DUAL-PARTON MODEL FOR MESONS AND BARYONS

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Guided by the theory of Neveu and Schwartz spin is introduced into the dual string model by equipping each parton with a Dirac spin index. A nearest-neighbour spin-spin coupling is introduced. The continuum limit results in dual covariant amplitudes. The solutions show different characteristics depending on whether the total number of partons is even or odd. The first case yields the meson theory of Neveu and Schwartz. The odd case leads to a baryon model. The simplest meson-baryon amplitude is computed. All claims made about the properties of the system are based on the exact solution of the finite problem.

Recently, a dual theory of mesons was introduced by Neveu and Schwartz [1] **, utilizing anti-commuting operators. These operators were shown by Virasoro [2] to obey a one-dimensional Dirac equation on the conventional dual model strip. In the N-S theory the pion is assumed to couple linearly to the anti-commuting objects, which identifies them as the axial vector current. On the other hand, due to their anti-commutation properties these fields cannot be thought of as local observable currents. Deep inelastic experiments suggests that hadrons are systems composed of many spin-$\frac{1}{2}$ partons, and thus the local currents and vertices should be built out of the degrees of freedom of these partons. Guided by these considerations we were led to the construction of a dual covariant theory of mesons and baryons. The meson sector is identical with the N-S theory. The theory has the additional feature that local currents, in particular the vector and axial vector currents may be defined and reconstructed from the unphysical anti-commuting fields which obey simple equations. In what follows we explicitly deal only with purely hadronic processes.

In the dual parton model [3] a hadron is described by a one-dimensional array of partons, occupying the segment $0 \leqslant \theta \leqslant \pi$ with a density $\rho(\theta)$ proportional to $(\sin\theta)^{-1}$. These partons have coordinates and momenta, and interact with their nearest neighbours via harmonic forces.

In what follows, we identify the partons as spin-$\frac{1}{2}$ Dirac particles, and add a nearest neighbour spin-spin interaction to the harmonic orbital Hamiltonian.

Specifically, the l'th parton is described by a four component Dirac spinor [4] and has attached to it Dirac matrices $\gamma_\mu(l)$. Note that for different l's these matrices *commute*:

$$[\gamma_\mu(l),\ \gamma_{\mu'}(l)]_- = 0, \qquad \text{for } l \neq l' \qquad [\gamma_\mu(l), \gamma_\nu(l)]_+ = 2g_{\mu\nu} \tag{1}$$

The Hamiltonian is taken to be ***:

$$H_{\text{spin}} = c\sum_l \frac{1}{\sin\theta(l)}\left[v_\mu(2l-2)\cdot v_\mu(2l-1) + a_\mu(2l-1)\cdot a_\mu(2l)\right] \tag{2}$$

The operators v_μ and a_μ are γ_μ and $\mathrm{i}\gamma_5\gamma_\mu$ respectively. The factor $[\sin\theta(l)]^{-1}$ is the time dilation factor [5].

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** The apparent left-right asymmetry of the interaction may be cured by the introduction of an extra quantum number $B = \pm 1$ and requiring an array of the form ...++−−++.... B may be called 'baryon number'.

*** After the conclusion of this work we received a preprint by A. Neveu and J. Schwartz in which a similar model of fermion boson dual systems was discussed. We also learned that P. Ramond has proposed a model for fermions based on similar considerations.

The remarkable property of the Hamiltonian [2] is that it is exactly soluble when the system is discrete, and the solutions have smooth limits when the parton density is increased to ∞ and the continuum limit is taken. The procedure we use is to construct a new set of operators:

$$\psi_\mu(l) = \frac{i}{\sqrt{2}}[\rho(\theta)]^{\frac{1}{2}}\left[\prod_{n=1}^{2l-2}\gamma_5(n)\right]\gamma_\mu(2l-1) \qquad \phi_\mu(l) = \frac{i}{\sqrt{2}}[\rho(\theta)]^{\frac{1}{2}}\left[\prod_{n=1}^{2l}\gamma_5(n)\right]\gamma_\mu(2l) \tag{3}$$

Coupling ψ and ϕ in a column spinor $\chi = \binom{\psi}{\phi}$, it is readily seen that $\chi(l)$ satisfies F.D. anti-commutation rules:

$$[\chi_\mu(l),\ \chi_{\mu'}(l')]_+ = -g_{\mu\mu'}\delta_{ll'}\rho(\theta_l) \tag{4}$$

Moreover, the equation of motion satisfied by χ, after adjusting the constant c turns out to be the one-dimensional Dirac difference equation.

The difference equation for χ can be solved exactly. It is found that the character of the solution depends on whether the total number of partons is even or odd. In both cases the continuum limit exists and turns the difference equation into the one-dimensional Dirac equation:

$$i\partial_\tau\chi(\theta,\tau) = i\alpha\partial_\theta\chi; \tag{5}$$

$$[\chi_\mu(\theta),\ \chi_\nu(\theta')]_+ = -g_{\mu\nu}\delta(\theta-\theta') \tag{6}$$

Here α is the two-dimensional matrix $\begin{pmatrix}0 & -i\\ i & -0\end{pmatrix}$. The difference between the even and odd cases lies in the boundary conditions, which in turn emerge from the behaviour of the discrete operators at the edges.

In the even case the boundary conditions are:

$$\partial_\theta\,\psi(0) = \phi(0) = \partial_\theta\,\phi(\pi) = \psi(\pi) = 0 \tag{7}$$

The normal mode expansion of χ is therefore:

$$\begin{aligned}\psi &= \frac{1}{\sqrt{\pi}}\sum_{n=1}^{\infty}\cos(n-\tfrac{1}{2})\theta\left[b_n\exp\{-i(n-\tfrac{1}{2})\tau\} + b_n^\dagger\exp\{i(n-\tfrac{1}{2})\tau\}\right]\\ \phi &= -\frac{1}{\sqrt{\pi}}\sum_{n=1}^{\infty}\sin(n-\tfrac{1}{2})\theta\left[b_n\exp\{-i(n-\tfrac{1}{2})\tau\} - b_n^\dagger\exp\{i(n-\tfrac{1}{2})\tau\}\right]\end{aligned} \tag{8}$$

Noting that an even number of partons correspond to integer total spin value, we conclude that the systems considered are mesons.

The ground state is that state which is annihilated by all the $b_{n\mu}$'s and is readily seen to be the lowest energy state of the spin Hamiltonian.

Finally, the pion vertex is identified with $\sum k\cdot\gamma_5(l)\gamma(l)\exp\{ik\cdot s(l)\}$, where k is the pion momentum. At the edges, which are the only significant points for hadronic processes, $\gamma_5\gamma_\mu$ becomes identical to the relevant non-zero component of χ, thus:

$$k\cdot\gamma_5\gamma(0) \to k\cdot\sum(b_n + b_n^\dagger) \tag{9}$$

With these identifications, this meson model is seen to be identical with the dual meson model introduced by Neveu and Schwartz.

The odd case, which corresponds to half integral total spin, leads to the following boundary conditions:

$$\partial_\theta\psi(0) = \partial_\theta\psi(\pi) = \phi(0) = \phi(\pi) = 0 \tag{10}$$

The normal mode expansion becomes:

$$\begin{aligned}\psi_\mu &= \pi^{-\frac{1}{2}}\left(\Gamma_\mu + \sum_{n=1}^{\infty}\cos n\theta\left[c_{n\mu}\exp(-in\tau) + c_{n\mu}^\dagger\exp(in\tau)\right]\right)\\ \phi_\mu &= -\pi^{-\frac{1}{2}}\sum_{n=1}^{\infty}\sin n\theta\left[c_{n\mu}\exp\{-in\tau) - c_{n\mu}^\dagger\exp\{in\tau\}\right]\end{aligned} \tag{11}$$

The zero-frequency mode Γ_μ commutes with the Hamiltonian and satisfies the anticommutation rules:

$$[\Gamma_\mu , \Gamma_\nu]_+ = -g_{\mu\nu} \tag{12}$$

The ground state satisfies $c|0\rangle = 0$, as the c's lower the energy, but need not be annihilated by Γ_μ. In fact, the operators Γ_μ evidently generate a finite dimensional ground state space and must be identified with the Dirac matrices of the total hadron. The behaviour of the state under Γ_μ describes the centre of mass motion, while the $c_n^\dagger$'s excite the internal degrees of freedom. Thus, we are to postulate a subsidiary condition in the odd case, which corresponds to the usual $P^2 = M^2$, namely, the free particle Dirac equation:

$$\sqrt{2}\, P\cdot\Gamma|\rangle = \mathrm{i}M\Gamma_5|\rangle \tag{13}$$

where P is the total baryon momentum and M its mass.

The pion-parton vertex $-k\cdot\gamma_\mu\gamma_5$, remains of course the same, but acquires an additional zero frequency term when $\theta \to 0, \pi$:

$$k\cdot[\textstyle\sum(c_n + c_n^\dagger) + \Gamma] \to k\cdot\sum(c_n + c_n^\dagger) + 2M\Gamma_5 \tag{14}$$

A detailed inspection of the ground state baryon reveals that it couples to the 'unphysical' ground state scalar. We have not yet found which excited states decouple. Still, we shall illustrate the theory by computing the pion-ground state baryon scattering amplitude. The relevant terms are gotten by multiplying the usual dual integrand by the factors arising from the contraction of the two vertices defined by eq. (14). The $s-t$ term acquires the factor:

$$\Gamma\cdot k_1\, \Gamma\cdot k_2 + 2k_1\cdot k_2 \exp(\mathrm{i}\tau)(1-\exp(\mathrm{i}\tau))^{-1} \tag{15}$$

where τ is the 'time' difference between the two points acted upon by the vertices. Defining $x = \exp(\mathrm{i}\,\tau)$ and multiplying by the standard integrand we find:

$$\Gamma\cdot k_1\, \Gamma\cdot k_2 \int \mathrm{d}x\, x^{-s+M^2-1}(1-x)^{-t-1} + 2k_1\cdot k_2 \int \mathrm{d}x\, x^{-s+M^2}(1-x)^{-t-2} \tag{16}$$

We have used here $m_\pi^2 = -\frac{1}{2}$ [4]. Using the Dirac equation and some properties of the integrals we get an amplitude of the form $A+B$ where

$$A = 0, \qquad B = \int x^{-s+M^2-1}(1-x)^{-t-1} \tag{17}$$

The $t-u$ is obtained by s, u interchange. The s, u term is found to be:

$$A = 0, \qquad B = -\int x^{-s+M^2-1}(1-x)^{-u^2+M^2-1} \tag{18}$$

The model is obviously incomplete and also suffers from a disease. To complete it, baryonic states which decouple from the spurious states have to be found. To cure the model one has to detachionize it. We conjecture that detachionization has to do with isospin and possible chiral constraints.

We thank M. A. Virasoro for explaining the two-dimensional Dirac equation interpretation to us. The idea of an antiferromagnetic lattice with boundary conditions corresponding to $\frac{1}{2}$ integer frequencies was suggested to us by H. B. Nielsen even before the N-S model. We thank Dr. Nielsen for discussions about the application of spin lattices to hadron models.

References

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