodology.

3 The Model

I delineate a partial equilibrium model which serves as the basis for estimation.[5] There are identical workers and identical firms, who live forever and have rational expectations. All variables are expressed in terms of the output price level. Firms make gross investment (i) and gross hiring (h) decisions.[6] Once a new worker is hired, the firm pays her a per-period wage w. Firms use physical capital (k) and labor (n) as inputs in order to produce output goods y according to a constant-returns-to-scale production function f with

[5] The parts concerned with the labor market are consistent with the prototypical search and matching model within a stochastic framework. See, for example, Pissarides (2000) and Yashiv (2007).

[6] In the standard search and matching model, gross hires are labeled new job-matches.

productivity shock z:

$$y_t = f(z_t, n_t, k_t), \tag{1}$$

Gross hiring and gross investment are subject to frictions, spelled out below, and hence are costly activities. I represent these costs by a function $g[i_t, k_t, h_t, n_t]$ which is convex in the firm's decision variables and exhibits constant returns-to-scale, allowing hiring costs and investment costs to interact.

In every period t, the capital stock depreciates at the rate δ_t and is augmented by new investment i_t. Similarly, workers separate at the rate ψ_t and the employment stock is augmented by new hires h_t.The laws of motion are:

$$k_{t+1} = (1 - \delta_t)k_t + i_t, \quad 0 \leq \delta_t \leq 1. \tag{2}$$

$$n_{t+1} = (1 - \psi_t)n_t + h_t, \quad 0 \leq \psi_t \leq 1. \tag{3}$$

The representative firm chooses sequences of i_t and h_t in order to maximize its profits as follows:

$$\max_{\{i_{t+j}, h_{t+j}\}} E_t \begin{array}{r} \sum_{j=0}^{\infty} \left(\prod_{i=0}^{j} \rho_{t+i}\right) [(1 - \tau_{t+j}) \left(f(z_{t+j}, n_{t+j}, k_{t+j}) - g\left(i_{t+j}, k_{t+j}, h_{t+j}, n_{t+j}\right) - w_{t+j} n_{t+j}\right) \\ - \left(1 - \chi_{t+j} - \tau_{t+j} D_{t+j}\right) \tilde{p}^I_{t+j}\, i_{t+j} \,] \end{array} \tag{4}$$

subject to the constraints (2) and (3), and where τ_t is the corporate income tax rate, w_t is the wage, χ_t the investment tax credit, D_t the present discounted value of capital depreciation allowances, $\tilde{p}^I_t$ the real pre-tax price of investment goods, and ρ_{t+j} is a time-varying discount factor. The firm takes the paths of the variables $w, p^I, \delta, \psi, \tau$ and ρ as given. This is consistent with the standard models in the search and matching and Tobin's Q literatures. The Lagrange multipliers associated with these two constraints are Q^K_{t+j} and Q^N_{t+j}, respectively. These Lagrange multipliers can be interpreted as marginal Q for physical capital, and marginal Q for employment, respectively. I shall use the term capital value or present value of investment for the former and job value or present value of hiring for the latter.

The first-order conditions for dynamic optimality are:[7]

$$Q_t^K = E_t\left[\rho_{t+1}\left[\left(1-\tau_{t+1}\right)\left(f_{k_{t+1}} - g_{k_{t+1}}\right) + \left(1-\delta_{t+1}\right)Q_{t+1}^K\right]\right] \tag{5}$$

$$Q_t^K = \left(1-\tau_t\right)\left(g_{i_t} + p_t^I\right) \tag{6}$$

$$Q_t^N = E_t\left[\rho_{t+1}\left[\left(1-\tau_{t+1}\right)\left(f_{n_{t+1}} - g_{n_{t+1}} - w_{t+1}\right) + \left(1-\psi_{t+1}\right)Q_{t+1}^N\right]\right] \tag{7}$$

$$Q_t^N = \left(1-\tau_t\right)g_{h_t} \tag{8}$$

I can summarize the firm's first-order necessary conditions from equations (5)-(8) by the following two expressions:

$$\left(1-\tau_t\right)\left(g_{i_t} + p_t^I\right) = E_t\left[\rho_{t+1}\left(1-\tau_{t+1}\right)\left[\begin{array}{c} f_{k_{t+1}} - g_{k_{t+1}} \\ +(1-\delta_{t+1})(g_{i_{t+1}} + p_{t+1}^I) \end{array}\right]\right] \tag{9}$$

$$\left(1-\tau_t\right)g_{h_t} = E_t\left[\rho_{t+1}\left(1-\tau_{t+1}\right)\left[\begin{array}{c} f_{n_{t+1}} - g_{n_{t+1}} - w_{t+1} \\ +(1-\psi_{t+1})g_{h_{t+1}} \end{array}\right]\right]. \tag{10}$$

4 Estimation

I estimate alternative versions of the model. The alternatives pertain to the degree of convexity of the costs function, the existence of linear terms in this function, the examination of standard specifications, and the set of instruments used. I estimate equations (9) and (10), using structural estimation. In what follows I present the parameterization of this function (as well as of the production function), the econometric methodology, the data and estimation results.

[7] where I use the real after-tax price of investment goods, given by:

$$p_{t+j}^I = \frac{1-\chi_{t+j} - \tau_{t+j}D_{t+j}}{1-\tau_{t+j}}\widetilde{p}_{t+j}^I.$$

4.1 Methodology and Data

To estimate the model I need to parameterize the relevant functions. For the *production function* I use a standard Cobb-Douglas formulation:

$$f(z_{t,}n_t, k_t) = e^{z_t} {n_t}^{\alpha} k_t^{1-\alpha},\ 0 < \alpha < 1. \tag{11}$$

The *costs function* g, capturing the different frictions in the hiring and investment processes, is at the focus of the estimation work and merits discussion. Specifically, hiring costs include search costs for workers, costs of advertising, screening and testing, matching frictions, training costs and more. Investment involves implementation costs, financial premia on certain projects, capital installation costs, learning the use of new equipment, etc. Both activities may involve, in addition to production disruption, the implementation of new organizational structures within the firm and new production techniques; see Alexopoulos (2011) and Alexopoulos and Tombe (2012). In sum g is meant to capture all the frictions involved in getting workers to work and capital to operate in production, and not, say, just capital adjustment costs or vacancy costs. One should keep in mind that this is formulated as the costs function of the representative firm within a macroeconomic model, and not one of a single firm in a heterogenous firms micro set-up.

Functional Form. The parametric form I use is the following, generalized convex function.

$$g(\cdot) = \left[f_1 \frac{i_t}{k_t} + f_2 \frac{h_t}{n_t} + \frac{e_1}{\eta_1} (\frac{i_t}{k_t})^{\eta_1} + \frac{e_2}{\eta_2} (\frac{h_t}{n_t})^{\eta_2} + \frac{e_3}{\eta_3} \left(\frac{i_t}{k_t} \frac{h_t}{n_t} \right)^{\eta_3} \right] f(z_t, n_t, k_t). \tag{12}$$

This function is linearly homogenous in its arguments $i, k, h, n.$ The parameters $f_1, f_2, e_l, l = 1, 2, 3$ express scale, and the parameters η_1, η_2, η_3 express the elasticity of costs with respect to the different arguments. I rationalize the use of this form in what follows.

Arguments of the function. This specification captures the idea that frictions or costs increase with the extent of the activity in question, hiring or investment. The latter needs to be modelled relative to the size of the firm. The intuition is that hiring 10 workers, for example, means different levels of hiring activity for firms with 100 workers or for firms with 10,000 workers. Hence firm size, as measured by its physical capital stock or its level of employment, is taken into account and the costs function is increasing in the investment and hiring rates, $\frac{i}{k}$ and $\frac{h}{n}$. The function used postulates that costs are proportional to output, i.e., the results can be stated in terms of lost output. More specifically, the terms in the function presented above may be justified as follows (drawing on Garibaldi and Moen (2009)): suppose each worker i makes a recruiting and training effort h_i; as this is to be modelled as a convex function, it is optimal to spread out the efforts equally across workers so $h_i = \frac{h}{n}$; formulating the costs as a function of these efforts and putting them in terms of output per worker one gets $c\left(\frac{h}{n}\right)\frac{f}{n}$; as n workers do it then the aggregate cost function is given by $c\left(\frac{h}{n}\right)f$.

Convexity. I use a convex function, allowing for free estimation of the degree of convexity. The use of such a function may be questioned at the micro-level, as non-convexities were found to be significant at that level (plant, establishment, or firm). But a number of recent papers have given empirical support to the use of a convex function in the aggregate, showing that such a formulation is appropriate at the macroeconomic level.[8]

[8]Thus, Thomas (2002) and Kahn and Thomas (2008, see in particular their discussion on pages 417-421) study a dynamic, stochastic, general equilibrium model with nonconvex capital adjustment costs. One key idea which emerges from their analysis is that there are smoothing effects that result from equilibrium price changes. Favilukis and Lin (2011) use data on asset prices as additional restrictions when examining firm investment behavior and find that "...within such a model, non-convex frictions are unnecessary to match important features of aggregate investment...a model with convex costs alone does nearly as good of a job at matching firm level micro data as our preferred model with both convex and non-convex costs" (page 26).

Interaction.. The term $\frac{e_3}{\eta_3}\left(\frac{i_t}{k_t}\frac{h_t}{n_t}\right)^{\eta_3}$ expresses the interaction of investment and hiring costs. This term, absent in many studies, has important implications for the complementarity of investment and hiring. It, too, is estimated without constraints.

Relation to Known Cases. The function above encompasses widely-used cases as special cases. For example, the quadratic case has $\eta_1 = \eta_2 = 2$; a standard Tobin's Q model of investment has $e_2 = e_3 = 0$ and $\eta_1 = 2$; a Pissarides-type matching model would have $e_1 = e_3 = 0, \eta_2 = 1$.

Alternative specifications. In estimation, I explore a number of alternative specifications: the degree of convexity of the g function (I examine free and restricted estimation of the power parameters η_1, η_2 and η_3); existence of linear terms in the g function, i.e. whether f_1, f_2 are needed; standard specifications (for e.g. I set $e_2 = e_3 = 0$ and look at investment costs only and I set $e_1 = e_3 = 0$ and look at hiring costs only); and instrument sets. Estimation of the parameters in these functions allows for the quantification of the derivatives g_{i_t} and g_{h_t} that appear in the firms' optimality equations (9) and (10). I structurally estimate the firms' first-order conditions (9) and (10), using Hansen's (1982) generalized method of moments (GMM). The moment conditions estimated are those obtained under rational expectations. I formulate the equations in stationary terms by dividing (9) by $\frac{f_t}{k_t}$ and (10) by $\frac{f_t}{n_t}$. Appendix A spells out the first derivatives included in these equations. Importantly, I check whether the estimated g function fulfills the convexity requirement.

The data are quarterly, pertain to the private sector of the U.S. economy, and cover the period 1976-2011.The start date of 1976 is due to the lack of availability of credible monthly CPS data from which the gross hiring flow series is derived. This sample period covers five NBER-dated recessions, including the Great Recession of 2007-2009 and its aftermath. The data include NIPA data on GDP and its deflator, capital, investment, the price of investment goods and depreciation, BLS CPS data on employment and on worker flows, and Fed data computations on tax and depreciation allowances. Appendix B elaborates on the sources and on data construction. These data have the following distinctive features: (i) they pertain to the U.S. private sector; (ii) both hiring h and investment i refer to gross flows; likewise, separation of workers ψ and depreciation of capital δ are gross flows; (iii) the estimating equations take into account taxes and depreciation allowances. Table 1 presents key sample statistics. **Table 1.**

4.2 Results

Table 2 reports the results of estimation. The table reports the estimates and their standard errors, Hansen's (1982) J-statistic and its p-value. **Table 2 a,b.**

While typically one assumes a particular convex function, say a quadratic, I begin by looking at unrestricted estimates, in row 1 of panel a. In this specification all nine parameters are freely estimated, including α of the production function (11), and the scale $(f_1, f_2, e_1, e_2, e_3)$ and power parameters (η_1, η_2 and η_3) of the costs function (12). The results suggest that α is around the conventional estimate of 0.67, that the degree of convexity is around the cubic for the investment rate term, quadratic for the hiring rate term and linear for the interaction term ($\eta_3 = 1$). While there are low standard errors for these four power parameters, the five scale parameters are imprecisely estimated. Holding α fixed at 0.67 and setting the linear terms to zero ($f_1 = f_2 = 0$), as reported in row 2, yields similar results for the powers and precise estimates for the scale parameters (e_1, e_2, e_3). Following these results, rows 3 and 4 of panel a restrict the convexity to be either cubic-quadratic with linear interaction ($\eta_1 = 3, \eta_2 = 2$ and $\eta_3 = 1$) or quadratic with linear interaction ($\eta_1 = \eta_2 = 2$ and $\eta_3 = 1$). In these cases the scale parameters are precisely estimated and the p-value indicates that the model is not rejected. Over the relevant ranges both row3 and row 4 appear linear in the first derivatives of the costs function (i.e., marginal costs). The specification of row 4 is positive throughout, somewhat higher for the investment case and somewhat lower for the hiring case. When verifying that the resulting costs function satisfies first and second order conditions for convexity, only row 4 yields a convex costs function all through the sample period. This suggests that the specification of row 4 – quadratic with linear interaction – is the one to be preferred, and is, in any case, quite close to the cubic-quadratic specification of row 3. Appendix C reports variations on these specifications, mostly in terms of the instrument set, as a check for robustness. The results there are in line with those of panel (a) of Table 2.

Panel (b) of Table 2 looks at standard specifications in the literature. Column 1 sets $\eta_1 = 2, e_2 = e_3 = 0$, i.e., quadratic investment costs, with no role for hiring, as is typical in the Tobin's Q/investment literature. Column 2 sets $\eta_2 = 1, e_1 = e_3 = 0$, i.e., linear hiring costs with no role for investment, as used in the search and matching literature. Column 3 uses a quadratic function for both hiring and investment costs but no interaction ($\eta_1 = \eta_2 = 2, e_3 = 0$). The panel reports precise estimates and reasonable p-values for the J statistic. However, the reasons not to prefer these standard specifications become clear below, when studying various implications of the estimates.

The conclusions thus far are as follows, taking into account the alternative specifications discussed in Appendix C: quadratic costs and linear interaction of investment and hiring costs generate a good fit of the data; the interaction is significant and is negatively signed, implying complementarity between investment and hiring (to be discussed below). In what follows I shall refer to the results of row 4 in panel (a) as the preferred specification, adding some of the other specifications for comparison, where relevant.

The estimated costs are interesting and important by and of themselves, as many models rely on their existence. Hence, the results of Table 2 merit inspection for plausibility and the derivation of the time series for the frictions they imply. This is done by constructing the time series for total and marginal costs implied by the point estimates of the parameters of the g function and relating them to what is known on these issues. Key moments are presented in Tables 3a and 3b. **Table 3 a,b.**

For the preferred estimates, total costs are about 1.4% of GDP on average, with a standard deviation of 0.2%. Marginal investment costs add about 6% on average to the price p_t^I of a unit of capital. Marginal hiring costs are on average the equivalent of 1.6 weeks of wages. To gain a better grasp of the implications of these estimates, the following comparisons place them in context.

Total costs as a fraction of GDP (i.e., $\frac{g}{f}$) are around 1.4% of output according to the preferred specification (row 4 of Table 3a), a reasonable estimate. The specifications, which are the standard ones in the literature, reported in Table 3b, posit higher costs, up to 3% of output.

Marginal costs of hiring in terms of average output per worker ($\frac{g_h}{\frac{f}{n}}$) have a sample mean of 0.08 in row 4 of Table 3a, the preferred specification. This is roughly equivalent to 12% of quarterly wages.[9] In other words, firms pay the equivalent of about 1. 6 weeks of wages to hire the *marginal* worker.

How does one evaluate this estimate? There is little direct empirical evidence on these costs in the literature. The literature has some estimates of *average* hiring costs, which are typically based on linear vacancy costs. Note that the results here do not refer only to vacancy costs and pertain to the marginal hire with convex costs. It turns out that the current results are consistent with the literature estimates. Appendix D1 elaborates on the comparisons.

The marginal costs of investment (i.e. g_i) in terms of average output per unit of capital ($\frac{f}{k}$) have a sample mean of 0.75 in row 4 of Table 3a.

[9] Wages are 65% of output per worker on average, see Table 1.

To give another, more intuitive, perspective on these numbers, consider how much one needs to add to the price of one unit of the investment good p^I in marginal costs: this mean represents 5.6% on average. By contrast, the estimate of row 1 of Table 3b with only quadratic investment costs – the standard specification in the Tobin's Q literature – has a sample mean of 2.33 in terms of average output per unit of capital $(\frac{f}{k})$ or 17% to be added to the price of the investment good, an implausible result. Beyond this comparison, how reasonable are the preferred parameter estimates? The most natural place to look for comparisons is the Q-literature. Appendix D1 discusses this comparison, concluding that the preferred estimates here are within the lowest range of costs estimates in that literature.

Overall, then, the frictions implied by the estimates are low and reasonable, in light of what is known from the literature.

5 Hiring, Investment and Their Present Values

This section examines the implications of the estimates for the co-movement of hiring and investment and capital and job values in the context of business cycle behavior.

5.1 Hiring and Investment Rates as Functions of the Present Values

Taking equations (6)-(8), using the definitions of the derivatives of the g function spelled out in Appendix A, and the results of row 4 in Table 2a whereby $\eta_1 = \eta_2 = 2, \eta_3 = 1$, and $e_1e_2 - e_3^2 > 0$, the following relations are derived:

$$\frac{h_t}{n_t} = \frac{1}{(1-\tau_t)(e_1e_2-e_3^2)}\left(e_1\frac{Q_t^N}{\frac{f_t}{n_t}} - e_3\frac{Q_t^K}{\frac{f_t}{k_t}} + e_3(1-\tau_t)\frac{p_t^I}{\frac{f_t}{k_t}}\right) \quad (13)$$

$$\frac{i_t}{k_t} = \frac{1}{(1-\tau_t)(e_1e_2-e_3^2)}\left(-e_3\frac{Q_t^N}{\frac{f_t}{n_t}} + e_2\frac{Q_t^K}{\frac{f_t}{k_t}} - e_2(1-\tau_t)\frac{p_t^I}{\frac{f_t}{k_t}}\right) \quad (14)$$

The implications of these relations are that, as the estimates of Table 2 indicate that $e_1, e_2 > 0, e_3 < 0$, the hiring and investment rates, $\frac{h_t}{n_t}$ and $\frac{i_t}{k_t}$, are positive linear functions of both their present values, Q_t^N and Q_t^K,and negative functions of p_t^I, taking into account taxes. This can be easily quantified from re-writing (13) and (14) as the following linear equations:[10]

$$\frac{h_t}{n_t} = a\frac{g_{h_t}}{\frac{f_t}{n_t}} - c\frac{g_{i_t}}{\frac{f_t}{k_t}}; \quad \frac{i_t}{k_t} = -c\frac{g_{h_t}}{\frac{f_t}{n_t}} + b\frac{g_{i_t}}{\frac{f_t}{k_t}} \quad (15)$$

It is therefore apparent that models which ignore the present value of the other factor are incorrect as long as $e_3 \neq 0$ (and so $c \neq 0$). Table 4 shows the first and second moments of the decomposition of the RHS of the equations in (15). **Table 4.**

Of the mean hiring rate of 13%, a fraction of 58% is due to the present value of hiring term ($a\frac{g_{h_t}}{\frac{f_t}{n_t}}$) and the remaining 42% are due to the investment term ($c\frac{g_{i_t}}{\frac{f_t}{k_t}}$). The variance of of the hiring rate (std of 1%) is decomposed in rows 2 and 3, which sum up to 1. The investment term again plays a substantial role – its variance is half of that of the hiring term and the covariance of the two terms is substantial. Overall, these results imply that the present value of investment $\frac{g_{i_t}}{\frac{f_t}{k_t}}$ plays a substantial role in the determination of hiring rates. The mean investment rate of 2% is due to the present value of hiring term (32%) and the investment term (68%). The variance of the

[10] where

$$a = \frac{e_1}{e_1e_2-e_3^2}; \; b = \frac{e_2}{e_1e_2-e_3^2}; \; c = \frac{e_3}{e_1e_2-e_3^2}$$

investment rate (std of 0.3%) is decomposed into a small part due to the hiring term and the big part played by the variance of the investment term $(\frac{g_{i_t}}{\frac{f_t}{k_t}})$ and the large co-variation with hiring. It ensues that the cross effects are asymmetric: the investment terms play a bigger role in hiring than the hiring terms in investment.

5.2 Negative Interaction Engenders Simultaneity

Across all specifications of Table 2a, the estimate of the coefficient of the interaction term, e_3, is negative. This negative point estimate implies a negative value for g_{hi} and, therefore, as can be seen in equations (13)-(14), a positive sign for $\partial(\frac{h_t}{n_t})/\partial Q^K$ and for $\partial(\frac{i_t}{k_t})/\partial Q^N$ (for the full derivations of these derivatives, as well as the relevant elasticities, see Appendix A.) Note that $\partial(\frac{i_t}{k_t})/\partial Q^K$ and $\partial(\frac{h_t}{n_t})/\partial Q^N$ are positive due to convexity. Hence, when the marginal value of investment Q^K rises, both investment and hiring rise. A similar argument shows that they both rise when the marginal value of hiring Q^N rises.

The signs of these elasticities and derivatives imply that for given levels of investment, total and marginal costs of investment decline as hiring increases. Similarly, for given levels of hiring, total and marginal costs of hiring decline as investment increases. This finding of complementarity between investment and hiring is to be expected as it implies that they should be simultaneous. One interpretation of this result is that simultaneous hiring and investment is less costly than sequential hiring and investment of the same magnitude. This may be due to the fact that simultaneous action by the firm is less disruptive to production than sequential action. This feature is quantified by the 'scope' statistic $\frac{g(0,\frac{h}{n})+g(\frac{i}{k},0)-g(\frac{i}{k},\frac{h}{n})}{g(\frac{i}{k},\frac{h}{n})}$. The statistic measures how much – in percentage terms – is simultaneous investment and hiring cheaper than non-simultaneous action. Its sample mean and standard deviation are presented in the first column of Table 5. **Table 5.**

The scope is 0 by construction in any specification without a cost interaction. For the preferred specification, it is on average a multiple 1.4 of total costs, with a standard deviation of 0.08. The cost of doing investment and hiring sequentially $(g(0, \frac{h}{n}) + g(\frac{i}{k}, 0))$ sums up to about 3.3% of GDP; the cost of doing them simultaneously sums up to about 1.4% of GDP, i.e., it is 1.9% of GDP cheaper. This is a multiple 1.4 of costs $(\frac{1.9\%}{1.4\%} = 1.4)$. It means that there are substantial savings of costs when investing and hiring at the same time. Hence the preferred estimates of row 4 in Table 2a imply that there is meaningful inter-relation between hiring and investment costs. The decision by the firm on one factor is strongly dependent on the other.

5.3 The Elasticities of Hiring and Investment w.r.t Present Values

Table 5 further quantifies the relations between hiring and investment and their present values. It presents the mean and standard deviation of the elasticities of investment i and of hiring h with respect to the present values Q^K and Q^N. The table shows that investment is very highly elastic with respect to the present value of investing Q^K. Hiring has much lower elasticity, lower than unitary, with respect to its own present value Q^N. The cross elasticities are low for investment w.r.t Q^N and high for hiring w.r.t Q^K. These results are of course consistent with those of sub-section 5.1 reported above, which implied a great sensitivity of hiring to Q^K and lower sensitivity of investment to Q^N. The more standard formulation of Table 4b row 3 – quadratic in investment and hiring rates – which leaves out the interaction, implies an investment elasticity that is somewhat lower relative to the preferred case and a unitary elasticity for hiring, which is almost double that implied by the preferred specification. By construction, this specification does not admit cross-elasticities. Thus it can be concluded that omitting the interaction term distorts the elasticities picture.

The following distinction, however, is important. The preceding sub-section has shown that optimal behavior includes simultaneous hiring and investment, i.e., positive levels of both ($\frac{i}{k}, \frac{h}{n} > 0$). Thus the representative firm is hiring and investing at the same time. But it does **not** necessarily imply highly positive co-movement or correlation between hiring and investment rates. In other words, investment and hiring take place at the same time, but it is possible to have one rise while the other rises, stays the same or even declines. This has to do with the elasticities discussed above. Suppose, for example, Q^K rises while Q^N declines. The rise in Q^K will lead to higher investment and higher hiring, while the fall in Q^N will lead to lower investment and lower hiring. The elasticity estimates of Table 5 imply that the Q^K movements and the Q^N movements engender different responses. Therefore it is possible that investment will rise with the rise in Q^K while hiring falls with the fall in Q^N. These are indeed the patterns found in this U.S. data sample, as discussed in the following sub-section.

5.4 Co-Movement and Cyclical Analysis

The analysis focuses on the gross hiring rate $\frac{h}{n}$ and the gross investment rate $\frac{i}{k}$ of the aggregate private sector of the U.S. economy. In what follows I examine their cyclical behavior and their co-movement, over the data sample 1976-2011, which includes the Great Recession period. I then look at the cyclical behavior of marginal costs, which are equivalent to expected present values.

5.4.1 The Data Facts

Figure 1a plots *the raw series* and Table 6a reports their key moments. **Figure 1a and Table 6a.**

The figure and the table indicate that the rate of investment has higher volatility (in terms of the coefficient of variation) and somewhat higher persistence relative to the hiring rate. While the rate of investment has gone up in the early 1990s and has stayed up, albeit with a lot of fluctuations, the hiring rate has gradually declined and has stayed down since the mid 1990s. The correlation between them is negative.

Figure 1b and Table 6b look at *the cyclical behavior* of the two series. The graphs depict the logged series in levels and using the Hodrick-Prescott (HP) and Baxter-King (BK) band pass filters and displays NBER-dated recessions. The table presents co-movement with three cyclical measures – real business sector GDP f, labor productivity $\frac{f}{n}$ and capital productivity $\frac{f}{k}$. **Figure 1b and Table 6b.**

While the investment rate is clearly pro-cyclical, the hiring rate is counter-cyclical. Both contemporaneously and dynamically, hiring is counter-cyclical with respect to the three cyclical variables. These correlations are somewhat stronger when using the BK filter, relative to the HP filter. With respect to the same cyclical measures, investment is pro-cyclical, sometimes strongly so. This is so both contemporaneously and at some leads and lags.

Note that in recessions the rate of hiring rises while the rate of investment falls. Two years ahead of the recession investment rises and hiring falls. Judging by the strength of the correlation measures, investment rates are stronger leading indicators of the cycle.

Figure 1c and Table 6c show *the co-movement* of the two series over the cycle, referring again to logged, HP-filtered and BK-filtered series of investment and hiring with NBER-dated recessions. The table reports their dynamic correlations. **Figure 1c and Table 6c.**

The investment and hiring rates series do not move together, consistently with their afore-mentioned, markedly different cyclical behavior. They exhibit negative correlation, contemporaneously and at most leads and lags.

5.4.2 Examining the Counter-Cyclicality of Hiring

The counter-cyclicality of the gross hiring rate may appear counter-intuitive. To put this behavior in further perspective and show how it relates to other known labor market facts, I look at labor market variables which are often discussed in the literature. Appendix D2 spells out several relations in the labor market and looks at the co-movement of key variables. The appendix shows that the employment stock n and the job finding rate $\frac{h_t}{u_t+o_t}$ are pro-cyclical, as is well known. Steady state non-employment $\frac{\psi}{\frac{h}{u+o}+\psi}$ and the inverse of the employment ratio $\frac{1}{\frac{n}{pop}}$ are counter-cyclical, as widely known too. At the same time the gross hiring rate $\frac{h_t}{n_t}$ is counter-cyclical, as shown above. The analysis in Appendix D2 makes clear that the hiring rate is counter-cyclical as the counter-cyclicality of the last two variables dominates the pro-cyclicality of the job-finding rate.

5.4.3 The Cyclicality of Marginal Costs and Present Values

What is the cyclical behavior of marginal costs and therefore also of expected present values? Table 7 and Figure 1d report the relevant statistics. **Figure 1d and Table 7.**

Marginal costs of investment are pro-cyclical, and, as implied by equation (14), co-move positively with the investment rate. Marginal costs of hiring are counter-cyclical, and, as implied by equation (13), co-move positively with the hiring rate. This is true across the three cyclical measures and the two filtering methods. The relationships go beyond the contemporaneous ones and usually extend at least four quarters back (i.e., the cyclical indicator is lagged four quarters) and at least one quarter ahead.

The results imply the following cyclical patterns: in a boom investment rates rise while hiring rates decline. This is so because the rates move together with their marginal costs, which themselves represent expected present values. Specifically, in the U.S. data sample examined here, the present value of investment (capital value) was pro-cyclical while that of hiring (job values) was counter-cyclical. As the marginal productivity of capital rises in booms and in subsequent quarters, g_i rises and with it the investment rate. By contrast, the hiring rate falls with the decrease in g_h, as future labor profitability falls. The latter falls due to the fact that while the labor share first falls in a boom (thereby increasing profitability), it subsequently rises for a substantial period of time (see Rios-Rull and Santaeulalia-Llopis (2010)). Following the same logic, in recessionary times, firms, looking into the future, expect higher profitability from employing labor. Hence, they increase the rate at which they hire workers.

5.4.4 Job Values Across Models

The standard search and matching model (see Pissarides (2000), Yashiv (2007) and Rogerson and Shimer (2011) for surveys) also posits a formulation of Q^N, which is the value of the job match. This would be given by:

$$Q^N_{t,search} = (1-\tau_t)c\frac{1}{q_t} \tag{16}$$

where c are marginal vacancy costs, q is the rate at which vacancies are filled (so $\frac{1}{q}$ is expected vacancy duration) and τ is the corporate tax rate. See, for example, equation (1.7) in Pissarides (2000, p.11). The vacancy matching rate is given by $q_t = \frac{h_t}{v_t}$, where v are job vacancies. This means that the value of the (single) job is given by $Q^N_{t,search} = (1-\tau_t)c\frac{v_t}{h_t}$.

In the current set-up the formulation is given by (in terms of average output $\frac{f_t}{n_t}$ so as to make it consistent with the above):

$$\frac{Q^N_t}{\frac{f_t}{n_t}} = (1-\tau_t)\, g_{h_t} = (1-\tau_t)\left[e_2(\frac{h_t}{n_t})^{\eta_{2-1}} + e_3\left(\frac{i_t}{k_t}\right)^{\eta_3}\frac{h_t}{n_t}^{\eta_3-1}\right] \tag{17}$$

It is already clear from the comparison of (16) and (17) and from the discussion in 5.4.2 above that $Q^N_{t,search}$ and $\frac{Q^N_t}{\frac{f_t}{n_t}}$ behave differently. The former is a positive function of $\frac{v_t}{h_t}$ and is likely to be pro-cyclical. The latter is a positive function of $\frac{h_t}{n_t}$ and a negative function of $\frac{i_t}{k_t}$, given the estimates of a negative e_3. It will thus be counter-cyclical, as $\frac{h_t}{n_t}$ is counter-cyclical and $\frac{i_t}{k_t}$ is pro-cyclical.

The reason for this difference is that the standard search model formulates vacancy costs as a function of their duration $\frac{1}{q_t}$, without assigning any variability to marginal costs – they are fixed at c. It ignores capital and any other variable that varies over time. Vacancy duration is pro-cyclical, hence job values are too.

The current model captures the entire recruiting process (from vacancy creation to the training of the hired workers) in the convex g function defined over the hiring rate and the investment rate. Importantly it is defined over the actual hiring rate. Hence, given that $\frac{h_t}{n_t} = \frac{q_t v_t}{n_t}$, a rise in q_t, ceteris paribus, means more hiring and therefore higher costs. This formulation of costs follows the "tradition" of the Lucas-Prescott and of the Tobin-Brainard (Q) models, whereby costs and expected values rise with the activity in question. Hence counter-cyclical hiring rates are consistent with counter-cyclical job values.

Figure 2 quantifies these values as follows. For $\frac{Q_t^N}{\frac{f_t}{n_t}}$ it shows two series: the time series implied by the preferred specification of Table 2a row 4 as well as that implied by the restricted case of Table 2b row 2 (linear hiring costs, where $e_1 = e_3 = 0$ and $\eta_2 = 1$). For $Q_{t,search}^N$ it shows $(1-\tau_t)c\frac{v_t}{h_t}$:[11]**Figure 2.**

As analyzed above, the figure shows that the time series of the standard search and matching Q^N is pro-cyclical, while the preferred specification here is negatively correlated (-0.28) with it and is counter-cyclical. In Section 7 below I show how this fits in with the explanation of labor market experience

[11] I use a calibrated value of c derived as follows: I solve this term out of (16), where $Q_{t,search}^N$ is the sample average value of $\frac{Q_t^N}{\frac{f_t}{n_t}}$(after tax) in the case of Table 2b row 2 and $\frac{v_t}{h_t}$ and τ are set at their sample average values. The vacancy series is defined in Appendix B.

in U.S. data.

6 The Determinants of Capital and Job Values

I have derived, through structural estimation, the costs function (g), from which one can derive the value of the job (i.e., the expected present value of hiring (Q^N)) and the value of capital (i.e., the expected present value investment (Q^K)). How are these values related to their expected future determinants, given that both hiring and investment are forward-looking variables? In other words, what in the future drives hiring and investment today? In this section, I follow the empirical methodology of the asset pricing literature in Finance and examine the present value relationships governing hiring and investment. This involves the use of a forecasting VAR. The analysis is based on the framework proposed by Campbell and Shiller (1988) and its more recent elaboration by Cochrane (2011), whose notation I follow.[12] Note that I do not consider stock prices or any financial data here; rather, I apply the empirical framework developed in the cited Finance literature to the current context. The results in the Finance literature do, however, provide a natural benchmark against which to compare the current results.[13]

6.1 An Asset Pricing Model

The model begins with the following two-period representation for the stock price (P) and dividends (D):

[12] The importance of this approach and its wider significance was noted in the Nobel Economics Prize for 2013 (see in particular pp.17-20 in Nobel Prize (2013)). This model is often referred to as the dynamic, dividend-growth model. Cochrane (2011) provides a discussion of empirical findings and their implications for asset pricing.

[13] See Jermann (1998, 2010).

$$P_t = E_t\left(R_{t+1}^{-1}[D_{t+1} + P_{t+1}]\right)$$

where R is the gross return. Dividing by dividends and iterated forward this yields:

$$\frac{P_t}{D_t} = E_t\left(\sum_{j=0}^{\infty}\left(\prod_{k=1}^{j+1} R_{t+k}^{-1}\frac{D_{t+k}}{D_{t+k-1}}\right)\right) \tag{18}$$

These relationships hold true also ex-post if one defines the gross return as:

$$R_t \equiv \frac{D_{t+1} + P_{t+1}}{P_t} \tag{19}$$

Using logs, this asset pricing relationship can be approximated as:[14]

$$p_t - d_t = k + E_t\left(d_{t+1} - d_t - r_{t+1} + \rho(p_{t+1} - d_{t+1})\right) \tag{20}$$

Equation (20) is an ex-ante formulation using conditional expectations. The ex-post equation, omitting E_t in the above, holds true as well, when using (19).

The current price-dividend ratio $(p_t - d_t))$ is related to future dividend growth $(d_{t+j+1} - d_{t+j})$ and to future returns (r_{t+j+1}), with the relevant discounting (using ρ^j). The price-dividend ratio will be higher when future dividend growth is higher and/or when future returns are lower.

6.2 Implementing the Forecasting Model for Hiring and Investment

I cast the estimated model of hiring and investment into this asset pricing framework by defining P and D for the optimal investment equation and for

[14] where:

$$p_t \equiv \ln P_t,\ d_t = \ln D_t,\ r_t = \ln R_t \quad k = \ln(1 + \frac{P}{D}) - \rho(p - d); \quad \rho = \frac{\frac{P}{D}}{1 + \frac{P}{D}}$$

and where P, D are steady state or long-term average values.

the optimal hiring equation. The "price" P is the value of capital or the value of jobs; this is essentially marginal Q for capital investment (Q^K) and marginal Q for labor hiring (Q^N), each divided by the relevant productivity ($\frac{f}{k}$ or $\frac{f}{n}$); the "dividend" D is the flow of net income from capital or from labor. As shown below, additional terms come into play here. These prices and "dividends" are not observed on the market, as in the Finance literature. Rather, they represent what the firm actually gets from its use of capital and labor in production. Thus, the "dividend" in the investment case is the net marginal productivity of capital; in the hiring case it is net labor profitability, i.e., the net marginal product of labor less the wage. These "dividends" do not depend on institutional or financial considerations of firms as dividends do in the Finance context.

Define, using $G^{f/k}_{t+1} = \frac{\frac{f_{t+1}}{k_{t+1}}}{\frac{f_t}{k_t}}$:

$$P^1_t \equiv (1-\tau_t)\left(\frac{g_{i_t} + p^I_t}{\frac{f_t}{k_t}}\right) = \frac{Q^K_t}{\frac{f_t}{k_t}}; \quad D^1_t = (1-\tau_t)\frac{(f_{k_t} - g_{k_t})}{\frac{f_t}{k_t}}; \quad R^1_t = \frac{G^{f/k}_{t+1}\left[(1-\delta_t)P^1_t + D^1_t\right]}{P^1_{t-1}} \tag{21}$$

Comparing equation (21) to (19), one can see that two additional terms in the current context are the one involving capital depreciation (δ_t) and one involving productivity growth ($G^{f/k}_{t+1}$). Note, too, that D^1_t expresses the share in capital productivity received by the firm, which without taxes and investment costs would be $\frac{f_{k_t}}{\frac{f_t}{k_t}} = 1-\alpha$. The term $G^{f/k}_{t+1}$ captures the gross rate of growth of this productivity.

Appendix E shows that this formulation yields the following log-linear approximation for log capital values:

$$p^1_{t-1} \cong c_3 + \ln G^{f/k}_t + \rho^k \ln(1-\delta_t) + \rho^k p^1_t + (1-\rho^k) d^1_t - r^1_t \tag{22}$$

where small letters denote variables in logs and where $\rho^k = \frac{\frac{(1-\delta)P^1}{D^1}}{1+\frac{(1-\delta)P^1}{D^1}}$.

Below it is shown that the resulting return series, R^1_t, plays a significant role in determining capital values. How can this return series be evaluated?

It turns out to be consistent with the return on capital series computed by McGrattan and Prescott (2003) and by Gomme, Ravikumar and Rupert (2011) for the U.S., using NIPA data. Note, though, that the three series are not the same as they treat taxes differently, the McGrattan Prescott is annual and uses the non-corporate sector, and the current one features (inter alia) marginal investment costs g_{i_t}, which are absent in the other series. The following tables summarize the key moments of these series. **Tables 8a and 8b.**

The three series are quite close in terms of means and medians and the skewness statistics. The McGrattan and Prescott series and the R_t^1 series have similar kurtosis as well. The series differ on second moments, with the McGrattan and Prescott series the least volatile and the R_t^1 series the most volatile. The latter is probably due to the role of g_i, which is absent in the other two series. The series are positively correlated with each other. The strongest correlation is between R_t^1 and the McGrattan and Prescott series (0.56 in 1976-2000 annual data).

For labor, define, using $G_{t+1}^{f/n} = \frac{\frac{f_{t+1}}{n_{t+1}}}{\frac{f_t}{n_t}}$:

$$P_t^2 \equiv \frac{(1-\tau_t)\, g_{h_t}}{\frac{f_t}{n_t}} \equiv \frac{Q_t^N}{\frac{f_t}{n_t}}; \quad D_t^2 = (1-\tau_t)\left(\alpha - \frac{g_{n_t}}{\frac{f_t}{n_t}} - \frac{w_t}{\frac{f_t}{n_t}}\right); \quad R_t^2 = \frac{G_{t+1}^{f/n}\left[(1-\psi_t)P_t^2 + D_t^2\right]}{P_{t-1}^2} \tag{23}$$

$$D_t^2 = D_t^{2,1} - D_t^{2,2}; \quad D_t^{2,1} = (1-\tau_t)\left(\alpha - \frac{g_{n_t}}{\frac{f_t}{n_t}}\right); \quad D_t^{2,2} = (1-\tau_t)\frac{w_t}{\frac{f_t}{n_t}}$$

Note that D_t^2 expresses the share in labor productivity received by the firm, which, without taxes, hiring costs and wages would be equal to α. Dividends are the actual receipts or profits from labor, once taxes, costs and wages have been deducted. The term $G_{t+1}^{f/n}$ captures the gross rate of growth of labor productivity. I further decompose D_t^2 into the share of the firm in net, after-tax productivity ($D_t^{2,1}$) and the share of wages in productivity, paid to workers ($D_t^{2,2}$). Appendix E shows that this yields the following log-linear approximation of job values:

$$\begin{aligned}
p_{t-1}^2 &= c_8 + \ln G_t^{f/n} + \rho^{n2}\ln(1-\psi_t) + \rho^{n2} p_t^2 \\
&\quad + d_t^{2,1}(1-\rho^{n1})(1-\rho^{n2}) + d_t^{2,2}(\rho^{n1}(1-\rho^{n2})) - r_t^2 \\
\rho^{n1} &= \frac{-\frac{D^{2,2}}{D^{2,1}}}{1-\frac{D^{2,2}}{D^{2,1}}}, \quad \rho^{n2} = \frac{\frac{(1-\psi)P^2}{D^2}}{1+\frac{(1-\psi)P^2}{D^2}}
\end{aligned} \tag{24}$$

6.3 Empirical Methodology

I use a restricted VAR to examine these relationships. Consider, first, the log-linear pricing equations in the non-stochastic steady state. These are given by:

$$p^1 \cong \frac{c_3}{1-\rho^k} + \frac{\ln G^{f/k}}{1-\rho^k} + \frac{\rho^k}{1-\rho^k}\ln(1-\delta) + d^1 - \frac{r^1}{1-\rho^k}$$

$$p^2 \simeq \frac{c_8}{1-\rho^{n2}} + \frac{\ln G^{f/n}}{1-\rho^{n2}} + \frac{\rho^{n2}}{1-\rho^{n2}}\ln(1-\psi) + d^{2,1}(1-\rho^{n1}) + d^{2,2}\rho^{n1} - \frac{r^2}{1-\rho^{n2}}$$

These equations state that, in the non-stochastic steady state, the value of capital (p^1) and of jobs (p^2) can each be decomposed (using log-linear approximation) into parts due to dividends (d) or shares in net productivity, returns (r), productivity growth ($\ln G^{f/k}$ or $\ln G^{f/n}$) and deprecation (δ) or separation (ψ).

Thus I estimate the following structural VAR:

$$\mathbf{x}_{t+1} = A + B\mathbf{x}_t + \varepsilon_t \tag{25}$$

where $\mathbf{x}_{t+1} = (p_{t+1}^1, d_{t+1}^1, r_{t+1}^1, \ln\left(G_{t+1}^{f/k}\right), \ln(1-\delta_{t+1}))$ for capital, $\mathbf{x}_{t+1} = (p_{t+1}^2, d_{t+1}^{2,1}, d_{t+1}^{2,2}, r_{t+1}^2, \ln\left(G_{t+1}^{f/n}\right), \ln(1-\psi_{t+1}))$ for labor, under the restrictions implied by the above steady state equations. Following estimation I compute the relevant long run coefficients (see Appendix E).

6.4 VAR Results

Table 9 reports the results of the VAR for selected coefficients in the B matrix and the implied long run coefficients. **Table 9.**

For investment, the most substantial role is played by returns (a long run coefficient of -1.05), while the other determinants have much smaller effects. Among the latter, productivity growth has a somewhat stronger effect but it is imprecisely estimated. The adjusted R^2 of the return regression (r^1 on the lagged values of all the other variables) is not high, though at 0.11 it is basically the same as in the results reported in the Finance literature for return regressions using stock prices.

For hiring, the most substantial role is again played by returns (a long run coefficient of -0.90), while the other determinants have smaller effects. Productivity (the d_{21} term) has a substantial effect ($b^{lr}_{d21\_p2} = 0.18$) but it is imprecisely estimated. The adjusted R^2 of the regressions, but for the productivity growth regression, are high, including the return regression and the productivity level regression.

Repeating the analysis for the alternative estimates of row 3 in Table 2a yields very similar findings.

What, then, do we learn about the various future determinants of investment and hiring values?

First, *returns* play the dominant role, as also found in the empirical Finance literature. Their VAR coefficients (b_{r1_p1} and b_{r2_p2}) are precisely estimated and the implied long run coefficients are sizeable. The adjusted R^2 in the investment case of the return regression (0.11) resembles that of regressions in Finance while for hiring it is even much bigger (0.66). Note that these coefficients are negative, implying that a rise in log prices is associated with future declines in returns (r), for both investment and hiring, i.e., high prices predict low subsequent returns, as found in the Finance literature. A similar result is obtained when computing the relation between the log price-dividend ratio $(p-d)$ of investment and of hiring with their subsequent returns. This result has also been observed for stock prices and dividends and for house prices and rents (see Cochrane (2011, pp. 1051-1052)).

Second, *dividends* play a role in the hiring case, although smaller than returns. In this case, higher prices are associated with subsequent higher dividends and the adjusted R^2 is very high (0.95). The analysis indicates that if wages do not move closely with labor productivity there is a meaningful effect to productivity changes, in line with the "Shimer puzzle" findings.

Third, *productivity growth*, does not appear to play a role in both cases: the VAR coefficients (b_{gk_p1} and b_{gn_p2}) are not significantly different from zero and the long run coefficients are small. This is akin to the finding in Finance that dividend growth does not matter much.

Fourth, *prices* – the values of investment and hiring – are persistent (as measured by ϕ_1 and ϕ_2), which is consistent with the persistence of the investment and hiring rates themselves.

Fifth, *the rates of separation and depreciation* do not appear to play a meaningful role. This means that the variable that determines the length of the hire (ψ determines job duration) does not have much effect on the value of the hire, relative to the other determinants. It is the discounting of future streams which plays the overwhelming role.

7 U.S. Labor Market Experience

In this section I embed the afore-going set-up in a matching framework which facilitates the analysis of unemployment, including the recent Great Recession experience. The essential idea is to incorporate the firms' F.O.C into a Pissarides-style model of vacancies and unemployment with a matching function and relate the model's steady state formulation to U.S. data. Then U.S. experience is analyzed. This exercise does not entail estimation or calibration in the full sense of these methodologies. Rather, it uses the estimates of Table 2 to embed the hiring F.O.C. in a wider framework, albeit still a partial equilibrium one. Then, by calibrating key parameters and using data averages, the steady state of this framework is derived and compared to actual data using graphical analysis. This allows one to see how movements in the data over three sub-periods can be approximated by movements in the steady state curves over the same sub-periods. The changes in unemployment and vacancies/hiring over time can be understood in terms of changes in variables that were discussed above, in particular in terms of job values.

7.1 Incorporating the Analysis in a Matching Framework

Following Pissarides (2000) a matching function defines the hiring rate $\frac{h_t}{n_t}$ as a CRS function of the unemployment rate $\frac{u_t}{n_t}$ and the vacancy rate $\frac{v_t}{n_t}$. Specifically I shall use the following Cobb-Douglas form:[15]

$$\frac{h_t}{n_t} = \mu_t \left(\frac{u_t}{n_t}\right)^{\sigma} \left(\frac{v_t}{n_t}\right)^{1-\sigma} \tag{26}$$

Consider now a modification of the hiring costs function used above to accommodate vacancies. The cost function is now:

$$g(\cdot) = \left[\frac{e_1}{\eta_1}(\frac{i_t}{k_t})^{\eta_1} + \frac{e_2}{\eta_2}(\lambda\frac{v_t}{n_t} + (1-\lambda)\frac{q_t v_t}{n_t})^{\eta_2} + \frac{e_3}{\eta_3}\left(\frac{i_t}{k_t}\frac{q_t v_t}{n_t}\right)^{\eta_3}\right] f(z_t, n_t, k_t). \tag{27}$$

The modification is that now some costs relate to the vacancy rate $\frac{v_t}{n_t}$, with a share λ, and the hiring rate $\frac{h_t}{n_t} = \frac{q_t v_t}{n_t}$ enters with the complementary share $1-\lambda$.

In this set-up the firm decides on investment i and on vacancies v so the two FOC are given by, using steady state formulations:

[15] Hence the firm matching rate is given by $q_t = \frac{h_t}{v_t} = \mu_t \left(\frac{v_t}{u_t}\right)^{-\sigma}$

$$(1-\tau)\left(\frac{p^I}{\frac{f}{k}}+\frac{g_i(\frac{qv}{n},\frac{i}{k})}{\frac{f}{k}}\right)=\frac{Q^K}{\frac{f}{k}} \tag{28}$$

$$(1-\tau)\frac{g_v(\frac{v}{n},q,\frac{i}{k})}{\frac{f}{n}}=\frac{Q^N}{\frac{f}{n}} \tag{29}$$

Making use of the above equations steady state equilibrium can be presented as a plot in $\frac{u}{n}$ and $\frac{v}{n}$ space as follows, noting that $q=\frac{\mu\left(\frac{v}{n}\right)^{1-\sigma}\left(\frac{u}{n}\right)^{\sigma}}{v/n}$:

$$\left[\begin{array}{c} e_2(\lambda\frac{v}{n}+(1-\lambda)\left(\frac{\mu\left(\frac{v}{n}\right)^{1-\sigma}\left(\frac{u}{n}\right)^{\sigma}}{\frac{v}{n}}\right)\frac{v}{n})\left(\lambda+(1-\lambda)\left(\frac{\mu\left(\frac{v}{n}\right)^{1-\sigma}\left(\frac{u}{n}\right)^{\sigma}}{\frac{v}{n}}\right)\right) \\ +e_3\left(\frac{i}{k}\right)\left(\frac{\mu\left(\frac{v}{n}\right)^{1-\sigma}\left(\frac{u}{n}\right)^{\sigma}}{\frac{v}{n}}\right) \end{array}\right]=\frac{1}{(1-\tau)}\frac{Q^N}{\frac{f}{n}} \tag{30}$$

$$\mu\left(\frac{v}{n}\right)^{1-\sigma}\left(\frac{u}{n}\right)^{\sigma}=\psi+g \tag{31}$$

where the last equation makes use of the fact that in steady state the flows from and to unemployment are equal and where g is the rate of growth of the labor force $(n+u)$. Using (30), the vacancy creation curve, and (31), the steady state flows curve, one solves for $\frac{u}{n}$ and $\frac{v}{n}$ given $\frac{1}{(1-\tau)}\frac{Q^N}{\frac{f}{n}}, \frac{i}{k}, \psi, g,\ \mu$ and the parameter values λ and σ.

7.2 Relating the Matching Model to U.S. Data

The idea is to relate the steady state relationships (30) and (31) to the actual data. The aim is to find a region in $\frac{u}{n}-\frac{v}{n}$ space where these equations are a reasonable approximation of the steady state around which the data points are scattered. Hence this is a “stylized exercise” which needs to be understood as such.

In order to do so one needs to use the relevant unemployment pool u. The hiring series used here includes worker flows to employment from both the out of the labor force pool and the official unemployment pool. In what follows I present three alternative formulations for u: in one it is the official unemployment pool; in a second, it is the official unemployment pool plus marginally attached workers; and in a third it is the official unemployment pool plus workers who "want a job." Using these variables, and a vacancy series, Figure 3 plots the data and the model steady state equations (30) and (31) in $\frac{u}{n} - \frac{v}{n}$ space for official unemployment while Appendix F, which elaborates on the data and the procedure, does the same for the other two formulations of unemployment.

The figure shows actual U.S. data points of $\frac{u}{n}$ and $\frac{v}{n}$ as well as the curves implied by the two steady state equations in three sub-periods: 1976-1991; 1992 (or 1994) - 2006; 2007-2011.[16] Table 10 presents average sample values of all relevant variables in the three sub-periods using official unemployment. Appendix F does the same for the other two formulations of unemployment.
Figure 3 and Table 10.

The data points are fairly well distributed around the steady state curves. By construction the intersection of the curves lies at the relevant sample average values. It turns out that the three alternative unemployment pools yield the same qualitative conclusions. The figure and the table suggest the following interpretation of U.S. labor market developments:

(i) Both curves shifted down going to the 1990s, thereby lowering unemployment and vacancies.

(ii) With the Great Recession, both curves shifted up in a way that the unemployment rate increased considerably while the vacancy rate fell somewhat.

What movements in variables generated these changes? The emerging partial equilibrium "story" is as follows:

(i) Going from the 1976-1991 sub-period to the 1992/4-2006 sub-period, both curves shift down. For the vacancy creation curve this is due to the

[16] The "marginally attached" and "want a job" worker series are available only from 1994. Appendix F discusses the reasons underlying the choice of the three sub-periods.

decline in job values $\frac{1}{1-\tau}\frac{Q^N}{\frac{f}{n}}$ and to the increase in the investment rate $\frac{i}{k}$. For the steady state flows curve this is due to the decline in the separation rate ψ and in the labor force growth rate g. Equilibrium unemployment and vacancy rates both decline, as do hiring and separation rates.

(ii) Going from 1992/4-2006 to 2007- 2011 both curves shift up. For the vacancy creation curve this is due to a rise in job values $\frac{1}{1-\tau}\frac{Q^N}{\frac{f}{n}}$ and a decline in investment $\frac{i}{k}$. For the steady state flows curve this is due to the rise in the separation rate ψ, and despite a further decline in the labor force growth rate g. Equilibrium unemployment rises (as do hiring and separation rates) while vacancy rates fall. The intuition is clear: the higher the job value, the higher is vacancy creation, and the curve for the latter moves up. The less intuitive aspect is the rise in job values in recession. This aspect was documented and explained around Figure 2 in Section 5 above.

How would these $\frac{u}{n} - \frac{v}{n}$ developments look like in the typical search and matching model? Figure 4 shows the three sub-periods changes in a prototypical Pissarides (2000) model which may be compared to Figure 3 of the current model. **Figure 4.**

Essentially the curve of the steady state flows equation (31) remains the same across models and it therefore moves identically over the three sub-periods across models. But the equation for vacancy creation is now equation (16), as discussed above, and it replaces equation (30)). It is depicted as a straight, positively sloped line. Re-writing (16) in the steady state, using the same matching function, it is given by:

$$Q^N_{search} = (1-\tau)c\mu\left(\frac{v}{u}\right)^{\sigma} \tag{32}$$

Going from the 1976-1991 sub-period to the 1992/4-2006 sub-period it hardly moves in the Pissarides (2000) framework. As the $\frac{v}{u}$ ratio in the data hardly changes, this lack of movement means that unless there are substantial changes in τ, c or μ then Q^N_{search} is little changed. The data inform us that the corporate tax τ is little changed and Figure 2 (in sub-section 5.4.4 above) indicates that on average Q^N_{search} is indeed little changed across these sub-periods. Hence the typical search and matching model basically attributes the changes in this time frame to the declines in the separation rate ψ and in the labor force growth rate g, moving the steady state flows curve downwards along a mostly unchanged vacancy creation curve. Were it not for this latter movement, unemployment and vacancies would be little changed in the Pissarides model. In contrast, the current model would predict a big decline in unemployment and a big rise in vacancies were the steady state flows curve unchanged. In terms of Figure 3 consider the case whereby the dashed steady state flows curve does not move but the solid vacancy creation curve moves down. As in sub-section 5.4.4 above, the two models tell different "stories" about job values and their effects.

Going from the sub-period 1992/4-2006 to 2007-2011, including the Great Recession, the interpretations differ again and once more job values are key. In the current model the vacancy creation curve underlying (30) shifts up as explained above, implying higher vacancy creation for a given rate of unemployment. In the Pissarides (2000) framework the vacancy creation curve underlying (32) moves down, implying lower vacancy creation for a given rate of unemployment. This implies that in the Pissarides model the job value Q^N_{search} has gone down, while in the current model the job value has gone up; both of these movements in job values may be seen in Figure 2 of Section 5 above. Hence, while both models account for the developments in u and v they attribute different reasons to the changes that took place.

7.3 The Determinants of U.S. Unemployment and Vacancies

In order to determine the specific role played by the different variables which shift equations (30) and (31) in $\frac{u}{n} - \frac{v}{n}$ space, namely $\frac{1}{1-\tau}\frac{Q^N}{\frac{f}{n}}, \frac{i}{k}, \psi, g$ and μ,

Table 10 and Appendix F offer a comparison between the actual, total change across sub-periods and counter-factual changes induced when one variable only changes at any one time. In what follows, note that each variable has its own effect on $\frac{u}{n}$ and $\frac{v}{n}$; sometimes the effect is dominated by the effects of other variables.

(i) The job value $\frac{1}{1-\tau}\frac{Q^N}{\frac{f}{n}}$ went down from the 1976-1991 sub-period to the 1992-2006 sub-period thereby contributing to the fall in unemployment. But this actually ran counter to the fall in vacancy rates. Going from the sub-period 1992/4-2006 to 2007- 2011 it rose, contributing to both the rise in unemployment in the Great Recession and the continued fall in the vacancy rate.

(ii) The role of the investment rate $\frac{i}{k}$ turns out not to be dominant. Its rise operates to induce lower vacancies and higher unemployment and its fall is supposed to induce the opposite. However it only contributed to the fall in vacancy rates going from the 1970s and 1980s to the 1990s and 2000s but it failed to influence the other changes.

(iii) The roles of the flow rates – separation ψ and labor force growth $g-$ can be summed up as follows: first, going down from the 1976-1991 sub-period to the 1992-2006 sub-period they contributed to the fall in the vacancy rate but did not bring about a rise in unemployment. Second, going to the Great Recession period, ψ rose and g fell. The latter contributed to the continued fall in the vacancy rate and the rise in the unemployment rate but the effects of ψ operated in the other direction and did not prevail.

Overall, the changes in job values $\frac{1}{1-\tau}\frac{Q^N}{\frac{f}{n}}$ and in the labor force growth rate g played the dominant role. In particular, with job values rising and labor force growth falling ahead of the Great Recession, they engendered the fall in the vacancy rate and the big rise in unemployment, i.e., the shift out of the Beveridge curve.[17]

[17] The Beveridge curve is the term used for the empirical relationship between v and u. In the search and matching literature, this term is often used to designate the steady

Another lesson from this analysis is that implied matching efficiency (μ) first rises and then falls over the sample period (see Table 10 and Appendix F). The matching efficiency parameter is solved out of equation (31) in each sub-period. In particular, the Great Recession period is characterized by lower matching efficiency or by higher mismatch. The analysis above, which includes the relevant steady-state value of μ in each sub-period, incorporates these matching efficiency changes. It thus shows movements of the relevant curves after already taking into account matching efficiency changes.

8 Conclusions

The key notions in this paper are the forward-looking aspect of investment and hiring and their joint determination. More specifically, the results indicate three sets of key implications:

One is the complementarity between hiring and investment, with the hiring rate heavily influenced by the present value of investment, while the rate of investment is less influenced by the present value of hiring. A second is that in the sample period, U.S. investment rates and their present value (the value of capital) are pro-cyclical while hiring and job values are counter-cyclical. Estimated job values here were shown to differ from those derived from the standard search and matching model. The main determinant of these capital values and job values are future returns, in line with what has been found in the Finance literature for asset prices. The third is that U.S. labor market experience, including the Great Recession, can be depicted in a stylized way using the estimated model. Going from the 1970s and 1980s to the 1990s and 2000s, job values declined as did labor force growth rates. Hence there ensued a decline in vacancy and hiring rates, and, concurrently, in unemployment rates. Moving from the last period to the Great Recession,

state flows equation, given by (31).

job values went up while labor force growth rates continued to decline, leading to a rise in unemployment and a decline in vacancy rates.

The particular role of job values (Q^N) merits emphasis. It was shown to be different from the standard search and matching value (see Figure 2 and the discussion in sub-section 5.4.4); it exhibited counter-cyclical behavior over the sample period, rising in recessions (see Figures 1d and 2); and it was dominant in the stylized explanation of unemployment changes – both the fall, going into the 1990s and 2000s, and the subsequent rise in the Great Recession of 2007-2009.

This paper, intentionally, did not specify a full DSGE model. This was done in order to focus on firms' investment and hiring decisions and not let the analysis be affected by possible mis-specifications or problematics in other parts of the macroeconomy. To account for firm investment and hiring behavior, one does not need to get into issues such as optimal intertemporal consumption and labor choices of the individual, with all the associated empirical difficulties. Future research may, nonetheless, take up such a model in an attempt to map the linkages between the structural shocks to the economy and the differential evolution of the relevant present values.

References

[1] Alexopoulos, Michelle, 2011. "Read All About It! What Happens Following a Technology Shock," **American Economic Review** 101, 1144-1179.

[2] Alexopoulos, Michelle and Trevor Tombe, 2012. "Management Matters," **Journal of Monetary Economics** 59, 269-285.

[3] Campbell, John .Y., and Robert J. Shiller, 1988. "The Dividend–Price Ratio and Expectations of Future Dividends and Discount Factors," **Review of Financial Studies** 1, 195–227.

[4] Chirinko, Robert S. 1993. "Business Fixed Investment Spending: Modeling Strategies, Empirical Results, and Policy Implications," **Journal of Economic Literature** XXXI:1875-1911.

[5] Christiano, Lawrence J., Mathias Trabandt and Karl Walentin, 2010. "DSGE Models for Monetary Policy Analysis," Chapter 7 in B. Friedman and M. Woodford (eds.) **Handbook of Monetary Economics,vol. 3,** North Holland, Amsterdam.

[6] Cochrane, John H. 2011. "Discount Rates," **Journal of Finance** 66, 1047-1108.

[7] Coles Melvyn G., and Dale T. Mortensen, 2013a. "Equilibrium Labor Turnover, Firm Growth and Unemployment," NBER WP no. 18022.

[8] Coles Melvyn G., and Dale T. Mortensen, 2013b."Search-on-the-Job and Aggregate Employment and Wage Fluctuations," working paper.

[9] Favilukis, Jack and Xiaoji Lin, 2011. "Micro Frictions, Asset Pricing and Aggregate Implications," LSE mimeo.

[10] Gali, Jordi, and Thijs van Rens, 2014. "The Vanishing Procyclicality of Labor Productivity," CREI working paper.

[11] Garibaldi, Pietro and Espen R. Moen, 2009. "Industry Dynamics and Search in the Labor Market," working paper.

[12] Gomme, Paul, B. Ravikumar and Peter Rupert, 2011. "The Return to Capital and the Business Cycle," **Review of Economic Dynamics** 14, 262-278.

[13] Hall, Robert E. 2004. "Measuring Factor Adjustment Costs," **Quarterly Journal of Economics,** August, 119(3), 899-927.

[14] Hall, Robert E., 2007. " How Much Do We Understand about the Modern Recession?" **Brookings Papers on Economic Activity** 2, 13-28.

[15] Hall, Robert E., 2014. "High Discounts and High Unemployment," available at stanford.edu/ rehall.

[16] Hansen, Lars P. 1982. "Large Sample Properties of Generalized Method of Moments Estimators," **Econometrica** 50:1029-1054.

[17] Hayashi, Fumio 1982. "Tobin's Marginal q and Average q: A Neoclassical Interpretation," **Econometrica** 50:213-224.

[18] Jermann, Urban J., 1998. "Asset Pricing in Production Economies," **Journal of Monetary Economics** 41, 257–275.

[19] Jermann, Urban J., 2010. "The Equity Premium Implied by Production," **Journal of Financial Economics** 98, 279–296.

[20] Kahn, Aubhik and Julia K. Thomas, 2008. "Idiosyncratic Shocks and the Role of Nonconvexities in Plant and Aggregate Investment Dynamics," **Econometrica** 76 (2), 395–436.

[21] Lucas, Robert E. and Edward C. Prescott, 1971. "Investment Under Uncertainty," **Econometrica** 39, 5, 659-681.

[22] McGrattan , Ellen and Edward C. Prescott, 2003. "Average Debt and Equity Returns: Puzzling?" **AER Papers and Proceedings,** 93, 2, 392-397.

[23] Merz, Monika and Eran Yashiv, 2007. "Labor and the Market Value of the Firm," **American Economic Review,** 97, 4, 1419-1431.

[24] Mortensen, Dale T., 1973. "Generalized Costs of Adjustment and Dynamic Factor Demand Theory," **Econometrica** 41, 4, 657-665.

[25] Nadiri, M. Ishag and Sherwin Rosen, 1969. "Interrelated Factor Demand Functions," **American Economic Review** 59, 4, Part 1, 457-471.

[26] Nobel Prize, 2013. "Understanding Asset Prices," **the Sveriges Riksbank Prize in Economic Sciences** 2013.

[27] Pissarides, Christopher A. 2000. **Equilibrium Unemployment Theory,** 2nd edition, Cambridge, MIT Press.

[28] Rios-Rull, Jose-Victor and Raul Santaeulalia-Llopis, 2010. "Redistributive Shocks and Productivity Shocks,"**Journal of Monetary Economics** 57, 931–948.

[29] Rogerson, Richard and Robert Shimer, 2011. "Search in Macroeconomic Models of the Labor Market," in O.Ashenfelter and D.Card (eds.) **Handbook of Labor Economics Vol 4A,** Amsterdam, Elsevier – North Holland.

[30] Shapiro, Matthew .D. 1986. "The Dynamic Demand for Capital and Labor," **The Quarterly Journal of Economics** 101:513-542.

[31] Smith, Gary, 2008. "Tobin's q " in Steven N. Durlauf and Lawrence E. Blume (eds.) **The New Palgrave Dictionary of Economics** Second Edition, Palgrave Macmillan.

[32] Thomas, Julia K., 2002. "Is Lumpy Investment Relevant for the Busines Cycle?" **Journal of Political Economy** 110, 3, 508-534.

[33] Tobin, James 1969. "A General Equilibrium Approach to Monetary Theory," **Journal of Money, Credit, and Banking** 1:15-29.

[34] Yashiv, Eran 2007. "Labor Search and Matching in Macroeconomics," **European Economic Review** 51, 8, 1859-1895,

Table 1

Descriptive Sample Statistics
Quarterly, U.S. data 1976-2011

Variable	$\frac{f}{k}$	τ	$\frac{i}{k}$	δ	$\frac{wn}{f}$	$\frac{h}{n}$	ψ	β
Mean	0.153	0.380	0.022	0.015	0.652	0.132	0.131	0.994
Standard Deviation	0.013	0.053	0.003	0.003	0.017	0.012	0.012	0.004

Table 2a

GMM estimates

	e_1	e_2	e_3	η_1	η_2	η_3	f_1	f_2	α	J
1	$57,166$	20.5	-98.6	3.16	1.93	1.00	-0.98	0.005	0.67	80.5
	$(94,598)$	(18.2)	(57.7)	(0.34)	(0.54)	(0.05)	(22.8)	(0.15)	(0.05)	(0.009)
2	$54,299$	7.9	-73.5	3.25	1.71	1.00	0	0	0.67	88.0
	$(36,173)$	(1.7)	(9.2)	(0.11)	(0.16)	(0.01)	–	–	–	(0.004)
3	1585	2.0	-3.9	3	2	1	0	0	0.67	75.1
	(328)	(0.3)	(1.3)	–	–	–	–	–	–	(0.08)
4	76	1.8	-6.9	2	2	1	0	0	0.67	75.1
	(12)	(0.3)	(1.4)	–	–	–	–	–	–	(0.08)

Table 2b

GMM estimates, Standard Specifications

	e$_1$	**e**$_2$	**e**$_3$	**J-Statistic**	**fixed parameters**
1	107	0	0	77.4	$\eta_1 = 2$
	(4)	–	–	(0.08)	
2	0	0.16	0	75.7	$\eta_2 = 1$
	–	(0.01)	–	(0.10)	
3	64	0.84	0	76.3	$\eta_1 = 2, \eta_2 = 2$
	(10)	(0.26)	–	(0.08)	

Notes:

1. The tables report point estimates with standard errors in parantheses. The J-statistic is reported with p value in parantheses.
2. The instrument set is $\frac{h}{n}, \frac{w}{n/f}, \frac{i}{k}$ with 10 lags.
3. When fixed, α is set at 0.67.

Table 3a
Costs Implied by the GMM Estimation Results

	specification	$\frac{g}{f}$		$\frac{g_i}{\frac{f}{k}}$		$\frac{g_h}{\frac{f}{n}}$	
		mean	std.	mean	std.	mean	std.
1	all free	0.026	0.015	1.05	5.67	0.97	0.50
2	partially constrained	0.011	0.009	0.67	4.07	0.29	0.32
3	$\eta_1 = 3, \eta_2 = 2, \eta_3 = 1$	0.012	0.002	0.25	0.27	0.18	0.03
4	$\eta_1 = \eta_2 = 2, \eta_3 = 1$	0.014	0.002	0.75	0.30	0.08	0.04

Notes:
1. Mean and std. refer to sample statistics.
2. The functions were computed using the point estimates in Table 2a.

Table 3b
Costs Implied by the GMM Estimation Results
Standard Specifications

	specification	$\frac{g}{f}$		$\frac{g_i}{\frac{f}{k}}$		$\frac{g_h}{\frac{f}{n}}$	
		mean	std.	mean	std.	mean	std.
1	$\eta_1 = 2, e_2 = e_3 = 0$	0.03	0.008	2.33	0.36	–	
2	$e_1 = e_3 = 0$	0.02	0.002	–	–	0.16	–
3	$e_3 = 0$	0.02	0.004	1.39	0.21	0.11	0.01

Notes:
1. Mean and std. refer to sample statistics.
2. The functions were computed using the point estimates in Table 2b.

Table 4
Decomposition of the Hiring Rate and Investment Rate Equations

First Two Moments

a. Hiring Equation

$$\frac{h_t}{n_t} = \frac{1}{(e_1 e_2 - e_3^2)} \left(e_1 \frac{g_{h_t}}{\frac{f_t}{n_t}} - e_3 \frac{g_{i_t}}{\frac{f_t}{k_t}} \right)$$

		$\frac{h_t}{n_t}$		1 $\left(\frac{e_1}{e_1 e_2 - e_3^2}\right) \frac{g_{h_t}}{\frac{f_t}{n_t}}$	2 $\left(\frac{-e_3}{e_1 e_2 - e_3^2}\right) \frac{g_{i_t}}{\frac{f_t}{k_t}}$
1	mean	0.13	relative mean	0.58	0.42
2	std	0.01	relative var	7.9	3.9
3			relative cova	−5.38	

b. Investment Equation

$$\frac{i_t}{k_t} = \frac{1}{(e_1 e_2 - e_3^2)} \left(-e_3 \frac{g_{h_t}}{\frac{f_t}{n_t}} + e_2 \frac{g_{i_t}}{\frac{f_t}{k_t}} \right)$$

		$\frac{i_t}{k_t}$		1 $\left(\frac{-e_3}{e_1 e_2 - e_3^2}\right) \frac{g_{h_t}}{\frac{f_t}{n_t}}$	2 $\left(\frac{e_2}{e_1 e_2 - e_3^2}\right) \frac{g_{i_t}}{\frac{f_t}{k_t}}$
1	mean	0.02	relative mean	0.32	0.68
2	std	0.003	relative var	0.85	3.50
3			relative cova	−1.67	

Notes:

1. The equations include the following terms:

$$\frac{g_{h_t}}{\frac{f_t}{n_t}} = \left[e_2 (\frac{h_t}{n_t})^{\eta_{2-1}} + e_3 \left(\frac{i_t}{k_t}\right)^{\eta_3} \frac{h_t}{n_t}^{\eta_3 - 1} \right]$$

$$\frac{g_{i_t}}{\frac{f_t}{k_t}} = \left[e_1 (\frac{i_t}{k_t})^{\eta_1 - 1} + e_3 \left(\frac{h_t}{n_t}\right)^{\eta_3} \frac{i_t}{k_t}^{\eta_3 - 1} \right]$$

2. Row 1 reports the mean hiring or investment rate and the relative means of the two decomposition terms indicated in columns 1 and 2.

3. Row 2 reports the std. of the hiring or investment rate and the relative variances of the two decomposition terms indicated in columns 1 and 2.

4. Row 3 reports the relative co-variance of the two decomposition terms indicated in columns 1 and 2.

5. All results are based on the point estimates of row 4 in Table 2a.

Table 5

Scope and Elasticities Implied by the GMM Estimation Results

	specification	**scope**	$\frac{\partial i_t}{\partial Q^K} \frac{Q^K}{i_t}$	$\frac{\partial i_t}{\partial Q^N} \frac{Q^N}{i_t}$	$\frac{\partial h_t}{\partial Q^k} \frac{Q^K}{h_t}$	$\frac{\partial h_t}{\partial Q^N} \frac{Q^N}{h_t}$
Table 2b row 3	both, no interaction	0	11.1	–	–	1
			(2.6)			–
Table 2a row 4	preferred	1.36	13.7	0.35	8.32	0.56
		(0.08)	(3.2)	(0.18)	(0.51)	(0.21)

Notes:

1. All computations are based on the point estimates of Table 2a and 2b.

2. The scope statistic is defined as

$$\frac{g(0, \frac{h}{n}) + g(\frac{i}{k}, 0) - g(\frac{i}{k}, \frac{h}{n})}{g(\frac{i}{k}, \frac{h}{n})}$$

3. The elasticities are derived in Appendix A.

Table 6

Stochastic Behavior of Hiring and Investment

a. The Raw Series – Data Moments

	$\frac{i}{k}$	$\frac{h}{n}$
mean	0.02	0.13
median	0.02	0.13
std.	0.003	0.010
coefficient of variation	0.15	0.08
auto-correlation	0.98	0.93
correlation	−0.58	

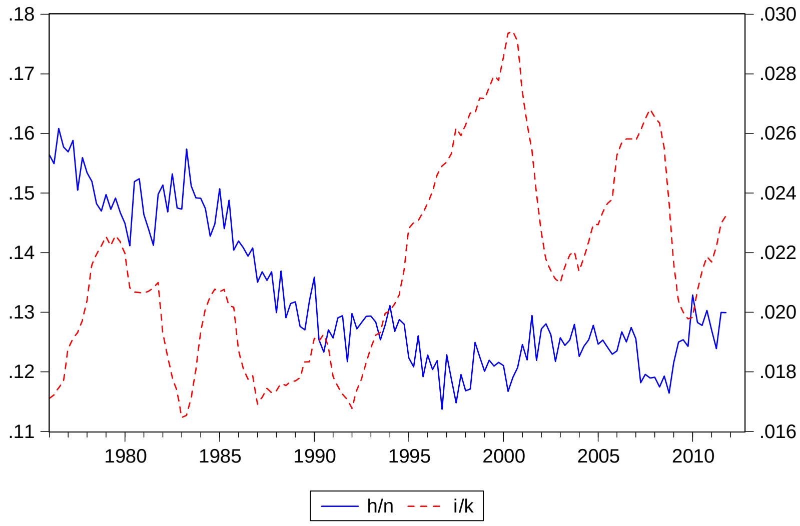


Figure 1a: Hiring $\frac{h}{n}$ (left axis) and investment $\frac{i}{k}$ (right axis), raw data

b. Cyclicality

Hiring $\rho(\frac{h_t}{n_t}, y_{t+i})$

HP filtered ($\lambda = 1600$)

lag/lead	-8	-4	-1	0	1	4	8
f	-0.15	-0.30	-0.34	-0.25	-0.12	0.17	0.20
$\frac{f}{n}$	-0.13	-0.20	-0.11	-0.04	0.05	0.21	0.09
$\frac{f}{k}$	-0.18	-0.31	-0.30	-0.19	-0.07	0.22	0.19

BK filtered (Baxter-King, 6-32)

lag/lead	-8	-4	-1	0	1	4	8
f	-0.23	-0.34	-0.45	-0.36	-0.24	0.11	0.13
$\frac{f}{n}$	-0.09	-0.19	-0.20	-0.08	0.01	0.17	0.03
$\frac{f}{k}$	-0.29	-0.35	-0.40	-0.29	-0.17	0.17	0.13

Investment $\rho(\frac{i_t}{k_t}, y_{t+i})$

HP filtered ($\lambda = 1600$)

lag/lead	-8	-4	-1	0	1	4	8
f	0.10	0.50	0.84	0.79	0.63	-0.03	-0.40
$\frac{f}{n}$	0.10	0.62	0.63	0.50	0.29	-0.34	-0.44
$\frac{f}{k}$	-0.06	0.60	0.84	0.75	0.55	-0.17	-0.49

BK filtered (Baxter-King, 6-32)

lag/lead	-8	-4	-1	0	1	4	8
f	-0.12	0.51	0.84	0.79	0.62	0.00	-0.27
$\frac{f}{n}$	0.12	0.49	0.63	0.49	0.27	-0.28	-0.37
$\frac{f}{k}$	0.01	0.62	0.84	0.73	0.51	-0.16	-0.39

Notes:

1. The variable y denotes the cyclical indicator which is f (NFCB GDP), or $\frac{f}{n}$ (labor productivity), or $\frac{f}{k}$ (capital productivity).

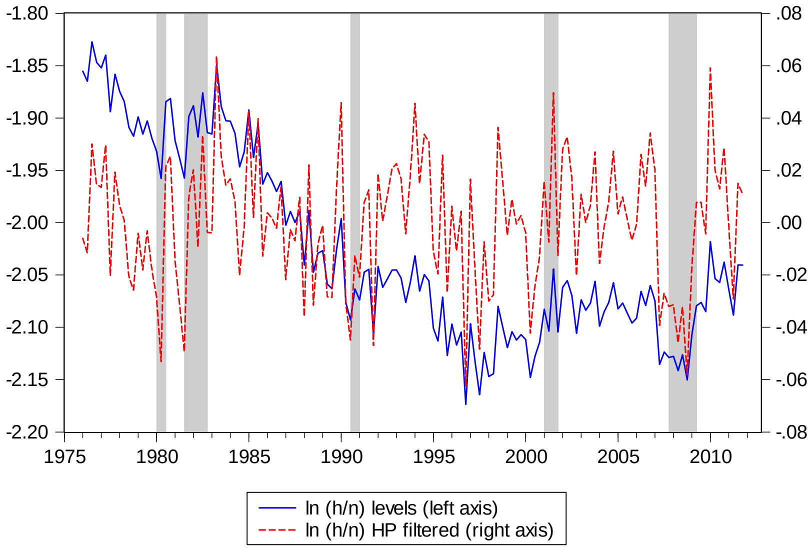


Figure 1b, Panel A: Log Hiring Rates (levels and HP filtered).

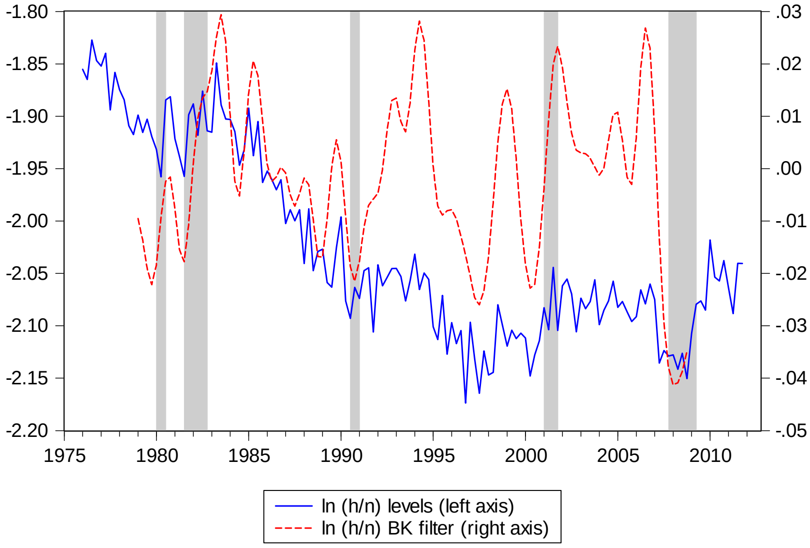


Figure 1b, Panel B: Log Hiring Rates (levels and BK filtered).

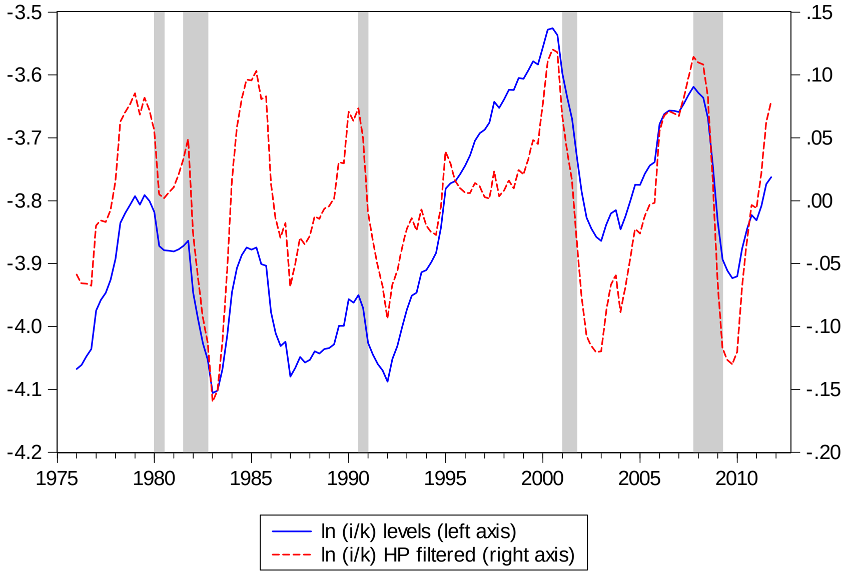


Figure 1b, Panel C: Log Investment Rates (levels and HP filtered).

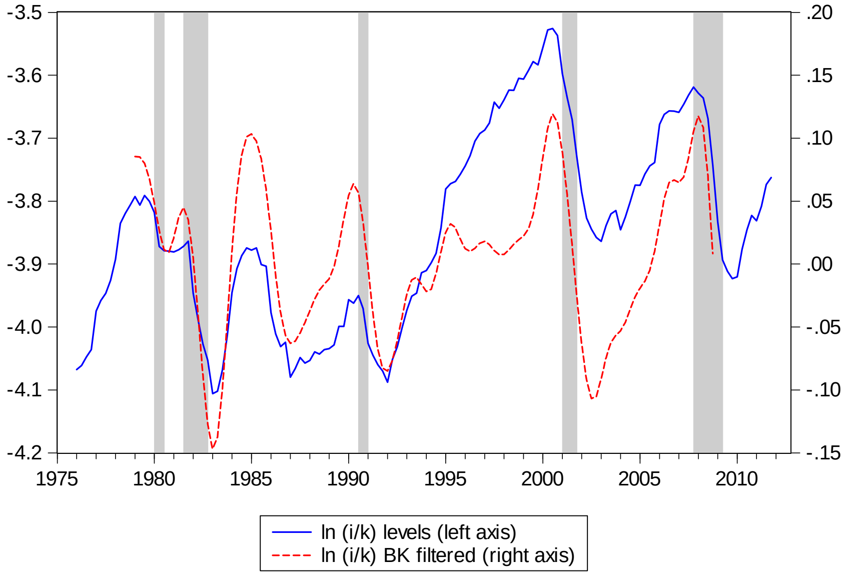


Figure 1b, Panel D: Log Investment Rates (levels and BK filtered).

c Investment and Hiring Co-Movement $\rho(\ln \frac{h_t}{n_t}, \ln \frac{i_{t+i}}{k_{t+i}})$

HP filtered ($\lambda = 1600$)

lag/lead	-8	-4	-1	0	1	4	8
	-0.08	-0.24	-0.35	-0.30	-0.22	0.10	0.21

BP filtered (Baxter-King, 6-32)

lag/lead	-8	-4	-1	0	1	4	8
	-0.20	-0.26	-0.44	-0.42	-0.35	0.02	0.19

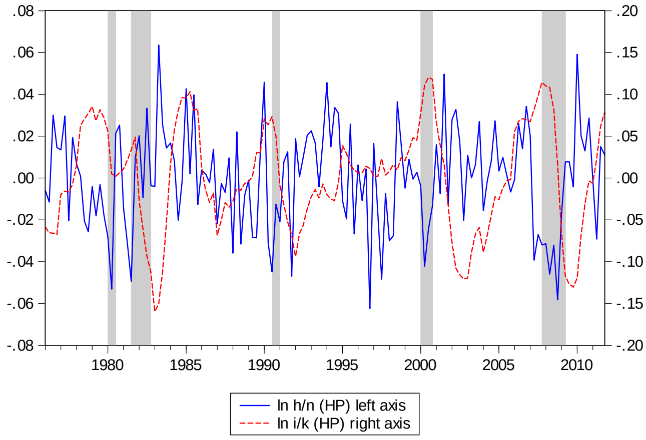


Figure 1c, Panel A: Hiring $\frac{h}{n}$ and investment $\frac{i}{k}$ rates (logged, HP filtered).

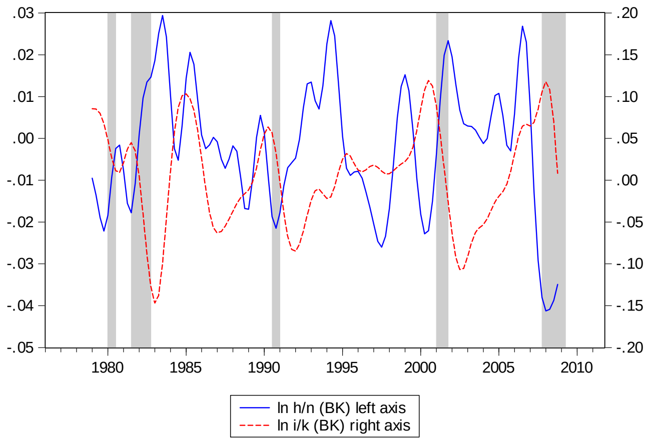


Figure 1c, Panel B: Hiring $\frac{h}{n}$ and investment $\frac{i}{k}$ rates (logged, BK filtered).

Table 7
Cyclicality of Marginal Costs and the Expected Present Values

Investment Value $\rho(\frac{g_{i_t}}{\frac{f_t}{k_t}}, y_{t+i})$

HP filtered ($\lambda = 1600$)

lag/lead	−8	−4	−1	0	1	4	8
f	−0.13	0.46	0.77	0.64	0.51	−0.11	−0.38
$\frac{f}{n}$	0.09	0.53	0.52	0.39	0.18	−0.34	−0.38
$\frac{f}{k}$	−0.04	0.55	0.76	0.66	0.44	−0.22	−0.45
$\frac{i}{k}$	−0.33	0.25	0.84	0.92	0.80	0.15	−0.44

BK filtered (Baxter-King, 6-32)

lag/lead	−8	−4	−1	0	1	4	8
f	−0.20	0.46	0.80	0.71	0.53	−0.08	−0.34
$\frac{f}{n}$	0.03	0.51	0.55	0.40	0.19	−0.29	−0.38
$\frac{f}{k}$	−0.08	0.57	0.79	0.66	0.43	−0.22	−0.42
$\frac{i}{k}$	−0.36	0.24	0.89	0.94	0.86	0.18	−0.43

Hiring Value $\rho(\frac{g_{h_t}}{\frac{f_t}{n_{t0.64}}}, y_{t+i})$

HP filtered ($\lambda = 1600$)

lag/lead	−8	−4	−1	0	1	4	8
f	−0.01	−0.41	−0.65	−0.61	−0.49	0.08	0.36
$\frac{f}{n}$	−0.13	−0.48	−0.50	−0.41	−0.25	0.31	0.35
$\frac{f}{k}$	−0.10	−0.46	−0.62	−0.55	−0.39	0.22	0.44
$\frac{h}{n}$	−0.16	−0.05	0.26	0.58	0.31	0.13	−0.13

BK filtered (Baxter-King, 6-32)

lag/lead	−8	−4	−1	0	1	4	8
f	−0.03	−0.41	−0.69	−0.67	−0.56	−0.02	0.24
$\frac{f}{n}$	−0.12	−0.50	−0.57	−0.48	−0.32	0.21	0.30
$\frac{f}{k}$	−0.14	−0.48	−0.66	−0.60	−0.45	0.13	0.34
$\frac{h}{n}$	−0.12	−0.03	0.55	0.63	0.58	0.09	−0.04

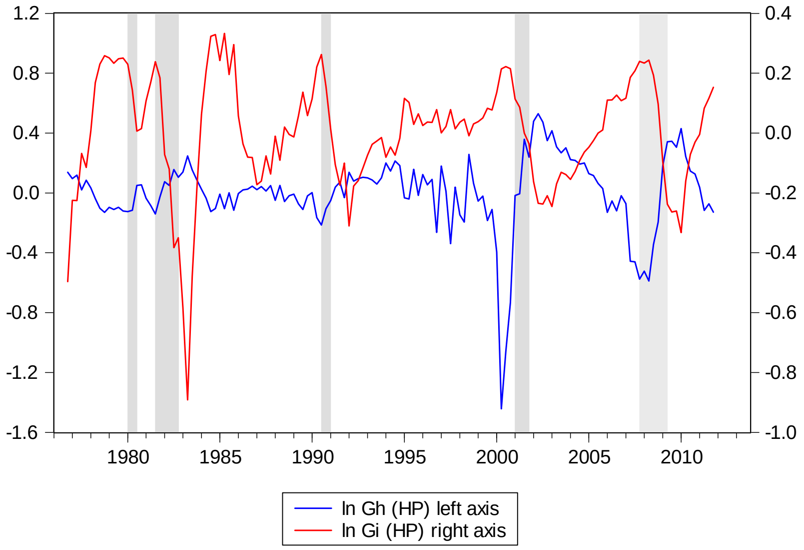


Figure 1d, Panel A: Marginal Costs of Hiring $\frac{g_h}{\frac{f}{n}}$ and of Investment $\frac{g_i}{\frac{f}{k}}$(logged, HP filtered)

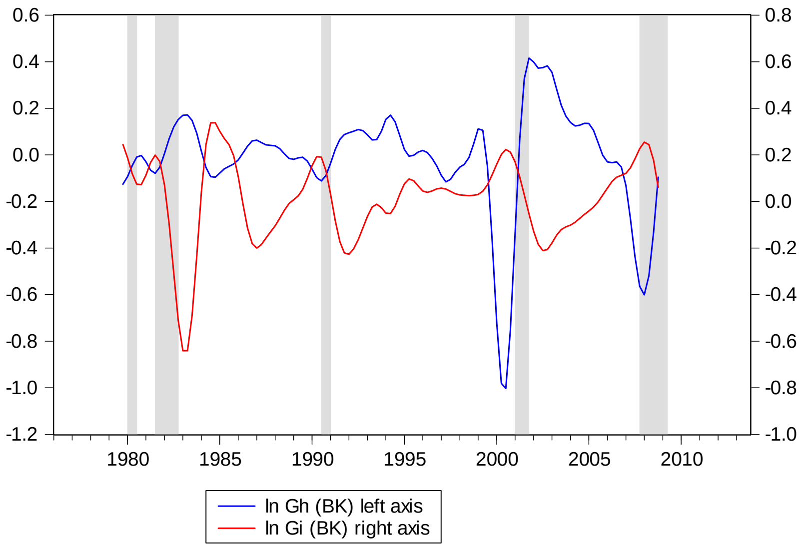


Figure 1d, Panel B: Marginal Costs of Hiring $\frac{g_h}{\frac{f}{n}}$ and of Investment $\frac{g_i}{\frac{f}{k}}$(logged, BK filtered)

Table 8
Investment Returns Series

a. Moments, 1976-2000, annual data

	MP	**GRR**	$\mathbf{R}_t^1$
mean	3.78	3.74	3.64
median	3.87	3.80	4.09
std	0.25	1.33	2.49
skewness	−0.56	−0.62	−0.65
kurtosis	1.86	5.04	2.13

b. Correlations I, 1976-2000, annual data

	MP	**GRR**	$\mathbf{R}_t^1$
MP	1		
GRR	0.14	1	
$\mathbf{R}_t^1$	0.56	0.19	1

Correlation II, 1976-2008, quarterly data

$$\rho(\mathbf{GRR},\ R_t^1) = 0.37$$

Notes:

1. MP is the McGrattan and Prescott (2003) series, described on their page 393 and available on
ftp://ftp.mpls.frb.fed.us/pub/research/mcgrattan/sr313/data/nipar.dat.

2. GRR is the Gomme, Ravikumar and Rupert (2011) series, described on their pages 269-270 and delineated in their Table 2 (page 270).

3. Table 8a drops 3 annual observations and Table 8b drops 3 quarterly observations where returns exhibit big spikes.

Table 9
VAR Results
Investment

	coef.	std.	$\overline{R}^2$		LR coef.
ϕ_1	0.89	0.03	0.90		
b_{d_p1}	-0.07	0.03	0.97	$b^{lr}_{d\_p1}$	-0.01
b_{r_p1}	-0.14	0.03	0.11	$b^{lr}_{r\_p1}$	-1.05
b_{gk_p1}	-0.005	0.02	0.16	$b^{lr}_{gk\_p1}$	-0.04
b_{δ_p1}	0.0004	0.0002	0.998	$b^{lr}_{\delta\_p1}$	0.003

Hiring

	coef.	std.	$\overline{R}^2$		coef.
ϕ_2	0.80	0.02	0.92		
b_{d21_p2}	0.01	0.01	0.95	$b^{lr}_{d21\_p2}$	0.18
b_{d22_p2}	0.004	0.01	0.94	$b^{lr}_{d22\_p2}$	-0.07
b_{r2_p2}	-0.34	0.02	0.66	$b^{lr}_{r2\_p2}$	-0.90
b_{gn_p2}	0.001	0.02	0.001	$b^{lr}_{gn\_p2}$	0.003
b_{ψ_p2}	-0.004	0.02	0.88	$b^{lr}_{\psi\_p2}$	-0.007

Notes:

1. The VAR formulation is given in Section 6.3, with full derivation provided in Appendix E.

2. The relevant long run coefficients, for capital are:

$$\begin{aligned} b^{lr}_{gk\_p1} &= \frac{b_{gk\_p1}}{1-\rho^k\phi_1}; \ b^{lr}_{\delta\_p1} = \frac{\rho^k b_{\delta\_p1}}{1-\rho^k\phi_1} \\ b^{lr}_{d\_p1} &= \frac{(1-\rho^k)b_{d\_p1}}{1-\rho^k\phi_1}; \ b^{lr}_{r\_p1} = \frac{b_{r\_p1}}{1-\rho^k\phi_1} \end{aligned}$$

For labor:

$$\begin{aligned} b^{lr}_{gn\_p2} &= \frac{b_{gn\_p2}}{1-\rho^{n2}\phi_2}; \ b^{lr}_{\psi p} = \frac{\rho^{n2} b_{\psi\_p2}}{1-\rho^{n2}\phi_2} \\ b^{lr}_{d21\_p2} &= \frac{(1-\rho^{n1})(1-\rho^{n2})b_{d21\_p2}}{1-\rho^{n2}\phi_2} \\ b^{lr}_{d22\_p2} &= \frac{\rho^{n1}(1-\rho^{n2})b_{d22\_p2}}{1-\rho^{n2}\phi_2}; \ b^{lr}_{r2\_p2} = \frac{b_{r2\_p2}}{1-\rho^{n2}\phi_2} \end{aligned}$$

where ϕ_1 is the AR coefficient on p^1, ϕ_2 is the AR coefficient on $p^2$the $b_{.\_p1,2}$ are the coefficients w.r.t $p^{1,2}$ and lr denotes the long-run.

Table 10
Variables in the $\frac{u}{n} - \frac{v}{n}$ Analysis

$u =$ **official unemployment**

a. Total

	1976 – 1991	**1992 – 2006**	**2007 – 2011**	**Full sample**
$\frac{v}{n}$	0.039	0.030	0.024	0.033
$\frac{u}{n}$	0.076	0.057	0.083	0.069
$\frac{h}{n}$	0.142	0.124	0.124	0.132
$\frac{i}{k}$	0.019	0.024	0.023	0.022
$\frac{1}{1-\tau}\frac{Q^N}{\frac{f}{n}}$	0.489	0.245	0.340	0.369
ψ	0.142	0.122	0.126	0.131
g	0.0049	0.0032	0.0006	0.0035
μ	2.72	3.01	2.82	2.82

b. Only $\frac{i}{k}$ changes

	1976 – 1991	**1992 – 2006**	**2007 – 2011**
$\frac{v}{n}$	0.039	0.028	0.029
$\frac{u}{n}$	0.058	0.082	0.079
$\frac{i}{k}$	0.019	0.024	0.023
$\frac{1}{1-\tau}\frac{Q^N}{\frac{f}{n}}$	full sample average		
ψ	full sample average		
g	full sample average		

c. Only $\frac{1}{1-\tau}\frac{Q^N}{\frac{f}{n}}$ changes

	1976 – 1991	**1992 – 2006**	**2007 – 2011**
$\frac{v}{n}$	0.025	0.050	0.036
$\frac{u}{n}$	0.092	0.045	0.064
$\frac{i}{k}$	full sample average		
$\frac{1}{1-\tau}\frac{Q^N}{\frac{f}{n}}$	0.489	0.245	0.340
ψ	full sample average		
g	full sample average		

d. Only ψ changes

	1976 – 1991	**1992 – 2006**	**2007 – 2011**
$\frac{v}{n}$	0.043	0.025	0.029
$\frac{u}{n}$	0.062	0.079	0.074
$\frac{i}{k}$	full sample average		
$\frac{h}{n}$	full sample average		
$\frac{1}{1-\tau}\frac{Q^N}{\frac{f}{n}}$	full sample average		
ψ	0.142	0.122	0.126
g	full sample average		

e. Only g changes

	1976 – 1991	**1992 – 2006**	**2007 – 2011**
$\frac{v}{n}$	0.034	0.033	0.031
$\frac{u}{n}$	0.068	0.069	0.072
$\frac{i}{k}$	full sample average		
$\frac{h}{n}$	full sample average		
$\frac{1}{1-\tau}\frac{Q^N}{\frac{f}{n}}$	full sample average		
ψ	full sample average		
g	0.0049	0.0032	0.0006

Notes:

1. For each sub -period one of the variables $\frac{i}{k}, \frac{1}{1-\tau}\frac{Q^N}{\frac{f}{n}}, \psi$ or g is taken to be at its **sub-sample** average while the rest are taken to be at their **full sample** average. This then is computed four times, each time picking another variable.

2. For each of the above permutations, $\frac{v}{n}$ and $\frac{u}{n}$ are **solved out** of the two equations:

$$\left[\begin{array}{c} e_2(\lambda\frac{v}{n} + (1-\lambda)\left(\frac{\mu\left(\frac{v}{n}\right)^{1-\sigma}\left(\frac{u}{n}\right)^{\sigma}}{\frac{v}{n}}\right)\frac{v}{n})\left(\lambda + (1-\lambda)\left(\frac{\mu\left(\frac{v}{n}\right)^{1-\sigma}\left(\frac{u}{n}\right)^{\sigma}}{\frac{v}{n}}\right)\right) \\ +e_3\left(\frac{i}{k}\right)\left(\frac{\mu\left(\frac{v}{n}\right)^{1-\sigma}\left(\frac{u}{n}\right)^{\sigma}}{\frac{v}{n}}\right) \end{array}\right] = \frac{1}{1-\tau}\frac{Q^N}{\frac{f}{n}}$$

$$\mu\left(\frac{v}{n}\right)^{1-\sigma}\left(\frac{u}{n}\right)^{\sigma} = \psi + g$$

3. The parameters $\lambda, e_2, e_3, \sigma$ are always constant (see Appendix F).

Figures 1 appear within Tables 6 and 7 above

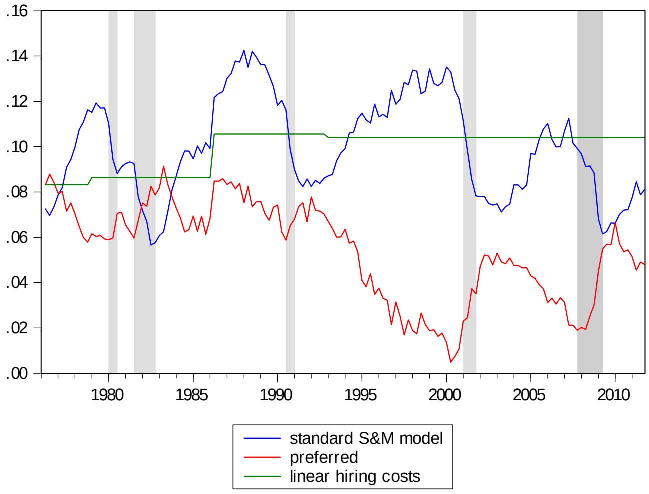
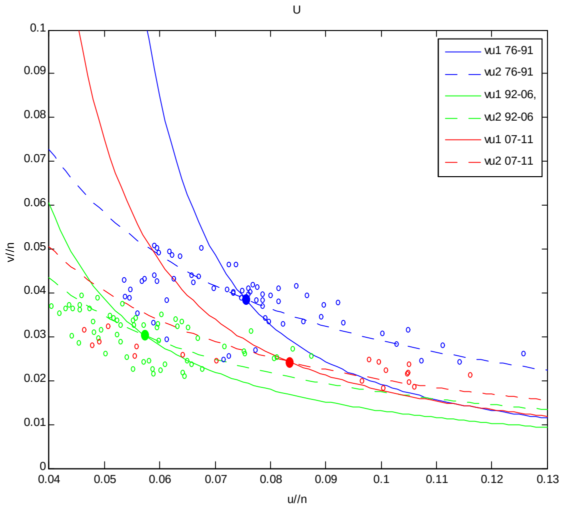


Figure 2: Job Values (Q^N) across models

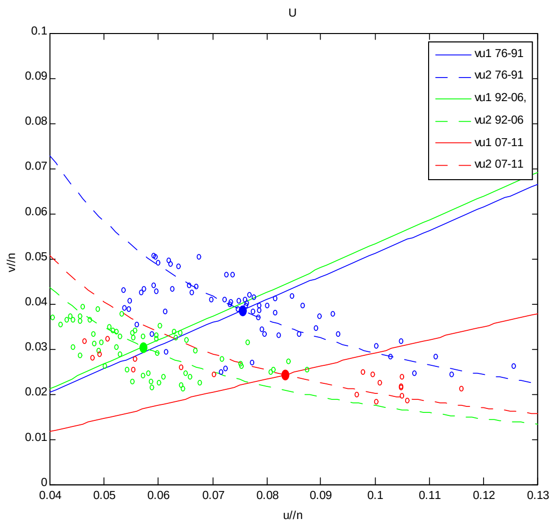
Figure 3
Unemployment-Vacancies Analysis



$u =$ **official unemployment**

Notes: The solid line is the vacancy creation curve (equation 30) and the dashed line the steady state flows curve (equation 31).

Figure 4
Unemployment-Vacancies Analysis of the Pissarides (2000) Model



$u =$ **official unemployment**

Notes: As in Figure 3, except that the vacancy creation curve is equation (32).

May 19, 2014

Capital Values, Job Values,
and the Joint Behavior of Hiring and Investment
by Eran Yashiv

APPENDICES –FOR ONLINE PUBLICATION

1 Appendix A

The Cost Function and its Derivatives; Elasticities

The Cost Function

$$g(\cdot)=\left[\frac{e_1}{\eta_1}(\frac{i_t}{k_t})^{\eta_1}+\frac{e_2}{\eta_2}(\frac{h_t}{n_t})^{\eta_2}+\frac{e_3}{\eta_3}\left(\frac{i_t}{k_t}\frac{h_t}{n_t}\right)^{\eta_3}\right]f(z_t,n_t,k_t). \tag{1}$$

First Derivatives

$$g_{i_t}=\left[e_1(\frac{i_t}{k_t})^{\eta_1-1}+e_3\left(\frac{h_t}{n_t}\right)^{\eta_3}\frac{i_t}{k_t}^{\eta_3-1}\right]\frac{f_t}{k_t} \tag{2}$$

$$g_{h_t}=\left[e_2(\frac{h_t}{n_t})^{\eta_{2-1}}+e_3\left(\frac{i_t}{k_t}\right)^{\eta_3}\frac{h_t}{n_t}^{\eta_3-1}\right]\frac{f_t}{n_t} \tag{3}$$

$$\begin{aligned} g_{k_t} &= -\left[e_1(\frac{i_t}{k_t})^{\eta_1}+e_3\left(\frac{h_t}{n_t}\frac{i_t}{k_t}\right)^{\eta_3}\right]\frac{f_t}{k_t} \\ &\quad +(1-\alpha)\left[\frac{e_1}{\eta_1}(\frac{i_t}{k_t})^{\eta_1}+\frac{e_2}{\eta_2}(\frac{h_t}{n_t})^{\eta_2}+\frac{e_3}{\eta_3}\left(\frac{i_t}{k_t}\frac{h_t}{n_t}\right)^{\eta_3}\right]\frac{f_t}{k_t} \end{aligned} \tag{4}$$

$$\begin{aligned} g_{n_t} &= -\left[e_2(\frac{h_t}{n_t})^{\eta_2}+e_3\left(\frac{h_t}{n_t}\frac{i_t}{k_t}\right)^{\eta_3}\right]\frac{f_t}{n_t} \\ &\quad +\alpha\left[\frac{e_1}{\eta_1}(\frac{i_t}{k_t})^{\eta_1}+\frac{e_2}{\eta_2}(\frac{h_t}{n_t})^{\eta_2}+\frac{e_3}{\eta_3}\left(\frac{i_t}{k_t}\frac{h_t}{n_t}\right)^{\eta_3}\right]\frac{f_t}{n_t} \end{aligned} \tag{5}$$

Second Derivatives

$$g_{ii_t} = \underbrace{\begin{bmatrix} e_1(\eta_1 - 1)\left(\frac{i_t}{k_t}\right)^{\eta_1 - 2} \\ +e_3(\eta_3 - 1)\left(\frac{i_t}{k_t}\frac{h_t}{n_t}\right)^{\eta_3 - 2}\left(\frac{h_t}{n_t}\right)^2 \end{bmatrix}}_{\widetilde{g}_{ii}} \frac{f(z_z, n_t, k_t)}{k_t^2} \tag{6}$$

$$g_{hh_t} = \underbrace{\begin{bmatrix} e_2(\eta_2 - 1)\left(\frac{h_t}{n_t}\right)^{\eta_2 - 2} \\ +e_3(\eta_3 - 1)\left(\frac{i_t}{k_t}\frac{h_t}{n_t}\right)^{\eta_3 - 2}\left(\frac{i_t}{k_t}\right)^2 \end{bmatrix}}_{\widetilde{g}_{hh}} \frac{f(z_z, n_t, k_t)}{n_t^2} \tag{7}$$

$$g_{ih_t} = g_{hi_t} = \underbrace{\left[e_3\eta_3 \left(\frac{i_t}{k_t}\frac{h_t}{n_t}\right)^{\eta_3 - 1} \right]}_{\widetilde{g}_{ih}} \frac{f(z_z, n_t, k_t)}{k_t n_t} \tag{8}$$

Elasticities
Starting from the F.O.C and differentiating the following is obtained:[1]

$$\frac{\partial i_t}{\partial Q^K}\frac{Q^K}{i_t} = \frac{\widetilde{g}_{hh}}{(1-\tau_t)\left[\widetilde{g}_{ii}\widetilde{g}_{hh} - \widetilde{g}_{ih}\widetilde{g}_{hi}\right]}\frac{\frac{Q^K}{\frac{f_t}{k_t}}}{\frac{i_t}{k_t}}$$

$$\frac{\partial h_t}{\partial Q^k}\frac{Q^K}{h_t} = -\frac{\widetilde{g}_{hi}}{(1-\tau_t)\left[\widetilde{g}_{ii}\widetilde{g}_{hh} - \widetilde{g}_{ih}\widetilde{g}_{hi}\right]}\frac{\frac{Q^K}{\frac{f_t}{k_t}}}{\frac{h_t}{n_t}}$$

$$\frac{\partial h_t}{\partial Q^N}\frac{Q^N}{h_t} = \frac{\widetilde{g}_{ii}}{(1-\tau_t)\left[\widetilde{g}_{ii}\widetilde{g}_{hh} - \widetilde{g}_{ih}\widetilde{g}_{hi}\right]}\frac{\frac{Q^N}{\frac{f_t}{n_t}}}{\frac{h_t}{n_t}}$$

$$\frac{\partial i_t}{\partial Q^N}\frac{Q^N}{i_t} = -\frac{\widetilde{g}_{ih}}{(1-\tau_t)\left[\widetilde{g}_{ii}\widetilde{g}_{hh} - \widetilde{g}_{ih}\widetilde{g}_{hi}\right]}\frac{\frac{Q^N}{\frac{f_t}{h_t}}}{\frac{i_t}{k_t}}$$

[1] The complete derivation is avaialble upon request.

2 Appendix B

The Data

variable	**symbol**	**definition**
GDP	f	gross value added of NFCB
GDP deflator	p^f	price per unit of gross value added of NFCB
wage share	$\frac{wn}{f}$	numerator: compensation of employees in NFCB
discount rate	r	the rate of non-durable consumption growth minus 1
employment	n	employment in nonfinancial corporate business sector
hiring	h	gross hires
separation rate	ψ	gross separations divided by employment
vacancies	v	adjusted Help Wanted Index
investment	i	gross investment in NFCB sector
capital stock	k	stock of private nonresidential fixed assets in NFCB sector
depreciation	δ	depreciation of the capital stock
price of capital goods	p^I	real price of new capital goods

variable	**symbol**	**source**
GDP	f	NIPA accounts, table 1.14, line 40
GDP deflator	p^f	NIPA table 1.15, line 1
wage share	$\frac{wn}{f}$	NIPA table 1.14, lines 17 and 20
discount rate	r	NIPA Table 2.3.5; see note 1
employment	n	CPS; see note 2
hiring	h	CPS; see note 2
separation rate	ψ	CPS; see note 2
vacancies	v	Conference Board; see note 3
investment	i	BEA and Fed Flow of Funds; see note 4
capital stock	k	BEA and Fed Flow of Funds; see note 4
depreciation	δ	BEA and Fed Flow of Funds; see note 4
price of capital goods	p^I	NIPA and U.S. tax foundation; see note 5

The sample period is 1976:2-2011:4 and all data are quarterly.

Notes:

1. *The discount rate and the discount factor*

The discount rate is based on a DSGE-type model with logarithmic utility $U(c_t) = \ln c_t$.

Then in general equilibrium:

$$U'(c_t) = U'(c_{t+1}) \cdot (1 + r_t)$$

Hence:

$$\rho_t = \frac{c_t}{c_{t+1}}$$

where c is non-durable consumption (goods and services) and 5% of durable consumption.

2. *Employment, hiring and separations*

As a measure of employment in nonfinancial corporate business sector (n) I take wage and salary workers in non-agricultural industries (series ID LNS12032187) less government workers (series ID LNS12032188), less self-employed workers (series ID LNS12032192), less unpaid family workers (series ID LNS12032193). All series originate from CPS databases. I do not subtract workers in private households (the unadjusted series ID LNU02032190) from the above due to lack of sufficient data on this variable.

To calculate hiring and separation rates for the whole economy I use the series kindly provided by Ofer Cornfeld. This computation first builds the flows between E (employment), U (unemployment) and N (not-in-the-labor-force) that correspond to the E, U, N stocks published by CPS. The methodology of adjusting flows to stocks is taken from BLS, and is given in Frazis et al (2005).[2] This methodology, applied by BLS for the period 1990 onward, produces a dataset that appears in **http://www.bls.gov/cps/cps_flows.htm**. Here the series have been extended back to 1976.

The quarterly separation rate (ψ) and the quarterly hiring rate (h/n) for the whole economy are defined as follows:

$$\begin{aligned} \psi &= \frac{EN + EU}{E} \\ h/n &= \frac{NE + UE}{E} \end{aligned}$$

[2] Frazis, Harley J., Edwin L. Robison, Thomas D. Evans and Martha A. Duff, 2005. "Estimating Gross Flows Consistent with Stocks in the CPS," **Monthly Labor Review**, September, 3-9.

where the employment (E) is the quarterly average of the original seasonally adjusted total employment series from BLS (LNS12000000).

3. *Vacancies and Market Tightness*

In order to compute $\frac{v}{n+o}$ I use:

(i) The vacancies series based on the Conference Board Composite Help-Wanted Index that takes into account both printed and web job advertisements, as computed by Barnichon. The updated series is available at **https://sites.google.com/site/regisbarnichon/research/publications.**

This index was multiplied by a constant to adjust its mean to the mean of the JOLTS vacancies series over the overlapping sample period (2001Q1–2011Q4).

(ii)The unemployment and the out of labor force series are the BLS CPS data.

4. *Investment, capital and depreciation*

The goal here is to construct the quarterly series for real investment flow i_t , real capital stock k_t , and depreciation rates δ_t. I proceed as follows:

- Construct end-of-year fixed-cost net stock of private nonresidential fixed assets in NFCB sector, K_t . In order to do this I use the quantity index for net stock of fixed assets in NFCB (FAA table 4.2, line 28, BEA).

- Construct annual fixed-cost depreciation of private nonresidential fixed assets in NFCB sector, D_t . The chain-type quantity index for depreciation originates from FAA table 4.5, line 28. The current-cost depreciation estimates are given in FAA table 4.4, line 28.

- Calculate the annual fixed-cost investment flow, I_t:

$$I_t = K_t - K_{t-1} + D_t$$

- Calculate implied annual depreciation rate, δ_a:

$$\delta_a = \frac{I_t - (K_t - K_{t-1})}{K_{t-1} + I_t/2}$$

- Calculate implied quarterly depreciation rate for each year, δ_{qt}:

$$\delta_q + (1-\delta_q)\delta_q + (1-\delta_q)^2\delta_q + (1-\delta_q)^3\delta_q = \delta_a$$

- Take historic-cost quarterly investment in private non-residential fixed assets by NFCB sector from the Flow of Funds accounts, atabs files, series FA105013005).

- Deflate it using the investment price index (the latter is calculated as consumption of fixed capital in domestic NFCB in current dollars (NIPA table 1.14, line 18) divided by consumption of fixed capital in domestic NFCB in chained 2000 dollars (NIPA table 1.14, line 41). This procedure yields the implicit price deflator for depreciation in NFCB. The resulting quarterly series, i_t_unadj, is thus in real terms.

- Perform Denton's procedure to adjust the quarterly series i_t_unadj from Federal Flow of Funds accounts to the implied annual series from BEA I_t, using the depreciation rate δ_{qt} from above. I use the simplest version of the adjustment procedure, when the discrepancies between the two series are equally spread over the quarters of each year. As a result of adjustment I get the fixed–cost quarterly series i_t.

- Simulate the quarterly real capital stock series k_t starting from k_0 (k_0 is actually the fixed-cost net stock of fixed assets in the end of 1975, this value is taken from the series K_t) , using the quarterly depreciation series δ_{qt} and investment series i_t from above:

$$k_{t+1} = k_t \cdot (1-\delta_{qt}) + i_t$$

5. *Real price of new capital goods*

In order to compute the real price of new capital goods, p^I, I use the price indices for output and for investment goods. Investment in NFCB Inv consists of equipment Eq and structures St. I define the time-t price-indices for good $j = Inv, Eq, St$ as p_t^j and their change between $t-1$ and t by Δp_t^j, $j = Inv, Eq, St$. These price indices are chain-weighted. Thus:

$$\frac{\Delta p_t^{Inv}}{p_{t-1}^{Inv}} = \omega_t \frac{\Delta p_t^{Eq}}{p_{t-1}^{Eq}} + (1-\omega_t)\,\frac{\Delta p_t^{St}}{p_{t-1}^{St}}$$

where

$$\omega_t = \frac{\begin{array}{c}(\text{nominal expenditure share of } Eq \text{ in } Inv)_{t-1} \\ + (\text{nominal expenditure share of } Eq \text{ in } Inv)_t\end{array}}{2}.$$

The weights ω_t are calculated from the NIPA table 1.1.5, lines 8,10. The price indices p_t^j for $j = Eq, St$ are from NIPA table 1.1.4, lines 9, 10. I divide the series by the price index for output, p_t^f, to obtain the real price of new capital goods, p^I.

Note that the price indices p^{Eq} and p^{St} and therefore p^I are actually adjusted for taxes. The parameter τ denotes the statutory corporate income tax rate as reported by the U.S. Tax Foundation.

Let ITC denote the investment tax credit on equipment and public utility structures, $ZPDE$ the present discounted value of capital depreciation allowances, and χ the percentage of the cost of equipment that cannot be depreciated if the firm takes the investment tax credit. Flint Brayton has kindly provided me with the data. Then

$$\begin{aligned} p^{Eq} &= \widetilde{p}^{Eq} (1 - \tau_{Eq}) \\ p^{St} &= \widetilde{p}^{St} (1 - \tau_{St}), \end{aligned}$$

$$\begin{aligned} 1 - \tau^{S_t} &= \frac{\left(1 - \tau \; ZPDE^{St}\right)}{1 - \tau} \\ 1 - \tau^{Eq} &= \frac{1 - ITC - \tau ZPDE^{Eq} (1 - \chi ITC)}{1 - \tau} \end{aligned}$$

3 Appendix C

Alternative Specifications

The following tables report variations on the specifications reported in Tables 2a and 2b.

Table C-1
GMM estimates

	$\mathbf{e_1}$	$\mathbf{e_2}$	$\mathbf{e_3}$	η_1	η_2	η_3	$\mathbf{f_1}$	$\mathbf{f_2}$	α
1	85,018	8.6	−41.8	3.88	2.20	1.06	2.74	0.02	0.68
	(234,098)	(9.2)	(12.2)	(0.34)	(0.52)	(0.01)	(6.60)	(0.25)	(0.009)
2	113,317	6.9	−33.0	3.98	2.08	1.02	3.56	−0.02	0.67
	(191,649)	(1.7)	(5.6)	(0.09)	(0.19)	(0.01)	(3.50)	(0.36)	–
3	62,637	8.4	−52.1	3.36	1.87	1.01	0	0	0.67
	(45,384)	(2.3)	(8.1)	(0.11)	(0.21)	(0.01)	–	–	–
4	30,378	7.1	−43.4	3.22	1.96	1.02	0	0	0.67
	(48,639)	(3.6)	(19.8)	(0.38)	(0.37)	(0.07)	–	–	–
5	1436	1.9	−2.5	3	2	1	0	0	0.67
	(355)	(0.4)	(1.3)	–	–	–	–	–	–
6	−469	0.6	7.6	3	2	1	0	0	0.67
	(327)	(0.3)	(1.5)	–	–	–	–	–	–
7	76	1.5	−4.8	2	2	1	0	0	0.67
	(12)	(0.4)	(1.5)	–	–	–	–	–	–
8	58	1.4	−4.2	2	2	1	0	0	0.67
	(10)	(0.3)	(1.5)	–	–	–	–	–	–

	J-Statistic	instrument set
1	68.7	$\frac{h}{n}, \frac{i}{k}, \frac{f}{k}$
	(0.07)	
2	72.9	$\frac{h}{n}, \frac{i}{k}, \frac{f}{k}$
	(0.04)	
3	82.9	$\frac{h}{n}, \frac{i}{k}, \frac{f}{k}$
	(0.01)	
4	85.9	$\frac{h}{n}, \frac{i}{k}, p^I$
	(0.01)	
5	73.5	$\frac{h}{n}, \frac{i}{k}, \frac{f}{k}$
	(0.10)	
6	71.6	$\frac{h}{n}, \frac{i}{k}, p^I$
	(0.13)	
7	71.7	$\frac{h}{n}, \frac{i}{k}, \frac{f}{k}$
	(0.12)	
8	77.1	$\frac{h}{n}, \frac{i}{k}, p^I$
	(0.06)	

Notes:

1. The table reports point estimates with standard errors in parantheses.
2. The J-statistic is reported with p value in parantheses.

Table C-2
Adjustment Costs Implied by the GMM Estimation Results

	specification	$\frac{g}{f}$		$\frac{g_i}{\frac{f}{k}}$		$\frac{g_h}{\frac{f}{n}}$	
1	all free	0.050	0.003	0.39	0.87	0.13	0.16
2	partially constrained	0.034	0.004	1.10	0.83	0.11	0.15
3	partially constrained	0.015	0.008	1.29	3.19	0.35	0.24
4	partially constrained	0.007	0.007	1.37	2.44	0.18	0.18
5	$\eta_1 = 3, \eta_{\mathbf{2}} = 2, \eta_3 = 1$	0.014	0.001	0.37	0.23	0.19	0.03
6	$\eta_1 = 3, \eta_{\mathbf{2}} = 2, \eta_3 = 1$	0.025	0.002	0.77	0.13	0.24	0.02
7	$\eta_1 = \eta_2 = 2, \eta_3 = 1$	0.018	0.003	1.01	0.29	0.09	0.03
8	$\eta_1 = \eta_2 = 2, \eta_3 = 1$	0.015	0.002	0.72	0.22	0.09	0.03

Notes:
1. Mean and std. refer to sample statistics.
2. The functions were computed using the point estimates in Table C-1.

The first four specifications, with no or few restrictions, have low p-values and imprecise estimates of the scale parameters, very much like those of rows 1 and 2 in Table 2a. As in the latter table, they seem to point to a power specification of $\eta_1 = 3, \eta_{\mathbf{2}} = 2, \eta_3 = 1$. The remaining four specifications, more restricted, have precise estimates and higher p-values. They imply cost functions that are similar to those of rows 3 and 4 in Table 2a.

4 Appendix D1

Comparison of the Frictions Estimates to the Literature

4.1 Hiring Costs

Mortensen and Nagypal (2006, page 30)[3] note that "Although there is a consensus that hiring costs are important, there is no authoritative estimate of their magnitude. Still, it is reasonable to assume that in order to recoup hiring costs, the firm needs to employ a worker for at least two to three quarters. When wages are equal to their median level in the standard model ($w = 0.983$),hiring costs of this magnitude correspond to less than a week of wages." The widely-cited Shimer (2005) paper[4] calibrates these costs at 0.213 in terms similar to g_h here, using a linear cost function, which is equivalent to 1.4 weeks of wages. Hagedorn and Manovskii (2008)[5] decompose this cost into two components: (i) the capital flow cost of posting a vacancy; they compute it to be – in steady state – 47.4 percent of the average weekly labor productivity; (ii) the labor cost of hiring one worker, which, relying on micro-evidence, they compute to be 3 percent to 4.5 percent of quarterly wages of a new hire. The first component would correspond to a figure of 0.037 here; the second component would correspond to a range of 0.02 to 0.03 in the terms used here; together this implies 0.057 to 0.067 in current terms or around 1.1 to 1.3 weeks of wages.

4.2 Investment Costs

The Q literature exhibits huge variation across studies over the past four decades. One finds estimates of marginal costs varying from as low as 0.04 to as high as 60 (in terms of $\frac{f}{k}$). These differences in marginal cost estimates are usually due to differences in the parameter estimates, and not just due to the diversity in the rate of investment used. One can divide the results into three

[3] Mortensen, Dale T. and Eva Nagypal, 2006. "More on Vacancy and Unemployment Fluctuations," working paper.

[4] Shimer, Robert, 2005. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," **American Economic Review,** 95,1, 25-49.

[5] Hagedorn, Marcus, and Iourii Manovskii, 2008. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited," **American Economic Review**, 98, 4, 1692–1706.

sets: (i) the earlier studies, from the 1980s, suggested high costs, whereby marginal costs range between 3 to 60 in terms of average output per unit of capital and the implied total costs range between 15% to 100% of output; (ii) more recent studies report moderate costs, whereby marginal costs are around 1 in terms of average output per unit of capital and total costs range between 0.5% to 6% of output; (iii) micro-based studies, using cross-sectional or panel data, report low costs, whereby marginal costs are 0.04 to 0.50 of average output per unit of capital and total costs range between 0.1% to 0.2% of output.

Coming back to the initial question of comparing these estimates to the current findings, two main conclusions emerge:

(i) The standard specification that I run that is closest to the one used in most Tobin's Q studies is the one reported in row 1 of Tables 2b and 3b. This is the specification positing a quadratic function and ignoring labor. The implied total costs are 3% of output (as in studies of the moderate costs set) and the implied marginal costs are 2.3 of average output per unit of capital (as in the high costs set). As indicated above, this is 17% of the price of a unit of investment good p^I. These implausible results are a major reason to reject these particular estimates here.

(ii) The results for the preferred specifications, i.e., the GMM results of the full model reported in row 4 of Tables 2a and 3a,[6] correspond to the third set, i.e., to low costs.

[6]Looking at marginal costs as a fraction of output per unit of capital $(\frac{g_i}{\frac{f}{k}})$, estimated at a mean of 0.75.

5 Appendix D2

Cyclical Behavior of Key Labor Market Variables

The following hold true in steady state:
Hiring to employment h equals separations from employment s:

$$h = s \tag{9}$$

Non-employment in the steady state, i.e., unemployment u plus the pool out of the labor force o, satisfies:

$$\frac{u+o}{pop} = \frac{\psi}{\frac{h}{u+o} + \psi} \tag{10}$$

where pop is the working age population and ψ is the separation rate from employment n (i.e., $s = \psi\ n$).

In steady state the hiring rate is the product of the job finding rate, steady state non-employment and the inverse of the employment rate:

$$\frac{h}{n} = \frac{h}{u+o} \times \frac{u+o}{pop} \times \frac{pop}{n} \tag{11}$$

Using the above formulation of steady-state non-employment::

$$\underbrace{\frac{h}{n}}_{\text{hiring rate}} = \underbrace{\frac{h}{u+o}}_{\text{job finding}} \times \underbrace{\frac{\psi}{\frac{h}{u+o} + \psi}}_{\text{ss non-emp}} \times \underbrace{\frac{1}{\frac{n}{pop}}}_{\text{inv emp ratio}} \tag{12}$$

The following table shows the co-movement statistics for these variables.

Table D
Stochastic Behavior of the Gross Hiring Rate and Other Labor Market Variables

Co-Movement (contemporaneous) with Cyclical Indicators

logged, HP filtered

	n_t	$\frac{h_t}{n_t}$	$\frac{h_t}{u_t+o_t}$	$\frac{\psi}{\frac{h_t}{u_t+o_t}+\psi}$	$\frac{1}{\frac{n_t}{POP_t}}$
with GDP f	0.81	−0.25	0.53	−0.39	−0.82
with labor productivity $\frac{f}{n}$	0.42	−0.04	0.38	−0.31	−0.46

logged, BK filtered

	n_t	$\frac{h_t}{n_t}$	$\frac{h_t}{u_t+o_t}$	$\frac{\psi}{\frac{h_t}{u_t+o_t}+\psi}$	$\frac{1}{\frac{n_t}{POP_t}}$
with GDP f	0.85	−0.36	0.69	−0.84	−0.86
with labor productivity $\frac{f}{n}$	0.46	−0.08	0.50	−0.75	−0.50

Notes:
1. o_t is the pool out of the labor force.
2. POP_t is the working-age population.

The employment stock n and the job finding rate $\frac{h_t}{u_t+o_t}$ are pro-cyclical, as is well known. Steady state non-employment $\frac{\psi}{\frac{h}{u+o}+\psi}$ and the inverse of the employment ratio $\frac{1}{\frac{n}{pop}}$ are counter-cyclical, as widely known too. At the same time the gross hiring rate $\frac{h_t}{n_t}$ is counter-cyclical, as shown above. Hence the hiring rate is counter-cyclical as the counter-cyclicality of the last two variables dominates the pro-cyclicality of the job-finding rate.

6 Appendix E

Derivation and Estimation of the Asset Pricing Model

6.1 Investment in Capital

Define:

$$P_t^1 \equiv (1-\tau_t)\left(\frac{g_{i_t}+p_t^I}{\frac{f_t}{k_t}}\right) = \frac{Q_t^K}{\frac{f_t}{k_t}} \tag{13}$$

$$D_t^1 = (1-\tau_t)\frac{(f_{k_t}-g_{k_t})}{\frac{f_t}{k_t}} \tag{14}$$

$$R_t^1 = \frac{\left(1+g_t^{f/k}\right)\left[(1-\delta_t)P_t^1+D_t^1\right]}{P_{t-1}^1} \tag{15}$$

Using:

$$G_{t+1}^{f/k} = \frac{\frac{f_{t+1}}{k_{t+1}}}{\frac{f_t}{k_t}}$$

Hence:

$$\begin{aligned}
R_t^1 &= \frac{G_t^{f/k}\left[(1-\delta_t)P_t^1+D_t^1\right]}{P_{t-1}^1} \\
&= G_t^{f/k}\frac{D_t^1(1+\frac{(1-\delta_t)P_t^1}{D_t^1})}{P_{t-1}^1} \\
\ln R_t^1 &= \ln\left(G_t^{f/k}\right) \\
&\quad +\ln\left(D_t^1(1+\frac{(1-\delta_t)P_t^1}{D_t^1})\right) \\
&\quad -\ln P_{t-1}^1
\end{aligned}$$

Looking into the second term:

$$\begin{aligned}
\ln\left(D_t^1(1+\frac{(1-\delta_t)P_t^1}{D_t^1})\right) &= \ln D_t^1 + \ln(1+\frac{(1-\delta_t)P_t^1}{D_t^1}) \\
&= \ln D_t^1 + \ln(1+e^{\ln(1-\delta_t)+p_t^1-d_t^1}) \\
&\cong d_t^1 + c_0 + \rho^k\left(\ln(1-\delta_t)+p_t^1-d_t^1\right)
\end{aligned}$$

where:

$$\rho^k = \frac{\frac{(1-\delta)P^1}{D^1}}{1+\frac{(1-\delta)P^1}{D^1}}$$

Hence:

$$\ln R_t^1 \cong c_2 + \ln\left(G_t^{f/k}\right) + d_t^1 + c_0 + \rho^k\left(\ln(1-\delta_t)+p_t^1-d_t^1\right) - p_{t-1}^1$$

So:

$$p_{t-1}^1 \cong c_3 + \ln G_t^{f/k} + \rho^k \ln(1-\delta_t) + \rho^k p_t^1 + (1-\rho^k)d_t^1 - r_t^1 \tag{16}$$

6.2 Hiring of Labor

Define:

$$P_t^2 \equiv \frac{(1-\tau_t)\,g_{h_t}}{\frac{f_t}{n_t}} \equiv \frac{Q_t^N}{\frac{f_t}{n_t}} \tag{17}$$

$$D_t^2 = (1-\tau_t)\left(\alpha - \frac{g_{n_t}}{\frac{f_t}{n_t}} - \frac{w_t}{\frac{f_t}{n_t}}\right) \tag{18}$$

$$D_t^{2,1} = (1-\tau_t)\left(\alpha - \frac{g_{n_t}}{\frac{f_t}{n_t}}\right) \tag{19}$$

$$D_t^{2,2} = (1-\tau_t)\frac{w_t}{\frac{f_t}{n_t}} \tag{20}$$

$$D_t^2 = D_t^{2,1} - D_t^{2,2}$$

$$R_t^2 = \frac{\left(1+g_t^{f/n}\right)\left[(1-\psi_t)P_t^2 + D_t^2\right]}{P_{t-1}^2} \tag{21}$$

where:

$$G_{t+1}^{f/n} = \frac{\frac{f_{t+1}}{n_{t+1}}}{\frac{f_t}{n_t}}$$

Hence:

$$\begin{aligned}
R_t^2 &= \frac{G_t^{f/n}\left[(1-\psi_t)P_t^2 + D_t^{2,1} - D_t^{2,2}\right]}{P_{t-1}^2} \\
&= \frac{G_t^{f/n}\left(D_t^{2,1} - D_t^{2,2}\right)\left[1 + \frac{(1-\psi_t)P_t^2}{D_t^{2,1}-D_t^{2,2}}\right]}{P_{t-1}^2} \\
&= \frac{G_t^{f/n}D_t^{2,1}\left(1 - \frac{D_t^{2,2}}{D_t^{2,1}}\right)\left[1 + \frac{(1-\psi_t)P_t^2}{D_t^{2,1}-D_t^{2,2}}\right]}{P_{t-1}^2} \\
\ln R_t^2 &= \ln G_t^{f/n} \\
&\quad + \ln D_t^{2,1} \\
&\quad + \ln\left(1 - \frac{D_t^{2,2}}{D_t^{2,1}}\right) \\
&\quad + \ln\left[1 + \frac{(1-\psi_t)P_t^2}{D_t^{2,1} - D_t^{2,2}}\right] \\
&\quad - \ln P_{t-1}^2
\end{aligned}$$

Looking into the third term on the RHS:

$$\begin{aligned}
\ln\left(1 - \frac{D_t^{2,2}}{D_t^{2,1}}\right) &= \ln(1 - e^{d_t^{2,2} - d_t^{2,1}}) \\
&\cong c_4 + \rho^{n1}(d_t^{2,2} - d_t^{2,1})
\end{aligned}$$

where:

$$\rho^{n1} = \frac{-\frac{D^{2,2}}{D^{2,1}}}{1 - \frac{D^{2,2}}{D^{2,1}}}$$

Looking into the fourth term on the RHS:

$$\ln\left(1+\frac{(1-\psi_t)P_t^2}{D_t^{2,1}-D_t^{2,2}}\right) = \ln(1-e^{\ln((1-\psi_t)+p_t^2-d_t^2})$$
$$\cong c_5+\rho^{n2}(\ln(1-\psi_t)+p_t^2-d_t^2)$$

where

$$d_t^2 = \ln D_t^2 = \ln(D_t^{2,1}-D_t^{2,2})$$
$$\rho^{n2} = \frac{\frac{(1-\psi)P^2}{D^2}}{1+\frac{(1-\psi)P^2}{D^2}}$$

Now note that:

$$d_t^2 = \ln D_t^{2,1}(1-\frac{D_t^{2,2}}{D_t^{2,1}})$$
$$\cong d_t^{2,1}+c_6+\rho^{n1}(d_t^{2,2}-d_t^{2,1})$$

So:

$$\ln\left(1+\frac{(1-\psi_t)P_t^2}{D_t^{2,1}-D_t^{2,2}}\right) \cong c_7+\rho^{n2}(\ln(1-\psi_t)+p_t^2-\left(d_t^{2,1}+\rho^{n1}(d_t^{2,2}-d_t^{2,1})\right))$$
$$\cong c_7+\rho^{n2}(\ln(1-\psi_t)+p_t^2-\left((1-\rho^{n1})d_t^{2,1}+\rho^{n1}d_t^{2,2}\right))$$

Collecting all terms:

$$\begin{aligned}\ln R_t^2 \cong\ & c_8+\ln G_t^{f/n}+d_t^{2,1}-p_{t-1}^2\\ &+\rho^{n1}(d_t^{2,2}-d_t^{2,1})\\ &+\rho^{n2}(\ln(1-\psi_t)+p_t^2-\left((1-\rho^{n1})d_t^{2,1}+\rho^{n1}d_t^{2,2}\right))\end{aligned}$$

So:

$$\begin{aligned}p_{t-1}^2 =\ & c_8+\ln G_t^{f/n}+\rho^{n2}\ln(1-\psi_t)+\rho^{n2}p_t^2\\ &+d_t^{2,1}(1-\rho^{n1})(1-\rho^{n2})\\ &+d_t^{2,2}(\rho^{n1}(1-\rho^{n2}))\\ &-r_t^2\end{aligned} \qquad (22)$$

6.3 The VAR

I estimate the following structural VAR:

$$(\mathbf{x}_{t+1}) = A + B\mathbf{x}_t + \varepsilon_t$$

For capital

$$\mathbf{x}_{t+1} = \begin{pmatrix} p^1_{t+1} \\ d^1_{t+1} \\ r^1_{t+1} \\ \ln\left(G^{f/k}_{t+1}\right) \\ \ln(1-\delta_{t+1}) \end{pmatrix}$$

The structural restrictions implied by (**??**):[7]

$$e_1(I - \rho^k B) = \left((1-\rho^k)e_2 - e_3 + e_4 + \rho^k e_5\right) B \tag{23}$$

For labor:

$$\mathbf{x}_{t+1} = \begin{pmatrix} p^2_{t+1} \\ d^{2,1}_{t+1} \\ d^{2,2}_{t+1} \\ r^2_{t+1} \\ \ln\left(G^{f/n}_{t+1}\right) \\ \ln(1-\psi_{t+1}) \end{pmatrix}$$

The structural restrictions implied by (**??**) are:[8]

---

[7] where
$e_1 = (1, 0, 0, 0, 0)$
$e_2 = (0, 1, 0, 0, 0)$
$e_3 = (0, 0, 1, 0, 0)$
$e_4 = (0, 0, 0, 1, 0)$
$e_5 = (0, 0, 0, 0, 1)$

[8] where
$e_1 = (1, 0, 0, 0, 0, 0)$
$e_{21} = (0, 1, 0, 0, 0, 0)$
$e_{22} = (0, 0, 1, 0, 0, 0)$
$e_3 = (0, 0, 0, 1, 0, 0)$
$e_4 = (0, 0, 0, 0, 1, 0)$
$e_5 = (0, 0, 0, 0, 0, 1)$

$$e_1(I - \rho^{n2}B) = \big((1-\rho^{n1})(1-\rho^{n2})e_{21} + \rho^{n1}(1-\rho^{n2}))e_{22} - e_3 + e_4 + \rho^{n2}e_5\big)\, B \tag{24}$$

with similar definitions and where ϕ_2 is the AR coefficient on p^2.

Following estimation I compute the relevant long run coefficients. For capital:

$$\begin{aligned} b^{lr}_{gk\_p1} &= \frac{b_{gk\_p1}}{1-\rho^k\phi_1};\ \ b^{lr}_{\delta\_p1} = \frac{\rho^k b_{\delta\_p1}}{1-\rho^k\phi_1} \\ b^{lr}_{d\_p1} &= \frac{(1-\rho^k)b_{d\_p1}}{1-\rho^k\phi_1};\ \ b^{lr}_{r\_p1} = \frac{b_{r\_p1}}{1-\rho^k\phi_1} \end{aligned}$$

where ϕ_1 is the AR coefficient on p^1, the $b_{.\_p1}$ are the coefficients w.r.t p^1 and lr denotes the long-run.

For labor:

$$\begin{aligned} b^{lr}_{gn\_p2} &= \frac{b_{gn\_p2}}{1-\rho^{n2}\phi_2};\ \ b^{lr}_{\psi p} = \frac{\rho^{n2} b_{\psi\_p2}}{1-\rho^{n2}\phi_2} \\ b^{lr}_{d21\_p2} &= \frac{(1-\rho^{n1})(1-\rho^{n2})b_{d21\_p2}}{1-\rho^{n2}\phi_2} \\ b^{lr}_{d22\_p2} &= \frac{\rho^{n1}(1-\rho^{n2})b_{d22\_p2}}{1-\rho^{n2}\phi_2};\ b^{lr}_{r2\_p2} = \frac{b_{r2\_p2}}{1-\rho^{n2}\phi_2} \end{aligned}$$

with similar definitions and where ϕ_2 is the AR coefficient on p^2.

7 Appendix F

Relating the Model to the Data in $\frac{u}{n}-\frac{v}{n}$ Space

7.1 The Data

The unemployment data include the following three alternatives: In one it is the official unemployment pool. In a second, it is the official unemployment pool plus marginally attached workers; these are defined as persons who want a job, have searched for work during the prior 12 months, and were available to take a job during the reference week, but had not looked for work in the past 4 weeks.[9] In a third it is the official unemployment pool plus workers who "want a job;" these are workers who are out of the labor force but replied (in the CPS) in the affirmative to the question if they want a job now.[10] Using these variables, and a vacancy series,[11] Figure 3 in the main text plots the data and the model steady state equations (30) and (31) in $\frac{u}{n} - \frac{v}{n}$ space for official unemployment.

7.2 Construction of Figure 3

To see how Figure 3 is constructed start off from the equations:

$$\left[\begin{array}{c} e_2(\lambda\frac{v}{n} + (1-\lambda)\left(\frac{\mu\left(\frac{v}{n}\right)^{1-\sigma}\left(\frac{u}{n}\right)^{\sigma}}{\frac{v}{n}}\right)\frac{v}{n})\left(\lambda + (1-\lambda)\left(\frac{\mu\left(\frac{v}{n}\right)^{1-\sigma}\left(\frac{u}{n}\right)^{\sigma}}{\frac{v}{n}}\right)\right) \\ +e_3\left(\frac{i}{k}\right)\left(\frac{\mu\left(\frac{v}{n}\right)^{1-\sigma}\left(\frac{u}{n}\right)^{\sigma}}{\frac{v}{n}}\right) \end{array}\right] = \frac{1}{1-\tau}\frac{Q^N}{\frac{f}{n}} \tag{25}$$

$$\mu\left(\frac{v}{n}\right)^{1-\sigma}\left(\frac{u}{n}\right)^{\sigma} = \psi + g \tag{26}$$

For each sub-sample period in Figure 3, I insert the average sample value of $\frac{1}{1-\tau}\frac{Q^N}{\frac{f}{n}} = (1-\tau_t)\, g_{v_t}, \frac{i_t}{k_t}, \psi_t$, and g_t, where the latter is labor force $(u_t + n_t)$ growth.

[9] FRED code is LNU05026642.
[10] FRED code NILFWJN.
[11] See Appendix B for the computation of the vacancy series.

I use the point estimates of the preferred specification (Table 2a row 4) for e_2 and e_3. As the GMM estimates pertain to a specification which implies $\lambda = 0$, I use here an arbitrary low value of λ, set at 0.01.

I use a conventional estimate of $\sigma = 0.5$ (see Yashiv (2007)) and I solve (26) for μ using the sample average values of $\frac{v}{n}$ and $\frac{u}{n}$.

This allows me to plot (25) and (26) to which I add the actual data points of $\frac{v}{n}$ and $\frac{u}{n}$ and get Figure 3. When doing so it turns out that the sample period can be sub-divided into three sub-periods (1976 – 1991, 1992 (or 1994, depending on data availability) – 2006, and 2007 – 2011) so that the data points are scattered in a reasonable way around the intersection of the two curves.

The above procedure is repeated for each sub-sample and for each definition of unemployment.

7.3 Additional Tables

Table F-1

$u =$ **official unemployment+marginally attached**

a. Total

	1994 – 2006	**2007 – 2011**	**Full sample average**
$\frac{v}{n}$	0.031	0.024	0.029
$\frac{u}{n}$	0.065	0.098	0.074
$\frac{h}{n}$	0.123	0.124	0.123
$\frac{i}{k}$	0.024	0.023	0.024
$\frac{1}{1-\tau}\frac{Q^N}{\frac{f}{n}}$	0.212	0.344	0.242
ψ	0.122	0.126	0.123
g	0.0031	0.0010	0.0024
μ	2.78	2.61	2.70

b. Only $\frac{i}{k}$ changes

	1994 – 2006	**2007 – 2011**
$\frac{v}{n}$	0.028	0.033
$\frac{u}{n}$	0.078	0.066
μ	full sample average	
$\frac{i}{k}$	0.024	0.023
$\frac{1}{1-\tau}\frac{Q^N}{\frac{f}{n}}$	full sample average	
ψ	full sample average	
g	full sample average	

c. Only $\frac{1}{1-\tau}\frac{Q^N}{\frac{f}{n}}$ changes

	1994 – 2006	**2007 – 2011**
$\frac{v}{n}$	0.033	0.020
$\frac{u}{n}$	0.065	0.106
μ	full sample average	
$\frac{i}{k}$	full sample average	
$\frac{1}{1-\tau}\frac{Q^N}{\frac{f}{n}}$	0.212	0.344
ψ	full sample average	
g	full sample average	

d. Only ψ changes

	1994 – 2006	**2007 – 2011**
$\frac{v}{n}$	0.028	0.033
$\frac{u}{n}$	0.077	0.069
μ	full sample average	
$\frac{i}{k}$	full sample average	
$\frac{h}{n}$	full sample average	
$\frac{1}{1-\tau}\frac{Q^N}{\frac{f}{n}}$	full sample average	
ψ	0.122	0.126
g	full sample average	

e. **Only g changes**

	1994 – 2006	**2007 – 2011**
$\frac{v}{n}$	0.030	0.027
$\frac{u}{n}$	0.073	0.077
μ	full sample average	
$\frac{i}{k}$	full sample average	
$\frac{h}{n}$	full sample average	
$\frac{1}{1-\tau}\frac{Q^N}{\frac{f}{n}}$	full sample average	
ψ	full sample average	
g	0.0031	0.0010

Table F-2

u = official unemployment + "want a job"

a. Total

	1994 – 2006	**2007 – 2011**	**Full sample average**
$\frac{v}{n}$	0.031	0.024	0.029
$\frac{u}{n}$	0.091	0.123	0.100
$\frac{h}{n}$	0.123	0.124	0.123
$\frac{i}{k}$	0.024	0.023	0.024
$\frac{1}{1-\tau}\frac{Q^N}{\frac{f}{n}}$	0.210	0.346	0.240
ψ	0.122	0.126	0.123
g	0.0028	0.0011	0.0023
μ	2.34	2.33	2.32

b. Only $\frac{i}{k}$ changes

	1994 – 2006	**2007 – 2011**
$\frac{v}{n}$	0.028	0.032
$\frac{u}{n}$	0.105	0.090
$\frac{i}{k}$	0.024	0.023
$\frac{1}{1-\tau}\frac{Q^N}{\frac{f}{n}}$	full sample average	
ψ	full sample average	
g	full sample average	

c. Only $\frac{1}{1-\tau}\frac{Q^N}{\frac{f}{n}}$ changes

	1994 – 2006	**2007 – 2011**
$\frac{v}{n}$	0.034	0.020
$\frac{u}{n}$	0.087	0.145
$\frac{i}{k}$	full sample average	
$\frac{1}{1-\tau}\frac{Q^N}{\frac{f}{n}}$	0.210	0.346
ψ	full sample average	
g	full sample average	

d. Only ψ changes

	1994 – 2006	**2007 – 2011**
$\frac{v}{n}$	0.028	0.033
$\frac{u}{n}$	0.103	0.093
$\frac{i}{k}$	full sample average	
$\frac{h}{n}$	full sample average	
$\frac{1}{1-\tau}\frac{Q^N}{\frac{f}{n}}$	full sample average	
ψ	0.122	0.126
g	full sample average	

e. Only g changes

	1994 – 2006	**2007 – 2011**
$\frac{v}{n}$	0.030	0.028
$\frac{u}{n}$	0.099	0.103
$\frac{i}{k}$	full sample average	
$\frac{h}{n}$	full sample average	
$\frac{1}{1-\tau}\frac{Q^N}{\frac{f}{n}}$	full sample average	
ψ	full sample average	
g	0.0028	0.0011

Notes:

1. For each sub -period one of the variables $\frac{i}{k}, \frac{1}{1-\tau}\frac{Q^N}{\frac{f}{n}}, \psi$ or g is taken to be at its **sub-sample** average while the rest are taken to be at their **full sample** average. This then is computed four times, each time picking another variable.

2. For each of the above permutations, $\frac{v}{n}$ and $\frac{u}{n}$ are **solved out** of the two equations:

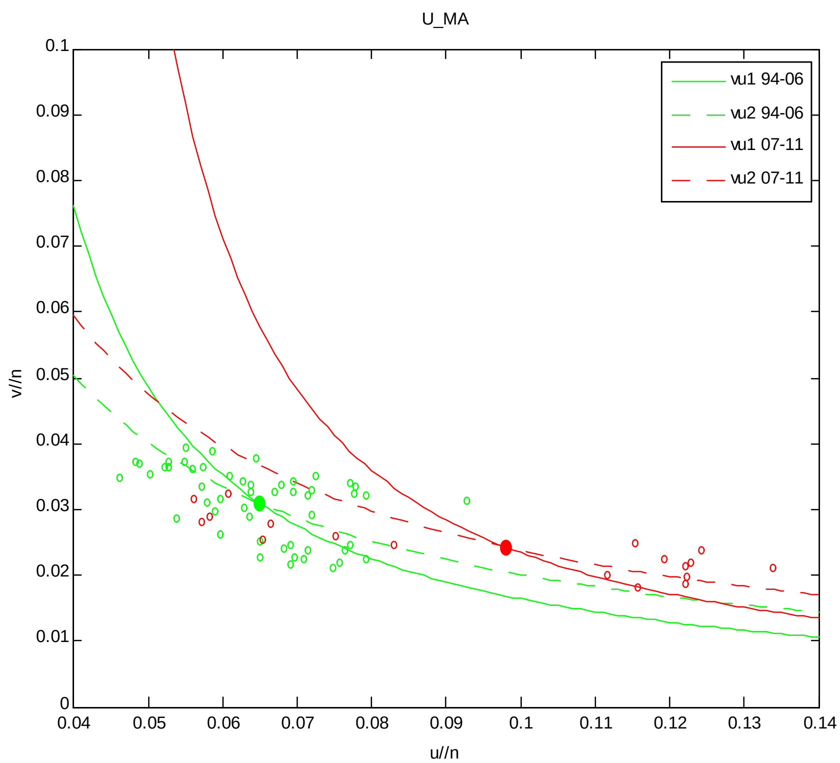
$$\left[\begin{array}{c} e_2(\lambda\frac{v}{n} + (1-\lambda)\left(\frac{\mu\left(\frac{v}{n}\right)^{1-\sigma}\left(\frac{u}{n}\right)^{\sigma}}{\frac{v}{n}}\right)\frac{v}{n})\left(\lambda + (1-\lambda)\left(\frac{\mu\left(\frac{v}{n}\right)^{1-\sigma}\left(\frac{u}{n}\right)^{\sigma}}{\frac{v}{n}}\right)\right) \\ +e_3\left(\frac{i}{k}\right)\left(\frac{\mu\left(\frac{v}{n}\right)^{1-\sigma}\left(\frac{u}{n}\right)^{\sigma}}{\frac{v}{n}}\right) \end{array}\right] = \frac{1}{1-\tau}\frac{Q^N}{\frac{f}{n}}$$

$$\mu\left(\frac{v}{n}\right)^{1-\sigma}\left(\frac{u}{n}\right)^{\sigma} = \psi + g$$

3. The parameters $\lambda, e_2, e_3, \sigma$ are always constant.

7.4 Additional Figures

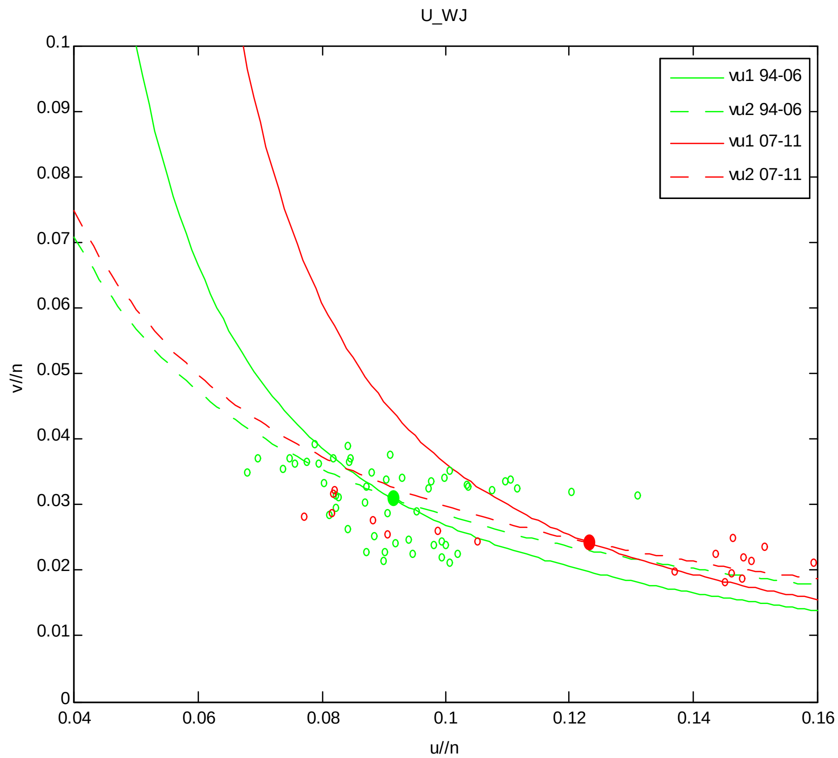
Figure F-1
Unemployment-Vacancies Analysis



$u =$ **official unemployment+marginally attached**

Notes:

1. The figure pertains to the sub-periods 94-06; 07-11.
2. The solid line is the vacancy creation curve (equation 30) and the dashed line the steady state flows curve (equation 31).



$u =$ **official unemployment + "want a job"**

Notes:

1. The figure pertains to the sub-periods 94-06; 07-11.

2. The solid line is the vacancy creation curve (equation 30) and the dashed line the steady state flows curve (equation 31).

Notes:

1. For each sub -period one of the variables $\frac{i}{k}$, $\frac{1}{1-\tau}\frac{Q^N}{\frac{f}{n}}$, ψ, g or μ is taken to be at its **sub-sample** average while the rest are taken to be at their **full sample** average. This then is computed four times, each time picking another variable.

2. For each of the above permutations, $\frac{v}{n}$ and $\frac{u}{n}$ are **solved out** of the two equations:

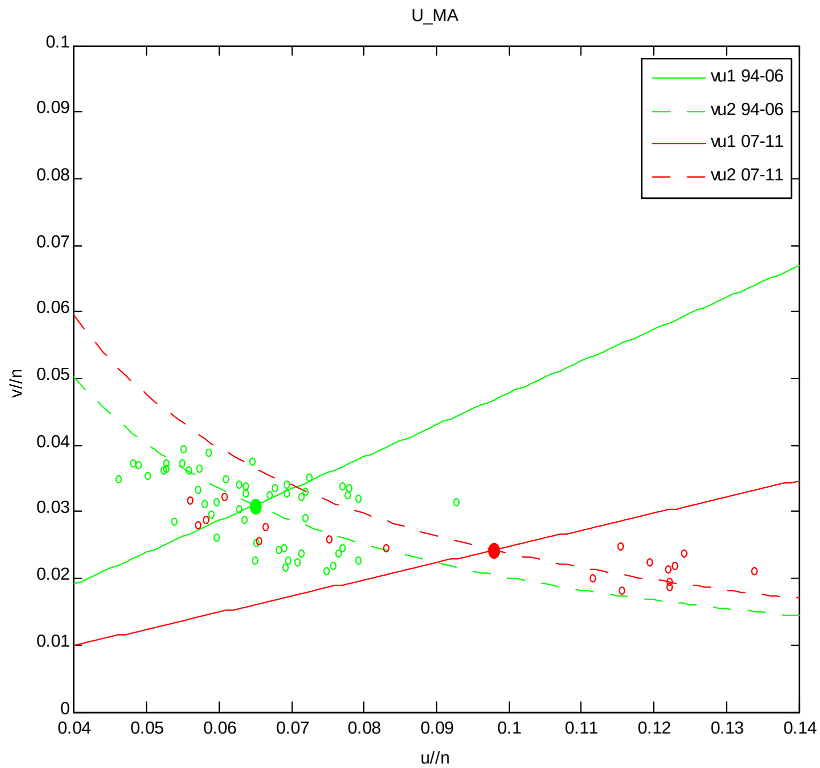
$$\left[\begin{array}{c} e_2(\lambda\frac{v}{n} + (1-\lambda)\left(\frac{\mu\left(\frac{v}{n}\right)^{1-\sigma}\left(\frac{u}{n}\right)^{\sigma}}{\frac{v}{n}}\right)\frac{v}{n})\left(\lambda + (1-\lambda)\left(\frac{\mu\left(\frac{v}{n}\right)^{1-\sigma}\left(\frac{u}{n}\right)^{\sigma}}{\frac{v}{n}}\right)\right) \\ +e_3\left(\frac{i}{k}\right)\left(\frac{\mu\left(\frac{v}{n}\right)^{1-\sigma}\left(\frac{u}{n}\right)^{\sigma}}{\frac{v}{n}}\right)\end{array}\right] = \frac{1}{1-\tau}\frac{Q^N}{\frac{f}{n}}$$

$$\mu\left(\frac{v}{n}\right)^{1-\sigma}\left(\frac{u}{n}\right)^{\sigma} = \psi + g$$

The first equation is labeled $uv1$ and the second equation is labeled $uv2$.

3. The parameters $\lambda, e_2, e_3, \sigma$ are always constant.

Figure F-2
Unemployment-Vacancies Analysis of the Pissarides (2000) Model

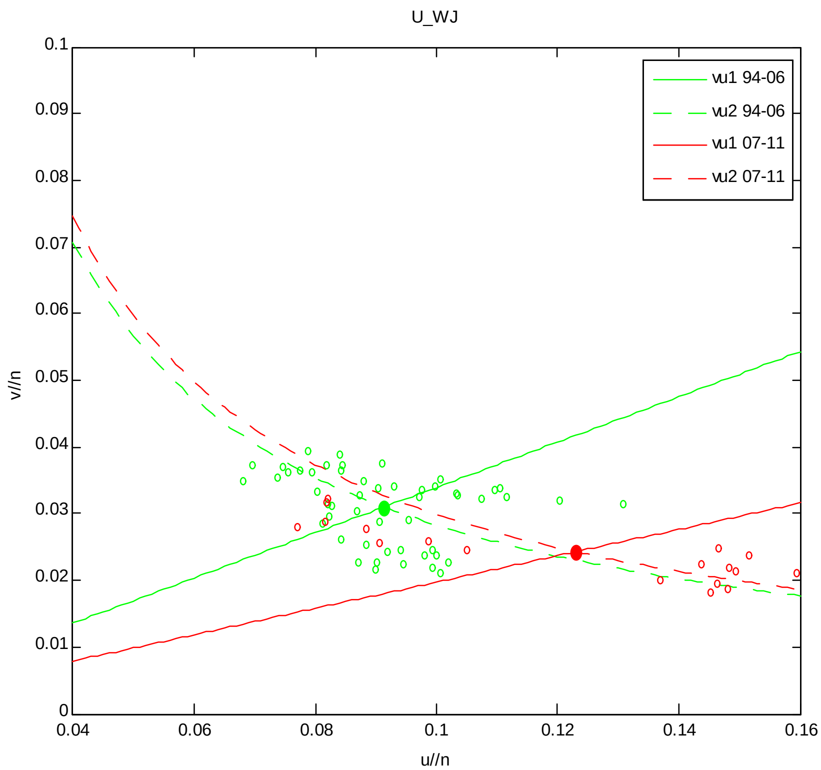


$u =$ **official unemployment+marginally attached**

Notes:

1. The figure pertains to the sub-periods 94-06; 07-11.

2. The solid line is the vacancy creation curve (equation 32) and the dashed line the steady state flows curve (equation 31).



$u =$ **official unemployment + "want a job"**

Notes:

1. The figure pertains to the sub-periods 94-06; 07-11.

2. The solid line is the vacancy creation curve (equation 32) and the dashed line the steady state flows curve (equation 31).

Notes:

1. The figure shows the Piassarides (2000) model, with the vacancy creation curve (labeled $uv1$):

$$\begin{aligned} Q^N_{t,search} &= (1-\tau_t)c\frac{v_t}{h_t} \\ Q &= (1-\tau)c\mu\left(\frac{v}{u}\right)^\sigma \end{aligned}$$

and the steady state flow equation:

$$\mu\left(\frac{v}{n}\right)^{1-\sigma}\left(\frac{u}{n}\right)^\sigma = \psi + g$$

This equation is labeled $uv2$.

2. It uses use average data values of τ, ψ, g for each period and $\sigma = 0.5$.

3. The parameter μ is first solved out from the second equation using average values for $\frac{v}{n}, \frac{u}{n}$ each sub-period; then c is solved out the first equation using average values for $\frac{v}{n}, \frac{u}{n}$ each sub-period and Q^N from the current estimates