TOWARDS A TWO VECTOR FORMULATION OF QUANTUM MECHANICS

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Abstract

A two-vector formulation of Quantum Mechanics is proposed. The new formalism permits a covariant description of the "collapse" of the wave function. A number of surprising predictions of the new formalism are discussed.

Quantum systems are generally described via wave-functions, or states. The state is supposed to represent all our knowledge about a quantum system; form the state we predict its behavior. However, this standard description via states has serious shortcomings. Indeed, the standard formulation fails to accommodated basic symmetries of spacetime: Lorentz covariance and time-reversal symmetry at the microscopic level. These flaws are usually glossed over, but efforts to deal with them are gradually yielding clues how quantum mechanics might better be formulated. We find that a description of quantum systems in terms of two state vectors is needed. One state evolves forward from initial conditions, as usually, while the second evolves backwards in time from future boundary conditions. Such a formulation generates all the results of standard quantum

mechanics, and in addition leads to surprising new results that are hard to obtain using the conventional description. We will present the two vector formulation here, and also summarize the research that has inspired it.

1. The standard formulation of quantum mechanics is not time-symmetric even at the microscopic level. The breakdown of time symmetry reflects our habit of predicting future events on the basis of initial conditions; the latter have occurred and are given data, the former have not occurred and fully depend on the initial conditions. We inherit this habit of thinking from classical mechanics. There, it entails no violation of time symmetry, because classical physics is deterministic. But quantum physics is not deterministic, and indeed the usual dichotomy of initial conditions vs. equations of motion is not particularly natural there, for quantum mechanics sets strict limits to what we can know about initial conditions. We have the additional problem of inferring, from given data, what values dynamical variables took at earlier times. When such retrodictions are considered, along with predictions, time symmetry is restored, as the following example shows.

Consider a sequence of measurements, at successive time t_1, t_2, and t_3, on a quantum system. At time t_1 we measure the operator A so the system is prepared in an eigenstate which we will denote $|a\rangle$. At time t_3 we measure the operator B and find the system in the state $|b\rangle$. (A and B need not commute.) In between, at time t_2, the operator C is measured; we are informed of this measurement but not told the result. Nevertheless, we can make a good prediction about the outcome of the measurement. We know that the probability of an outcome $|c_i\rangle$ for the measurement C, given the initial state $|a\rangle$, is

$$\left|\langle c_i|U(t_2,t_1)|a\rangle\right|^2$$

where $U(t_2,t_1)$ is the unitary operator for time evolution from t_1 to t_2. Likewise the probability of the outcome $|b\rangle$ given an initial state $|c_i\rangle$ is

$$\left|\langle b|U(t_3,t_2)|c_i\rangle\right|^2$$

These are the only quantum mechanics results we need to know. It now follows[1] from classical laws of probability that the probability that the state $|c_i\rangle$ was found at t_2 is

$$P_i=\frac{|\langle b|U(t_3,t_2)|c_i\rangle|^2|\langle c_i|U(t_2,t_1)|a\rangle|^2}{\sum_j|\langle b|U(t_3,t_2)|c_j\rangle|^2|\langle c_j|U(t_2,t_1)|a\rangle|^2} \qquad (1)$$

Why? Suppose we begin with an ensemble of N identical systems. Each system begins in the state $|a\rangle$, but we do not always end up in the state $|b\rangle$; that only happens in

$N\cdot\sum p(b|c_j)p(c_j|a)$ cases, on the average. (We use the notation p (f|g) to denote the probability of f given g.) In this smaller sample that remains, the outcome $|c_i\rangle$ is found Np $(b|c_i)$ p $(c_i|a)$ times on average. Hence the probability p_i is given by Eq. (1).

With a slight rewriting, Eq. (1) becomes manifestly time-symmetric:

$$P_i = \frac{|<c_i|U(t_2,t_3)|b>|^2 |<c_i|U(t_2,t_1)|a>|^2}{\sum_j |<c_j|U(t_2,t_3)|b>|^2 |<c_j|U(t_2,t_1)|a>|^2} \qquad (1')$$

where $U(t_i, t_j) = U^{-1}(t_j,t_i) = U^{\dagger}(t_j, t_i)$. Eq. (1′) suggests the following description: the outcome of a measurement of C at t_2 can be calculated in terms of two vectors |a> and |b>, which represent initial and final boundary conditions, respectively. Evolving |a> forwards in time from t_1 to t_2, and |b> backwards in time from t_3 to t_2, we project $|c_i>$ onto these two vectors to find the amplitude for the outcome $|c_i>$ (up to a normalization). Such an interpretation [2] is just as tenable as the conventional one, at least in the present example, and has the added virtue of being time -symmetric.

Eq. (1) suggests something more. (For simplicity let the operators $U(t_i, t_j)$ equal 1 here.) Note that if the operator C had been chosen to be A, the outcome |a> would have been certain at t_2; whereas if the operator B had been chosen for C, the outcome |b> would have been certain. Even if A and B are non-commuting observables, we seem to be able to assign a definite value to both these operators at the intermediate times. Could there be some sense in which both have simultaneous reality? That question will be explored below. However, we are so far dealing with the special case where a measurement was definitely made between t_1 and t_3, only we do not know what the measurement was. Eqs. (1) and (1′) do not entitle us to talk about values of an operator C at t_2 if no measurement was made then. As long as only particular operator was measured, we cannot say much about values of other operators, for the measurement of C represents a disturbance of the system. But can the time-symmetric description be extended to a broader class of interactions that do not disturb the system between initial and final states? We will see that an extension is indeed possible. But we defer the question and focus on another shortcoming of the conventional description.

2. The problem of relativistic covariance in quantum mechanics[3] arises as soon as we consider as simple a system as two spin-1/2 particles prepared in a spin-0 state:

$$x \equiv \frac{1}{\sqrt{2}}\left[|\uparrow>_r|\downarrow>_s - |\downarrow>_r|\uparrow>_s\right]$$

for particles located to points r and s, spacelike separated. Since the total angular momentum is zero, a measurement of any component of spin on either of them represents a measurement of that component on both of them. Therefore the conventional (Copenhagen) interpretation of quantum mechanics asserts that a measurement on either particle instantly reduces their combined state X to a direct product of one-particle states, regardless of how far apart the particles may be. We are led directly into a paradox, for what is simultaneous in one Lorentz frame is not simultaneous in another; while a measurement of the spin of either particle acts locally and so is covariant. Thus it seems each Lorentz frame implies a different version of when the two particles decouple, and we are left without any covariant state definition.

More formally, the paradox is this: on the one hand, we have different assertions about what is the actual quantum state for these two particles, by different observers; on the other hand these different assertions correspond to physically different worlds. What makes these worlds physically different? It is the fact that, in principle, we have ways to measure non-local properties.[3] For example, the state X is characterized by J = 0, where J

is the spin quantum number for the spin-correlated two -particle system. This property of X is non-local: it belongs to neither particle alone. A verification that J = 0 constitutes a non-local measurement. And it can be accomplished by means of two local devices, on at r and the other at s. Consider an interaction Hamiltonian

$$H_{\text{int}} = -g(t) \sum_{i=x,y,z} (\sigma_i^r \; q_i^r + \sigma_i^s \; q_i^s).$$

Here g(t) is a function with support only in a short interval (t_1, t_2); its integral over the interval is one. The matrices σ_i^r represent the i-th component of spin for a particle located at r, etc. The variables q_i^r belong to the measuring device located at r, etc.; their conjugate variables, π_i^r correspond to actual pointer or indicator positions on the device. Now choose the initial state of the two devices to be as follows:

$$q_i^r - q_i^s = 0, \qquad \pi_i^r + \pi_i^s = 0. \tag{2}$$

This choice is always available since $[q_i^r - q_i^s \; , \pi_i^r + \pi_i^s \;] = 0$. After completion of the measurement, we have

$$(\text{change in } \pi_i^r) \;=\; i\int dt\left[H_{\text{int}}, \pi_i^r\right] = \sigma_i^r, \tag{3}$$

and similarly for the change in π_i^s. So imagine that immediate after t_2, each local device records the final value of π_i^r and π_i^s for posterity. Even without any further lapse of time (necessary for comparing the results at r and s) a measurement that J = 0 is essentially complete. From Eqs. (2-3) we deduce that

$$(\text{change in } \pi_i^r + \pi_i^s) = \left[\pi_i^r + \pi_i^s\right]_{t_2} = \sigma_i^r + \sigma_i^s = 0, \quad \text{if J } = \; 0 \tag{4}$$

Conversely, if Eq. (4) is found true when the measurements at r and s are compared, we have verified (in retrospect) that the two particles were in the combined state J = 0. The interval (t_1,t_2) can in principle be made arbitrarily short, so the verification of J=0 can be practically instantaneous. Furthermore, if in the initial state J = 0 then the measurement leaves the state unchanged: $q_i^r + q_i^s = 0$ remains true throughout the measurements and so is true at all times as well, for all components i.

When one observer, say in the lab frame, measures one particle of a pair, he concludes that the state of both reduces instantly, The reduced state must be $|\uparrow\rangle_r|\downarrow\rangle_s$ or $|\downarrow\rangle_r|\uparrow\rangle_s$ after a measurement of either σ_z^r or σ_z^s. Furthermore, he could have determined, an instant earlier, that the pair was in the non-local state X. For an observer in a rocket frame, neither of these conclusions is acceptable. The first conclusion fails because state reduction of the two systems cannot be simultaneous to both observers. The second conclusion fails because in the rocket frame, the non-local measurement of the state X is not instantaneous: the devices at r and s act during different time intervals. These

actions thus disturb the state of the system from the point of view of the rocket frame. Either observer alone can verify the state of an extended system in his own Lorentz frame; but if both want to verify their independent descriptions, they will interfere with each other. There is no covariant description of the state that can be confirmed by experiment on both frames. the state is not something covariant: two observers cannot both verify their independent versions of the state history.

The two-vector formulation offers a way out of the paradox. We have two vectors, one propagating forwards in time from an experiment in the past, and a second propagating backward in time from an experiment in the future. The only reduction takes place locally, when an experiment is made. Therefore, in the example of the two correlated spins, we can always appeal to future measurement of the second particle which will complete the definition of the future boundary conditions. If the initial state was X and experimenters later find the particle at r polarized along the positive z-axis and the particle at s polarized along the positive x-axis, then the final boundary condition for the system is

$$\frac{1}{\sqrt{2}}|\uparrow>_r\left[|\uparrow>_s+|\downarrow>_s\right].$$

But even without knowing the result of the measurement a s, we know that any measurement of the z-component of spin at s will find it polarized along the negative z-axis. The boundary condition at r enforces this correlation, without our needing to specify a moment when it becomes true as a result of non-local state reduction. It is easy to check, from Eq. (1′), that between these boundary values the state $|\downarrow>_s$ has probability 1. And since the past and future boundary values the state $|\downarrow>_s$ has probability 1. And since the past and future vectors used in this description are determined by events (local experiments) in spacetime, the two-vector formulation is automatically covariant: the two vectors transform covariantly along with the events that define them.

Here we begin to see how the language of two vectors may actually be more consistent that the conventional language. In non-relativistic quantum mechanics, the state is a useful tool for calculating all possible outcomes--not only of actual experiments, but also of experiments that could be or could have been performed. It transforms covariantly under the appropriate group, the group of Galilean transformation. When we consider Lorentz transformations, we find that the state does not exist at all in a covariant way. The two-vector formulation restores covariance and at the same time answers the same need: it allows us to predict what will result, or what would result, or what would have resulted, from an experiment in between the initial and final boundary conditions. These predictions will in general be uncertain, and they will agree in every respect with predictions of the conventional formulation. The advantage is simply that the two-vector description will itself be covariant, not just its predictions.

The reader may object: predictions for actual or potential experiments can be made using the two vectors, but if these experiments are performed, they will in general change the evolution of the system. Since the future boundary condition depends on this evolution, the analogy with the conventional formulation does not work! Let us consider this objection in relation to the two-spin system described above. There, we saw that once the spin of one of the particles had been measured, the two-vector formalism predicts the other to have the opposite spin (if the same component is measured); this result holds no matter what the future boundary condition on the second particle may be. True, if the measurement is actually performed, it disturbs the two vectors that previously described the system, by inserting new intermediate conditions. But these intermediate conditions do not

destroy the original two vectors; they only limit their propagation. Thus, if we are given boundary conditions at t_1 and t_3 and insert an experiment at t_2, the state at t_3 still propagates forwards and backwards, but it only propagates backwards as far as t_2, not to t_1. Similarly the state at t_1 propagates to t_2 and not to t_3. The experiment at t_2 now propagates forwards and backwards. However, the post selection of the ensemble at t_3 may look different now, because the future boundary condition is being applied to a disturbed ensemble. In contrast with conventional quantum mechanics, the ensemble depends on the experiment chosen. A departure from the conventional notion of an ensemble is apparent, but the ensemble of the two-vector formulation is in many cases the natural one, e.g. for any experiment that is described by an S-matrix.

3. Another objection may be raised: Perhaps the two-vector formulation provides a more flexible language in cases where the conventional formulation has problems. But does it justify introduction of a new formalism? Is it not just a matter of taste, how a theory is best stated? This point is well taken. The introduction of the two vectors in place of one is a big step, and it is not obvious that it leads to a useful formalism. In particular, we have the following question. Conventional quantum mechanics defines expectation values of operators in states via

$$< A > \equiv \frac{\langle \psi | A \psi \rangle}{\langle \psi | \psi \rangle}$$

where $<A>$ is the expected value of the operator A in the state ψ. In a two-vector formalism, this expression would naturally generalize to [4]

$$< A >_w \equiv \frac{\langle \psi_2 | A \psi_1 \rangle}{\langle \psi_2 | \psi_1 \rangle}$$

in a interval between states ψ_1 and ψ_2. But conventional quantum mechanics does not assign any particular meaning to this expression.

Remarkable, a physical meaning can indeed be given to the expression in Eq. (5). This kind of expectation value, measured between two different states, will be referred to as the weak value of an operator A. (Of course, when $\psi_2 = \psi_1$ the weak value becomes strong.) It turns out that by 'weakening' the measurement of A, we can strengthen the claim that the two vectors actually describe a system during the intervening period. To 'weaken' in this context means to diminish interference with the system, at the cost of deceased accuracy. This notion of a 'weak measurement' complements the two-vector formalism by introducing a broad class of measurements that can be made without significantly disturbing boundary conditions. It sounds paradoxical, and leads to apparently paradoxical behavior, but in fact accords with conventional quantum mechanics. Let us take an example:

We want to measure the orientation of a ferromagnet made of N spins.[5] To do this we use a measuring device which interacts with the ferromagnet according to a Hamiltonian of the von Neumann type:

$$H_{int} = -g(t) g A.$$

A is an operator which we wish to measure; in this case, a spin component. As before, q is canonically conjugate to π, the pointer setting, and g(t) is a normalized function with compact support. But this time we choose the initial state of the device to be a gaussian in q, with spread equal to $1/\sqrt{N/2}$. In the π-representation the initial state will also be a gaussian, with spread equal to $\sqrt{N/2}$. We select for an ensemble in which all the spins were initially polarized along the positive x-axis, and finally polarized along the positive y-axis. Naturally, for large N this is an unlikely set of boundary conditions, but there is no limit to how many tries we can make until we get the ensemble, so let us assume we have it. Because we have chosen a diffuse initial state for the measuring device, a measurement of L_x induces a change in L_y of magnitude

$$\Delta L_y = -i\int dt\, g(t) q\left[L_{x_1} L_y\right] = qL_z \leq \frac{1}{\sqrt{N/2}} \cdot \frac{N}{2}$$

which is of order $\sqrt{N/2}$, or less. Similarly, a measurement of L_y entails a disturbance in the magnitude of L_x of this order. Given the initial and final boundary conditions, we are justified in taking the value of both L_x and L_y to be N/2 (up to corrections of order $\sqrt{N/2}$) throughout the entire period.

However, we can draw from these results another one which looks quite paradoxical: Let us measure the component of spin along the line bisecting the two axes, i. e. choose the interaction Hamiltonian to be

$$H_{int} = -g(t) q (L_x + L_y)/\sqrt{2};$$

then the state of the system after the measurement is

$$\exp\left\{iq(L_x + L_y)/\sqrt{2}\right\} \left| L_x = \frac{N}{2} \right\rangle \left| \pi \approx 0 \right\rangle,$$

where $|\pi \approx 0\rangle$ represents the gaussian in π centered about zero. Including the post-selection of states with $L_y = N/2$, the state of the apparatus after the measurement is

$$\left\langle L_y = \frac{N}{2} \left| e^{iq(L_x + L_y)/\sqrt{2}} \right| L_x = \frac{N}{2} \right\rangle \left| \pi \approx 0 \right\rangle$$

$$\approx e^{iqN/\sqrt{2}} |\pi \approx 0\rangle$$

up to a term of order 1 in the exponent, which is otherwise of order $\sqrt{N}$. (The exponential of $L_x + L_y$ has been written as a product of exponential, using the Baker-Campbell-Hausdorf relation and estimates of the size of q.) The result, then, is that the gaussian in p has been shifted by approximately $N/\sqrt{2}$, exactly as if L_x and L_y added classically to yield a vector $\sqrt{2}$ times larger than possible!

Thus, by 'weakening' the disturbance induced by successive measurements of L_x and L_y, we have shown that the weak value

$$\left\langle \frac{L_x + L_y}{\sqrt{2}} \right\rangle_w = \frac{\left\langle L_y = \frac{N}{2} \middle| \frac{L_x + L_y}{\sqrt{2}} \middle| L_x = N/2 \right\rangle}{\left\langle L_y = N/2 \middle| L_x = N/2 \right\rangle} = \frac{N}{\sqrt{2}}$$

corresponds to a genuine (and surprising) physical process, albeit an infrequent one. Clearly, we are just getting started; this is just one of many possible phenomena. To exhibit a similar result involving a single spin, we consider a weak measurement of a z-component of spin, σ_z, between an initial state

$$|\xi\rangle \equiv \cos\frac{\alpha}{2}\left|\sigma_x = \frac{1}{2}\right\rangle + \sin\frac{\alpha}{2}\left|\sigma_x = -\frac{1}{2}\right\rangle$$

and a final state polarized along the positive x-axis. The expected weak value of σ_z is then arbitrarily large:

$$\langle \sigma_z \rangle_w = \frac{\left\langle \sigma_x = \frac{1}{2} \middle| \sigma_z \middle| \xi \right\rangle}{\left\langle \sigma_x = \frac{1}{2} \middle| \xi \right\rangle} = \tan\frac{\alpha}{2}.$$

Such a result looks far removed from anything physical. Nevertheless, an experiment designed to measure σ_z under these conditions will indeed yield this 'impossible' value! The result can be derived rigorously,[4] but it may be more enlightening to follow what happens intuitively: A beam of spin-1/2 particles passes through a stern-Gerlach device which polarizes them alone the z-axis, splitting the up and down components which then impinge on a screen. However, the polarization is so weak that the width of the beam is much larger than the separation induced by the polarizer. The wave function near the screen, for the variable z, would look like two very broad gaussian peaks slightly separated one from the other. But the effect of the boundary conditions is to give those two peaks a relative phase that makes them cancel almost every where, except for a tiny peak far from either of the original peaks, at a deflection corresponding to the predicted absurd value of $\tan\frac{\alpha}{2}$. Of course, to get these results requires a large beam population, since many of the trials will not pass the boundary conditions. However, the experiment can proceed as slowly as desired, so that at any moment only one particle is present in the apparatus, and thus this is really an effect belonging to the particles in the deflected peak as individuals.

This effect may be regarded as a quantum stunt, a straightforward if unanticipated consequence of standard quantum mechanics, or it may be that we are seeing a qualitatively new aspect of quantum behavior that was never understood before. In either case, the weak measurement supplied by the two-vector description proves itself physically meaningful, and the tow vector formulation makes it easier to discover such effects. A number of other examples provide further reason to accept these effects as physically meaningful and not just quantum stunts. As a final example, let us consider a particle trapped in a finite potential well. As is well known, such a particle can 'tunnel' to a classically forbidden region where its total energy is necessarily less than its potential energy V. We ask in what sense we can observe that such tunneling takes place. If we

measure the position well enough to localize the particle strictly outside the well, it will be in a superposition of energy eigenstates, some with energies greater than V. On the other hand, if the particle is in a definite eigenstate of energy with E < V, and a measurement of position is made, the particle may turn up either in the well or in the forbidden region. A genuine tunneling situation has the particle in the forbidden region with energy E < V, but quantum mechanics apparently forbids us from observing such tunneling.

Now let us consider an ensemble of particles pre-selected to an energy eigenstate and post-selected by a measurement of position. In the intermediate time we make a measurement of the kinetic energy. The measurement proceeds via an interaction Hamiltonian

$$H_{\text{int}} = -g(t)q\frac{p^2}{2m}$$

where the canonical conjugate of q is π, a pointer setting on the device. In an ideal measurement the initial and final settings would be free of error, but any realistic experiment contains errors. Thus we can take the initial state of the pointer to be

$$\phi_{in} = \exp\left(-\pi^2/\delta^2\right)$$

introducing errors of order δ in the kinetic energy. The pre-selected state of the particle will be, by assumption, $|E_0\rangle$. The post-selected state will be a position eigenstate $|x_0\rangle$ deep inside the classically forbidden region., Realistically, this will also be a gaussian centered around x_0 with dispersion Δ. The final state of the apparatus will thus be (taking into account the interaction Hamiltonian):

$$\phi_{fin} = \left\langle x = x_o \middle| e^{iqp^2/2m} \middle| E = E_o \right\rangle \phi_{in}.$$

Now comes the logic of the weak measurement. We would like to choose and x_0 so far from the potential well that the particle must have been outside the potential well before the measurement of kinetic energy. An explicit calculation shows that the probability of finding particles inside the potential well becomes negligible when

$$x_0 >> \frac{1}{m\delta\Delta}.$$

When the precision of the kinetic energy measurement is increased ($\delta \rightarrow 0$), the post-selected position must be ever farther out ($x_0 \rightarrow \infty$). We now ask what is the final state of the pointer p for particles satisfying this condition. Another explicit calculation shows that the final state is again a gaussian with the same spread, but with the peak at the value E - V, a negative value, Thus precisely in the case of particles found so far from the well that they could not have been there before the kinetic energy measurement we find the 'impossible' negative values for the kinetic energy.

From the point of view of standard quantum mechanics all this is a game of errors of measurement. An ideal measurement of kinetic energy can yield only positive values, since the eigenvalues of $p^2/2m$ are positive. Realistic measurements, however, have errors

and thus negative values are possible. They also disturb particle positions, but the disturbance is bounded along with the accuracy of the measurement. Hence it is possible to localize a tunneling particle far enough from the well while getting negative readings for the kinetic energy. But this fantastic conspiracy of cancellations seems to be telling us, in straight forward language, that genuine tunneling can be observed if we design subtle experiments, as prescribed by a two-vector formalism involving weak measurements.

To conclude, this paper has pointed out some shortcomings of the conventional formulation of quantum mechanics, relating to Lorentz covariance of states and time symmetry of predictions and measurements.

A new approach, based on pairs of vectors evolving forwards and backwards in time, has been offered to remove these shortcomings. But the two-vector description offers much more than a reformulation of familiar results. As a formalism of recalculations, it introduces a host of unexpected and peculiar phenomena. We have seen examples of spins vectors adding classically, 'impossible' values of spin projections and observable negative kinetic energy. The new formalism paints an entirely new picture of quantum reality for us. This new picture of quantum reality is not yet fully clear to us, but we may hope it will involve a new understanding of time, since in this formalism quantum states propagate in both time directions.

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