Wage Contracts When Output Grows Stochastically: The Roles of Mobility Costs and Capital Market Imperfections

Yoram Weiss, *Tel-Aviv University*

The paper considers an industry in which individual output follows a stochastic growth process with a cumulative effect. It analyzes the roles of labor mobility and capital market conditions in the determination of wage contracts. Positive costs of mobility are shown to be necessary for the provision of wage and employment insurance when workers have no access to the capital market. When insurance is provided, wages grow less than average productivity. If the capital market is perfect, wage insurance will be provided even in the absence of costs of mobility. In this case, wages grow faster than average productivity.

I. Introduction

The purpose of this paper is to analyze the life-cycle development of wages in industries in which workers' output grows stochastically. In particular, I wish to investigate the roles of costs of mobility (e.g., loss of specific human capital), capital market imperfections, and properties of the stochastic production process as determinants of the degree of separation of wages from individual performance.[1]

I am grateful for helpful comments by E. Berkovitch, K. Burdett, and C. Wilson. Financial aid from the Ford Foundation, grant no. 10, Project D-4, and from the Hoover Institution is gratefully acknowledged.

[1] Thus formulated the problem is a special case of Reder's (1968) insightful analysis of the interaction between the worker's desire to avert risks and the firm's

[*Journal of Labor Economics*, 1984, vol. 2, no. 2]

0734-306X/84/0202-0001$01.50

In the following section of this paper, I describe the stochastic process which generates output. This description has been motivated by an application to the accumulation of scientific output as measured by published papers (see Weiss and Lillard 1982). A number of observers have noted that this stochastic process is characterized by a cumulative advantage (see Simon 1957; Price 1976). The more papers an author has already published, the easier it becomes to publish an additional paper. Learning by doing, recognition, and the accumulation of research-related resources combine to produce this "Matthew effect" (Merton 1968). The processes analyzed in this paper are thus the stochastic extension of the idea, common in conventional models of human capital, that human capital is self-productive.

Section III provides a 2-period analysis of the wage contract associated with the above class of stochastic processes. Information in this model is symmetric and public, but there are asymmetries in risk aversion and, possibly, in the access to the capital market. These asymmetries lead to potential wage exchanges between the worker and the firm over time and over productivity states. The amount of insurance provided by the wage contract is, however, limited by labor mobility. A successful worker who is mobile cannot be forced to give up any part of his earning power to the less successful. The only feasible transfer is from the present toward less favorable states (i.e., with lower productivity) in the future. Equalization of incomes across productivity states requires an increase in the variability of income over time. The demand for such insurance depends on the access of workers to the capital market and on the development of output over time. For instance, I show that for a class of stochastic processes in the absence of costs of mobility and under imperfect market conditions, no insurance will be provided. This means that, in this case, piece rate will be the dominating arrangement. Generally, the higher the costs of mobility, the flatter and less variable will be the wage profile.

Section IV extends the 2-period model to incorporate selection for

---

wish to avoid repetitive costs of training and screening. Those forces combine to "increase the fraction of labor force hired [for the bottom rung] by firms offering life time careers, but the effect of risk hedging on the dispersion of earnings prospects is not clear a priori" (p. 605). In Reder's analysis the latter ambiguity arises from the interaction between the provision of insurance and firm size. He also notes the role of external market constraints in limiting the provision of insurance, concluding that "the very imperfect attempt at mutual insurance represented by policy of promotion from within enormously complicates the relation between the value of current marginal contribution of a productive agent and its reward and alters the distribution of earnings in a variety of ways" (pp. 607–8). Reder strongly recommended this view of the labor market when I became his student at Stanford in the mid-sixties. This was the time when a human capital approach which strongly linked productivity and wages became dominant. It was only later that I (and others) learned to appreciate the role of insurance and capital market imperfections in partially separating wages from productivity.

tenure. It is assumed that workers can move to an alternate sector. They agree, ex ante, to delegate the power of terminating the employment contract to the firm. Workers who stay with the firm throughout their career (i.e., attain tenure) may be promoted (i.e., obtain a higher wage) depending upon their success record. The higher the costs of mobility across firms, the higher will be the proportion tenured and the fewer promotions there will be.

Section V contains a multiperiod analysis of the wage contract for the case in which all workers are assumed to have tenure and no access to the capital market. A multiperiod analysis is clearly important for any application to actual data on life-cycle wage development. It also brings up some new theoretical issues which cannot be analyzed in the restricted 2-period context, such as differential treatment of early and late success in the wage contract. It is shown that the wage contract can be characterized by three phases of random duration: an initial phase with constant wages, a second phase in which wages are increased as successes occur, and a final phase in which wages become constant again. Workers with few or late successes will have constant wages throughout their whole career. Again, the higher are the costs of mobility, the longer will be the expected duration of the phases with constant wages.

It is shown in the paper that asymmetries in risk aversion and in access to the capital market combined with costs of mobility can generate optimal contracts (more precisely, competitive contracts) with features similar to those actually observed. However, the usual caveats apply. One cannot conclude that real-life contracts are indeed generated in the way suggested by the model since many important aspects of the problem are not included in the present framework. The most obvious omissions are the moral hazard issue, which arises from the influence that workers exercise on the outcomes of the production process, and the possible interaction among workers in the production process.[2]

II. The Production Process

Consider a stochastic process in discrete state and in discrete time. At each period t, $t = 0, 1, 2, \ldots, T$, the individual may either produce a unit of output (success) or produce nothing (a failure). Let $\mathbf{h}_t$ be a sample path, that is, a vector of size t consisting of zeros and ones. Let $N_t(\mathbf{h}_t)$ be the number of successes accumulated up to t. We assume that the production process is Markovian and denote $\text{pr}\{N_{t+1} - N_t = 1 | N_t = y\} = P(y)$.[3] The Matthew effect can be represented by the assumption that

[2] The role of capital market imperfections in the determination of wages when effort is variable is discussed in Lazear (1981) and Rogerson (1983).

[3] This is clearly a restrictive aspect of the formulation, since it does not allow the duration of time in any given state to affect the probability of "birth" for a new success.

$P'(y) > 0$, that is, the probability of success is an increasing function of past output. Let $Q_t(y)$ denote the expected output from t to T of a worker with $N_t = y$ successes at time t. The expected output satisfies the difference equation:

$$Q_t(y) = P(y) + P(y)Q_{t+1}(y + 1) + [1 - P(y)]Q_{t+1}(y). \tag{1}$$

Since $P'(y) > 0$, $Q_t(y + 1) > Q_t(y)$, that is, at any point in time, expected output is an increasing function of past accumulated output. Since life is finite $Q_t(y) > Q_{t+1}(y)$, a worker who has accumulated y successes is expected to produce less the shorter is the production period.

There is an alternative specification of the production process in which each worker has a fixed output rate but workers are heterogeneous and their (fixed) individual parameters are known. A Bayesian rule is used to update the estimated output capacity per unit of time (see Freeman 1977; Harris and Holmstrom 1982; Harris and Weiss 1982). These models appear to be inconsistent with at least one piece of data, namely, the observed increase in both the mean and the variance of output (e.g., publication rates within a cohort; see Weiss and Lillard 1982), while the present specification is consistent with these findings. This suggests that the reinforcement hypothesis, reflected in $P'(y) > 0$, merits some attention. While in principle it is possible to combine heterogeneity with reinforcement, my objective here is to complement the available literature by focusing on the latter issue.

III. A 2-Period Analysis of the Wage Contract

A 2-period model, similar to the one discussed by Freeman (1977), can be used to highlight the role of the various determinants of the wage contract. In particular, I wish to discuss the interaction among capital market conditions, mobility costs, and stochastic growth.

Let there be two periods indexed by zero and one, respectively. The production process starts at zero at the beginning of the first period and can reach any point in {0, 1, 2} by the end of the second period when the worker stops producing. Assume that wages in each period are paid before success or failure is observed. One can simplify the notation considerably noting the one-to-one correspondence between state and history at the beginning of the second period (i.e., time of last payment). Let s_0 be the wage at the first period and s_1^i, $i = 0, 1$, the wage at the second period associated with i successes in the first period. Similarly, let p_i be the probability of success at any period given that i successes have occurred. Let a denote the costs of changing employers.

In competition each firm must offer a wage contract such that no other firm can make positive expected profits by offering an alternative contract which is considered superior by a worker with any success record. Under the assumption that workers cannot borrow or lend, the "optimal," that is, competitive, contract solves[4]

$$\max_{s_0, s_1^0, s_1^1} U(s_0) + (1 - p_0)U(s_1^0) + p_0 U(s_1^1) \tag{2}$$

subject to

$$s_0 - p_0 + (1 - p_0)(s_1^0 - p_0) + p_0(s_1^1 - p_1) = 0 \tag{3}$$

$$s_1^i \geq p_i - a, \quad i = 0, 1. \tag{4}$$

The first-order conditions for the maximization are

$$U^1(s_1^i) \leq U^1(s_0), \tag{5}$$

with equality if (4) is ineffective, $i = 0, 1$.

As noted by Freeman (1977), it follows immediately from (5) that the wage is rigid downward. $s_1^i \geq s_0$ for $i = 1, 2$. The special nature of the reinforcing stochastic process is that productivity per period cannot decline. In this case it can be shown that wage insurance is provided by the firm only if the costs of mobility are positive, $a > 0$. More specifically, one can prove the following proposition:

PROPOSITION 1: Under imperfect capital markets and for $p_1 > p_0$, the optimal wage contract satisfies

$$s_1^1 = s_1^0 = s_0 = p_0\left(1 + \frac{p_1 - p_0}{2}\right) \quad \text{for} \quad a \geq (p_1 - p_0)\left(1 - \frac{p_0}{2}\right);$$

$$s_1^1 = p_1 - a > s_0 = s_1^0 = p_0\left(1 + \frac{a}{2 - p_0}\right) \quad \text{for} \quad (p_1 - p_0)\left(1 - \frac{p_0}{2}\right) > a \geq 0.$$

PROOF: The proposition follows from the fact that for each of the two possible cases $s_1^1 > p_1 - a$ and $s_1^1 = p_1 - a$ allowed by (4), one must have $s_1^0 > p_0 - a$, unless $a = 0$. Assume first $s_1^1 > p_1 - a$ and suppose $s_1^0 = p_0 - a$. From the first-order conditions, $s_1^1 = s^0 < s_1^0$. Thus $p_0 - a > p_1 - a$, a contradiction. Assume next $s_1^1 = p_1 - a$ and $s_1^0 = p_0 - a$. From the first-order conditions $s_1^0 > s_0$, from the zero profit condition if $a > 0$, $s_0 > p_0$, thus $s_1^0 = p_0 - a > s_0 > p_0$, again a contradiction. It is

[4] All workers are assumed identical and there are constant returns to scale in production. We can thus normalize the talk about utility per worker and profits per worker (or per contract). We simplify by assuming zero subjective discount and market rates. The assumption of imperfect capital markets is reflected in the appearance of s_i in the direct utility function. Workers are assumed risk averse. Thus $U'(\ ) > 0$ and $U''(\ ) < 0$.

never optimal to pay $s_1^0 = p_0 - a$ since it is feasible, if $a > 0$, to reduce the variance in future earnings by increasing the payments to the less successful, taking income away from the more successful. To complete the proof, note that $s_1^0 > p_0 - a$ implies $s_1^0 = s_0$, that is, the wage in the case of failure is the same as in the first period. It thus remains to check if it is feasible to set $s_1^1 = s_0$ as well. Substituting $s_1^0 = s_1^1 = s_0$ into (3), we get $s_1^1 = p_0[1 + (p_1 - p_0)/2]$. This will be feasible if $p_0[1 + (p_1 - p_0)/2] \geqslant p_1 - a$ or $a \geqslant (1 - p_0/2)(p_1 - p_0)$, which establishes the first part of the proposition. If the inequality is reversed then by setting $s_1^1 = p_1 - a$, and $s_1^0 = s^0 = p_0[1 + a/(2 - p_0)]$, one satisfies the constraints (3), (4), and the first-order condition (5). This completes the proof.

It is remarked that the wages determined in proposition 1 are independent of the parameters of the utility function.

There are two important empirical implications of proposition 1. First, the variance in future wages is a decreasing function of the size of the costs of mobility across firms. Second, wages grow on the average at a lower rate than productivity. More specifically, for high costs of mobility the contract provides perfect insurance and fixed wages over time. If the mobility costs are positive but not "too high," wages of the successful will be higher than those of the less successful, but the discrepancy in wages will be smaller than in the productivity of the two groups, thus providing partial insurance. If the costs of mobility are zero, the contract provides no insurance at all; wages will be equal to the (expected) productivity in each period.

The above results are sensitive to the specification of the stochastic process and capital market conditions. If future productivity can decline then insurance against such states will be bought even under imperfect capital markets and in the absence of costs of mobility. However, if no future state leads to lower productivity than the current one, then, under imperfect capital markets, the worker will be unwilling to sacrifice current income in order to reduce the gap between the realizations of future wages. Risk aversion alone is *not* sufficient to generate insurance. When the population is heterogeneous, the perceived productivity of a worker can decline as a result of a poor success record, even though the worker's actual productivity remains the same. The risk-averse worker will buy insurance against this event, sacrificing some current income. In the production process specified in Section II, this motivation for insurance is absent, and the only way to finance insurance through the firm is to take income from the worker in the event that he is successful. Clearly this is not feasible if $a = 0$.

If the worker has access to a perfect capital market, the wage contract solves

$$\max_{c, s_0, s_1^0, s_1^1} U(c) + (1 - p_0)U(s_0 + s_1^0 - c) + p_0 U(s_0 + s_1^1 - c) \qquad (6)$$

subject to

$$U'(c) = (1 - p_0)U'(s_0 + s_1^0 - c) + p_0U'(s_0 + s_1^1 - c) \tag{7}$$

and the constraints (3) and (4), where c, first-period consumption, is chosen by workers according to rule (7) as a function of the wages specified in the contract. It is easy to verify that the solution is of the form $s_1^1 = s_1^0 \geqslant p_1 - a$ for all a where s_0 is determined from (4) as a residual. While the income stream is not uniquely determined, full insurance across states is attained. Under perfect capital markets insurance is bought even if $a = 0$. Moreover, in this case, the expected growth in wages will *exceed* the expected growth in productivity, contrary to the case of imperfect capital markets.[5]

The reason for these differences is easy to explain. Under imperfect capital markets, the worker will seek a contract which spreads consumption evenly over time, and not only over states. If productivity grows and workers are mobile, wage insurance can be bought only if income is spread unevenly over time. This limits the demand for wage insurance under imperfect capital markets. However, if workers have access to the capital market they are not affected by wage variation over time and, being risk averse, they will choose to buy full wage insurance.

A somewhat paradoxical aspect of the model is that an increase in costs of mobility can generate a Pareto-superior allocation of wages. The reason is twofold: First, it is always possible to avoid the costs of moving by selecting wages which will not induce mobility. Second, the potential costs of mobility enlarge the set of feasible trades between the workers and the firm.

IV. Selection for Tenure

The discussion so far has assumed that all workers can obtain the same wage contract. In practice this is not the case. Employers offer a lifetime wage contract only to some of the workers; others are forced to leave. Since all employers are assumed identical and information is public, the workers who are not tenured must leave the industry. Let us assume that an alternative employment exists in which the wage is certain and stable

[5] The fact that under perfect capital markets (and in the absence of moving costs), young workers are paid less than the value of their current output raises a question concerning the completeness of my characterization of a competitive contract. Firms can pay each worker the expected value of his lifetime output and still make positive profits if they can increase their labor force over time without a bound (see Stiglitz and Weiss 1982, p. 15). In a stationary economy, of course, it is impossible for all firms to grow in such a manner, and a firm attempting such a policy will eventually be unable to pay the wages it promises the newly hired workers. One can thus argue that in a rational expectations equilibrium such unbounded hiring policies will not be pursued.

over time. In the absence of risk aversion and costs of mobility, workers will leave voluntarily when their productivity falls below their opportunity wage. However, risk aversion, combined with costs of mobility, induces labor contracts in which firms set employment and promise wages sufficient to retain the workers they choose to employ.

We can use the 2-period model of Section III to illustrate the joint determination of employment and wages. Let q_1^i be the probabilities of employment in period 1 conditioned on the number of successes in period 0. By definition these retention probabilities satisfy $0 \leqslant q_1^i \leqslant 1$. Let b denote the wage outside the industry. Assume for simplicity that costs of mobility, denoted by a, are identical within and across industries. Then the optimal wage and employment contract, under imperfect capital markets, solves the problem:

$$2U(b) = \max_{q_1^0, q_1^1, s_0, s_1^0, s_1^1} U(s_0) + p_0[U(s_1^1)q_1^1 + (1 - q_1^1)U(b - a)] + (1 - p_0)[U(s_1^0)q_1^0 + (1 - q_1^0)U(b - a)] \tag{8}$$

subject to

$$p_0 - s_0 + p_0(p_1 - s_1^1)q_1^1 + (1 - p_0)(p_0 - s_1^0)q_1^0 = 0 \tag{9}$$

$$s_1^1 \geqslant p_1 - a \tag{10}$$

$$s_1^0 \geqslant b - a. \tag{11}$$

Implicit in this formulation is a price for the output which adjusts to make the zero profit condition consistent with the requirement that workers enjoy the same expected lifetime utility in both industries. We assume that such an equilibrium is attained when the price of output is unity. This implies $p_1 > b > p_0$ and explains the appearance of p_1 and b in the constraints (10) and (11), respectively.

The first-order conditions for maximum can be written in the following form:

$$q_1^i[U'(s_1^i) - U'(s_0)] \leqslant 0, \quad i = 0, 1, \tag{12}$$

equality if (10) or (11) is ineffective.

$$\begin{aligned} U(s_1^i) - U(b - a) + (p_i - s_1^i)U'(s_0) &= 0 \quad \text{if} \quad 0 < q_1^i < 1, i = 0, 1, \\ &\geqslant 0 \quad \text{if} \quad q_1^i = 1, \\ &\leqslant 0 \quad \text{if} \quad q_1^i = 0. \end{aligned} \tag{13}$$

Condition (12) implies that the wages of workers who stay with the firm are nondecreasing. Condition (13) states that a worker will be retained only if the utility loss from forcing him out to the alternative sector, given the wage contract, is less than the resulting gain in profits.

Propositions 2, 3, and 4 describe properties of the solution.

PROPOSITION 2: If some unsuccessful workers obtain tenure, they will also obtain wage insurance. That is, $q_1^0 > 0 \longrightarrow s_1^0 = s_0 > b - a$.

PROOF: Suppose $q_1^0 > 0$ and $s_1^0 = b - a$. From (13) we must have $p_0 \geqslant s_0^1$. Since, due to (12), $s_1^0 > s_1$, the zero profit condition implies $p_1 < s_1^1$. Hence, due to (12) again, $s_1^1 = s_0$, which implies $p_0 > p_1$, a contradiction.

A consequence of proposition 2 is that those who did not receive tenure suffer (ex post) a real loss. The departure of those workers cannot be voluntary. They must be forced to leave (i.e., tenure is denied).[6] This result will not hold in models of tenure and job turnover without risk aversion, such as that of Jovanovic (1979), where separations are always voluntary given the workers' realized record. In the simple discrete model presented here, this requires an arbitrary selection (e.g., lottery) among identical workers. If we allow output to be continuous, selection can be based on the production record alone.

PROPOSITION 3: All successful workers will be offered tenure. That is, $q_1^1 = 1$.

PROOF: Suppose $q_1^1 < 1$. Then, due to (10) and (13), $s_1^1 > p_1$. That is, it is optimal to deny tenure to some successful workers only if, given the wage contract, their retention causes losses ex post. We first show that this is impossible under an optimal wage contract with $0 < q_1^1$. Using proposition 2 and the zero profits condition (9), we see that $p_1 - s_1^1$ and $p_0 - s_0$ must have opposite signs, thus the assumption that $s_1^1 > p_1$ implies $s_0 < p_0$. But from (12), the assumption that $s_1^1 > p_1$ implies $s_1^1 = s_0$ when $q_1^1 > 0$. Thus $s_1^1 > p_1$ and $s_1^1 = s_0 < p_0 < p_1$; this contradiction establishes the impossibility of $0 < q_1^1 < 1$. It remains to show that $q_1^1 = 0$ cannot be optimal. But this follows directly from the assumption that $p_1 > b > p_0$. If instead of setting $q_1^1 = 0$ one sets $s_1^1 = p_1$ and $q_1^1 > 0$, it is still possible to choose the same values for s_0, s_1^0, q_1^0 and thus increase expected utility.

Since the successful workers produce more than in the alternative sector it is always efficient to retain them. The demand for wage insurance can then be satisfied by transferring some of their product to the unsuccessful. The interesting question is under what conditions will it be optimal to retain the unsuccessful. This depends on the costs of mobility as shown in proposition 4.

[6] Since $p_1 > b > p_0$ in equilibrium and all firms are assumed identical, no firm can offer the worker who is denied tenure a better wage than a firm in the fixed-wage sector. A laid-off worker must switch to the fixed-wage sector.

PROPOSITION 4:

i. In the absence of mobility costs it will be optimal to deny tenure to all unsuccessful workers. That is, $a = 0 \longrightarrow q_1^0 = 0$.

ii. If the mobility costs exceed the difference between the productivity in the alternative sector and the productivity in the event of failure all workers will be retained. That is, $a > b - p_0 \longrightarrow q_1^0 = 1$.

PROOF:

i. Suppose $a = 0$ and $q_1^0 > 0$. From (11) $s_1^0 \geq b$. Consider first the case that $s_1^0 > b$. From (12), $s_1^0 = s_0$. Thus both s_1^0 and s_0 exceed b which exceeds p_0. For $a = 0$ (10) implies $s_1^1 \geq p_1$. Therefore the zero profit condition (9) is violated and $s_1^0 > b$ leads to a contradiction. Now suppose $s_1^0 = b$ from (13) $p_0 \geq s_1^0$, which implies $p_0 \geq b$, a contradiction to the condition $p_1 > b > p_0$, which is required for equilibrium. We thus conclude that $q_1^0 = 0$.

ii. From proposition 2, $q_1^0 > 0$ implies $s_1^0 = s_0$. For $q_1^0 = 0$ we may also set $s_1^0 = s_0$. Thus, the right-hand side of (13) for $i = 0$ is $u(s_0) - u(b - a) + u'(s_0)(s_0 - p_0)$. From the concavity of $u(\ )$, $u(s_0) + u'(s_0)(s_0 - p_0) \geq u(p_0)$. Therefore, if $p_0 > b - a$, the right-hand side of (13) is strictly positive and it is optimal to set $q_1^0 = 1$.

It was shown in Section III, proposition 1, that in the absence of mobility costs there will be no wage insurance. Proposition 4 demonstrates that there will be no employment insurance either. These two results are related. If firms pay no wage insurance, the unsuccessful worker will prefer not to stay with the firm even if tenure is offered. On the other hand, when the costs of mobility are sufficiently high the optimal contract offers full wage and employment insurance. That is, the first-order conditions are satisfied with $s_0 = s_1^0 = s_1^1$ and $q_1^0 = q_1^1 = 1$. For intermediate values of a, the optimal contract offers partial wage and employment insurance, that is, $s_0 = s_1^0$, $s_1^1 = p_1 - a$ and $0 < q_1^0 < 1$.[7] It can be shown that partial wage insurance can also be associated with full employment insurance, but not vice versa.

[7] In the comparisons in the text, the costs of mobility are low or high *relative* to the productivity parameters. This is not a simple comparative static experiment in which only a changes and all other parameters remain constant. Such a variation would violate condition (8), which states that the expected utility from the optimal contract is uniquely determined by the opportunity wage in the alternative sector. As long as (8) holds as an equality, a change in a alone will change the price of the product. If one limits attention only to the properties of contracts supplied at any *fixed* price of output, γ, satisfying, $\gamma p_1 > b > \gamma p_0$, allowing inequality at (8), then it is easy to show that $dq_1^0/da \geq 0$. Specifically,

$$\frac{dq_1^0}{da} = \frac{(1 + (1 - p_0)q_1^0)u'(b - a) + (\gamma p_0 - s_0)u''(s_0)}{-(\gamma p_0 - s_0)^2 u''(s_0)(1 - p_0)} > 0$$

for $0 < q_1^0 < 1$.

Similar results can be established for the case of perfect capital markets. The fact that employment in the second period is uncertain considerably restricts the wage profiles which the firm can offer. In particular, s_0 must be sufficiently high to allow the worker who is not tenured to mitigate the reduction in consumption due to the loss of earnings in the event of nontenure. Under perfect capital markets, a full insurance contract may be dominant even if there are no costs of mobility provided the workers are sufficiently risk averse.

To summarize, risk-averse workers will voluntarily enter agreements which allow firms to set employment. The optimal employment contract conditions tenure on the success record of the worker. It offers a higher retention probability for the successful worker. It also provides insurance by keeping some workers whose output turns out to be less than in the alternative sector (clearly, the allocation of labor does not maximize output). Not all tenured workers are promoted. In the simple 2-period model only the successful worker obtains a wage increase. However, the contract provides a minimum wage guarantee for all retained workers. Under imperfect capital markets, costs of mobility are essential for the provision of wage and employment insurance. Under perfect capital markets, insurance will be provided even in the absence of such costs.

V. The Wage Contract: A Multiperiod Analysis

The 2-period model is often used to derive qualitative properties of lifetime wage contracts. There are, however, some major issues which cannot be discussed within the restricted 2-period framework. For example, what precisely is the information which is used in the wage contract? Are two workers with the same number of successes always treated in the same way irrespective of the history of accumulation? Is there some advantage in early accumulation of a given number of successes? In the 2-period model, there is a unique path associated with each state. Consequently such questions do not arise. We thus turn to a multiperiod analysis of the wage contract. As in Section III, we assume that no alternative sector exists and abstract from selection for tenure.[8]

To fix the main ideas, it will be useful to review, first, the roles of productivity growth, mobility costs, and capital market imperfections under conditions of certainty. When output is not random, the wage contract for the case of imperfect capital markets is derived from

$$\max_{\{s(t)\}} \int_0^T U[s(t)]dt \tag{14}$$

$$\dot{X}(t) = -s(t) \tag{15}$$

[8] The combined determination of employment and wages in a multiperiod context is the natural next step. (See Harris and Weiss 1982.)

$$X(t) \geqslant Q(t) - a, \quad X(0) = Q(0), \tag{16}$$

where X and Q denote expected payments and output from t to T, respectively, and a is the cost of changing employment. If the productivity per unit of time, $q(t) \equiv -\dot{Q}(t)$, is increasing, the solution for this problem, $s^*(t)$, will be of the form $s^*(t) = s_0 > q(t)$ for $t \in [0, t)$, $s^*(t) = q(t)$ for $t \in [t_0, t_1]$, $s^*(t) = s_1 < q(t)$ for $t \in (t_1, T]$ (see fig. 1). The firm provides the worker with a lifetime earnings profile which is flatter than his productivity profile and enables him to spread his consumption more evenly, thus increasing his lifetime utility. Clearly, such an exchange between the firm and the worker is feasible only if the costs of mobility are positive, $a > 0$. With positive costs of mobility the worker can be forced to repay, via reduced wages, the amounts he received from the firm in the past. If the worker can borrow and lend freely outside the firm, the contract will not determine a unique wage profile. There are many wage profiles which satisfy the constraint (16). All those profiles are equivalent from the worker's point of view, since the worker can rearrange his income streams by borrowing and lending. There is no particular reason for the wage to increase at a lower average rate than the worker's productivity.[9]

It is also important to note the role of growth in productivity in this context. If $q(t)$ is monotone decreasing, rather than increasing, the optimal contract will yield stable wages $s(t) = s_0$ even if costs of mobility are zero. It is the combination of growth in productivity and imperfect capital markets which makes costs of mobility a necessary condition for the provision of a stabilizing wage policy.

Let us return to the case in which output is uncertain and assume that the workers' output follows the stochastic process described in Section II. Initially all workers are identical. As the process evolves, different

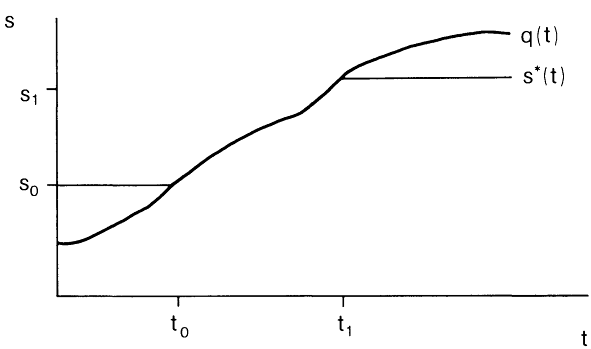

FIG. 1.—Productivity and wages as functions of age

[9] Nor is there any reason, under certainty, for the emergence of binding wage contracts. A piece-rate scheme is perfectly sensible in this case.

workers accumulate a different number of successes, and their expected future output differs accordingly. There is perfect information on the past performance of each worker.

Firms compete for workers with a given record by offering contingent contracts which specify the wage profile for each prospective path of accumulated successes. Wages in each period are paid before the output within the period is observed. Let $S_t(\mathbf{h}_t)$ be the wage paid at time t given a sample path $\mathbf{h}_t$, and let $X_t(\mathbf{h}_t)$ be the *expected* value of wages from t to T for an individual with record $\mathbf{h}_t$.

These functions are related by the recurrence relation:

$$X_t(\mathbf{h}_t) = S_t(\mathbf{h}_t) + P[N_t(\mathbf{h}_t)]X_{t+1}(\mathbf{h}_t, 1) + \{1 - P[N_t(\mathbf{h}_t)]\}X_{t+1}(\mathbf{h}_t, 0). \tag{17}$$

That is, the expected value of wages from t to T equals the wage offer for the period t and the expected value of the future wage prospects from $t + 1$ to T associated with failure and success, respectively. In contrast to the case of certainty (see eq. [15]), the change in future wage prospects depends not only on the current salary rate but also on the realization of the random process. As information changes due to a success or a failure, expected payments change accordingly. A stochastic adjustment rule for X_t arises. To describe the adjustment rule, we define the functions

$$Z_t^1(\mathbf{h}_t) \equiv X_{t+1}(\mathbf{h}_t, 1) - X_t(\mathbf{h}_t), \; Z_t^2(\mathbf{h}_t) \equiv X_{t+1}(\mathbf{h}_t, 0) - X_t(\mathbf{h}_t). \tag{18}$$

These functions, together with an initial condition on X_0, fully characterize the wage contract.

At any point in time the firm is indifferent among all contracts with the same expected payments to the worker. An "optimal," that is, Pareto efficient, contract must maximize the expected utility for the worker over all feasible contracts among which the firm is indifferent. Competition among firms will eliminate nonefficient contracts.

Consider a worker at time t with a particular given history. Let him possess a contract with value x and assume that he accumulated y successes.[10] Again, denote the costs of mobility between firms by a (a constant independent of age or the success record) and assume that the firm is willing to finance the mobility costs of the worker. Denote the maximal expected utility from t to T by $V(x, y, t)$. Assume that the worker cannot borrow or lend outside the firm and that only the firm has access to a

[10] I use lowercase letters to denote specific realizations of the functions $s_t(h_t)$, $X_t(h_t)$, and $N_t(h_t)$. When there is no risk of confusion the time subscript, t, is deleted.

perfect capital market. Then the optimal value function must satisfy the recurrence solution

$$V(x, y, t) = \max_{s, z_1, z_2} \{U(s) + P(y)V(x + z_1, y + 1, t + 1) + [1 - P(y)]V(x + z_2, y, t + 1)\} \tag{19}$$

subject to

$$s + P(y)z_1 + [1 - P(y)]\, z_2 = 0, \tag{20}$$

$$x + z_1 \geqslant Q_{t+1}(y + 1) - a, \tag{21}$$

and

$$x + z_2 \geqslant Q_{t+1}(y) - a. \tag{22}$$

The constraint (20) describes the possible revisions in the contract which do not affect the expected payments by the firm. All the choices of s, z_1, z_2 which satisfy (20) guarantee the worker with $\mathbf{h}_t$ at t, the same expected payments, x, as the contract he currently holds. If the contract is indeed optimal, the expected utility of the worker cannot be increased by such revisions.

The constraints (21) and (22) restrict the potential revisions of the contract to the set consistent with no mobility across firms. That is, the optimal contract satisfies

$$X_t^*(\mathbf{h}_t) \geqslant Q_t\, [N_t\, (h_t)] - a. \tag{23}$$

Condition (23) is necessary to retain the worker. If the inequality is reversed, then a competing firm can attract the worker by paying him at least the same wage at each state and make positive profits. It is also sufficient to retain the worker, since wages, for any given x, are distributed optimally. Since all the available information is public and known to all firms, there is no potential gain from mobility. The optimal contract will be structured so as to avoid the unnecessary moving costs.

The first-order conditions for the maximization of (19) subject to (20), (21), and (22) are

$$U'(s) - \lambda = 0, \tag{24}$$

$$V_x(x + z_1, y + 1, t + 1) - \lambda + \frac{\mu_1}{P(y)} = 0, \tag{25}$$

$$V_x(x + z_1, y, t + 1) - \lambda + \frac{\mu_2}{1 - P(y)} = 0, \tag{26}$$

where λ, μ_1, and μ_2 are nonnegative Lagrange multipliers for the constraints (20), (21), and (22), respectively. Differentiating (19) with respect to x using conditions (20)–(22), we also have

$$V_x(x, y, t) = \lambda. \tag{27}$$

To solve the recurrence relation (19), one uses the end conditions $V(x, y, T + 1) = 0$ for all x, y, and an initial condition specifying X_0. For the problem under analysis, the initial condition is given by the zero profits condition:

$$X_0 = Q_0. \tag{28}$$

Free entry (exit) into the industry will prevent the firms from making positive (negative) profits on the optimal lifetime contract.

It follows directly from the first-order conditions that the optimal solution $S_t^*(\mathbf{h}_t)$ satisfying conditions (24)–(27) is nondecreasing. That is, both $S_{t+1}^*(\mathbf{h}_t, 1)$ and $S_{t+1}^*(\mathbf{h}_t, 0)$ are larger than or equal to $S_t^*(\mathbf{h}_t)$. This is seen by solving for s from (24) and (27) as a function of the state variables x, y, and t and noting that (25) and (26) imply that under any change in information $V_x(\ )$ is nonincreasing along the optimal path.

Further properties of the solution depend crucially on the magnitude of the costs of mobility across firms. When the costs of mobility, a, are sufficiently high to make the constraints (21) and (22) ineffective, the necessary conditions (24)–(27) imply that V_x remains constant whether a success or a failure occurs. That is, $S_t^*(\mathbf{h}_t) = S_{t+1}^*(\mathbf{h}_t, 1) = S_{t+1}^*(\mathbf{h}_t, 0)$ for all t and $\mathbf{h}_t$. The contract provides perfect insurance against individual failure and a stable salary stream over time. All the variability in output over state and over time is shifted to the firm which is risk neutral and faces perfect capital markets. In the absence of mobility costs (i.e., $a = 0$), the optimal contract provides insurance only against the uncertainty in output within each period but no insurance at all against future uncertainty. That is, salary at period t equals the expected output in this period. We formalize this result in proposition 5:

PROPOSITION 5: In the absence of costs of mobility, $a = 0$, the optimal contract is to pay for all t and $\mathbf{h}_t$ the immediate expected output. That is, $S_t^*(\mathbf{h}_t) = P[N_t(\mathbf{h}_t)]$ or equivalently $X_t(\mathbf{h}_t) = Q_t[N_t(\mathbf{h}_t)]$, for all t and $\mathbf{h}_t$.

PROOF: See Weiss and Lillard (1982).

In the absence of costs of mobility, the wage contract is thus seen to be almost identical to a piece-rate arrangement. The only difference from strict piece rate arises from the assumption that wages are paid before production occurs, which leads to within-period wage insurance. As in the 2-period case, the lack of redistribution across periods is due to two interrelated causes: first, the production process is characterized by growth, that is, the immediate (per period) expected output never decreases (this is in contrast to the sorting model of Freeman [1977]). Second, the worker is assumed to have no access to the capital market. Since a transfer from a successful worker to the less successful is not feasible under perfect labor mobility, the only way to pay for the wage insurance is by giving up wages at the beginning of one's career. For a growing process this means that a lower variability over state can only be achieved through higher variability in wages over time. Under imperfect capital markets the worker will prefer not to engage in such an exchange.

The empirically relevant case seems to be the one in which the costs of mobility across firms are positive, but not high enough to eliminate mobility completely. Thus, if wages are held constant, favorable runs of success will increase expected productivity to a level at which the threat of moving into another firm becomes effective. To retain the successful workers, the firm must increase their wage. In contrast to the case of zero mobility costs, not every success calls for a wage increase. In particular, toward the end of the worker's life, mobility costs will be sufficiently high (compared to the potential gain from mobility) to prevent mobility. Similarly, at the beginning of the contract period, due to (28), the worker's expected earnings are close to his expected output and the costs of changing employer are also relatively high. In these phases the constraints (21) and (22) are ineffective and the wage remains constant. Thus the effect of mobility costs is not uniform over the life cycle. Given any fixed mobility costs, the incentive to leave the firm either very early or very late in the contract period is relatively weak. This fact can be exploited to smooth the wage payment profile by offering the worker constant wages at the initial and final phases.

To complete the description of the wage profile, we note that even in the interim phase of the life cycle in which the mobility threat is of relevance, wages do not increase unless the worker's record is improved.

PROPOSITION 6: For all t and $\mathbf{h}_t$, $S_t^*(\mathbf{h}_t) = S_{t+1}^*(\mathbf{h}_t, 0)$. That is, the wage always remains constant in the case of failure.

PROOF: See Weiss and Lillard (1982).

The result is quite intuitive. Since the worker's expected output per period does not change, his earning power outside the firm remains fixed, and it is feasible to smooth his income within the firm. There is no danger that the worker will quit in such circumstances.

To summarize, the optimal wage contract consists of three successive phases: an initial phase with constant wages, an interim phase in which wages are increased as successes occur, and a final phase in which wages are stabilized again. The duration of each phase is random. For some workers the second phase will be empty and their wage will never increase. This group includes all those with complete runs of failure and also some who only had late success.

An important aspect of the optimal wage contract is that the wage generally depends on the whole history of successes and not simply on the total number of accumulated successes. In particular, it is possible that $S_t^*(\mathbf{h}_{t-2}, 1, 0) > S_t^*(\mathbf{h}_{t-2}, 0, 1)$. While in both cases the worker has the same number of successes at time t, there is an advantage in gaining the successes at an early stage. This is because an early success provides an opportunity for sharing risks with potentially more productive realizations. Since at time $t - 1$ $Q_{t-1}(\mathbf{h}_{t-2}, 1) > Q_{t-1}(\mathbf{h}_{t-2}, 0)$, the worker who was successful can obtain a wage insurance which is not available to the worker with failure. Realizations of the wages at time t will reflect these differences in past opportunities.

The expected rate of increase in wages throughout the life of the contract is lower than the expected rate of increase in output. In the first phase, expected wages exceed expected output; in the last phase, the converse is true. During the interim phase, expected output and salary move together. The existence of mobility costs across firms implies that the individual success record, while perfectly public, has in it some element of specific human capital. The result that on the average wages grow at a slower rate than productivity can be interpreted as a sharing arrangement of a joint investment in acquiring information about the worker's future output.

The multiperiod analysis of wages under perfect capital markets is essentially the same as in the 2-period case. It is feasible to provide the worker with perfect insurance by paying him wages which equal the highest possible productivity level in each period, except for the initial point of entry (see Harris and Holmstrom 1982, p. 322). When the worker enters the firm, he pays for the insurance by receiving a low, possibly negative, wage. Again the arrangement is not unique, since a worker with access to the capital market can also be retired with higher wages in the future and a lower initial wage. If the costs of mobility are zero, it is clear that the growth in wages exceeds the expected (i.e., average) growth in productivity. (With sufficiently high positive costs of moving, it may be feasible to pay a sequence of wages which grows at a lower rate than output.)

VI. Conclusions

This paper analyzed the case in which the variation in output is a consequence of stochastic accumulation of productive capacity. It has been shown that the workers can shift away some of the risk and obtain wage and employment profiles with lower variations than in per period output. The amount of insurance actually purchased depends on the costs of changing employers, on the capital market, and on the nature of the stochastic process. The role of mobility costs is particularly crucial if one jointly assumes a growing stochastic process and no access to the capital market by workers. In this case positive mobility costs are necessary to generate wage and employment insurance. Wage insurance is then reflected in a wage profile which is flatter and less variable than the productivity profile. On the other hand, under perfect capital markets, mobility costs play no particular role and wage insurance is always provided; and though the wage profiles cannot be uniquely determined, wages will tend to grow faster than the average productivity profile. The crucial role of the capital market in the determination of wages is a special case of the general principle that intragroup exchanges (e.g., within a family) can substitute and be substituted by outside (market) exchanges. By the same principle, the assumption that workers have no access to a wage insurance market, an assumption which was not challenged in this paper, also plays a crucial role. Indeed, with sufficient outside opportunities the development of wages within the contract is largely indeterminate, unless additional considerations, such as the inducement of effort, are introduced. I view the capital market imperfection considered in this paper (i.e., limited borrowing possibilities against future wages) as rather plausible. It is important, nevertheless, to realize the sensitivity of the wage contract to this assumption.

References

Freeman, S. "An Economic Model of Academic Early Retirement." RCA Technical Report. September 1975.

———. "Wage Trends as Performance Displays Productive Potential: A Model and Application to Academic Early Retirement." *Bell Journal of Economics* 8 (Autumn 1977): 419–43.

Harris, M., and Holmstrom, B. "A Theory of Wage Dynamics." *Review of Economic Studies* 64 (April 1982): 315–33.

Harris, M., and Weiss, Y. "Job Matching, Risk Aversion and Tenure." Discussion Paper no. 515. Evanston, Ill.: Northwestern University, Kellogg Graduate School of Management, 1982.

Jovanovic, B. "Job Matching and the Theory of Turnover." *Journal of Political Economy* 87 (October 1979): 972–90.

Lazear, E. "Agency, Earnings Profiles, Productivity and Hours Restrictions." *American Economic Review* 71 (September 1981): 606–20.

Merton, R. "The Matthew Effect in Science." *Science* 159 (January 1968): 56–63.

Price, D. "A General Theory of Bibliometric and Other Cumulative Advantage Processes." *Journal of American Society for Information Science* 26 (September–October 1976): 292–306.

Reder, M. "The Size Distribution of Earnings." In *The Distribution of*

National Income, edited by J. Marchal and B. Ducro. New York: Macmillan, 1968.
Rogerson, W. "Repeated Moral Hazard." Mimeographed. Stanford, Calif.: Stanford University, Department of Economics, 1983.
Simon, H. "On a Class of Skew Distribution Functions." In *Models of Man.* New York: Wiley, 1957.
Stiglitz, J., and Weiss, A. "Cutting Off Credit: An Application of Constraint as Incentive Devices." Mimeographed. Princeton, N.J.: Princeton University, Department of Economics, 1982.
Weiss, Y., and Lillard, L. "Output Variability, Academic Labor Contracts and Waiting Times for Promotion." In *Research in Labor Economics,* edited by R. Ehrenberg. Greenwich, Conn.: JAI, 1982. Vol. 5.