Abstract

The paper examines optimal policy responses to COVID 19 by building on an epidemiological model based on analysis of the biology of the SARS-CoV-2 virus. The epidemiological model maps the distributions of serial and generation intervals of the coronavirus into a SEIR model with Erlang distributions. This model is then used in an optimizing social planner problem.

Among a number of policy strategies considered, special attention is given to cyclical strategies of $k$ days of work and $14 - k$ days of lockdown in fortnight batches, coming after a period of full lockdown. The planner optimally chooses the strategy in terms of $k$ days and in terms of three starting-time points (lockdown, implementation of the $k$ strategy, and release).

The results, applied using calibrated values for the U.S. economy, indicate that cyclical strategies provide for significant improvement in the trade-offs of health outcomes (deaths, breaches of ICU capacity) and economic outcomes (loss of output, employment, and consumption).

The paper explores the underlying mechanism. A key aspect is that there is a fundamental choice between aiming for “effective $R$” herd immunity and waiting for a vaccine.

Inter alia, we show that some prevalent SIR modeling in the recent COVID 19 Economics literature derives disease dynamics, which are erroneous, and which leads to a mis-specification of the planner problem.
COVID-19: Looking for the Exit

1 Introduction

As of mid-2020, countries are facing difficult choices in terms of health and economic outcomes, with lockdowns seriously disrupting economic and social activity while the COVID-19 pandemic risks lives. The difficulty is finding a strategy that lessens this trade-off, a task made all the more difficult as large-scale testing is not available in many affected countries, making it challenging to identify sub-populations that may be allowed out of the lockdown, and to determine how long other groups need to remain sequestered. The idea examined in this paper is to exploit timescales of the virus and formulate a cyclical strategy of work and lockdown, as proposed by Karin et al (2020). This paper examines the idea in economic terms.

The paper examines optimal policy responses to COVID 19 by building on an epidemiological model based on analysis of the biology of the SARS-CoV-2 virus, the timescales associated with the infection, and key epidemiological parameters. The epidemiological model maps the distributions of serial and generation intervals of the coronavirus into a SEIR model with Erlang distributions. This model is then used in an optimizing social planner problem. Special attention is given to cyclical strategies, whereby the economy undergoes 14 days cycles of \( k \) days of work and 14 – \( k \) days of lockdown. The planner optimally chooses the strategy in terms of days worked and in terms of the starting time points of (i) lockdown, (ii) implementation of the cyclical strategy, and (iii) release. We apply the model to calibrated values of the U.S. economy.

We find that trade-offs of health outcomes (deaths, breaches of ICU capacity) and economic outcomes (loss of output, employment, and consumption) are substantial. Cyclical strategies provide for significant improvement. Relative to no intervention, they save almost 60% of annual GDP in PDV terms over 18 months; relative to a full lockdown they save about 20% of annual GDP over 18 months in PDV terms.

A fundamental dilemma exists between aiming for “effective \( R \)” herd immunity and waiting for a vaccine. The resolution of the dilemma depends on the cost of lockdowns in terms of output relative to the cost of lost lives, and on the set of instruments available to the planner.

In the benchmark calibration, the cyclical strategies with low \( k \) provide for 20 times lower death rates than high \( k \) strategies. This happens at the price of increased loss – as much as a multiple of three – in foregone output.

The idea of a cyclical exit strategy has been brought to the attention of policymakers (see Yashiv (2020), Alon and Yashiv (2020), and Alon, Milo

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1We thank seminar participants at UCL for comments. Any errors are our own.
and Yashiv (2020)) and is being considered or implemented by firms and institutions, as of June 2020, mostly in the U.S.

The paper proceeds as follows: in Section 2 we present some key data facts on COVID 19 lockdowns and their economic effects and discuss the relevant literature. Section 3 presents the biological analysis of the SARS-CoV-2 virus. Section 4 discusses the model, while Section 5 presents the calibration and the solution methodology. Section 6 presents the results and Section 7 explores the mechanism. Section 8 offers a discussion of (i) the relation between the model planner solution and what we observe in the real world; (ii) problematic modelling of the disease in some papers in Economics. Section 9 elaborates on issues to be studied in future versions of the paper. Section 10 briefly concludes.

2 Background

2.1 Data Facts

Figure 1\(^2\) shows the time path of a lockdown index computed by Goldman Sachs for seven countries from early February to early June 2020. It combines Google global mobility reports of daily data on smartphone user behavior\(^3\) with the government response stringency index complied by Oxford University\(^4\). The latter is based on the following categories: school closings, workplace closings, public event cancellations, closure of public transportation, public information campaigns, internal movement restrictions, and international travel controls.

\(^{2}\)Using data taken from Goldman Sachs.
\(^{3}\)See https://www.google.com/covid19/mobility/
\(^{4}\)See https://covidtracker.bsg.ox.ac.uk/stringency-scatter
Figure 1: GS Effective Lockdown Index

The figure shows that there was a fast rise in lockdown measures, then stability over a few weeks, and subsequently a slow release.

Figures 2-4 show the associated economic performance. Figure 2 shows four major advanced economies, India, and China\textsuperscript{5}, with double digit declines in GDP. The lows range from 15\% to 25\%. Figure 3 shows the OECD June 2020 projections of GDP and unemployment for the U.S. economy,\textsuperscript{6} with a decline of up to 40\% in GDP (Q to Q annualized terms) and a jump of unemployment from 4\% to 15\%. Figure 4 shows ONS monthly GDP data for the U.K.,\textsuperscript{7} indicating drops of 20.4\% in April following a 5.8\% drop in March.

\textsuperscript{5}Taken from Goldman Sachs, Global Views, June 8, 2020.
\textsuperscript{6}See https://www.oecd.org/economic-outlook/ and https://www.oecd-ilibrary.org/sites/0d1d1e2e-en/1/3/3/47/index.html?itemId=/content/publication/0d1d1e2e-en&_csp_=bfaa0426ac4b641531f0226ccc9a886&ItemGO=oecd&itemContentType=#
\textsuperscript{7}See https://www.ons.gov.uk/economy/grossdomesticproductgdp/bulletins/gdpmonthlyestimateuk/april2020#gdp-fell-by-204-in-april-2020
Figure 2: Goldman Sachs Analysis of Key Economies

Figure 3: U.S. GDP and unemployment projections, OECD, June 2020
2.2 Literature

There has been an explosion of research on COVID-19. The papers most related to the current one are those that examine an optimal planner. These include Acemoglu, Chernozhukov, Werning, and Whinston (2020), Alvarez, Argente, and Lippi (2020), Chari, Kirpalani, and Phelan (2020), Jones, Philippon, Venkateswaran (2020), and Farboodi, Jarosch, and Shimer (2020). Two differences of the current paper with respect to these papers are: (i) they do not examine the cyclical strategy at the focal point here; (ii) some of the papers model disease dynamics erroneously (in terms of the parameters used), as discussed below.

3 Biological Analysis of the SARS-CoV-2 Virus.

At the core of many mathematical frameworks for modeling the spread of infectious diseases such as COVID-19, lie key timescales which characterize the transmission of the disease between individuals, as well as its progression within an infected individual. The most basic relevant timescale is known as the generation interval, which is defined for an infector-infectee pair. The generation interval is defined as the time between the infection of the infector and the infection of the infectee. This duration is, however, hard to directly quantify, as it is hard to pinpoint the exact time in which transmission occurred. The more commonly observed quantity is known as the serial interval, which measures the time between symptom onset in the infector and symptom onset in the infectee. To infer the generation in-
terval from the observed serial interval, a third timescale is needed. This
timescale, known as the incubation period, is defined as the time from in-
fecction with the virus until symptom onset.

It is clear that these three timescales could vary significantly between
individuals, and thus they are better represented at the population level as
probability density functions. Once the distribution of generation intervals
is inferred, it can be used in the context of a common mathematical frame-
work for modeling the disease – the renewal equation. In its simplest form,
the renewal equation dictates that:

$$I(t) = \int I(t - \tau) \cdot \beta(\tau) d\tau$$  \hspace{1cm} (1)

Where $I(t)$ is the number of infected people at time $t$, and $\beta(\tau)$ is the
transmission rate of people in day $\tau$ after their infection (in units of days$^{-1}$).
The function $\beta(\tau)$ is also known as the infectiousness profile, and is defined
to be the same as the probability distribution function of the generation in-
terval, scaled by the reproduction number $R(t)$, which is the average num-
ber of people an infected person infects over the course of their infection:

$$\beta(\tau) = R(t) \cdot g(\tau)$$  \hspace{1cm} (2)

An alternative modeling approach attempts to discretize the infectious-
ness profile by splitting up the infectiousness profile of an infected individual
into distinct compartments – the Exposed ($E$), Infectious ($I$), and Re-
covered ($R$) compartments. This discretization is at the heart of the widely
used Susceptible-Exposed-Infectious-Recovered ($SEIR$) model. In both the
$E$ and $R$ compartments, infectiousness is zero, whereas in the $I$ com-
partment, the transmission rate is $\beta$, which is constant. An infected individual
spends a specific amount of time in each compartment, before moving to
the next compartment. The time spent in the $E$ and $I$ compartment are
known as the latent and infectious periods, respectively. The overall infec-
tiousness across the entire disease progression for an infected individual,
which is the reproduction number, is just $\beta$ times the infectious period. The
$SEIR$ model can be defined in terms of differential equations in the follow-
ing manner:

$$\dot{S}(t) = -\beta(t) \cdot I(t) \cdot S(t)$$  \hspace{1cm} (3)
$$\dot{E}(t) = \beta(t) \cdot I(t) \cdot S(t) - \sigma E(t)$$  \hspace{1cm} (4)
$$\dot{I}(t) = \sigma E(t) - \gamma I(t)$$  \hspace{1cm} (5)
$$\dot{R}(t) = \gamma I(t)$$  \hspace{1cm} (6)
Where $S$, $E$, $I$ and $R$ are the number of people in the respective compartments, $\beta$ is the constant transmission rate during the infectious period, $\sigma$ is the rate at which a person moves from the $E$ to the $I$ compartment and $\gamma$ is the rate at which a person moves from the $I$ to the $R$ compartment. Because $\sigma$ and $\gamma$ are constant, this implies that the time spend in either the $E$ and $I$ compartments is exponentially distributed with a mean of $1/\sigma$ and $1/\gamma$. Because the time spent in the $E$ and $I$ compartments are the latent and infectious periods, this implies the shape of these periods is distributed according to the following formula:

$$T_L(t) = \sigma \exp(-\sigma t); \quad T_I(t) = \gamma \exp(-\gamma t)$$

(7)

Where $T_L(t)$ and $T_I(t)$ are the probability density functions of the latent and infectious periods, respectively. Exponentially distributed latent and infectious periods imply that most people spend zero amount of time being either in their latent period or in their infectious period, which is not very accurate biologically. Therefore, to produce more accurate distributions for the exposed and infectious periods, with a mode near their mean, people can employ a mathematical trick in which the $E$ and $I$ compartments are split into two compartments each, with a double the rate of transfer between them. In terms of differential equations this corresponds to:

$$S(t) = -\beta(t) \cdot (I_1(t) + I_2(t)) \cdot S(t)$$

(8)

$$\dot{E}_1(t) = \beta(t) \cdot (I_1(t) + I_2(t)) \cdot S(t) - 2\sigma E_1(t)$$

(9)

$$\dot{E}_2(t) = 2\sigma E_1(t) - 2\sigma E_2(t)$$

(10)

$$\dot{I}_1(t) = 2\sigma E_2(t) - 2\gamma I_1(t)$$

(11)

$$\dot{I}_2(t) = 2\gamma I_1(t) - 2\gamma I_2(t)$$

(12)

$$R(t) = 2\gamma I_2(t)$$

(13)

Now, the latent and infectious periods are the sum of the time spent in the $E_1$ and $E_2$ or $I_1$ and $I_2$ compartments, respectively, and their distribution is thus the same as that of a sum of exponentially distributed random variables. The distribution of the sum of $m$ exponentially distributed random variables is also known as the Erlang distribution, which is a special case of the Gamma distribution with an integer as the shape of the distribution. This type of augmented model is known as the SEIR-Erlang model, and the corresponding probability density distributions for the latent and infectious periods are now described by the following formulas:

$$T_L(t) = (2\sigma)^2 t \exp(-2\sigma t); \quad T_I(t) = (2\gamma)^2 t \exp(-2\gamma t)$$

(14)
The mean of these distributions is still $1/\sigma$ and $1/\gamma$, but now the mode of the distribution also near $1/\sigma$ and $1/\gamma$.

4 The Model

We rely on the afore-going analysis to model an optimizing social planner who operates within a SEIR model of the epidemic and a model of the macroeconomy. The planner minimizes a loss function defined over output loss and value of lost life. All units of time $t$ are in days since outbreak. The planner can lock down the economy at an optimally chosen point in time $T_0$. Starting from another optimally chosen point, $T_1$, we allow the planner to undertake a cyclical strategy, whereby for every 14 day period, people work for $k$ days and are in lockdown for $14 - k$ days. As one of the options is $k = 0$, remaining in lockdown in a special case. At optimally chosen date $T_2$, the planner releases the economy. These plans are made under the perfect foresight of vaccine arrival at $T = 540$.

4.1 The Planner Problem

The planner chooses the following:
(i) the timing of policies – $T_0$ lockdown of the economy; $T_1$ implementation of a cyclical strategy; $T_2$ release of the economy. Throughout we assume that at $T = 540$ a vaccine arrives.
(ii) fraction of days locked when implementing the cyclical strategy, $k, 14 - k$ with $k \in \mathbb{Z}^+$.

The loss function is given by:

$$
\min_{\{T_0,T_1,T_2\}} V = \int_0^\infty e^{-rt} \cdot \left( \frac{Y(t)}{N(t)} (N^{SS} - N(t)) + \chi \cdot D(t) \right) \, dt
$$

4.2 Cyclical Strategies

The cyclical strategy posits $k$ days of work and $14 - k$ days of lockdown. On work days, people are released from lockdown with strict hygiene and physical distancing measures on the same $k$ weekdays for everyone. On lockdown days, people are kept away from work places as well as from other public spaces. Epidemiological measures need to be used and improved throughout, including rapid testing, contact isolation and compartmentalization of workplaces and regions. The strategies are analyzed and
elaborated in Karin et al. (2020). The cyclical strategies available to the planner are described in Table 1.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Name</th>
<th>Workdays Closed</th>
<th>Days, starting Monday</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>In Total</td>
<td>Week 1</td>
</tr>
<tr>
<td>k, 14-k</td>
<td>Full lockdown</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>3-11</td>
<td>3-11</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>4-10</td>
<td>4-10</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>5-9</td>
<td>Week on/off</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>6-8</td>
<td>3 days week</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>7-7</td>
<td>7-7</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>8-6</td>
<td>4 days week</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>14-0</td>
<td>No intervention</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 1: Menu of Cyclical Strategies

4.3 The Epidemiological Model

The planner operates within the Erlang SEIR model described above and which is extended to a number of stages after the resolution stage \( R \). We note that the reproduction number is given by \( R(t) = \frac{\beta(t)}{\gamma} \).

Following resolution, people flow out of \( R \) to recovery (\( C \)) or to hospital (\( H \)). From hospital they may recover or move to ICU (\( X \)), from which they can recover or die. We assume that ICU capacity has an upper bound at \( X \). The flow out of \( R \) is at rate \( \theta_R \); part \( (1 - \xi) \) recover (go to \( C \)) and part \( (\xi) \) go to \( H \). People flow out of \( H \) at rate \( \theta_H \); part \( (\pi) \) go to ICU (\( X \)) and part \( (1 - \pi) \) go to recovery (\( C \)). People flow out of \( X \) at rate \( \theta_X \); part \( (\delta) \) die (\( D \)) and part \( (1 - \delta) \) go to recovery (\( C \)).

The model may be described as follows, with all groups expressed as fractions out of \( POP_t \), the total population. Figure 5 illustrates this set up.

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8The cyclical strategy can be considered as a component of an evolving policy toolkit that can be combined with other interventions. The exact implementation of the cyclical strategy depends on the rate of increase or decrease of the number of infected people in workdays and lockdown days, respectively, as expressed by the effective reproduction number, to be discussed below.
The following equations formalize this structure.

\[ S(t) = I(t \leq T) \cdot S(t) \]  

At \( T \) the vaccine arrives and \( S(t) \) drops to zero.

\[
\begin{align*}
\dot{S}(t) &= -\beta(t) \cdot (I_1(t) + I_2(t)) \cdot S(t) \\
\dot{E}_1(t) &= \beta(t) \cdot (I_1(t) + I_2(t)) \cdot S(t) - 2\sigma E_1(t) \\
\dot{E}_2(t) &= 2\sigma E_1(t) - 2\sigma E_2(t) \\
\dot{I}_1(t) &= 2\sigma E_2(t) - 2\gamma I_1(t) \\
\dot{I}_2(t) &= 2\gamma I_1(t) - 2\gamma I_2(t) \\
\dot{R}(t) &= 2\gamma I_2(t) - \theta R \cdot R(t)
\end{align*}
\]
\[ \dot{H}(t) = \xi \theta_R \cdot R(t) - \theta_H \cdot H(t) \]  
(23)

\[ \dot{X}(t) = \pi \theta_H \cdot H(t) - \theta_X \cdot X(t) \]  
(24)

\[ \dot{C}(t) = (1 - \xi) \cdot \theta_R \cdot R(t) + (1 - \pi) \theta_H \cdot H(t) + (1 - \delta(X(t))) \cdot \theta_X \cdot X(t) \]  
(25)

\[ \dot{D}(t) = \delta(X(t)) \cdot \theta_X \cdot X(t) \]  
(26)

where

\[ \delta(X(t), X) = \delta_1 + \delta_2 \cdot \frac{\mathbb{I}(X(t) > \overline{X}) \cdot (X(t) - \overline{X})}{X(t)} \]  
(27)

### 4.4 The Economy

The economy is described as follows. Population is normalized to 1.

\[ \text{POP} = 1 \]  
(28)

Prior to COVID-19, there is a steady state employment-population ratio of \( l < 1 \):

\[ N^{SS} = l \]  
(29)

During COVID-19 we assume that the pool of available workers is given by \( L(t) \) as follows:

\[ L(t) = S(t) + E(t) + (1 - \phi) \cdot [I(t) + R(t)] + C(t) \]

Two groups do not work: a fraction \( 0 < \phi < 1 \) of the infected and the resolved, and all the people in hospital, including in ICU. Hence workers (on week days) are given by:

\[ N^{\text{week-day}}(t) = \begin{cases} 
L(t) & 0 \leq t < T_0 \\
\rho \cdot L(t) & T_0 \leq t < T_1 \\
\rho \cdot L(t) & T_1 \leq t < T_2; 14 - k \\
L(t) & t \geq T_2 
\end{cases} \]  
(30)

In lockdown only \( \rho \) of \( L(t) \) work. We use a linear production technology:

\[ Y(t) = AN(t) \]  
(31)

In steady state:

\[ Y^{SS}(t) = Al \]  
(32)
We normalize $Y^SS = 1$ so:

$$A = \frac{1}{I}$$ (33)

In the planner problem, loss of output is thus given by:

$$\frac{Y(t)}{N(t)}(N^SS - N(t)) = A (I - N(t)) = 1 - \frac{N(t)}{I}$$ (34)

$$= 1 - \begin{cases} L(t) & 0 \leq t < T_0 \\ \rho L(t) & T_0 \leq t < T_1 \\ L(t) & T_1 \leq t < T_2 \ ; k \\ \rho L(t) & T_2 \leq t < T_2 \ ; 14 - k \\ L(t) & t \geq T_2 \end{cases}$$

5 Calibration and Solution Methodology

We calibrate the model as follows. Throughout we work in daily terms.

5.1 The Epidemiological Model

We use the following values based on Bar-On et al (2020). For the SEIR part we postulate:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Benchmark</th>
<th>Definition (inverse of average duration)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>1/3</td>
<td>exposed stage</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1/4</td>
<td>infectious stage</td>
</tr>
<tr>
<td>$\theta_R$</td>
<td>1/5</td>
<td>resolved state</td>
</tr>
</tbody>
</table>

For the HXDC part we postulate:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Benchmark</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_H$</td>
<td>1/2</td>
<td>non-ICU hospitalization</td>
</tr>
<tr>
<td>$\theta_X$</td>
<td>1/5.5</td>
<td>ICU stay, till one recovers or dies</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.04</td>
<td>$H$ probability</td>
</tr>
<tr>
<td>$\pi$</td>
<td>0.40</td>
<td>$X$ probability</td>
</tr>
<tr>
<td>$\delta(X(t), X)$</td>
<td>$D$ probability in $X$</td>
<td></td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$1.8 \times 10^{-4}$</td>
<td>ICU capacity.</td>
</tr>
</tbody>
</table>
where $\bar{X} = \frac{58,094}{329,329 + 10^6} = 1.8 \times 10^{-4}$ is based on an estimate of 58,094 ICU beds estimate by the Harvard Global Health Institute.\(^9\) We get $IFR = \xi \cdot \pi \cdot \delta_1 = 0.008$, consistent with the estimates of the Imperial College COVID-19 Response Team (2020).

For the path of the reproduction parameter, we assume initially

$$R_0 = 2.50 \tag{35}$$

Subsequently we posit

$$R(t) = \begin{cases} 1.50 & \text{work} \\ 0.80 & \text{lockdown} \end{cases} \tag{36}$$

We assume the decline to 1.50 happens only if preceded by a 14 day minimum of lockdown.

We justify these values as follows, assuming throughout that rational individual behavior\(^10\) and suppression policy underlie the declines in $R(t)$ values.

*Initial levels.* We get the value of 2.50 in (35) from U.S. estimates by Jones and Villaverde (2020). Their data analysis yields $R_0$ values of 2.59 on March 17, 2020 and 2.42 on March 18, 2020.

*Subsequent values.* Bar-On et al (2020) review the literature and estimate values for $R(t)$ in work and lockdown periods. Figure 6 reports their estimates.

\(^9\)See https://globalepidemics.org/our-data/hospital-capacity/

We therefore use 1.50 for work periods and 0.80 for lockdown periods.

**Dynamics of the reproduction parameter.** We look at two sources.

a. Jones and Villaverde (2020) infer $R_t$ from daily death flow data in the U.S. taken from Johns Hopkins University CSSE (2020). They use the following equations.

$$\beta_t = \frac{\text{POP}}{S_t} \cdot \left( \gamma + \frac{1}{\delta} \Delta d_{t+3} + \Delta d_{t+2} \right)$$

$$S_{t+1} = S_t \cdot \left( 1 - \frac{\beta_t}{\delta} \text{POP} \cdot \left( \frac{1}{\theta} \Delta d_{t+2} + d_{t+1} \right) \right)$$

We follow these authors here in positing $1/\gamma = 5$ (Infectious stage), $1/\theta = 10$ (Resolved stage), $\delta = 0.01$ (Infection Fatality Rate), and initial $S_0 \approx \text{POP}$. Using these equations, data, and calibrated values, it takes 9 days to get from $R_t = 2.42$ to $R_t = 1.5$.

b. Imperial College COVID-19 Response Team (2020) estimates $R_t$ for U.S. states since the start of the epidemic using a log-linear growth function

$$\ln R(T_s^1) = \ln R(T_s^0) - \alpha_{\log-linear} \cdot (T_s^1 - T_s^0)$$

We obtain two alternative estimates for the average speed of a $R_t$ decline from 2.5 to 1.5 in the period $T_s^0$ to $T_s^1$:

a. $T_s^0$ = first day in the state; $T_s^1$ = end of the decline (i.e., the point where $R$ is not statistically different from 1 at 5%):

$$\ln \left( \frac{2.5}{1.5} \right) = 0.027 \Rightarrow 19.2 \text{ days}$$

b. $T_s^0$ = highest $R_t$ in the state; $T_s^1$ as in (a):

$$\ln \left( \frac{2.5}{1.5} \right) = 0.065 \Rightarrow 7.9 \text{ days}$$

The decay time of $R_t$ is thus 9 days based on Jones and Villaverde (2020) national death data, or 8 or 19 days based on the state-level data, Imperial model estimates. We adopt a conservative calibration and assume that 14 days must pass before $R_t$ declines from 2.50 to 1.50.

### 5.2 The Economy

We posit a 4% annual discount rate for the planner ($r = 0.04$), converted to daily terms. We calibrate the fraction of non-employed in $L(t)$ to be zero ($\phi = 0$).

**The value of $\rho$.** We use a number of sources to try to determine $\rho$, the fraction of people working out of $L(t)$ when in lockdown.

a. BLS data on the employment-population ratio is the following for the relevant period.
This implies the following for April-May 2020:

\[
\rho = \frac{N(t)}{l} = \frac{0.513}{0.61} = 0.84 \text{ (April)}
\]

\[
= \frac{0.528}{0.61} = 0.87 \text{ (May)}
\]

b. Studying remote work, Dingel and Neiman (2020) find that 37 percent of jobs in the United States can be performed entirely at home, with significant variation across cities and industries. The analysis by Yasenov (2020) across population groups suggests this number may be higher on average.

c. An Economic Policy Institute (EPI) analysis\(^{11}\) puts essential workers at 55,217,845. February employment was 158,759,000; hence \(\frac{55,217,845}{158,759,000} = 0.35\).

d. A McKinsey Global Institute analysis\(^{12}\) of all 804 occupations tracked by the BLS (O*NET data) assign each one a vulnerability rating of low, medium, or high. Low-vulnerability jobs are the essential ones, require no physical proximity to others, or are likely to guarantee pay even if workers are furloughed. Medium-vulnerability jobs require workers to be in proximity to coworkers but not the public; shutdowns affect 30 to 50 percent of these jobs. High-vulnerability jobs are nonessential roles that involve exposure to the public; shutdowns affect 70 to 90 percent of these jobs. This analysis estimates that a nationwide shutdown could leave 44 million to 57 million jobs vulnerable. Note that this analysis covers both remote work

\(^{11}\)See https://www.epi.org/blog/who-are-essential-workers-a-comprehensive-look-at-their-wages-demographics-and-unionization-rates/

and essential workers. Hence $\rho = 1 - \frac{44}{158.759} = 0.72$ to $\rho = 1 - \frac{57}{158.759} = 0.64$.

From this discussion a reasonable conjecture is that $\rho \in [0.65, 0.80]$; we take $\rho = 0.7$ at the baseline. Subsequently we also examine $\rho = 0.60, 0.80$

**Value of Statistical Life.**

We follow the analysis in Hall, Jones, and Klenow (2020), who state that estimates of the Value of Statistical Life per year range from $100,000 to $400,000, with a benchmark of $270,000, and that COVID deceased have an expected average of 14 years of life remaining. With GDP per capita at $65,351 this yields:

$$\chi = \frac{\text{expected years remaining} \cdot VSLY}{\text{POP}}$$

$$= \frac{14 \times 270,000}{65,351} = 14 \times 4.13 = 57.8$$

We also study two variations, using the range of estimates given above.

a. $\chi = \frac{14 \times 100,000}{65,351} = 21.423$

b. $\chi = \frac{14 \times 400,000}{65,351} = 85.691$

We note that the point (a) estimate is consistent with Alvarez et al (2020) and point (b) with Greenstone and Nigam (2020).

### 5.3 Solution Methodology

We use a numerical solver in Mathematica (NDSolve) to solve the continuous time system of ODE with initial conditions and the calibrated values discussed above.

Initial values are set from an initial seed of 0.01% of the population (100 people per million). Stages of the disease ($E, I, R$) are initialized pro-rata according to their length in days. Compartments under $E$ and $I$ are initialized to be equal in number each. $H$ and $X$ are initialized according to their expected probabilities. Susceptibles are the rest of the pool (initially, there are no $C$ or $D$).

The solution of the system of ODE is a set of interpolated functions of stocks (of population) dynamics. One of the equations is the planner’s objective; its value at end of simulation is the target of minimization. For the three discrete control variables ($T_0, T_1, T_2$), we evaluate the objective function using ‘brute force.’ This is done first over a coarse grid of integer values. To find the global minimum, we maintain a set of best possible minima, spanning the control variables space. We then recursively refine the grid, until the desired granularity (of 1 day) is reached.
6 Results

Table 4 and Figures 8 and 9 report the results with the baseline calibration for (i) a no intervention scenario; (ii) a full lockdown scenario; (iii) seven variants of the $k$ strategies. The table reports the planner optimal $T_0, T_1, T_2$, the resulting values $V$ (in annual GDP terms evaluated over the entire horizon), decomposed into $V_Y$ the value of foregone output and $V_D$, the value of lost life, and the cumulative number of dead (per 1 million people). The figures show the time evolution of $I$ and of $X$.

Table 4: Baseline Results

<table>
<thead>
<tr>
<th>Baseline</th>
<th>$T_0$</th>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$V$</th>
<th>$V_Y$</th>
<th>$V_D$</th>
<th>$D$ (per 10$^6$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Intervention</td>
<td>540</td>
<td>540</td>
<td>540</td>
<td>0.78</td>
<td>0.023</td>
<td>0.752</td>
<td>13,113</td>
</tr>
<tr>
<td>Full lockdown</td>
<td>0</td>
<td>540</td>
<td>540</td>
<td>0.43</td>
<td>0.431</td>
<td>0.000</td>
<td>3</td>
</tr>
</tbody>
</table>

$k = 0$
- $\begin{bmatrix} 40 & 91 & 91 \\ 0 & 14 & 413 \\ 0 & 14 & 455 \\ 0 & 14 & 497 \\ 0 & 71 & 528 \\ 33 & 50 & 359 \\ 33 & 52 & 331 \end{bmatrix}$
- $\begin{bmatrix} 0.29 & 0.047 & 0.248 \\ 0.24 & 0.234 & 0.007 \\ 0.23 & 0.221 & 0.007 \\ 0.21 & 0.202 & 0.013 \\ 0.22 & 0.206 & 0.015 \\ 0.21 & 0.090 & 0.121 \\ 0.20 & 0.064 & 0.141 \end{bmatrix}$
- $\begin{bmatrix} 4,344 \\ 121 \\ 121 \\ 231 \\ 275 \\ 2,121 \\ 2,471 \end{bmatrix}$

$k = 3$
- $\begin{bmatrix} 40 & 91 & 91 \\ 0 & 14 & 413 \\ 0 & 14 & 455 \\ 0 & 14 & 497 \\ 0 & 71 & 528 \\ 33 & 50 & 359 \\ 33 & 52 & 331 \end{bmatrix}$
- $\begin{bmatrix} 0.29 & 0.047 & 0.248 \\ 0.24 & 0.234 & 0.007 \\ 0.23 & 0.221 & 0.007 \\ 0.21 & 0.202 & 0.013 \\ 0.22 & 0.206 & 0.015 \\ 0.21 & 0.090 & 0.121 \\ 0.20 & 0.064 & 0.141 \end{bmatrix}$
- $\begin{bmatrix} 4,344 \\ 121 \\ 121 \\ 231 \\ 275 \\ 2,121 \\ 2,471 \end{bmatrix}$
Figure 8: Dynamics of $I$ (% of the population)

Note that the graphs in Figure 8 have very different vertical scales, all in percent out of the population. The mid-range of the graph on the upper left, the case of no intervention, is 8%, while that of the full lockdown on the upper right is 0.002%. Thus, the former is 4,000 times bigger than the latter. The mid-range of the other four graphs (of the $k$ strategies) is between 0.15% and 0.60%.
The scale of the vertical axes here have mid ranges as follows: no intervention at 0.16%; full lockdown + $k$ strategies, at 0.01%.

The results can be divided into four groups, with the first two being non-optimizing, extremal benchmarks:

(i) The no intervention case has the disease erupt, breach the ICU capacity constraint $X$, and reach herd immunity at $S = 0.34$ by day 57. It leads to a high number of deceased (about 4.3 million people) and has a huge cost, at 78% of annual GDP in PDV terms over 18 months, most of it coming from the value of lost life ($V = 0.78; V_D = 0.75$).

(ii) The full lockdown case has the disease under control. It leads to a very low number of deceased (less than 1,000 people) and has a substantial cost, at 43% of annual GDP in PDV terms over 18 months, coming from the value of foregone output ($V = V_Y = 0.43$).

(iii) The three strategies with low $k$ ($= 4, 5, 6$) also have the disease under control and ICU capacity is not breached. They lead to relatively
low numbers of deceased (between 121 and 275 per million, i.e., between 40,000 and 91,000 people). Total cost is a little over 20% of annual GDP in PDV terms over 18 months, coming mostly from the value of foregone output (the difference between $V$ and $V_F$ is around 0.01).

(iv) The strategy with high $k$ ($= 8$) does not have the disease under control. There are two waves, whereby ICU capacity is breached in the second one. It leads to a relatively high number of deceased (2,471 per million, i.e., around 814,000 people). Total cost is still 20% of annual GDP in PDV terms over 18 months, but now it comes mostly from the value of lost life ($V_D = 0.14$).

We turn now to explore the mechanism underlying these results.

7 Exploring the Mechanism

We discuss the mechanism underlying the results from the perspective of the planner choices.

7.1 The Principles

There are two major ways to end the pandemic – herd immunity or vaccine. If no vaccine is expected, the planner’s room for action is only to flatten the curve order to avoid exceeding ICU capacity and to avoid overshooting the herd immunity threshold. This can be done under the serious limitations, whereby no intervention entails a huge cost of life, and that the cost of permanent lockdown is prohibitive.

We take the other route and model vaccine arrival at $T = 540$. The optimizing planner chooses how long to wait, when to lockdown, when to implement a cyclical strategy (including continued lockdown as an option) and when to release. Lockdowns serve to minimize the $\chi \cdot D(t)$ term in the planner loss function. It is dependent, inter-alia, on ICU capacity via $\delta(X(t), X)$ which affects $D(t)$. This consideration needs to be balanced against output loss driven by the term $(1 - \rho)$. The planner decides on timing according to (i) the initial state ($I_0$); (ii) the path of $R(t)$; (iii) the tool (strategy) available to lockdown at $T_1$ (a given $k$).

The point about the cyclical strategies is that they average $k$ working days and $(14 - k)$ lockdown days to reduce $R$. Thus approximation of the average $R$ is given by:\(^{13}\)

$$R^a(k) \approx \frac{k \cdot R^W + (14 - k) \cdot R^L}{14} < R^W$$

\(^{13}\)The average is not exactly a linear function of $R^W, R^L$ and so this is an approximation.
We should note that there are two concepts of herd immunity when we apply this cyclical strategy:

a. $S_{RW} = 1/R_W$ is relevant after the release

b. $S_{Ra} = 1/R_a$ is relevant during the cyclical phase

with $S_{Ra} > S_{RW}, T_{Ra} < T_{RW}$

### 7.2 The Dynamics

Given these concepts, we can describe two general scenarios for the planner dynamics, noting our assumption that $R_W = 1.50$.

**Scenario A.** The planner keeps the disease under control and keeps $S$ above herd immunity till vaccine arrival. For this scenario, the instruments at hand should be able to keep $R$ around 1; this happens when $k < 7$. This scenario holds true all the more so, when $\rho$ or $\chi$ are high (i.e., the relative value of lost life is high). In this case the planner will implement the cyclical strategy for a long time and $S$ will remain above herd immunity till vaccine arrival.

**Scenario B.** The planner lets $S$ reach some form of herd immunity. This happens (i) when $k < 7$ and $\rho$ or $\chi$ are low (i.e., the relative value of lost life is low), and (ii) when $k \geq 7$, and the instruments are not strong enough to contain $R_a$ in the region of 1. The planner will implement the cyclical strategies for a short time, at the price of loss of lives and getting to herd immunity. The latter takes on one of two forms:

1) the herd immunity of $R_W = 1.50$

2) the herd immunity of $R_e = R_a \cdot S(t)$

As examples consider the dynamics depicted in Figures 8 and 9. First note the extreme benchmarks. In the case of no intervention, the economy reaches herd immunity at the initial $R = 2.50$. Under full lockdown, there is a wait for vaccine arrival. Next, consider the cyclical strategies With $k = 4, 5, 6$ scenario A obtains. With $k = 8$ scenario B obtains and the economy reaches herd immunity of $R_e$.

One can further classify the dynamics as follows, noting that they depend on the value of $R_e(T_1) = R_a \cdot S(T_1)$ relative to 1. From this perspective three types of dynamics emerge in the cyclical phase and after it, depending on what state the economy is at time $T_1$:

1. $R_a < 1$, such that $R_e(T_1) = R_a \cdot S(T_1) < 1$. In this case:
   a. There is very efficient suppression.
   b. $I$ keeps declining during the cyclical phase.
   c. $S$ remains high and thus there is an eruption upon release.
   d. Strategies 3, 11 and 4, 10 belong in this class.

2. $R_a > 1$, but $R_e(T_1) = R_a \cdot S(T_1) \sim 1$. In this case:
   a. There is relatively efficient suppression.
b. $I$ rises slowly (may even decline towards the end) during the cyclical phase.
c. $S$ remains high and thus there is eruption upon release.
d. Strategies 5, 9 and 6, 8 belong in this class.

3. $\mathcal{R}^a >> 1$, so that $\mathcal{R}^e(T_1) = \mathcal{R}^a \cdot S(T_1) >> 1$. In this case:
   a. No suppression is possible.
   b. $I$ erupts and then declines during the cyclical stage.
   c. $S$ drops fast, passes $\mathcal{S}^{R^a}$ and approaches $\mathcal{S}^{R^w}$.
   d. If the planner would have aimed for quicker arrival of the economy to this herd immunity, the ICU limit will have been breached and the death toll would have been higher.
   e. Strategies 7, 7 and 8, 6 belong in this class.

Hence we can conclude that under a low $k$ strategy, $S$ is relatively high upon release, because the disease is under control and $I$ does not grow. It is way above herd immunity $\mathcal{S}(R^w) = 0.67$, and therefore $\mathcal{R}^e >> 1$ when the planner releases and $I$ erupts. We end up with a relatively high $S(T_{vaccine})$. Under a high $k$ strategy, $S$ is much lower upon release, as there was an eruption during the cyclical phase, $I$ rose and so $S$ fell, and it fell below the herd immunity of $\mathcal{S}(R^a)$; this is overshooting, though still higher than $\mathcal{S}(R^w)$. Upon release $S$ is quite close, from above, to herd immunity $\mathcal{S}(R^w)$, so $\mathcal{R}^e$ is close to 1. In this case $I$ grows very slowly when the planner releases. We end up with a relatively low $S$ when the vaccine arrives. Table 5 shows these dynamics.

### Table 5: $S$ and $\mathcal{R}$ Dynamics Under Cyclical Strategies

<table>
<thead>
<tr>
<th>Strategy</th>
<th>$S(T_0)$</th>
<th>$S(T_1)$</th>
<th>$S(T_2)$</th>
<th>$\mathcal{R}^e(T_2)$</th>
<th>$S(T_3)$</th>
<th>$\mathcal{R}^a$</th>
<th>$\mathcal{S}(R^a)$</th>
<th>$\mathcal{S}(R^w)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3, 11</td>
<td>0.999</td>
<td>0.999</td>
<td>0.999</td>
<td>1.499</td>
<td>0.985</td>
<td>0.95</td>
<td>1.052</td>
<td>0.67</td>
</tr>
<tr>
<td>4, 10</td>
<td>0.999</td>
<td>0.999</td>
<td>0.998</td>
<td>1.498</td>
<td>0.985</td>
<td>1</td>
<td>1</td>
<td>0.67</td>
</tr>
<tr>
<td>5, 9</td>
<td>0.999</td>
<td>0.999</td>
<td>0.989</td>
<td>1.484</td>
<td>0.971</td>
<td>1.05</td>
<td>0.952</td>
<td>0.67</td>
</tr>
<tr>
<td>6, 8</td>
<td>0.999</td>
<td>0.999</td>
<td>0.974</td>
<td>1.460</td>
<td>0.966</td>
<td>1.1</td>
<td>0.909</td>
<td>0.67</td>
</tr>
</tbody>
</table>

**7.3 Taking Stock**

We can conclude that some instruments ($k = 7$ and $k = 8$) are unable to effectively suppress the disease. With these instruments, the optimum is always to do relatively short interventions and to get close to herd immunity before the vaccine arrives. This is not sensitive to the parameters $\rho$ and
Other instruments ($k < 7$) allow for efficient suppression. When lockdowns are relatively cheap (or the value of lost life is relatively high) the planner will choose relatively long interventions, and $S$ will remain above herd immunity till vaccine arrival. When lockdowns become expensive (or the value of lost life relatively low), the planner will switch to relatively short interventions, at the price of loss of lives and gets closer to herd immunity before the vaccine arrives.

7.4 Parameter Variations

We turn to look at the effects of variations in $\chi$ and in $\rho$.

7.4.1 Value of Lost Life

Table 6 presents the results with the baseline $\chi$ value as well as with the two alternatives discussed in sub-section 5.2. The high level of $\chi$ prices $6.1$ annual GDP per capita for every lost life year, and the low level of $\chi$ prices $1.5$ for every lost life year. The benchmark is $4.1$.

<table>
<thead>
<tr>
<th>Value of Lost Life</th>
<th>$T_0$</th>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$V$</th>
<th>$V_\gamma$</th>
<th>$V_D$</th>
<th>$D$ (per 10$^6$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Intervention</td>
<td>540</td>
<td>540</td>
<td>540</td>
<td>0.78</td>
<td>0.02</td>
<td>0.75</td>
<td>13,113</td>
</tr>
<tr>
<td>Full lockdown</td>
<td>0</td>
<td>540</td>
<td>540</td>
<td>0.43</td>
<td>0.43</td>
<td>0.00</td>
<td>3</td>
</tr>
<tr>
<td>Baseline VSL $\chi = 57.8$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k = 5$</td>
<td>0</td>
<td>21</td>
<td>504</td>
<td>0.21</td>
<td>0.20</td>
<td>0.01</td>
<td>231</td>
</tr>
<tr>
<td>$k = 6$</td>
<td>0</td>
<td>71</td>
<td>528</td>
<td>0.22</td>
<td>0.21</td>
<td>0.02</td>
<td>275</td>
</tr>
<tr>
<td>$k = 8$</td>
<td>33</td>
<td>57</td>
<td>331</td>
<td>0.20</td>
<td>0.06</td>
<td>0.14</td>
<td>2,471</td>
</tr>
<tr>
<td>high VSL $\chi = 85.7$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k = 5$</td>
<td>0</td>
<td>21</td>
<td>504</td>
<td>0.22</td>
<td>0.21</td>
<td>0.01</td>
<td>142</td>
</tr>
<tr>
<td>$k = 6$</td>
<td>0</td>
<td>71</td>
<td>528</td>
<td>0.23</td>
<td>0.21</td>
<td>0.02</td>
<td>275</td>
</tr>
<tr>
<td>$k = 8$</td>
<td>33</td>
<td>57</td>
<td>344</td>
<td>0.27</td>
<td>0.07</td>
<td>0.20</td>
<td>2,401</td>
</tr>
<tr>
<td>low VSL $\chi = 21.4$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k = 5$</td>
<td>40</td>
<td>56</td>
<td>91</td>
<td>0.13</td>
<td>0.03</td>
<td>0.10</td>
<td>4,526</td>
</tr>
<tr>
<td>$k = 6$</td>
<td>38</td>
<td>56</td>
<td>113</td>
<td>0.13</td>
<td>0.04</td>
<td>0.09</td>
<td>4,118</td>
</tr>
<tr>
<td>$k = 8$</td>
<td>33</td>
<td>49</td>
<td>196</td>
<td>0.11</td>
<td>0.04</td>
<td>0.07</td>
<td>3,281</td>
</tr>
</tbody>
</table>
Figures 10-12 illustrate.

Figure 10: Baseline $\chi$ Value

Figure 11: Low $\chi$ Value
The change in the value of lost life moves the planner along a downward sloping frontier; it is a change in price/preferences, not in the state of the world. The planner moves to the SE in Figure 11 with the low \( \chi \). The death toll rises very substantially and with it the share of \( V_D \) in total loss \( V \). The latter, however, declines relative to the baseline.

The planner moves to the NW in Figure 12 with the high \( \chi \). In value terms there is relatively little change with respect to the baseline, noting an asymmetry between the rise and fall in \( \chi \).

### 7.4.2 Value of The Fraction Employed During Lockdown

Table 7 presents the results with the baseline \( \rho \) value as well as with the two alternatives discussed in sub-section 5.2. The high level has 0.8 of \( L(t) \) working during lockdown, while the low level has 0.6. The benchmark is 0.7.
Table 7: Alternative Values of $\rho$

<table>
<thead>
<tr>
<th>Employment Loss</th>
<th>$T_0$</th>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$V$</th>
<th>$V_Y$</th>
<th>$V_D$</th>
<th>$D$ (per 10^6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Intervention</td>
<td>540</td>
<td>540</td>
<td>540</td>
<td>0.78</td>
<td>0.023</td>
<td>0.752</td>
<td>13,113</td>
</tr>
<tr>
<td>Full lockdown</td>
<td>0</td>
<td>540</td>
<td>540</td>
<td>0.43</td>
<td>0.431</td>
<td>0.000</td>
<td>3</td>
</tr>
<tr>
<td>Baseline $\rho = 0.7$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k = 5$</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$k = 6$</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$k = 8$</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>high $\rho = 0.8$</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$k = 5$</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>$k = 6$</td>
<td></td>
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<td></td>
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<tr>
<td>$k = 8$</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>low $\rho = 0.6$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k = 5$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k = 6$</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$k = 8$</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figures 13-15 illustrate.

Figure 13: Benchmark $\rho$ Value

Figure 14: Low $\rho$ Value
Figure 15: High $\rho$ Value
The change in \( \rho \) moves the frontier out or in. It is a change in the state of the world – out (in) when \( \rho \) is low (high). The table shows that the changes in total loss, \( V \), are sizeable, coming mostly from \( V_Y \), with little change in the death toll for a given \( k \) strategy. The table shows that total loss \( V \) (in PDV, GDP per annum terms) varies between 15\% for low \( k \) and high \( \rho \) to 28\% for low \( k \) and low \( \rho \), i.e., almost doubling.

8 Further Discussion

We discuss two further issues: how the planner model compares with real world experience, and how the model compares with other models, which have been proposed recently.

8.1 The Planner vs Real World Experience

Trying to relate real world experience with the model one can think of the following description. In the real-world governments impose lockdowns, in order to get \( R\text{-effective} \) below 1, contain the epidemic, and reduce its size. When releasing, they aim at a value of \( R\text{-effective} \) which may slightly be higher than 1, in the order of 1.05 or 1.10. In this case herd immunity \((S = 1/\mathcal{R})\) is achieved when \( S \) is 0.95 \((R^W = 1.05)\) or 0.91 \((R^W = 1.10)\). This means that between 5\% and 9\% of the population is infected. With a mortality rate of 0.8\%, the death toll out of total population is 0.04\% – 0.07\%. Applied to the U.S population it is 132,000 – 231,000 deceased until a vaccine is found.

A problem will arise if \( R^W \) rises to 1.2. This doubles the deceased. Now \( S = \frac{1}{1.2} = 0.83; I = 1 - 0.83 = 0.17; D = 448, 120 \). Evidently, the problem will be aggravated as \( R^W \) rises.

The model here suggests another tool. Whatever \( R^W \) is, one can reduce it through the cyclical strategy. Hence, for example, if \( R^W \) surges to 1.6, the planner can use \( S_{5-9} \) to control the disease; if \( R^W \) surges to 1.7 the planner can use \( S_{4-10} \); if \( R^W \) surges to 1.8 the planner can use \( S_{4-10} \) or \( S_{3-11} \). Table 8 shows these possibilities.\(^{14}\)

\(^{14}\)The table was derived from numerical simulations yielding average \( R^a \) values that are only approximately linear in \( R^W, R^L \).
### Table 8: $R^a$ Values

<table>
<thead>
<tr>
<th>$R^w$ current</th>
<th>S10-4</th>
<th>S9-3</th>
<th>S8-6</th>
<th>S7-7</th>
<th>S6-8</th>
<th>S5-9</th>
<th>S4-10</th>
<th>S3-11</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>1.01</td>
<td>0.96</td>
<td>0.96</td>
<td>0.94</td>
<td>0.91</td>
<td>0.89</td>
<td>0.86</td>
<td>0.84</td>
</tr>
<tr>
<td>1.2</td>
<td>1.03</td>
<td>1.03</td>
<td>1.03</td>
<td>1.00</td>
<td>0.96</td>
<td>0.93</td>
<td>0.90</td>
<td>0.86</td>
</tr>
<tr>
<td>1.3</td>
<td>1.16</td>
<td>1.09</td>
<td>1.09</td>
<td>1.05</td>
<td>1.01</td>
<td>0.97</td>
<td>0.93</td>
<td>0.89</td>
</tr>
<tr>
<td>1.4</td>
<td>1.23</td>
<td>1.15</td>
<td>1.15</td>
<td>1.10</td>
<td>1.06</td>
<td>1.01</td>
<td>0.96</td>
<td>0.92</td>
</tr>
<tr>
<td>1.5</td>
<td>1.30</td>
<td>1.21</td>
<td>1.20</td>
<td>1.16</td>
<td>1.10</td>
<td>1.05</td>
<td>1.00</td>
<td>0.94</td>
</tr>
<tr>
<td>1.6</td>
<td>1.37</td>
<td>1.26</td>
<td>1.26</td>
<td>1.21</td>
<td>1.15</td>
<td>1.09</td>
<td>1.02</td>
<td>0.96</td>
</tr>
<tr>
<td>1.7</td>
<td>1.43</td>
<td>1.32</td>
<td>1.31</td>
<td>1.25</td>
<td>1.19</td>
<td>1.12</td>
<td>1.05</td>
<td>0.99</td>
</tr>
<tr>
<td>1.8</td>
<td>1.49</td>
<td>1.37</td>
<td>1.36</td>
<td>1.30</td>
<td>1.23</td>
<td>1.15</td>
<td>1.08</td>
<td>1.01</td>
</tr>
<tr>
<td>1.9</td>
<td>1.55</td>
<td>1.42</td>
<td>1.41</td>
<td>1.35</td>
<td>1.27</td>
<td>1.19</td>
<td>1.11</td>
<td>1.03</td>
</tr>
<tr>
<td>2</td>
<td>1.61</td>
<td>1.47</td>
<td>1.46</td>
<td>1.39</td>
<td>1.31</td>
<td>1.22</td>
<td>1.14</td>
<td>1.05</td>
</tr>
</tbody>
</table>

The left column shows the current $R^w$ value. The top row shows the cyclical strategy which can be used. Each cell presents the approximate $R^a$ when using a cyclical strategy given $R^w$. The cells marked in green show the place where $R^a$ is around 1.1 and the cheapest strategy (lowest $V$) to get there.

#### 8.2 Some Economists’ SIR vs The Current SEIR

The shorter version of the model, the SIR model, is very prevalent in recent economic modelling it has the following dynamic equation for $I$:

\[
\dot{I}(t) = \beta S(t)I(t) - \gamma I(t) = R(t)I(t) - \gamma I(t) = \gamma I(t)(R(t)S(t) - 1)
\]

At the start $S(0) \approx 1$, $R(t) = R_0$ so

\[
\begin{align*}
\dot{I}(t) &\approx \gamma I(t)(R_0 - 1) \\
I(t) &= I(0) \exp \gamma(R_0 - 1)t
\end{align*}
\]

Some papers in the emerging Economics literature posit a SIR model with $\gamma^L = \frac{1}{18}$. The current framework, collapsed into a SIR model, implies $\gamma^S = \frac{1}{\tau}$.\(^{15}\)

Plugging these values into a log-linear version of (44)

\[
\ln I(t) = \ln I(0) + 1.5\gamma t
\]

---

\(^{15}\)Where $\gamma^S$ is set to the current model $\gamma + \sigma$.  

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One can solve equation 45 for the number of days it takes for $I(0)$ to double. Using $R_0 - 1 = 1.50$, $\gamma^S = \frac{1}{17}$ and $\gamma^L = \frac{1}{18}$ yields the following:

<table>
<thead>
<tr>
<th>$\gamma^S$</th>
<th>$t$ days</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{17}$</td>
<td>3.3</td>
</tr>
<tr>
<td>$\frac{1}{18}$</td>
<td>8.3</td>
</tr>
</tbody>
</table>

Hence $\gamma^L$ means a slow moving disease, taking 8.3 days to double. As a consequence, these models need to start with a counter-factually high $I(0)$, such as $I(0) = 0.01$, or 3.3 million infected in the U.S. This implies that the planner problem is mis-specified.

9 Next Stages

In the next drafts we hope to expand the analysis in the following directions.

a. Within the current framework: model a probability distribution of vaccine arrival time rather than a determinate time; quadratic loss in the planner function; trigger strategies dependent on $\{I, X, \bar{X}, \ldots\}$; modelling of labor supply and expectations.

b. Within a richer framework: endogenize $R$ and the planner reaction to it, allowing for uncertainty about $R$ and other key parameters. Possible frameworks to deal with such a set-up are:

1. Robust control models; e.g., the ones explored by Hansen and Sargent and Brock and Hansen. The optimal solution is found by (i) solving for the optimal policy for each possible value of the state; (ii) policy is chosen assuming the worst possible state.

2. General signal extraction; e.g., Hauk, Lanteri, and Marcet (2019). This derives optimal policy when the planner has partial information in a setup where observed signals are endogenous to policy. Signal extraction and policy are determined jointly.

10 Conclusions

As both data facts and model show, there are substantial trade-offs of health outcomes (deaths, breaches of ICU capacity) and economic outcomes (loss of output, employment, and consumption). Cyclical strategies provide for significant improvement in terms of social welfare evaluated in PDV, annual GDP terms. Thus relative to no intervention, they save almost 60% of annual GDP in PDV terms over 18 months; relative to a full lockdown, they save about 20% of annual GDP over 18 months in PDV terms. Within the set of cyclical strategies itself, those strategies with low $k$ provide for 20 times lower death rates than high $k$ strategies. This happens at the price of
increased loss – as much as a multiple of 3 – in foregone output. A fundamental choice exists between aiming for “effective $R$” herd immunity and waiting for a vaccine. Inter alia, we show that some prevalent SIR modeling in the recent COVID 19 Economics literature derives disease dynamics, which are erroneous, and which distort the menu of choices available to the planner.

References


