Chapter 11

THE DETERMINATION OF LIFE CYCLE EARNINGS: A SURVEY

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1. Introduction

My purpose in this survey is to describe the theoretical work on the determination of life cycle earnings. The common thread in this work is the notion that workers can influence their earnings through various investment activities. A person who spends time in school or in on-the-job training sacrifices current earnings in the hope of increasing his future earning potential. Consequently, the observed life cycle earnings profiles reflect individual economic choices as well as purely technological or biological processes such as "depreciation" or "aging". Since the emergence of the influential work of Becker (1964), Mincer (1962, 1974), and Schultz (1963) this view has become widely accepted.

There is, however, considerable controversy on the market situation in which investment choices are made. As noted by Arrow (1973) and Spence (1973) the informational assumptions are particularly important. The welfare and policy implications are very different if schooling enhances productivity or is merely used as a mode of transferring income by signalling and screening. In this survey I will adhere mostly to the "human capital" approach but focus on its testable implications to individual earning profiles setting aside the aggregate and policy implications. Within this framework the discussion narrows on investments on the job.

The major stylized facts which the theory attempts to explain are: a life cycle earnings profile which is increasing at early ages and is declining towards the end of the working period. A wage profile which tends to increase over the life cycle with a weak tendency for wage reduction towards the end of the working period. An hours of work life cycle profile which is increasing at early ages and declining at older ages, with the peak occurring earlier than in the earnings or wage profiles [see Mincer (1974), Ghez and Becker (1975)]. In addition there are several

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important interactions between experience related earnings growth and individual characteristics such as sex, age, level of schooling, and perhaps vintage. Specifically, earnings growth (at a given level of experience) tends to be lower for women, for older workers, for workers with more years of schooling, and for workers of less recent vintages. These empirical regularities in the wage structure have been observed repeatedly at different points in time and in various countries and occupations. These findings are mostly from cross section data. However, longitudinal data, when available, also yield similar results [see Weiss and Lillard (1978), Lillard (1981)]. The observed stability of these broad patterns in the wage structure is the starting point of the human capital approach as an organizing framework. The theory attempts to explain *jointly* all the stylized facts mentioned above.

Most of the reviewed material is not new and was already covered in the excellent surveys by Rosen (1977) and Killingsworth (1983). I therefore choose to avoid both generalities and detailed enumeration of findings. Instead, my objective is to provide a relatively self-contained development of the main results in the area. I try to be quite explicit about the methods of analysis. Hopefully this will enable the readers, graduate students in particular, to reconstruct old results and produce new ones.

2. The human capital framework

The human capital approach can be applied at two different levels: at the market level it presents a set of restrictions on the equilibrium wage structure; at the individual level it analyzes the actions which workers can take to affect their current and future earnings taking market conditions as given. Most of the literature surveyed here focuses its attention on the individual experiment. A key element in the discussion is the assumed tradeoff between current and future earnings. In a complete analysis one must verify that this tradeoff indeed satisfies the restrictions imposed by market equilibrium. For this reason I begin the survey by describing the technology and the market structure in which individual decisions are embedded.

Each person in the economy is assumed to possess a certain amount of productive capacity or human capital. Human capital is not transferable but can be augmented by learning or training. The process of training generally requires individual inputs, mainly the worker's *own* time and knowledge and outside resources consisting of the knowledge and time of other workers. Outside resources can be obtained in two different ways, the worker may simply purchase the services of other knowledgeable workers or he may gain access to a job in which learning occurs jointly with work. The latter possibility arises because of the difficulties in the effective exclusion of information. In most work situations firms cannot prevent workers from learning on the job.

Firms are endowed with the technology of converting workers' time and human capital into flows of goods and new productive capacity for each of the workers. Thus, if a firm employs N workers, its inputs are specified as $K_1, K_2, \dots, K_N, h_1, h_2, \dots, h_N$, where K_i, h_i are, respectively, the human capital endowment of worker i and the time which he spends at the firm, while its outputs are specified as z, a composite good, and $\dot{K}_1, \dot{K}_2, \dots, \dot{K}_N$, where $\dot{K}_i$ is the rate of change of worker i's human capital. The addition to the earning capacity of each worker is treated as a different commodity since human capital is not transferable.

Two important simplifying assumptions are common to most of the literature:[1]

A.1.

Workers with different skills are perfect substitutes in the production of the composite good z, i.e. $z = F\left(\sum_{i=1}^{N} h_i K_i\right)$.[2]

A.2.

The amount of new earning capacity accruing to each worker depends only on his own inputs, i.e. $\dot{K}_i = G(K_i, h_i)$.

With this technology the market for work and training can be described very simply. To begin, assume that all firms are identical. Firms compete for workers by offering job opportunities. A job opportunity specifies both the wage and the time spent on the job. The worker needs to know both dimensions since the amount of human capital $\dot{K}$ which accrues to him depends on h. Since workers are distinguishable by their human capital endowment the job offer and the payment for it will depend on K. Given the terms of the contract each firm is free to select the number of workers of each type so as to maximize profits. Therefore the marginal product of a worker of type K must equal his cost, i.e. $h(K)w(K) = F'(\cdot)hK$, where $F'(\cdot)$ is common to all workers and can be defined as the rental rate of human capital, commonly denoted by R. With this payment structure the worker effectively faces an infinitely elastic demand for hours on the job. The firm can delegate the selection of hours to the worker at the equilibrium

[1] A notable exception is Mincer (1974, ch. 1). His exposition of the schooling model assumes, contrary to assumption A1 in the text that workers with different levels of schooling are essential in the production process. If (presumably) identical workers appear in the market with different levels of schooling *all* observed investment options must be equally attractive. The wage structure is then immediately determined, at the market level, by this indifference requirement. The individual investment pattern at this compensating wages equilibrium is indeterminate [see Rosen (1977)].

[2] The production function $F(\cdot)$ includes implicitly fixed factors such as capital. Assuming constant returns to scale we can interpret the short-term profits of the firm as normal profits required to compensate the fixed factors. The model outlined below therefore assumes zero profits and is thus consistent with free entry and exit of firms.

wage. The model can now be closed by determining the choice of hours by the workers given R. Workers' choices between leisure and work will fully determine the accumulation of human capital and development of wages over the life cycle. Full equilibrium is attained when R adjusts so as to make the aggregate accumulation consistent with $F'(\cdot) = R$.

The case just outlined describes the class of models which deal only with learning by doing. The distinguishing aspect is that *on-the-job* knowledge is provided freely. This does not mean that training is acquired costlessly since workers have opportunity costs to time spent on the job (i.e. the value of leisure). There is, however, an important generalization of the learning-by-doing idea which incorporates opportunity costs in the job market. According to this approach the worker can invest at a varying intensity on the job [see Becker (1975), Mincer (1974) and Killingsworth (1982)]. Such options arise either because firms differ in their capacity to provide training or because they can vary the proportions of z and $\dot{K}_i$ by shifting resources from production to training.

Similar to the ranking of workers by their human capital, K, we may introduce an index $0 \le x \le 1$ which ranks firms, or jobs within a firm, by the proportion of goods and training which they produce. Specifically, employing N workers with inputs $K_1, K_2, \ldots, K_N, h_1, h_2, \ldots, h_N$ in job x yields $(1-x)F(\sum_{i=1}^{N} K_i h_i)$ units of z and $G(K_i, h_i, x)$ units of $\dot{K}_i$ for $i = 1, 2, \ldots, N$. It is assumed that for given inputs, jobs with higher x produce relatively more knowledge ($\partial G/\partial x > 0$) and less goods. The loss of output reflects the real costs of providing training due to the involvement of the various productive inputs in the training process [see Rosen (1972)]. A special case of the above technology is one in which the costs are associated solely with the shift of the trainee's own time from work to training. In this case one may write $z = F(\sum_{i=1}^{N} K_i(1-x)h_i)$ and $\dot{K}_i = G(K_i, h_i, x)$ and interpret $(1-x)h$ as time on the job spent in work and hx as time on the job spent in training [see Ben-Porath (1967)].

A contract offered to a worker type K will now specify, in addition to the wage and the duration of work, the training content of the job x. (All the variables which determine $\dot{K}$ must be included.) For any given contract the firm can decide how many workers to employ. If a positive number of workers are employed under a particular contract their marginal cost to the firm must be equal to their marginal product. Hence a worker type K employed h hours at a job type x will earn $(1-x)RKh$. Under this payment scheme the firm can delegate to the worker the choice of both x and h. From the point of view of the worker we may interpret $G(K, h, x)$ as the production function of human capital on the job and $xRKh$ as the (opportunity) costs for acquiring training. Only the costs depend on market conditions as represented by R.

Schooling activities can be treated in precisely the same manner. Each student (worker) obtains a certain amount of additional knowledge and the costs depend on the total number of students (workers). The only difference is that a negative amount of aggregate good is produced. The price facing each student (worker) is the marginal costs of his training.

Each worker can allocate his time among jobs (firms) with different x and among various schooling activities. An efficient allocation maximizes $\dot{K}$ for given K, h and given current net earning. This maximization generates an efficient frontier which may be written, for given R, as

$$Y = Y(K, \dot{K}, h), \qquad (1)$$

where Y is current earnings net of explicit training costs, h is time spent at work or at school (i.e. non-leisure time), K is the amount of human capital which the worker possesses and $\dot{K}$ is the amount of new knowledge that accrues to him. The partial derivatives of Y are, respectively, positive with respect to K and negative with respect to $\dot{K}$. This reflects a basic tradeoff between current and future earnings. Choosing an activity which generates more training (or learning) reduces current earnings but enhances future earning capacity.

The current earning capacity of the worker can be defined as the maximal amount of net current earnings which is attainable given K and h. A worker who actually earns less than his earning potential is implicitly paying for acquiring knowledge. A worker may have low observed earnings, given his observed market time, either because of low earning capacity (i.e. low K) or because of high costs of investment (i.e. high $\dot{K}$). These alternatives cannot be separated empirically since neither K nor $\dot{K}$ can be directly observed. The human capital approach is in this respect reminiscent of the permanent income hypothesis. Observed behavior is guided by a variable which is observed only by the economic agents but not by the researchers.[3]

One can trace different specifications of the tradeoff in the literature to different assumptions on the function $G(K, h, x)$.

Using unified notation[4] one may cite the following.

[3]The difference from the permanent income hypotheses is that the error committed by using current earnings as a proxy for earning capacity is systematically determined by individual maximization.

[4]All of these authors except for Blinder and Weiss (1976) originally formulated their specification *without* considering variation in hours. Weizsacker (1967) and Sheshinski (1968) actually wrote

$$Y = RF(E)(1-x), \quad F'(E) > 0, \quad F''(E) < 0,$$
$$\dot{E} = x - \delta E.$$

The formula in the text is a result of the transformation $K = F(E)$. Oniki also considers a more general formulation, including direct costs. Formulation II is the Ben-Porath specification as adopted for the case of variable hours by Heckman (1976), who also includes direct costs, and Ryder, Stafford and Stephan (1976). Formulation III precisely agrees with Rosen (1976) only when hours are taken to be fixed. Blinder and Weiss (1976) actually wrote $Y = RKhg(y)$, $\dot{K} = \theta Kyh - \delta K, 0 \le y \le 1, g(0) = 1, g(1) = 0, g'(y) < 0, g''(y) < 0$. The formulation in the text is obtained by the transformation $x = 1 - g(y)$.

(I) Weizsacker (1967), Sheshinski (1968), Oniki (1968):

$$\dot{K} = g_1(K)Khx - \delta g_2(K), \qquad Y = R\left[hK - \dot{K}c_1(K) - \delta c_2(K)\right],$$
$$g_1(K) > 0, \quad g_1'(K) < 0, \quad g_2(0) = 0, \quad g_2'(K) > 0, \quad c_1(K) = 1/g_1(K),$$
$$c_2(K) = g_2(K)/g_1(K).$$

(II) Ben-Porath (1967):

$$\dot{K} = g(Khx) - \delta K, \quad Y = R(Kh - c(\dot{K} + \delta K)),$$
$$g(0) = 0, \quad g'(\cdot) > 0, \quad g''(\cdot) < 0, \quad c(\cdot) = g^{-1}(\cdot).$$

(III) Blinder and Weiss (1976), Rosen (1976):

$$\dot{K} = Khg(x) - \delta K, \qquad Y = RKhc\left(\frac{\dot{K} + \delta K}{Kh}\right),$$
$$g(0) = 0, \quad g'(\cdot) > 0, \quad g'' < 0, \quad c(\cdot) = 1 - g^{-1}(\cdot).$$

Specifications (II) and (III) imply increasing marginal costs, in terms of lost current earnings for acquiring training. In specification (I) the marginal costs for acquiring training are constant up to the boundary of feasible accumulation (where they become infinite). It is the need for individual non-purchased inputs which limits the rate of acquisition of training and produces costs of adjustment for investment in human capital at the individual level.[5]

In specifications (I) and (II), a worker who spends all his time at job x attains the same outcome as he would obtain by spending a proportion $1 - x$ of his time at the job which maximizes current earnings ($x = 0$) and a proportion x at the job which maximizes investment ($x = 1$). That is, on-the-job training is equivalent to a *mixture* of pure work and pure training (often defined as schooling). In specification (III) a mixture of pure work and pure training requires a larger sacrifice of current earnings and is strictly dominated by the acquisition of training in one job. This built-in advantage for on-the-job training leads to increasing returns with respect to market time. That is, given K and $\dot{K}$ an increase in h increases Y more than proportionally.

In specifications (II) and (III) human capital is assumed to be self-productive. This is more pronounced in specification (III) where constant returns to scale

[5]In the theory of investment of the firm costs of adjustment are introduced somewhat artificially at the level of the firm. Since a single firm can acquire an arbitrary amount of investment goods at a fixed price, it is only the internal difficulties in implementation that can cause cost of adjustment [see Eisner and Strotz (1963)].

with respect to K are assumed. In specification (I), on the other hand, it is assumed that a large stock hinders further accumulation of earning capacity. This is in addition to the natural "depreciation of human capital" (reflected in δ) common to all models. The marginal costs for acquiring additions to the stock of human capital, in terms of forgone current earnings, are increasing in K for specification (I), unaffected by K for specification (II) and diminishing in K for specification (III).

Notice that in all the above specifications, joint concavity in K, x, and h is *not* assumed. This is due to the appearance of the products such as xhK, indicating that with higher level of K each hour of work becomes more productive (in both training and earnings). This may be interpreted as dynamic increasing returns to scale, essential to models of human capital, where an increase in current effort either reduces the costs of, or increases the benefits from, future effort.

Since neither K nor $\dot{K}$ is observable, one cannot test these different specifications directly. It is only by the implications for the observed patterns of earnings and market time that one can, perhaps, separate such alternatives. Indeed, most of the research effort, at the theoretical level, was directed to yield such testable implications. I now turn to survey these attempts.

3. The wealth maximizing model

The focus of this class of models is on the allocation of time in the market, taking the total amount of non-leisure time as predetermined. The worker is assumed to have a fixed lifetime of length T and to operate in a static economy with a perfect capital market facing a fixed rate of interest, r. The labor and training market is summarized by the tradeoff function (1). The worker's problem, then, is the choice of an optimal path of accumulation for human capital under the above conditions. Formally, the problem is stated as[6]

$$\max_{\{x\}} \int_0^T RhK(1-x)e^{-rt}\,dt$$
$$\text{s.t.}$$
$$\dot{K} = G(K, h, x), \quad K(0) = K_0, \tag{2}$$
$$0 \le x \le 1,$$

where t is the worker's age, h is a predetermined function of t and the function $x(t)$ is the object of choice. This control problem is solved by maximizing the full

[6]The function $G(K, h, x)$ should be interpreted here as the envelope of all possible modes of generating knowledge (including investment in schooling activities). As such it may be non-differentiable with respect to x. This possibility is ignored in what follows, assuming essentially that the schooling option is identical to on the job training with $x = 1$.

(discounted) earnings of the worker at each age. The full earning of the worker consists of his current earnings and the value of the additional knowledge obtained on the job. One can write full earning (the Hamiltonian function) as:

$$H(K,h,x,\psi,t) = [RhK(1-x) + \psi G(K,h,x)]e^{-rt}, \tag{3}$$

where ψ is the marginal value to the worker of an additional unit of human capital. That is

$$\psi(t) = \int_t^T e^{-r(\tau-t)} Rh(1-x^*) + \psi(\tau) G_K(K,h,x^*)\,d\tau, \tag{4}$$

where $x^*(t)$ denotes an optimal choice for $x(t)$. By maximizing $H(\cdot)$ with respect to the control variable x, the worker takes into account the effects of investment on both current and future earnings. The optimal choice of x, if it is interior, equates the marginal cost from choosing a job with more intense training, RhK, to the marginal benefit $\psi G_x(K,x,h)$. The change in x over time is thus related to the time patterns of the endogenous variables K and ψ and to the predetermined profile of h. An increase in ψ will encourage the choice of jobs with higher training content. An increase in K will reduce the training intensity, provided that the degree of complementarity G_{Kx} is not too large. (In the analysis which follows I impose $KG_{Kx} \le G_x$ to ensure that x is non-increasing in K, this requirement is met by the three specifications mentioned in Section 2.)

For the special case in which $h(t)$ is a constant, say 1, one can use a phase diagram (see Figure 11.1) to describe some basic qualitative aspects of the solution. The line $\dot{\psi}=0$ in Figure 11.1 can be interpreted as the long-run (stock) demand for human capital. The line $\dot{K}=0$ is the long-run (stock) supply for human capital. A worker with infinite life may eventually reach the long-run level of capital which equates the stock demand and supply. But life is finite and the programs which are actually followed are dominated by this constraint. Since human capital cannot be transferred the marginal value of human capital becomes zero at the end of the worker's life. This fact provides the economic incentive for an eventual reduction in the investment in human capital.

A general saddlepoint property can be noted in Figure 11.1. The system $(\dot{K},\dot{\psi})$ is partially unstable with respect to ψ (if ψ is above the $\dot{\psi}=0$ line, $\dot{\psi}$ is positive and vice versa) and partially stable with respect to K (if K is to the right of the $\dot{K}=0$ line, $\dot{K}$ is negative and vice versa). This, together with the transversality condition $\psi(T)=0$, severely limits the admissible time patterns of ψ and K. In particular no trajectory can pass through the shaded area in the figure. It follows that whenever the worker increases his earnings capacity, $\dot{K}>0$, the shadow price of human capital must decrease, $\dot{\psi}<0$, and hence on such intervals, observed earnings must increase (unless $x=1$). The phase diagram also reveals

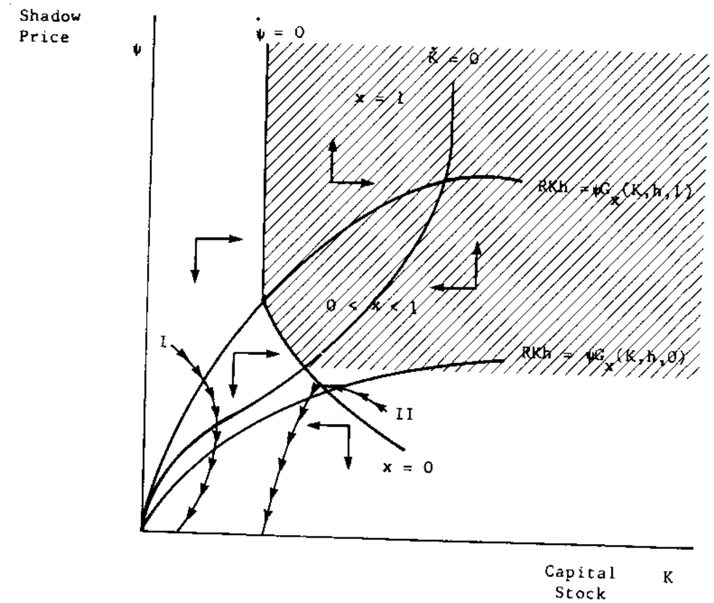

Figure 11.1

that such intervals with increasing earnings capacity, if they exist, must occur at the *beginning* of the worker's life. Trajectory I in Figure 11.1 represents the typical case.[7] Initially the worker invests at full intensity, $x=1$, then x declines, eventually reaching zero. Earning capacity increases at the range with high investment intensity and declines in the range with low investment intensity, due to a positive rate of depreciation. The increase in K and the reduction in x combine to increase observed earnings during the first part of the phase with $0<x<1$.[8]

Turning to the more special cases considered in the literature, one can make the following observations.

[7]Trajectory type II is a possible but less likely pattern. It reflects a situation in which, due to large initial stock of human capital the worker always chooses to reduce it (net investment is negative throughout the worker's life). Since a large initial K deters investment it may be optimal to postpone investment until the point in time in which K is sufficiently low. Earnings in this case will decline throughout the worker's life.

[8]Note that maximized full earnings, H^* never increases along an optimal path [provided that $h(t)$ is constant]. Specifically, the change in full earnings is proportional to actual (discounted) earnings, that is $\dot{H} = -rRKh(1-x)e^{-rt} \le 0$.

In the Weizsacker–Sheshinski–Oniki model the region in which $0 < x < 1$ degenerates into a single line.[9] The control variable x can therefore obtain only three values, the two extremes, zero and one, and a steady state rate of investment corresponding to maintenance of a fixed stock for some period of time. Assuming that the initial stock is zero, the pattern of investment depends entirely on the length of the working period T. If T is short, the worker will not invest at all and his stock of human capital (and earnings) will decay throughout the worker's life. As the horizon extends it becomes profitable to invest at the maximal rate for some period then to reduce investment to the steady state level, and finally reduce it again to zero sometime before the end of life. The optimal pattern of earning therefore contains an initial increasing segment, a flat middle segment and a final decreasing segment. Sheshinski (1968) notes a turnpike property: as the duration of the horizon extends the time spent at the steady state increases, and the duration of the flat segment in earnings will be relatively longer. This simple model illustrates very clearly the role of the finite life constraint in the accumulation process.

Under the Ben-Porath and the Blinder–Weiss–Rosen specifications, the long-run (stock) demand for human capital is perfectly elastic (i.e. the $\dot{\psi} = 0$ line is horizontal). The shadow price of human capital depends only on the remaining work horizon and not on the accumulated stock.[10] We can, therefore, partition the dynamic system and analyze the time pattern of ψ separately. In these cases, as the worker approaches the end of his working life, the demand price of human capital must decline monotonically, reflecting the fact that human capital will be used over a shorter period. Therefore, (gross) investment also declines monotonically (see Figure 11.2). The only difference between the models is that gross investment is measured in absolute terms, $\dot{K} + \delta K$, for the Ben-Porath specification and in proportional terms, $\dot{K}/K + \delta$, for the Blinder–Weiss–Rosen specification. As investment declines the amount (proportion) of earning capacity which is sacrificed declines. As long as net investment is positive, earnings capacity increases. These two forces combine to induce an increase in observed earnings. When net investment becomes sufficiently negative, observed earning declines. Since investment declines smoothly (and not in jumps as in the Weizsacker–Sheshinski–Oniki specification) there is no flat segment in the earning profile.

[9]As the boundary lines of the region $0 < x < 1$ in Figure 11.1 approach each other, the $\dot{\psi} = 0$ and $\dot{K} = 0$ locus in this model becomes disconnected. (Each includes a line and a disconnected point.) See Sheshinski (1968).

[10]These statements are correct for the Ben-Porath specification only in the regions where $0 < x \leq 1$. For the Blinder–Weiss–Rosen specification the long-run supply of human capital is also horizontal. The same holds for the Ben-Porath specification if $\delta = 0$. There is no long-run equilibrium stock (for the infinite horizon problem) in these cases.

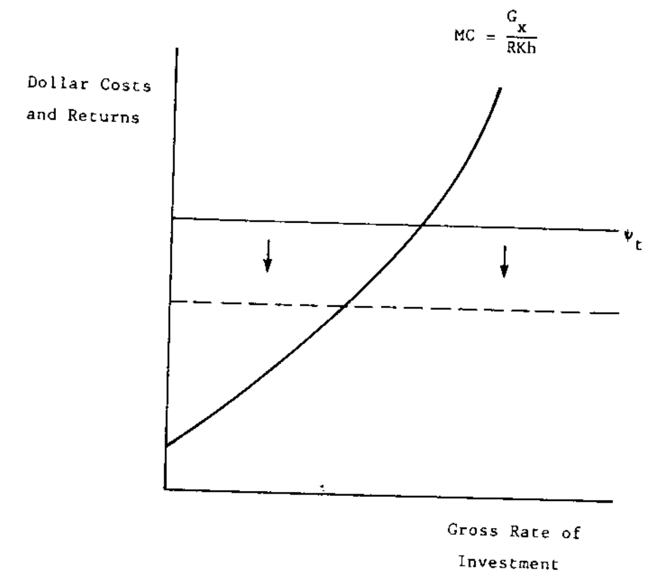

Figure 11.2

The wealth maximizing models of investment in human capital are thus capable of generating optimal patterns which imitate observed earnings profiles of full-time and continuous workers. The phenomenon of first increasing then decreasing earnings during a worker's life cycle is explained as an outcome of a voluntary economic decision, that is, a positive and declining investment rather than an exogenous natural development. This basic insight may be exploited further to explain several additional regularities in the earning structure. An important illustration is the issue of sex-related differences in earnings.

A robust empirical finding is that the (proportional) difference in male–female earnings is increasing with potential and actual work experience. To explain this phenomenon one has to relax the assumption that $h(t)$ is constant and take into account the male–female differentials in labor force participation [see Mincer and Polachek (1974)]. Consider an interruption in female labor force participation [i.e. an interval in which $h(t) = 0$] due to, say, childbirth. If the interruption is unexpected it will only affect earnings after the withdrawal from the labor force. The common hypothesis is that the accumulation of human capital requires active participation of the worker in the labor-training market, that is $G(K, 0, x) = -\delta K$. Therefore, human capital is actually lost and earnings capacity declines

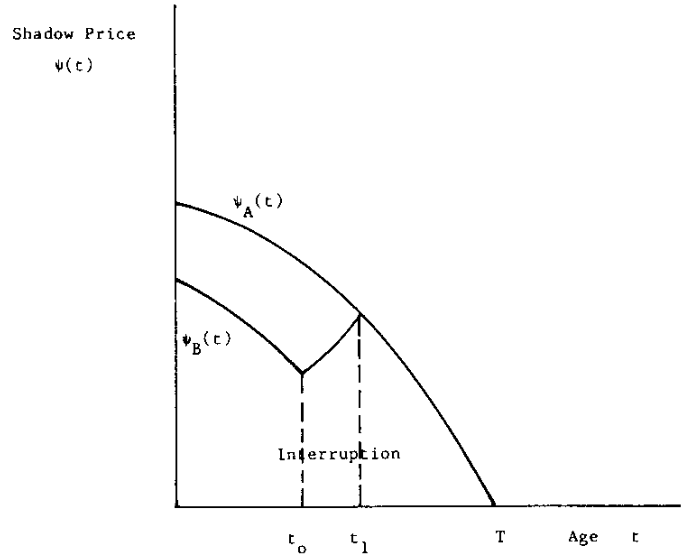

Figure 11.3

as a result of the departure from the labor force. Thus, if a woman accumulates several such episodes, an increasing gap between male and female earnings is created. To the extent that the interruption is expected it will also affect investment prior to the withdrawal from the labor force. It is convenient to apply in this case either the Ben-Porath specification [see Polachek (1975)] or the Blinder–Weiss–Rosen specification [see Weiss and Gronau (1981)], since, for these specifications, expectations are fully captured in the shadow price function $\psi(t)$. Figure 11.3 depicts the behavior of shadow price of human capital for two workers, one who participates continuously, and one who expects to withdraw during the period $[t_0, t_1]$. The two profiles $\psi_A(t)$ and $\psi_B(t)$, respectively, coincide after the interruption. However, during the interval $[t_0, t_1]$, $\psi_B(t)$ is *increasing*, reflecting the profitability of postponing investment to the time of re-entry into the labor force so as to avoid the depreciation and interest costs from unused capital. Consequently, $\psi_B(t) < \psi_A(t)$ prior to t_0, and investment will be lower. With some additional assumptions[11] the lower investment rate will be reflected in

[11] The required restrictions on tradeoff function are the same as those required for the *concavity* of the earnings (or log earnings) profiles. They involve third-order derivatives of the production, analogous to decreasing absolute (or relative) risk aversion [see Weiss and Gronau (1981) and Heckman (1976)].

lower earning growth prior to the *expected* withdrawal. This may explain why even after accounting for *past* interruptions, there is still a gap between male and female earnings growth [see Mincer and Polachek (1974)].

Another important implication of the wealth maximizing model is the potential impact of an income tax on (before-tax) earnings even when labor supply is held constant. For instance, an increase in a proportionate income tax is effectively equivalent in this model to a reduction in the interest rate. Such a change will generally encourage investment in human capital, and with some additional assumptions will be reflected in higher earnings growth [see Heckman (1976)].

In addition to these broad implications for the earnings profiles one may derive differential (or difference) equations for the observed earnings which summarize the whole process of earnings generation [see Rosen (1973)]. Such equations can be derived and their coefficients can be related to the basic parameters even when explicit solution in the extensive form $Y(t)$ is unattainable. They provide, therefore, an efficient method for distinguishing alternative specifications of the tradeoff function (1). For instance, if the tradeoff is quadratic in K and $\dot{K}$ (a special case of the Ben-Porath specification) then second-order, or higher, *linear* differential equations in observed earnings arise.[12] If one assumes the multiplicative form the tradeoff as in Blinder–Weiss–Rosen, a second-order, non-linear, differential equation in the *log* of earnings arises.

A final set of presumably testable implications of the wealth maximizing model apply to the duration of the specialization period with $x = 1$. It is common to identify this period with the observed schooling period of the worker. This interpretation is questionable since knowledge is produced in schools under different conditions than on the job and a more general model is therefore required [see Johnson (1978)]. The available wealth maximizing models all predict that specialization in training, if undertaken, will occur at the *beginning* of the working life. This provides a direct test of the more general implication that investment is declining in these models.[13] Though Oniki (1968) and Weiss (1971) provide proofs of the non-optimality of postponing schooling for slightly more general models, the precise conditions required to retain the result within the context of income maximizing models are not known. It should be remarked that postponement of investment in the period of specialization is not incompatible

[12] For instance, if the function $G(\cdot)$ is specified $\dot{K} = (Kx)^{1/2}$ [setting $h(t) = 1$], then $Y = R(\dot{K} - (K)^2)$ and $\ddot{Y} = 2r\dot{Y} - R$. The discrete time analogue is $Y_t = (2 + 2r + r^2)Y_{t-1} - (1 - 2r + r^2)Y_{t-2} - R(1 + r/2)$. The appearance of negative coefficients on lagged income also arises in more complicated examples [see Weiss (1974)]. This prediction is not supported by the findings of Ashenfelter (1978) who finds that *all* included lagged incomes (up to 5 years) have positive coefficients.

[13] Investment may be measured in a number of ways and therefore the statement that investment declines is slightly ambiguous. In the Ben-Porath specification $\dot{K}$ and "investment dollars" xK declines monotonically. This does not put a restriction on x when $\dot{K}$ is negative. In the Blinder–Weiss specification $\dot{K}/K + \delta$, and "investment time" x declines monotonically [see also Mincer (1974, ch. 5)].

with wealth maximization. If it is assumed that the intensity of investment decreases with K, then a worker with a large initial stock may find it profitable to wait until part of the existing stock depreciates.[14] Outside the scope of the model there are important potential causes for delayed investment such as changing market conditions, unknown ability (tastes) and borrowing constraints. Nevertheless, the observed life cycle pattern of investment in schooling is broadly consistent with the hypothesis of decreasing investment with age.

It is relatively easy to work out the comparative statics for the schooling period. Oniki (1968) has shown that for specification (I) (see Section 2 above), an increase in the length of the horizon or a reduction in the interest rate increases the schooling period. An increase in the initial stock reduces the duration of the schooling period but increases the final stock (attained upon exit from school). Similar results were obtained for the Ben-Porath specification [Ben-Porath (1970) and Wallace and Inhen (1975)]. A slight difference arises in the Blinder–Weiss–Rosen specification where the length of the schooling period is independent of the initial capital stock. The initial stock of human capital can be viewed as a measure of the worker's earning ability. This should be distinguished from another measure of ability, which is the worker's learning ability, usually modelled as a shift in the production function $G(\cdot)$. Such a shift is typically assumed to increase the marginal product of training and increase investment. Generally, the effect of ability on investment and on the duration of the schooling period is ambiguous. The reason is that higher ability can increase both the opportunity costs and the benefits from investment in human capital.

4. Life cycle earnings with endogenous labor supply

In this section I relax the assumption that the lifetime pattern of labor supply is predetermined. Endogenous labor supply affects the analysis in two basic ways: (1) future labor supply choices determine the utilization of human capital and thus the returns to the investment, and (2) past labor supply decisions influence

[14]A simple three-period example where

$$K_t = K_{t-1} + K_{t-1}^{\alpha} g(x_{t-1}), \quad 0<\alpha<1,$$
$$g(x_t) = ax_t - bx_t^2, \quad 0<a, \quad 0<b,$$

can be used to generate examples of postponement in investment. For instance, if $r=0$, $\delta=0.8$, $\alpha=0.8$, $a=1.65$, $b=0.15$, and $K_0=19$, the optimal policy is to set $x_0=0$ and $x_1=1$. That is, with these parameters the worker does not invest at all in the first period and specializes in investment in the second period. A milder type of postponement in investment is illustrated by trajectory II in Figure 11.1.

the current level of human capital and therefore the (opportunity) costs of investment.

It is useful to begin with a brief discussion of life cycle labor supply with exogenous wages. The worker's problem is to allocate his lifetime effort and consumption given a lifetime budget constraint. Assuming that preferences among consumption and work age profiles can be represented additively, the problem is stated as

$$\max_{\{c,h\}} \int_0^T e^{-\rho t} u(c, 1-h)\,dt$$
$$\text{s.t.} \tag{5}$$
$$\dot{A} = rA + wh - c, \quad A(0) = A_0, \quad A(T) = 0,$$
$$1 \geq h \geq 0, \quad c \geq 0,$$

where c denotes consumption, A is accumulated savings, $u(c,1-h)$ is a current utility index, and ρ a subjective discount factor for future utilities.

An optimal allocation must maximize, at each age, full utility:[15]

$$H(A,\mu,t) = e^{-\rho t}[u(c,1-h) + \mu\dot{A}], \tag{6}$$

where μ, the shadow price of current assets in utility terms, satisfies

$$\dot{\mu} = (\rho - r)\mu. \tag{7}$$

At an interior solution one obtains:

$$\frac{1}{\mu} u_l = w \tag{8}$$

and

$$\frac{1}{\mu} u_c = 1. \tag{9}$$

[15]The problem can be solved conveniently by *two-stage* maximization. Define the indirect utility $\phi(I,w) = \max_{c,h} u(c,1-h)$ subject to $c = wh + I$, and observe that

$$\max_{c,h} e^{-\rho t}[u(c,h) + \mu(rA + wh - c)] = \max_{I} e^{-\rho t}[\phi(I,w) + \mu(rA - I)].$$

Thus at the first stage one solves an optimal savings problem. Having solved for $I(t)$ one can use the regular static supply function, where work depends on the wage rate $w(t)$ and on the non-wage income $I(t)$, to find $h(t)$.

Formally, these conditions are the *profit* maximizing conditions for a worker who produces current utility with inputs l, c (where $l = 1 - h$) facing the price vector $1/\mu$, 1, and w.[16] Writing the demand function for the leisure input as

$$l = D(1/\mu, w), \tag{10}$$

we recall from the theory of production that $D_2 \equiv \partial l/\partial w < 0$. Moreover, if leisure is a normal input, $D_1 \equiv \partial l/\partial(1/\mu) > 0$. Differentiating (10) with respect to the worker's age using (7), we obtain:

$$\dot{l} = D_1 \frac{r-\rho}{\mu} + D_2 \dot{w}. \tag{11}$$

This formula shows that for a fixed wage the worker will choose a decreasing (increasing) profile for labor supply if the interest rate exceeds (is below) his subjective discount factor. If $r > \rho$, then starting from a stable work profile, lifetime utility can be raised by working more in the present, investing the proceeds of the wages, then working less in the future. A rising (exogenous) wage path is associated with an increasing labor supply profile. (It is efficient to allocate effort to periods with a relatively high wage.) If $w(t)$ is single peaked and $r > \rho$, the peak in hours will precede the peak in wages during the life cycle. [See Ghez and Becker (1975), Weiss (1972), Heckman (1974), and Macurdy (1981).]

Returning to the human capital framework, let us now assume that the worker can affect his earnings capacity. The simplest type of endogeneity arises when wages respond to a process of learning by doing on the job. One can then augment the model by adding the equations

$$w(t) = RK(1 - x_0) \tag{12}$$

and

$$\dot{K} = G(K, h, x_0), \tag{13}$$

where the index of training content, x, is taken as given.

An important implication of this extension is that the marginal rate of substitution between leisure and consumption generally *exceeds* the current wage [contrary to the case of exogenous wages where the marginal rate of substitution is equated to the wage; see eqs. (8) and (9)]. Specifically, in the presence of learning by doing, labor supply is determined by the condition

$$\frac{u_l}{u_c} = w + \frac{\psi}{\mu} G_h, \tag{14}$$

where ψ is now the shadow price of human capital in utility terms. This is a

[16] Note that the *current* utility index, like a production function, is arbitrary only up to linear transformation. It is only the lifetime functional $\int_0^T e^{-\rho t} u(c, l)\,dt$ that is ordinally scaled. It is therefore meaningful to assume, for instance, that $u(c, l)$ is strictly concave.

direct reflection of the fact that each hour spent on the job produces *jointly* earning and knowledge. The worker therefore takes into account both the current and future effects of his labor supply choices.

A characterization of the optimal labor supply profile and the corresponding endogenous wage profile requires some specific assumptions on the current utility index and the production function of human capital. I will illustrate here the analysis which follows from the assumptions that the utility indicator is additively separable in l and c, and the production function is given by specification (III) in Section 2. [For analyses corresponding to specifications (I) and (II), respectively, see Weiss (1972) and Killingsworth (1982).]

Under specification (III), with x predetermined at x_0, the optimal solution is characterized by the equations[17]

$$u_l(l) \le \mu RK(1 - x_0) + \psi g(x_0) K, \quad \text{with equality if } h > 0, \tag{15}$$

$$u_c(c) = \mu, \tag{16}$$

$$\dot{\mu} = (\rho - r)\mu, \tag{17}$$

$$\dot{\psi} = (\rho + \delta)\psi - \mu R(1 - x_0) h - \psi h g(x_0), \quad \psi(T) = 0, \tag{18}$$

$$\dot{K} = g(x_0) hK - \delta K, \qquad K(0) = K_0 > 0. \tag{19}$$

The solution can be analyzed by a phase diagram where μK and ψK are treated as the state variables (see Figure 11.4). From eq. (15) it is seen that a straight line with a slope $-R(1 - x_0)/g(x_0)$ defines combinations of these state variables which keep h constant at an interior. Therefore the $\dot{\mu K} = 0$ line form straight lines. From (18) and (19) it follows that the $\dot{\psi K} = 0$ line (which may have positive or negative slope) has a larger slope than any constant h line. Trajectory I in Figure 11.4 describes the typical pattern.[18] Along the optimal path h initially increases and then declines. The same holds for μK, but it starts to decline after hours have peaked. Recall that wages are proportional to K. Assuming $r > \rho$, μK will peak before K does. Hence, along trajectory I wages peak later than hours. This is the same pattern as in the case of exogenous wages.

The simple learning-by-doing model is thus capable of explaining the main stylized facts on wage and work profiles. An increase in wages followed by a

[17] It is assumed that $u_c(0) = \infty$, $u_l(0) = \infty$, therefore only a corner with $h = 0, l = 1$ is considered.

[18] Depending upon initial conditions the trajectory may start at a point such that both μK and h are decreasing throughout the work life. It has been shown by Driffill (1980) that a trajectory which ends with retirement (i.e. enters the $h = 0$ region) cannot start below the $\dot{\mu K} = 0$ locus. Thus a cycling trajectory in which μK decreases then rises and then decreases again is not optimal. The argument is based on the observation that in the Blinder–Weiss specification [see Blinder and Weiss (1976)] $K_0(\psi_0/\mu_0)$ equals lifetime earnings under the optimal policy. Thus, moving along a ray from the origin he shows that a cycling path can be replaced by one starting on the same ray which provides the same lifetime earnings but requires less lifetime disutility from work.

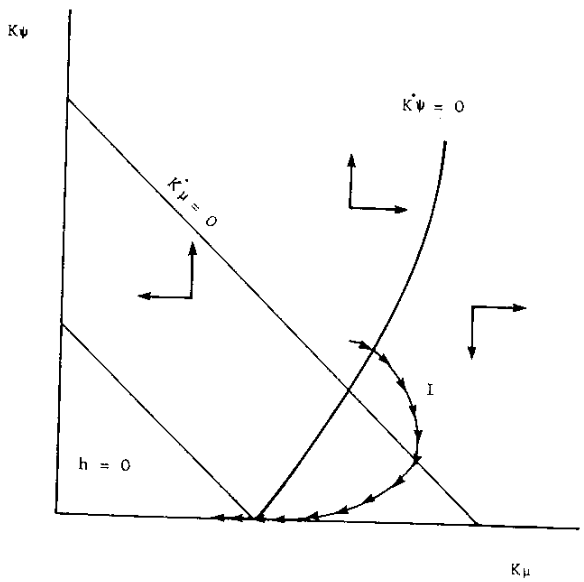

Figure 11.4

decline, an increase in hours of work followed by a decline, and a peak in hours which precedes the peak in wages [see Ghez and Becker (1975)].

The economic intuition behind these results is quite simple. The value of the investment component in work declines as the worker approaches the end of his work horizon. Thus, other things being equal, he would work more hours early in life. The high work intensity at young ages generates growth in the worker's stock of human capital. As the worker ages, hours and wage growth decline. Therefore the earning profile will first increase and then decline. The main difference from the earning maximizing models is that the patterns of accumulation are affected by the worker's *tastes*. With variable leisure, the costs of acquiring human capital (forgone leisure time) depend on the consumption state and not only on earning capacity. The production and consumption decisions cannot be separated. Consequently, taste parameters such as the subjective discount factor ρ influence the development of wages and earnings over the life cycle. If ρ is high, the worker has an incentive to work more in the future. This may lead to a pattern of increasing hours, and eliminate the final segment of reduced wages.[19]

[19] Killingsworth (1982) points out that this added flexibility is potentially beneficial. For instance, the constant decay in earnings late in life implied by the wealth maximizing model is modified to allow a variety of decay patterns when hours are flexible.

The analysis is in some respects simpler if the worker faces a variety of training options in the labor market. One may write the problem in the following form:

$$\max_{\{c,h,x\}}\left\{\int_0^T e^{-\rho t}u(c,1-h)\,dt+\mu_0\left[\int_0^T e^{-rt}(RKh(1-x)-c)\,dt+A_0\right]\right\}$$
$$\text{s.t.}$$
$$\dot K=G(K,h,x),\quad K(0)=K_0, \tag{20}$$
$$0\le x\le 1,$$
$$0\le h\le 1,$$

where μ_0, the marginal utility of wealth at time 0, is a constant to be determined. Since the occupational index does not appear in the utility function directly, it will be chosen so as to maximize lifetime earnings *conditioned* on the choice of the work profile. Investment therefore is governed by the same formulas as in the income maximizing model, except that the rental rate R is replaced everywhere by its utility equivalent $\mu_0 R$. [With this modification ψ is still given by (4), and $\mu_0 RhK$ is equated to $\psi G_x(K,x,h)$ at an interior solution.]

It has been observed by Heckman (1976) that the problem can be separated further, and thus simplified if one adopts the Ben-Porath specification (specification II in Section 2) and if one further assumes that utility depends on "effective leisure", Kl, rather than on actual leisure time. Define $hx=y$, and assume that the constraints $0\le x\le 1$ and $0\le h\le 1$ are not binding, then the solution to (20) is equal to the solution of

$$\max_{\{c,l\}}\left\{\int_0^T e^{-\rho t}u(c,Kl)\,dt+\mu_0\left[A_0-\int_0^T e^{-rt}(RKl+c)\,dt\right]\right\}$$
$$+\mu_0\left[\max_{\{y\}}\int_0^T e^{-rt}RK(1-y)\,dt\right.$$
$$\left.\text{s.t. } \dot K=g(Ky)-\delta K\right]. \tag{21}$$

Notice that the dynamic constraint is associated only with the second maximization in (21). This reflects the fact that the optimal solution for the first problem in (21) is at each age *locally independent* of K. The problem (20) can therefore be solved in stages. Given μ_0 the worker chooses an optimal investment program. This program has all the properties discussed in Section 3. In particular, for specification (II) investment is a declining function of age, and the resulting accumulation path for human capital is single peaked and concave. Taking this path of accumulation as exogenous the worker chooses an optimal consumption and leisure program. This program will have all the properties of the optimal life cycle labor supply with exogenous wages discussed above. In particular, assuming that effective leisure Kl is a normal input, it will monotonically increase if $r > \rho$ and monotonically decrease if $r < \rho$. The behavior of actual leisure time and work now follow from the exogenous pattern of K. When K peaks, then, if $r > \rho$, leisure time must increase. Therefore hours of work peak earlier than the potential wage of the worker. Finally, μ_0 is adjusted to make the two programs consistent with the lifetime wealth constraint which requires that lifetime consumption equals lifetime earnings plus initial wealth.

The studies by Ryder, Stafford and Stephan (1976) and Blinder and Weiss (1976) assume that actual leisure time appears in the utility function and admit corner solutions. Contrary to Heckman (1976) where human capital is equally productive at home and at the market and therefore future work plans have no effect on the returns from investment, these models allow the shadow price of human capital to reflect the expected intensity of labor force participation. It is shown that in a "typical" lifetime program, the worker passes through four

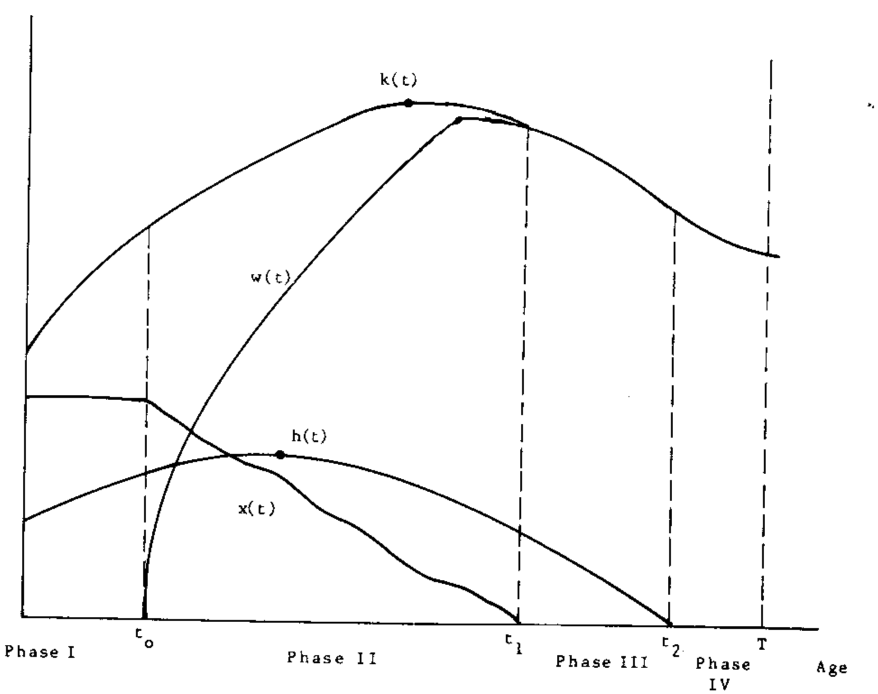

Figure 11.5. The age profiles of human capital, wages, investment and work.

different phases: schooling ($x = 1, h > 0$); on-the-job training ($0 < x < 1, h > 0$); work ($x \doteq 0, h > 0$); and retirement ($h = 0$). Normally the phases occur in this order, though, depending on initial condition, some may not arise. The behavior of the key variables is described in Figure 11.5. Again, these patterns are similar to those predicted by the models with exogenous wages.

The main difference from the earnings maximizing models [and Heckman (1976)] is that investment as measured by $\dot{K}$ or $(\dot{K}/K)$ *increases* during the schooling phase and also in the early part of the on-the-job training phase. This is a result of the initial increase in hours of work. While in Figure 11.5, investment intensity as measured by x *declines* throughout the worker's life career, this result is only true for $\rho < r + \delta$. Generally, the incentive to postpone investment is influenced by the subjective discount factor ρ. The higher is ρ, the more likely it is that the worker consumes his leisure rather early in life and thus postpones his investment in human capital. For sufficiently high rate of impatience the worker may decide to "retire" while young. It is then quite logical that he also postpones his investment to a period close to his entry into the labor force.

For a low rate of impatience the broad patterns of the optimal work and wage plans are similar in the different available models of endogenous labor supply. This is perhaps not surprising since they were all designed to fit the same stylized facts. There are, however, some marked and unexpected differences in the comparative statics and comparative dynamics. I now proceed to survey these issues.

5. Comparative statics and dynamics

So far I have only discussed the time patterns of investment in human capital and their implications. I described models which generate optimal work and wage profiles that imitate the observed life cycle patterns. The question naturally arises, how sensitive are the time patterns of the optimal programs to changes in parameters. This issue falls under the heading of comparative dynamics. A second question relates to the impact of various parameter changes on lifetime *aggregates* such as lifetime earnings, lifetime consumption or more narrowly the total time spent in a particular phase such as "schooling". These questions fall under the heading of comparative statics.

I will illustrate some of the issues in comparative statics analysis by focusing on the effect of changes in initial wealth, A_0, on lifetime earnings and consumption. For this purpose we need to examine the determination of μ_0 in more detail. Consider first the model by Heckman (1976). Define:

$$S = \int_0^T e^{-rt} RK(1-y)\,dt, \qquad E = \int_0^T e^{rt} RKh(1-x)\,dt, \tag{22}$$

where $y = hx$. We may refer to S as lifetime income and to E as lifetime earnings. Note that S exceeds E by the present value of effective leisure which the worker "buys back". Under the two stage procedure described above, investment policy is independent of μ_0. Therefore, the supply of lifetime income as a function of μ_0 is perfectly inelastic. The demand for lifetime income,

$$\int_0^T e^{-rt}(RKl + c)\,dt - A_0,$$

is determined by the solution to the first maximization in (21). For a given μ_0 this maximization is equivalent to an unconstrained profit maximization at each age, and an increase in μ_0 is equivalent to a reduction in the price of output (i.e. utility). Hence with a concave utility function expenditures (i.e. $c + RKl$) must increase with $1/\mu_0$. It follows that the demand for S as a function of μ_0 is downward sloping. The level of μ_0 is determined at the intersection of the demand and supply curves.

Now consider an increase in initial wealth A_0. The supply curve for lifetime income is unaffected by this change but the demand curve shifts (parallelly) to the left. It follows immediately that S is unaffected but μ_0 declines. Under the assumption that consumption and effective leisure are normal goods both increase, at every age, as μ_0 declines, hence lifetime earnings decline and lifetime consumption increases.

Different results can arise in the models proposed by Blinder and Weiss (1976) and Ryder, Stafford and Stephan (1976). In both cases leisure is measured in time units and the utility function is additive separable in leisure and consumption. These models can be separated in a different way from the Heckman model, and the solution to problem (20) equals to the solution of

$$\max_{\{c\}}\left[\int_0^T e^{-\rho t}u(c)\,dt - \mu_0\left(\int_0^T e^{-rt}c\,dt - A_0\right)\right] + \max_{\{x,h\}}\left[\int_0^T e^{-\rho t}v(1-h)\,dt + \mu_0\int_0^T e^{-rt}RKh(1-x)\,dt \quad \text{s.t. } \dot{K} = G(K,h,x)\right]. \tag{23}$$

By a standard argument one can show that for each of the separate maximization problems in (23) that the value of the optimal program is *convex* in μ_0. Since the demand and supply for lifetime earnings are the derivatives of the optimized value functions of these two problems with respect to μ_0, it follows that the demand is downward sloping while the supply is upwards sloping. If it can also be shown that for every μ_0 there corresponds a *unique* E (i.e. the two curves are continuous graphs) then by the same arguments used for the Heckman (1976) formulation, μ_0 is determined by the intersection of the demand and supply

curves and an increase in A_0 must lead to a reduction in lifetime earnings and in μ_0 and therefore to an increase in lifetime consumption.

A special aspect of the human capital problem is the presence of dynamic increasing returns to scale and therefore the concavity of the second maximization problem in (23) is generally *not* guaranteed. This will generally imply that the supply of lifetime earnings as a function of μ_0 need not be a continuous graph [see Brock and Dechert (1985)]. Not surprisingly, the question whether a unique level of lifetime earnings can be associated with any given μ_0 is closely related to the second-order conditions for dynamic maximization. The problem reduces to the question whether the first-order Euler or Pontryagin conditions for the second maximization in (23) identify a unique path. A well-known *sufficient* condition for uniqueness is that the corresponding Hamiltonian function be strictly concave in the control variables and that the maximized Hamiltonian is concave in the state variables [see Arrow and Kurz (1970)].

The models by Blinder and Weiss (1976) and Ryder, Stafford and Stephan (1976) do not satisfy this sufficiency condition and uniqueness cannot be guaranteed for these models.[20] Driffill (1980) has actually found a potentially wide class

[20]For the Blinder and Weiss (1976) model, the maximized Hamiltonian corresponding to the second maximization in (23) is

$$M(K,\psi) \equiv \max_{x,h}\left[v(1-h)e^{-\rho t} + \mu_0 RK(1-x)h e^{-rt} + \psi[hKg(x) - \delta K]\right]$$

and is convex in K for given ψ, since by the first-order conditions, x is independent of K and h is increasing in K, hence,

$$M_{KK} = \left[e^{-rt}\mu_0 R(1-x) + \psi g(x)\right]\frac{dh}{dK} > 0$$

[see also McCabe (1983)]. The convexity of the Hamiltonian in the state does *not* imply that the first-order conditions identify a minimum nor does it imply that the solution is not unique. That the condition is overly strong is immediately apparent from the fact that it is not independent of positive monotone transformation in the *state* variables. For example, define a new state variable Z such that $Z^\alpha = K$, $0 < \alpha < 1$. The new Hamiltonian function is

$$M(z,\psi) \equiv \max_{x,h}\left[v(1-h)e^{-\rho t} + \mu_0 Rz^\alpha(1-x)h e^{-rt} + \psi\alpha[hzg(x) - \delta z]\right]$$

and

$$M_{ZZ} = \frac{v'(1-h)h e^{-\rho t}}{z^2}\left[\alpha(\alpha-1)\frac{(1-x)g'(x)}{(1-x)g'(x)+g(x)} - \frac{v'(1-h)}{v''(1-h)h}\right].$$

If the optimal path is always interior [e.g. set $g'(1) = 0, g'(0) = \infty, v'(1) = 0, v'(0) = \infty$], then there may exist an α which yields $M_{ZZ} < 0$ along the optimal path. If such an α exists the solution to the Pontryagin conditions of the original problem is unique. Unfortunately, this sufficient condition can be verified only after a solution is found.

of cases in which an increase in initial wealth *reduces* lifetime consumption. This situation arises in the Blinder and Weiss (1976) model if the parameter configuration is such that the optimal life program starts with full time training and ends with retirement. McCabe (1983) notes that by modifying the Blinder–Weiss model, allowing effective leisure to enter the utility function [as in Heckman (1976)] uniqueness can be restored and therefore consumption *increases* with wealth.[21]

The economic explanation for these results is apparent if one considers the usual static leisure consumption problem with increasing returns (see Figure 11.6). This analogy is perfectly valid for comparative static analysis where all the dynamic effects are "maximized out". A change in initial wealth shifts the budget constraint in a way which keeps its slope constant along any vertical line. This means that with respect to leisure there is a pure income effect, and under the usual restrictions on preferences associated with normality, its consumption will increase. However, due to the increasing returns to scale the slope declines along any horizontal line. That is, holding consumption constant the increase in A leads to a *reduction* in wages. With respect to consumption both income and substitution effects operate. Since income increases but consumption becomes relatively more expensive, the income and substitution effect operate in opposite directions, and depending upon the relative strength of these effects consumption may increase or decrease. Thus, as noted by Driffill (1980), the reduction of consumption with initial wealth is a distinct possibility in any human capital model because of inherent increasing returns. The assumption of effective leisure eliminates the price effect on consumption by reducing the marginal rate of substitution together with the wage along a horizontal line. While this modification yields a more plausible relationship between lifetime consumption and wealth, it also has some undesirable implications. For instance, the effective leisure hypothesis is incapable of explaining the negative relationship between planned future withdrawals from the labor force and current investment. It seems that the two alternative hypotheses: that human capital, acquired at the market place, is equally productive at home and in the market [as in Heckman (1976)] or that it has no effect on home productivity [as in Blinder and Weiss (1976) and Ryder, Stafford and Stephan (1976)] are both rather extreme simplifications of the true situation.

Ryder, Stafford and Stephan (1976), who were the first to discuss the implications of the potential non-uniqueness of the solution to the Pontryagin necessary conditions, note that changes in the initial level of human capital may also cause discontinuous jumps in the optimal policy.

> A worker who starts near the "catastrophy set" but with slightly less initial human capital will find it optimal to go to work at once, do little training and retire early. If he had started with the same wealth but on the other side of the watershed with slightly more human capital it would have been optimal to start his career with training and then devote considerable time to labor continuing right to the end of his life.

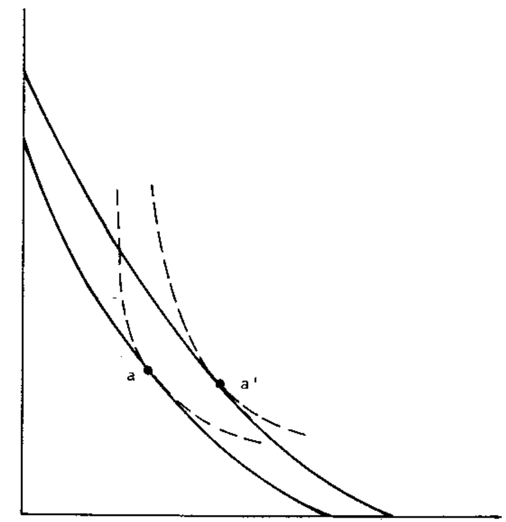

Figure 11.6

[21]A very simple example will help to illustrate these general statements. Consider the static leisure consumption problem but with increasing returns to scale.

$$\max_{c,h} V = c^a + \alpha(1-h), \quad 1 > \alpha > 2a, \quad 0 < a < 1/2,$$

s.t.

$$A + h^2 = c$$

substitute for h in the objective function and differentiate to obtain

$$V_c = ac^{a-1} - (\alpha/2)(c-A)^{-1/2}$$

$$V_{cc} = a(a-1)c^{a-2} + (\alpha/4)(c-A)^{-3/2}$$

If we set $A = 0$, then, due to the restrictions on the parameters, there is an interior solution with $1 > h > 0$, $c > 0$, which satisfies $V_c = 0$ and $V_{cc} < 0$. Since $V_{cA} < 0$ it follows that $\mathrm{d}c/\mathrm{d}A|_{A=0} < 0$. Note that the auxiliary problem $\max_h \alpha(1-h) + \mu_0 h^2$ has *two* solutions when $\mu_0 = \alpha$ (i.e. $h = 0$ and $h = 1$). Finally, if the second part of utility function is changed to $\alpha(1-h)h$ (as in the effective leisure hypothesis) then $V_{CA} > 0$ and $\mathrm{d}c/\mathrm{d}A > 0$.

Heckman (1976) derived several comparative dynamic results for his model. Some follow directly from the comparative statics exercise. For instance, the reduction in μ_0 as a result of an increase in A_0 implies that $\mu(t)$ is lower for all t and hence consumption and effective leisure will increase for every age. Heckman shows that an increase in the initial stock of human capital also reduces μ_0, and thus shifts upwards the demand for consumption and effective leisure at every age. Since investment policy in his model is completely independent of initial conditions, one can conclude that an increase in initial wealth will reduce hours of work at every age. An increase in initial human capital, on the other hand, will initially increase and later reduce labor supply.

The comparative dynamics for the investment profile in the Heckman (1976) model are the same as in the income maximizing model. This means that changes in tastes have *no* effect on the development of the worker's earning capacity. This is in sharp contrast to the models by Blinder and Weiss (1976) and Ryder, Stafford and Stephan (1976) where changes in taste parameters, such as the subjective discount factor ρ, affect investment policies in a substantive way. To analyze the effects of such a change on the optimal policy, under the Blinder and Weiss specification, consider again eqs. (15)–(19) and the added equation which determine optimal investment:

$$\begin{aligned} R\mu + \psi g'(x) &= 0, \quad \text{if } 0 < x < 1, \\ R\mu + \psi g'(x) &\le 0, \quad \text{if } x = 0, \\ R\mu + \psi g'(x) &\ge 0, \quad \text{if } x = 1. \end{aligned} \tag{24}$$

Notice that during the phase with on-the-job trainings (i.e. $0 < x < 1$) eqs. (24), together with (15)–(19) imply an autonomous system of differential equations in x and h, with boundary conditions $x(t_1) = 1$ and $x(t_2) = 0$, where t_1 and t_2 are the (variable) points of entry and exit into this phase. Differentiating this system with respect to ρ, one obtains a new system of differential equations for the *changes* in hours of work and training intensity (h_ρ and x_ρ, respectively) which result from the increase in the subjective discount factor [see Oniki (1973) and Epstein (1978)]. The boundary conditions for this new system are $x_\rho(t_1) = x_\rho(t_2) = 0$. Evaluating the system at $\rho = 0$ it can be shown, with some added assumptions,[22] that it satisfies the following sign pattern:

$$\begin{pmatrix} \dot{h}_p \\ \dot{x}_p \end{pmatrix} = \begin{pmatrix} + & + \\ - & + \end{pmatrix} \begin{pmatrix} h_p \\ x_p \end{pmatrix} + \begin{pmatrix} +, \\ 0 \end{pmatrix}. \tag{25}$$

[22] The added conditions are

$$\frac{\mathrm{d}}{\mathrm{d}h}\left(\frac{V''(1-h)}{V'(1-h)}\right) \le 0 \quad \text{and} \quad \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{g''(x)}{g'(x)}\right) \le 0.$$

These conditions are also related to the concavity of the hours and wage profiles.

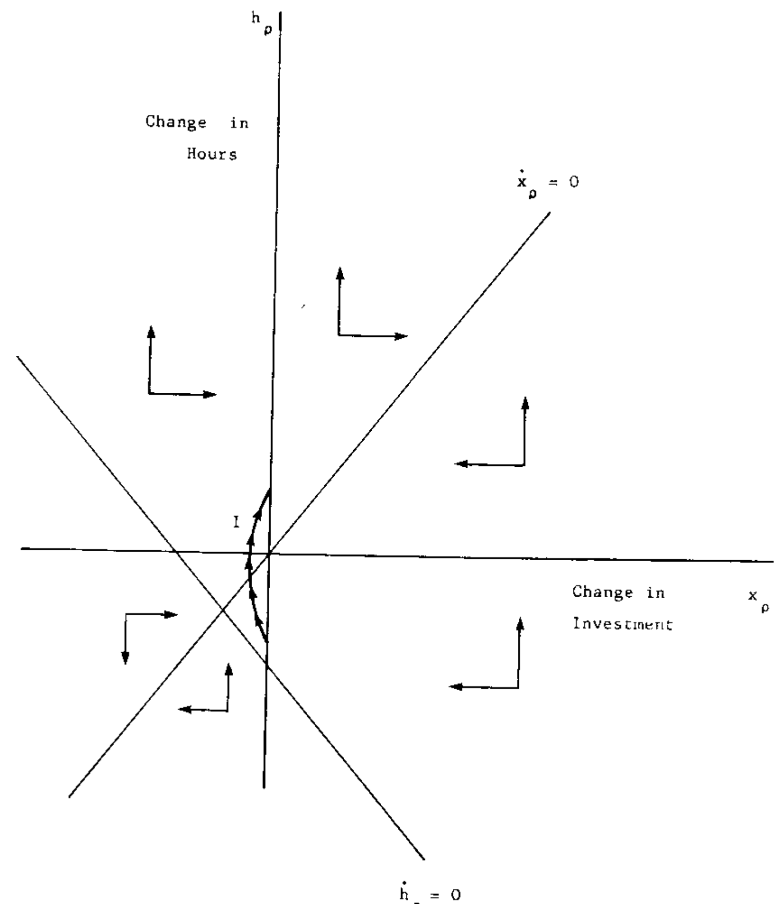

Figure 11.7

The phase diagram corresponding to (25) is presented in Figure 11.7. It is seen from examining the directional arrows that a trajectory satisfying $x_\rho(t_1) = x_\rho(t_2) = 0$ must start on the vertical axis at a point above the $\dot{h}_\rho = 0$ line and below the $\dot{x}_\rho = 0$ line, otherwise the trajectory will never return to the vertical axis. Trajectory I in Figure 11.8, therefore, describes the only admissible pattern. An increase in the subjective discount factor will reduce hours of work throughout the phase with on-the-job training. The investment intensity is initially reduced and then increased as one would expect given the incentive to postpone investment as ρ increases. The total amount of time spent at the phase with on-the-job training increases.

6. Extensions of the basic model

In this section I survey two important and related extensions of the basic model, imperfections in the capital market in the form of borrowing constraints and uncertainty with respect to the future earnings capacity of the worker. These two issues are related since potential lenders are more likely to be concerned about possible default when future earnings are random. I will follow the literature, however, and discuss each of the two issues separately.

6.1. *Borrowing constraints*

The simplest sort of borrowing constraint is one in which the worker cannot borrow to finance his educational costs but can borrow freely to finance his consumption. Such constraints were introduced into wealth maximizing models by Wallace and Inhen (1975), who impose non-negative net current earnings, and Oniki (1968), who requires that accumulated net earnings do not fall below some negative constant. Such constraints become effective only in the presence of *direct* costs of investment in human capital. It has been shown by these authors that the general time pattern of investment is not affected by such constraints, that is investment in human capital is *falling*, and a period of specialization if it occurs at all, will occur at the *beginning* of the worker's career.[23] The main difference is that the overall level of investment declines implying a lower (and with additional assumptions) flatter earnings profile.[24]

A more meaningful borrowing constraint is one which limits all borrowing, whether for consumption or training. Formally, this constraint can be written as $A(t) \geq 0$. This constraint can affect the worker in a more substantive way since it breaks the separation between consumption and investment decisions.

With the borrowing constraint the problem is restated as

$$\max_{\{x,c\}} \int_0^T e^{-\rho t} u(c)\,dt$$

s.t.

$$\begin{aligned} &\dot{A} = rA + RKh(1-x) - c, \quad A(t) \geq 0, \quad A(0) = A_0, \quad A(T) = 0, \\ &\dot{K} = G(K,h,x), \quad K(0) = K_0. \\ &0 \leq x \leq 1, \end{aligned} \tag{26}$$

[23] In Oniki (1968) this is strictly correct only if one adopts specification I in Section 2. Under a more general specification postponement of investment may arise.

[24] Oniki (1968) performs explicit comparative statics with respect to changes in the borrowing constraint. He shows that a tightening of the constraint leads to a *reduction* in the amount of human capital accumulated in the schooling phase. (The duration of this phase may increase or decrease.)

where, as in the wealth maximizing model, $h(t)$ is predetermined. An interior optimal path is characterized by the equations

$$u'(c) - \mu = 0, \tag{27}$$

$$-\mu RKh + \psi G_x = 0, \tag{28}$$

$$\dot{\psi} = \rho\psi - \mu Rh(1-x) - \psi G_K, \quad \psi(T) = 0, \tag{29}$$

$$\dot{\mu} = (\rho - r)\mu - \lambda, \quad \lambda \geq 0, \quad \lambda(t)A(t) = 0. \tag{30}$$

Combining eqs. (29) and (30) it is seen that the shadow price of human capital in dollar terms, $\eta \equiv \psi/\mu$, satisfies

$$\dot{\eta} = (r - G_K)\eta - Rh(1-x) + \frac{\lambda}{\mu}\eta, \quad \eta(T) = 0. \tag{31}$$

When the borrowing constraint is absent and $\lambda(t) \equiv 0$, eq. (31) is, of course, identical to eq. (4) in Section 2, and the optimal investment policy will maximize lifetime earnings. In the presence of the borrowing constraint the shadow price of human capital declines at a *slower* rate whenever the constraint is effective. This represents the incentive to shift investment towards periods in which an increase in investment does not require sacrifice of current consumption.

Recall that under specifications (II) and (III) in Section 2, eq. (31) involves only η and (λ/μ) [x and G_K can be eliminated using (28)]. In these cases it is still correct that η is decreasing whenever the borrowing constraint is not binding and $\lambda = 0$. When the constraint is binding, η may decrease at a slower rate or increase, depending upon the parameters of the utility function. The effect of a borrowing constraint is therefore to *reduce* the shadow price of human capital at all points during and prior to the phase (or phases) in which the borrowing constraint is effective. The worker will invest less in human capital and his earning profile will be generally lower (except at early ages) and flatter.

In the presence of borrowing constraints, investment need not decline monotonically, and the period of specialization may be postponed. For instance, if the worker starts with no initial assets, it is clear that he cannot train at a full rate at the beginning of his career, even though this might be efficient in terms of the maximization of lifetime earnings. He may choose to postpone his "schooling" period to a later point and accumulate sufficient savings to support such a program.

It is interesting to note that the general shape of the earnings profile may be unaffected by the introduction of a borrowing constraint. If, for instance, $r > \rho$, then it is seen from eq. (30) that consumption will increase on an optimal path.

Hence, when the borrowing constraint is effective earnings must increase. Whenever the constraint is not effective investment declines, and earnings will increase unless net investment becomes negative. Since the borrowing constraint is likely to be effective at the beginning of the worker's life, earnings will first increase and then decline. This is the same pattern as in the absence of the borrowing constraint. However, the rate of investment and the shape of the earnings profile will strongly depend on the worker's *tastes*.[25]

A common practice in the human capital literature is to acknowledge the practical importance of borrowing constraints only to ignore them in the analysis. The discussion above suggests that if the main purpose is to explain the general qualitative aspects of earning profiles the neglect of borrowing constraints can be, perhaps, justified. If, however, one's objective is to explain the level of investment, or to identify basic parameters such as the interest rate, from observed earnings, then the omission of the borrowing constraint may lead to serious biases.

6.2. *Uncertainty*

A worker who invests in human capital faces several risks. Both future market conditions and individual circumstances (such as health) are uncertain. The worker's capacity to learn on the job and its precise training content are not known when a job is accepted. The question arises to what extent does the presence of such risks affect the incentive to invest in human capital, and what is the effect on the time pattern of investment over the life cycle.

Consider again specifications (II) and (III) of the production function in Section 2, and assume that $g(x)$ is linear. We may then reduce *both* specifications to

$$\dot{K}/K = \theta x h - \delta, \tag{32}$$

where θ can be interpreted as training efficiency and δ is the depreciation rate of

[25] A simple example is the case in which the production function is linear and the rate of depreciation is zero. That is, $\dot{K} = aKxh$ and where a and h are constants and $a > r, \rho$. In this case it can be shown that if $\rho > r$ the borrowing constraint is *always* binding, while if $r > \rho$ it can *only* be binding during the investment period [see Weiss (1972)]. Working life is divided into two phases, an investment phase in which $0 < x < 1$ and a successive non-investment phase with $x = 0$. The duration of the investment phase t_0 is determined by $a\int_{t_0}^{T} e^{-r(t-t_0)}\,dt = 1$ if $r > \rho$ (the same condition as in the absence of a borrowing constraint) and by $a\int_{t_0}^{T} e^{-\rho(t-t_0)}\,dt = 1$ if $r < \rho$. Hence, the duration of the investment period is equal or shorter, and the intensity of investment is lower when a borrowing constraint is imposed.

human capital. Assume that θ follows a random process,

$$\theta(t) = \alpha + \gamma\varepsilon(t), \tag{33}$$

where $\varepsilon(t)$ is a white noise process.[26] Then the problem facing the worker is similar to the standard investor problem facing random returns [see Merton (1971)]. As in the case of certainty the difference remains that human capital cannot be bought or sold freely, a fact which introduces constraints on the rate of accumulation and introduces a potential direct link between utility and investment. Nevertheless, as shown by Williams (1979), the same techniques apply.

The worker's problem is now to choose an optimal strategy determining c, h, and x as functions of the state variables K and A (which are random) and t. The assumed objective is the *expected* lifetime utility. If one denotes the maximized value function by $J(K, A, t)$, then the optimal strategy satisfies, at an interior solution:

$$u_c = J_A, \tag{34}$$

$$u_l = RKJ_A, \tag{35}$$

$$xh = \left(\frac{-J_K}{J_{KK}K}\right)\left(\frac{\alpha - RJ_A/J_K}{\gamma^2}\right). \tag{36}$$

(For the case of certainty I used the notation $J_A = \mu$ and $J_K = \psi$.) Conditions (34) and (35) are of the same form as under certainty, and in particular (as is generally the under specification II) the marginal rate of substitution between consumption and leisure is equated to the *potential* wage of the worker [see also Macurdy (1983)]. The numerator of the second term on the right-hand side of (36) is the criterion which determined investment under certainty. Due to the assumed linearity, a "bang-bang" solution would arise in the absence of risk. But as noted by Williams (1979), this is modified under uncertainty due to the rise of a risk premium. If $\alpha < RJ_A/J_K$, then, as under certainty, the worker will not invest in human capital (i.e. $x = 0$). This will always hold towards the end of the worker's career since the transversality condition $J_K(K, A, T) = 0$ still applies. However, if $\alpha > RJ_A/J_K$, it does not follow that the worker plunges into investment in human capital since the riskiness of this investment is taken into account. Generally, the lower is the risk (as measured by γ^2) or the lower the degree of risk aversion (as measured by $(-J_{KK}K/J_K)$ the higher will be the investment. Just as in the case of imperfect capital markets the separation

[26] The process $\varepsilon(t)$ is the continuous time analogue of a sequence of independent random variables each normally distributed with zero means and unit variances [see Karlin and Taylor (1981, ch. 15, section 14)].

between investment and consumption breaks down. The investment decisions depend on *taste* parameters which are implicit in the value function $J(\cdot)$.

Since xh depends on the realizations of a random process, it is not as meaningful to ask whether it declines with age. One can enquire, though, whether the propensity to invest *given* the realizations of human capital and assets declines with age.

A precise statement can be made if it is assumed that the utility function is of the form

$$u(c, lK) = c^{\beta_1}(lK)^{\beta_2}, \quad \beta_1 + \beta_2 < 1, \tag{37}$$

where leisure is again measured in effective units. In this case conditions (34) and (35) imply that c and lK are in fixed proportions and utility as function of consumption has constant relative risk aversion. It is possible then to solve for $J(K, A, t)$ explicitly. At an interior solution, investment in human capital (in dollars) can be written as proportional to total wealth, where the factor of proportionality depends on the relative price of human capital. That is

$$\eta K x h = \Omega(\eta)(A + K\eta), \tag{38}$$

where $\eta \equiv (J_K/J_A)$ is the shadow price of human capital in dollar terms. Because of the assumption that leisure is measured in effective units and the specific form of the utility function (37), η depends only on t (and not on the realizations of K or A). It actually satisfies the *same* differential equation as under certainty [see Williams (1979, Appendix)] this means that η is ever decreasing. Since $\Omega(\eta)$ is an increasing function of η the conclusion is that investment in human capital as a proportion of total wealth is declining monotonically with the worker's age.

In the simple case outlined above an increase in risk, as measured by γ, reduces the propensity to invest in human capital. This may suggest that the general effect of uncertainty is to hinder the accumulation in human capital. This is not correct. In the more general model considered by Williams (1979) additional sources of uncertainty are included. In particular δ and R are also assumed random, and δ is allowed to be correlated with θ. This correlation introduces the potential for hedging against obsolesence thus encouraging the investment in human capital. In the simple two-period models [Levhari and Weiss (1974) and Williams (1978)] it is also shown that an increase in the investment in schooling may reduce the variability of earnings and total portfolio income. In such cases investment in human capital is encouraged when risk increases.

7. Specific human capital and binding labor contracts

The analysis up to this point was founded on the assumption that competition forces wages to equal the value of the marginal product which accrues to the firm.

As noted by Becker (1975) the forces of competition are often mitigated by the establishment of specific human capital. If training increases productivity within the firm more than elsewhere, then a bilateral monopoly situation arises, ex post. It is then in the interest of the parties, ex ante, to limit ex post bargaining. For this purpose they may seek a binding agreement which will set the division of the gains and the costs of the mutually beneficial investment. If it is indeed feasible to enforce such contracts the choice of the investment policy can be made jointly by the two parties, and the outcome will be independent of the division of the rents. The sharing rule can be determined, in principle, by some bargaining model, but since the transfers can take a variety of forms, there is little that can be said about the ensuing wage profiles. The fact is, however, that fully enforceable contracts are not observed in the labor market. A worker who wants to quit is rarely prevented legally or otherwise from doing so. Firms, on the other hand, do commit themselves quite often at least implicitly. This asymmetry can be exploited to put some bounds on the feasible wage profiles. Clearly, if the workers can leave the firm then the payment stream within the firm must at least match the outside opportunities of the worker. Further restrictions can be obtained if one adds asymmetry in the outside opportunities of the two parties, e.g. in the ability to rearrange payment schemes through the capital or insurance markets.

Consider the case in which a worker who joins the firm produces jointly output and knowledge which is purely specific to the firm. Suppose the worker has no access to an outside capital market and cannot rearrange the payments offered to him by the firm. At each point the worker has the option of leaving the firm, in which case his expected lifetime utility from that point on is given by $V(t, K_0)$ (V does not depend on $K(t)$ since human capital accumulated at the firm is purely specific).[27]

Suppose that the firm commits itself by offering a lifetime employment contract [with $h(t)$ set at 1 for all t] and a corresponding wage profile $w(t)$. How are the investment policy and wage profile determined? If one assumes that the bargaining between the worker and the firm leads to a *Pareto efficient* agreement, then the outcome of the bargaining process must solve

$$\begin{aligned} &\max_{\{x,w\}} \int_0^T \mathrm{e}^{-rt}(RK(1-x)-w)\,\mathrm{d}t \\ &\text{s.t.} \\ &\dot{K} = G(K,1,x), \quad K(0) = K_0, \\ &\dot{U} = \rho U - u(w), \quad U(T) = 0, \quad U \geq V(t, K_0) \quad \text{for all } t, \end{aligned} \tag{39}$$

where U is the discounted value of the worker's utility stream associated with

[27]It is also required that reentry into the firm is not optimal. Under static conditions and if, as assumed, experience is not transferable this will be the case [see Weiss (1971)].

$w(t)$, from t to T. That is to say, the firm's profits are maximized under the constraint that at no time can the worker's utility be improved either by quitting or by revising the investment policy and the wage offer.

It is immediately seen that the investment policy is the *same* as in the case in which human capital was perfectly general. The reason is, of course, that the collusion of the two parties allows them to jointly internalize the benefits of the investment, and it does not matter to whom the benefit accrues in the first place. The wage is determined separately by the conditions

$$-1-\mu u'(w)=0, \tag{40}$$

$$\dot{\mu}=(r-\rho)\mu-\lambda, \quad \lambda \geq 0, \quad \lambda(U-V(t, K_0))=0. \tag{41}$$

If the constraint $U \geq V(K_0, t)$ is never effective, wages are determined according to the desired *consumption* pattern of the worker. In this sense the firm acts as a bank on behalf of the worker. It follows from (40) and (41) that for $r \geq \rho$, the optimal, that is agreed upon, wage profile is *non-decreasing*.

With market imperfections there is an incentive for the provision of binding contracts even in the absence of any specific human capital. Several authors have noted that if productivity is uncertain and insurance of earnings is not available outside the firm, then a contingent wage agreement can be used to achieve risk sharing within the firm. Freeman (1977), Harris and Holmstrom (1982), and Weiss and Lillard (1983) consider extensions of the model outlined above when it is assumed that productivity both within and outside the firm evolve according to some stochastic process. If firms are assumed risk neutral, then again for $r \geq \rho$ the wage is *non-decreasing* along *any* sample path of the process. Thus in particular, average wages grow with age.

Hashimoto (1981) considers the case in which the productivity of the worker outside and inside the firm are random but not perfectly correlated. Under such circumstances it is not efficient to continue employment unless the occurrence of productivity within the firm turns out to exceed the worker's opportunity cost. The solution of the problem now requires an employment policy in addition to the wage and investment policy. If one could enforce a wage rate which is contingent on the ex post realized rents, allowing voluntary separations, given the wage rate, quits (or layoffs) will occur only if the separation is ex post efficient. (It is assumed that both parties are risk neutral.) However, since it is costly to verify the opportunities of the worker or the productivity of the firm, Hashimoto considers a non-contingent wage contract which is determined ex ante. Within a two-period model context, he shows that the predetermined wage profile will be *increasing*. The rate of increase is determined by its impact on the ex post incentives of workers to quit or firms to fire their workers. Lazear (1981) considers a similar model except that in his interpretation quitting is triggered by "shirking": a voluntary act which benefits the worker, imposes costs on the firm and leads to an immediate dismissal. In deciding whether to shirk the worker takes into account the endogenous probability that the firm terminates the contract unilaterally, sometime in the future. Lazear then shows that a Pareto efficient contract generates an upward sloping wage profile. The rise in the wage is used to discourage opportunistic behavior by the worker and, indirectly, mobility. It has been noted by several authors, e.g. Becker and Stigler (1974) and Kennan (1979), that if the worker has access to the capital market, alternative arrangements such as bonding can be used for this disciplinary purpose.

The analysis becomes considerably more complicated if one introduces variation in hours or effort. This is particularly true if effort cannot be monitored, which leads to an agency type problem. Holmstrom (1983) considers a special case where output is given by

$$Y_t=\theta+h_t+\varepsilon_t, \tag{42}$$

and the workers utility each period is

$$u_t=w_t-v(h_t), \tag{43}$$

where θ is unknown (but fixed) ability parameter, and ε_t a random transitory effect. Only output is directly observable. In equilibrium firms can infer from the workers past *output* on his ability and adjust wages correspondingly. Thus, by increasing effort the worker produces an individual specific human capital in the form of reputation. This type of human capital, however, has no direct effect on output. The model implies that effort *declines* monotonically toward zero over the life cycle. The reason is that effort is not rewarded directly, the sole return for effort is improved reputation, a return which diminishes towards the end of the work horizon.

Rogerson (1985) considers a case without learnings but with a utility function which is concave in w. The wage contract is, again, conditioned only on the outcome since effort is not observed. Because of the dynamic set-up, one can generally find more than one payment scheme which elicits the same effort and provides the same expected utility to the worker. This is accomplished by reducing utility in the present and increasing it uniformly at all future outcomes. An optimal wage contract must minimize the expected wage costs for the firm within this set. In a two-period context this leads to the condition:

$$\frac{1}{u'(w_0)}=E\left(\frac{1}{u'(w_1)}\right), \tag{44}$$

which states that the current marginal cost to the firm of increasing the worker's utility (with effort being the same) must equal the corresponding expected future cost. (It is assumed that $r = \rho$ and that the expectation is conditioned on the realized outcome at period 0.) By a direct application of Jensen's inequality it can be seen from (44) that the expected (unconditional) wages may increase or decrease depending upon the convexity of $1/u'(w)$. This is in contrast to the papers cited above which predict an increasing expected wage profile. The difference arises, in part, because of the assumed absence of quits in Rogerson's model.

To conclude, with specific human capital the wage and hours profiles are less closely tied to the accumulation of productive capacity. They also reflect the sharing of the costs and the benefits from the investment between the workers and the firm. The shares depend on the outside opportunities, mobility costs, information and attitude towards risk of the two parties. Depending upon the assumed role of these factors, one can obtain a variety of wage and work profiles. For this reason the results are considerably less robust than in the case of general human capital.

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