Matching: Finding a Partner for Life or Otherwise[1]

Dale T. Mortensen
Northwestern University

How do job-worker and husband-wife relationships form, and why do some matched pairs subsequently separate? Economists and game theorists are in the process of developing and applying an equilibrium explanation of match formation and separation based on the principle of voluntary pairing under competitive conditions. The purpose of this paper is to assess the current state of these developments. The general existence of partnership structures with the property that no one has an incentive to separate has been established. These equilibrium structures maximize aggregate "value" over the set of all feasible match structures in the transferable utility case. According to this pure theory of matching, competition for partners can be expected to yield equilibrium match structures that are both stable and socially efficient, at least under conditions of frictionless certainty. Search-theoretic models that take account of meeting costs and initial uncertainty about match values have been developed to explain separation behavior as a process of shopping for a "good match." These models suggest several possible social inefficiencies in the match formation and separation processes. Finally, labor economists have recently provided empirical evidence that supports this view of the process.

MARRIAGE AND EMPLOYMENT RELATIONSHIPS

The relationships with spouse and employer are among the most important partnerships in the lives of most people. Because of their contributions to the psychic and financial well-being of the principals and their families, considerable time and effort is spent in their formation and maintenance. In spite of this investment, a specific relationship of either kind is not necessarily permanent. Indeed, longevity of any particular marriage or job is an unrealized ideal for many. The topic of this paper

[1] The paper is a revised version of my inaugural lecture as the Ida C. Cook Professor of Consumer Economics at Northwestern University, presented on February 27, 1986, in Evanston, Illinois. This version has benefited from the helpful comments of a referee. Requests for reprints should be sent to Dale T. Mortensen, Department of Economics, Northwestern University, Evanston, Illinois 60201.

0002-9602/89/9407-0008$1.50

can be simply stated: How do relationships between complementary pairs, such as job-worker and husband-wife partnerships, form, and why do some matched pairs subsequently separate?

Economists and game theorists are in the process of developing and applying an equilibrium theory of match formation and separation designed to answer these questions. The purpose of this paper is to assess the current state of these developments. Although there are extensive and important contributions on the matching problem by other social scientists, the fact that the research by this subspecies is based on a common principle, that of voluntary pairing under competitive conditions, justifies an exclusive treatment.

The theoretical literature on the matching problem can be separated into two branches. The first, which might be called the "pure theory" of matching, abstracts from transaction costs and information problems. In this branch of the literature, a "solution" to the matching problem is viewed as a collection of pairs and singles that is *stable* in the sense that no matched individual has an incentive to separate and no unmatched pair has an incentive to form a new match. Formally, the set of stable match structures is the core of the underlying coalition-formation game. The principal results of the pure theory demonstrate the general existence of stable match structures and provide algorithms that find particular solutions to matching problems.

Because, in reality, the search for a partner takes time and because the value of any particular partnership is at least initially unknown, one might expect the matching process to be characterized by sequential decisions that do not generally yield stable match structures. From the point of view of the rational members of a potential pair engaged in a matching process characterized by transaction costs and uncertainty, a match that seems quite acceptable now may dissolve in the future either because a better alternative subsequently becomes available to one of the partners or because intitial expectations about match quality are disappointed. The second branch of the theoretical literature on matching attempts to model the outcome of the dynamics of matching processes characterized by imperfections.

Recently, labor economists have become interested in the implications of matching models for empirical phenomena related to job-worker separation behavior and the growth of individual earnings over the life cycle. The fact that the likelihood of job separation decreases as a worker gains experience in the world of work is consistent with the hypothesis. Recent empirical studies have also provided strong evidence that matching is an important explanation of the fact that the typical worker's earnings increase with length of work experience and tenure in a current job.

Whether or not these ideas and techniques might be useful in the empirical analysis of marriage and divorce as well is an exciting question that naturally arises.

The paper is organized around these three topics. The first section is an exposition of the pure theory of matching—theoretical structures that offer insight into the likely outcomes of any voluntary process of matching under ideal conditions. A review of the current state of the associated search-theoretic theory of separation constitutes the second section of the paper. Essentially, the latter is an extension of the former that recognizes the existence of matching costs and of uncertainty. Finally, the third section contains a brief report on a body of recent empirical work that attempts to measure the importance of matching as an explanation for empirical observations on job-separation behavior and earnings growth.

THE PURE ASSIGNMENT PROBLEM

The view that marriage and employment are essentially voluntary relationships is common to all the literature I review in this paper. Partnerships of both types are initiated by the principals and, by tradition and by law, continue for the most part only with their mutual consent. Under these conditions, one expects a specific relationship to form for the mutual benefit of both parties. In short, the relationship is a means by which joint opportunities that are not available to either separately can be exploited.

Of course, both good and bad potential match partners exist for any individual. Moreover, a good partner for one may be bad for another. Herein lies an assignment problem. How are workers and jobs or men and women of different types assigned to one another when specific matches are formed voluntarily and at the initiative of the two parties?

Although the general matching problem is analogous to that of voluntary market exchange, the standard (Walrasian supply and demand) model of competitive equilibrium is inappropriate for several reasons. Even under conditions of no uncertainty and transactions costs, the model applies only in the context of the exchange of some set of homogeneous and perfectly divisible commodities. In the case of an assignment problem, a "commodity" is the bundle of "traits" that each individual on one side of the market offers to individuals on the other side, and a particular "exchange" involves an all-or-nothing trade of the bundle supplied by one for the bundle provided by the other. In other words, heterogeneity and indivisibility of the commodities exchanged characterize a matching market.

The first formal model of equilibrium in a matching market was introduced by Gale and Shapley (1962). Mathematically, their model and concept of equilibrium can be characterized as follows: Suppose that an equal number of individuals, agents, or objects—n, say—of two types are to be matched. Let the numbers f_{ij} (m_{ij}) represent the preferences of individual i of type $f(m)$ for individual j of type $m(f)$ as a partner, where $j = 0$ in each case represents the single state. In other words, $f_{ij} > f_{ij'}$ means that individual i of type f would rather form a partnership with j than with j', both of whom are of type m, and $f_{i0} > f_{ij}$ means that individual i would rather remain single than form a partnership with individual j of type m. A feasible match structure is a partition of the set of $2n$ individuals into pairs of opposite types and singles such that no individual of either type is paired with more than one individual of the other. A feasible structure can be represented by a matrix $\alpha = [\alpha_{ij}]$, where $\alpha_{ij} = 1$ if and only if individual i of type f is assigned to individual j of type m and $\alpha_{ij} = 0$ otherwise. Hence, a solution is feasible if and only if the matrix $\alpha = [\alpha_{ij}]$ contains at most one "one" in each row and each column, with zero entries elsewhere. Remaining single is then represented as a row of zeros in the case of individual i of type f and a column of zeros in the case of individual j of type m. Obviously, if α is a feasible assignment, then one can assign the indices i and j to individuals of the two types so that α is a diagonal matrix; that is, $\alpha_{ij} = 0$ when $i \neq j$, without loss of generality.

Gale and Shapley regard any assignment that is stable relative to specified preferences as an equilibrium solution. A *stable match structure* is feasible and has the property that no matched individual prefers the single state and that no pair of two individuals of opposite types prefer a match with each other to their current assignments. In other words, a feasible match structure $\alpha = [\alpha_{ij}]$, where $\alpha_{ij} = 0$ for all $i \neq j$ by construction, is stable if and only if the following conditions hold:

$$f_{ii} \geq f_{i0} \text{ and } m_{ii} \geq m_{i0} \text{ for all } i \in \{1, \ldots, n\} \text{ such that } \alpha_{ii} = 1. \quad (1)$$

There exists no $i \neq j$ such that either

$$f_{ij} \geq \max[f_{ii}, f_{i0}] \text{ and } m_{ji} > \max[m_{jj}, m_{j0}], \quad (2a)$$

or

$$f_{ij} > \max[f_{ii}, f_{i0}] \text{ and } m_{ji} \geq \max[m_{jj}, m_{j0}]. \quad (2b)$$

A stable match structure is individually rational, condition (1), and Pareto optimal, condition (2). Because a two-person coalition of opposite types can accomplish anything for themselves that they could as members of a larger coalition, the set of stable match structures yields utility allocations equivalent to the core of an assignment game, as Gale and Shapley point out.

Gale and Shapley establish that the core of this game always exists, a result that is nontrivial for games with indivisibilities in general.[2] Indeed, they construct an adjustment process that converges to a stable match structure. Informally, the process can be described as follows: Each person on one side of the market—the men, say—makes an offer to his favorite individual on the other side, a woman, who either rejects the offer in favor of one already received (which includes the "offer" of remaining single) or tentatively accepts it by rejecting any previously accepted one. Each rejected man then makes an offer to his next most-preferred woman, and the process continues until each man either has been accepted by some woman or has been rejected by every woman whom he prefers to bachelorhood. As Roth (1984*a*) points out, this algorithm was actually in use before Gale and Shapley "invented" it for the purposes of their paper as the means of matching graduating medical students in the United States with positions as hospital interns and residents.

Although the set of stable match structures is not always a singleton, the concept has obvious appeal. Still, if one accepts the solution concept as useful for descriptive purposes, then it follows that separation or divorce is a "disequilibrium" phenomenon; that is, it occurs only when a "mistake" is made in the sense that condition (2) holds for some $i \neq j$. Hence, under the specified conditions, certainty and lack of transactions costs, "divorce" is irrational. In this sense, the theory is too strong. I will return to this point in the next section.

"Class" or "sex" conflict is a consequence of the multiplicity of stable solutions to the matching problem. Specifically, Gale and Shapley prove the following remarkable theorem: if no individual is indifferent between any two potential partners, then all individuals on a given side of the market agree on which is the best (and worst) stable match structure. Furthermore, the Gale-Shapley algorithm always converges to the stable match structure that is best (worst) from the point of view of the side of the market making (receiving) the offers. This result suggests the possibility that "custom" and "power" play nontrivial roles in determining the specific outcome in any real matching market. Indeed, Roth (1984*a*) proves that the stable match structure obtained in the market for interns is that which is best (worst) from the point of view of the hospitals (new doctors), as it is the hospitals that make the offers in this real-world example. In the same sense, men collectively do better than women in the

[2] A core assignment is a match structure with the property that no subset of individuals of any size or composition exists that can reassign its members in a way that all members of the coalition prefer. For an example of a coalition-formation game in which there is no core, see Dreze and Greenberg (1980).

marriage market to the extent that custom demands that men propose and women simply accept or reject.

Several inadequacies in the original Gale-Shapley formulation as a model of labor and marriage markets are immediately apparent. First, the numbers of individuals of the two types need not be equal in any real matching problem. Second, "polygamy" is of interest, particularly in labor markets where a given employer generally has need for more than one worker. Finally, the original formulation does not provide a means by which appropriate "contract terms" for the employment or marriage relationship can be negotiated. Subsequent contributors to the literature have established the two principal results of Gale and Shapley in the contexts of models that allow for all these complications.

Papers by Shapley and Shubik (1972) and by Becker (1973) analyze versions of the model that incorporate transferable utility in the form of a good (money) that can be used by one partner to compensate the other for participation in a match. Compensation has the natural interpretation of "salary" in a labor-market context. Becker argues that the idea applies in the marriage market as well. For example, a rich homely man and an attractive but poor woman are able to form a stable match when the former can compensate the latter for his deficient trait.

Assuming that the "principle of compensation" applies, one can incorporate negotiation of terms into the formal model outlined above by interpreting the rankings implicit in the matrices $[f_{ij}]$ and $[m_{ij}]$ introduced above in terms of the transferable good. Specifically, letting $f_{ij} - f_{i0}$ and $m_{ji} - m_{j0}$, respectively, represent the amounts that female i and male j would be willing to pay if positive and would have to be paid if negative in order to be married to each other, then the total surplus value of a match between male i and female j relative to the single state for both can be defined as the sum

$$s_{ij} = f_{ij} + m_{ji} - f_{i0} - m_{j0}. \quad (3)$$

Obviously, an acceptable match must compensate both parties for their forgone values of remaining single; that is, it must have a positive surplus value. Some potential matches are of sufficient value to compensate in this sense, and others are not.[3]

A bargaining problem exists in the case of any match with a positive surplus. How should the surplus be divided? Who should compensate whom and by how much? For the stability of a given relationship, the division generally depends on the surplus values associated with alternative matches available to the two parties. Specifically, a match can be

[3] See Becker (1973) for a detailed analysis of the sources of match surplus.

stabilized relative to alternatives only if a division of the match surplus exists such that neither party has an incentive to separate.

This idea, together with the concept of a feasible match structure previously introduced, yields the following definition of a stable match structure in the case of transferable utility: it is a feasible match structure and a division of the surplus of each match in the structure such that no matched individual prefers to be single and no pair of persons of opposite types prefer a match with each other for some feasible division of their surplus value. Formally, a feasible match structure, represented without loss of generality by a diagonal matrix $\alpha = [\alpha_{ij}]$, is stable if a vector of shares $\theta = [\theta_1, \ldots, \theta_i, \ldots, \theta_n]$ exists, where θ_i represents the share of the surplus associated with match i received by the member of the first type, such that both of the following conditions hold:

$$\theta_i s_{ii} \geq 0 \text{ and } (1 - \theta_i)s_{ii} \geq 0 \text{ for all } i \text{ such that } \alpha_{ii} = 1. \quad (4)$$

There exists no $i \neq j$ such that

$$s_{ij} > \theta_i s_{ii} + (1 - \theta_j)s_{jj}. \quad (5)$$

In other words, every match in a stable structure yields a positive surplus that is divided between the two parties in some manner, and the surplus associated with any potential match not in the structure has a surplus value that is less than or equal to the shares of the surpluses received in the matches to which the two are parties. In the case of transferable utility, then, a stable match structure is a pair composed of an assignment of partnerships represented by the matrix α and a vector of surplus shares θ that *supports* it in the sense of conditions (4) and (5).

The set of stable match structures is again the set of core assignments associated with the game. Shapley and Shubik (1972) and Becker (1973) prove that the set of stable match structures is also equivalent to the solution set associated with the problem of finding a match structure that maximizes aggregate surplus, the so-called optimal assignment problem found in the mathematical programming literature. In my terminology and notation, an optimal assignment is a match structure α^* such that

$$\Sigma \alpha^*_{ij} s_{ij} = \max_{\alpha} [\Sigma \alpha_{ij} s_{ij}], \quad (6)$$

where the maximization is over the set of feasible assignment matrices. Since this set is finite, existence follows trivially.

It should be emphasized that an appropriate division of match surpluses is required to stabilize an optimal assignment. As an example, consider the marriage game illustrated in table 1. The table records the

TABLE 1

MATCH SURPLUSES

	Mary	Sara	Violet
Sol	3	0	3
Sam	3	2	0
Rob	0	0	2

surplus value associated with every possible row-man and column-woman pair. It is obvious that the match structure associated with the diagonal entries is the only optimal assignment; that is, all other assignments among the $3! = 6$ possibilities yield an aggregate surplus less than seven, the sum of the diagonal entries. The optimal assignment can be stabilized but only with unequal shares that reflect the favorable bargaining position occupied by both Sam and Violet. Specifically, it is stable given that Mary and Sol share their surplus equally only if Sam and Violet receive at least $(3/4)2 = 3/2$ and Sara and Rob, respectively, receive the residual, which is no more than $(1/4)2 = 1/2$, since Sam and Violet must receive at least as much as they could obtain by proposing marriage to Mary and Sol, respectively.

The optimal assignment is not stable if each pair shares their surpluses equally for the following reason. Given equal shares, Sam gains one-half by marrying Mary, and Mary is indifferent between marriage to Sol and to Sam. Similarly, Violet would gain one-half in a match with Sol, and Sol is indifferent between Mary and Violet. Indeed, if equal sharing is required by custom, for ethical considerations, or because it is the law, the game is transformed into one of nontransferable utility. For this game, a pairing of Sol and Violet and of Sam and Mary, with Rob and Sara remaining single, is stable but does not maximize the aggregate surplus.

Again, the set of stable match structures is not generally a singleton. Indeed, the transferable-utility version of the model generates multiplicity of an additional kind. Even when the optimal assignment is unique, it is generally supported by more than one allocation of the match surpluses among the individuals, as the reader can easily verify by using the example illustrated in table 1. Furthermore, Shapley and Shubik (1972) establish that the conflict of interest noted by Gale and Shapley for the nontransferable-utility case continues to hold. Given no indifference over potential partners, best and worst core elements generally exist from the point of view of all individuals on a given side of a matching market. More recently, Crawford and Knoer (1981) and Kelso and Crawford (1982) show that a generalized version of the Gale-Shapley algorithm, set in a labor-market context—one that allows for "salary" negotiation, unequal numbers of "workers" and "jobs," and a multiple and endogenously determined number of "jobs" for each "employer"—converges in a finite number of steps to a stable match structure that is the best from the point of view of all "employers" when the "employers" make and "workers" accept or reject offers. Finally, Roth (1984*b*), in the context of an even more general bilateral matching market model, demonstrates that the interests of the two sides are polarized in the sense that the best core allocation from the point of view of employers is the worst in the view of workers. Conversely, there exists a worker-best core allocation that is the employer-worst element in the core, a mysterious result that the author contends still defies adequate explanation.

What is the state of the pure economic theory of voluntary matching? It seems fair to say that an appealing, quite general, and nonvacuous solution concept has been formulated. Furthermore, the Gale-Shapley algorithm and its generalization by Crawford and his coauthors provide an interesting example of a method that is capable of implementing at least one element in the solution set. Indeed, as Roth (1984*a*) points out, the method has been successfully applied in an important real-world context for some time.

However, the model, the solution concept, and the algorithm all are inadequate as a descriptive theory of how many real matching markets operate, for two possibility related reasons. First, the theory focuses only on match formation. It provides little useful insight into the separation process, one that is of obvious empirical significance in both employment and marriage relationships. Second, neither the cost of match formation nor the existence and resolution of uncertainty about match quality are addressed by the theory.

UNCERTAINTY AND FRICTION IN THE MATCHING PROCESS

It takes time to meet a partner and to learn the uncertain value of any partnership. These lags are acknowledged in the search-theoretic literature on matching. Indeed, the models are capital theoretic formulations of the problem of optimal search for a match partner. A principal implication of the models is that some *acceptable* matches form that will separate in the future even when the individuals involved are fully aware of the possibility. Furthermore, numerous dynamic externalities exist that suggest that self-interested decisions made by participants do not induce socially optimal outcomes.

In search-theoretic models of matching, potential partners meet sequentially at finite rates, and the identities of potential match partners are uncertain in the sense that no pair know their match quality before they meet. In more sophisticated versions of the theory, either the rates at which potential pairs meet are endogenously determined, or match quality can only be discovered through experience, or both. These complications provide a richer set of implications. The formal model sketched in this section, which is derivative of those developed by Mortensen (1978, 1982*a*), Diamond and Maskin (1979), and Diamond (1981), is an example of the simplest version of the theory. I will note some of the more interesting implications of sophisticated versions in passing.

There are two types of agents to be matched, "male" and "female," denoted as m and f, respectively, and every individual of each type is identical ex ante.[4] However, ex-post realized match qualities differ. Formally, the joint benefit to be shared, match quality, is characterized as a random variable that is observed only after a specific pair meet. Let X denote the random-match quality of any pair, and let $F(x) = Pr(X \leq x)$ denote its cumulative probability distribution. Over time, potential match partners continually meet at random. Meetings are generated by two independent Poisson arrival processes characterized by arrival rates λ_m and λ_f, where the magnitudes of each rate is the average frequency per unit time period with which an individual of the specified type generates meetings with individuals of the other type. For simplicity of exposition, I suppose that only unmatched agents search. Although this assumption is unwarranted in general, none of the results presented in the sequel are particularly sensitive to it.[5] Finally, it is assumed that any match has a random exponential life characterized by the exogenous constant hazard or turnover rate δ.

Each agent acts to maximize the expected capitalized value of his or her own future benefit stream—expected wealth. Capitalizing a future stream is simplified by assuming that the interest rate used, denoted as r, is stationary and the same for all and that every agent lives forever. Fortunately, none of these assumptions are crucial. The assumption that each individual is aware of the nature of the matching process, that is, knows the values of the arrival rate parameters, the distribution of match quality, the value of the exogenous turnover rate, and the interest rate, is important. Given these assumptions and a specification of how the income attributable to a match is divided between the partners, each agent of each type can compute the capital value associated with the unmatched state as well as the capital value of being matched, conditional on the realized value of the match.

[4] One can easily generalize the model to allow for observably different types of each sex, but the complexity of the representation increases accordingly.

[5] This statement is true provided that search while matched is more costly than search while single, in the sense that it is either more expensive or more difficult to generate a meeting.

Let $W(x)$ represent the capital value associated with a potential match known to be of quality x, and let V_f and V_m denote the capital value associated with the single state for each agent type. Two expected-wealth-maximizing individuals form a partnership given this information only if the capital value of their match exceeds the sum of the capital values associated with both remaining single. In other words, the surplus capital value of the match, denoted as

$$S(x) \equiv W(x) - V_f - V_m, \tag{7}$$

must be positive. However, in order to induce two singles to form a match, the surplus capital value must be shared. Given that any positive fraction, θ, is received by the female, the capital value of an acceptable match to each of its parties is

$$W_f(x) = V_f + \theta S(x) \tag{8a}$$

in the case of the single female and

$$W_m(x) = V_m + (1 - \theta)S(x) \tag{8b}$$

in the case of the single male. Since the division of the surplus capital value is a bilateral bargaining problem, little of interest could be said about the economic determination of a particular value for θ within the unit interval until recently.[6] It is interesting that none of the principal implications of the model depend on its value.

The definition of an acceptable partnership for two unmatched individuals is implicit in equations (8a) and (8b). Namely, given any positive share fraction, θ, both prefer a partnership with each other to the single state if and only if the surplus capital value of the match is positive. In other words, an acceptable match is any that yields a positive surplus-capital value. Endogenous turnover is a possibility in this model because a match of two singles that is acceptable in this sense at one point in time need not be stable relative to an alternative match prospect that might become available to one of the partners at some future date.

For example, consider a match of quality x and suppose that the male member of the pair is met by a single female and that the quality of their match is y. If both yield a positive surplus capital value but the capital value of the alternative is greater than the capital value of the existing match, that is, $W(y) > W(x)$, then a split of the alternative's surplus capital value always exists such that the male member prefers the alternative to the existing match. To see why, simply note that the female member in the original match is willing to offer no more than its surplus

[6] See Rubinstein and Wolinsky (1985) for a treatment of the problem. They apply strategic bargaining theory for the purpose of deriving a specific value for θ.

value to retain her partner, while her rival is willing to offer up to the surplus value of the alternative to gain a partner. Since $W(y) > W(x)$ implies $S(y) > S(x)$ by virtue of equation (7), the rival wins in an auction. Conversely, the existing partner is willing to match the offer of the rival rather than see the partnership dissolve if the surplus value of the alternative match is less than of equal to that of the existing match.

As an alternative, one might argue that the "deserted" partner should be "compensated" for her share of the surplus. Even so, the new partnership would form if and only if the surplus value of the alternative is greater, since the surplus capital value of the alternative, less her share of the surplus of the existing match, $S(y) - \theta S(x)$, exceeds the husband's share of the existing match, $(1 - \theta)S(x)$, if and only if $S(y) > S(x)$. Notice that turnover of this form always yields an aggregate improvement in the existing solution to the assignment problem; that is, it is socially efficient in the sense that the aggregate surplus-capital value is increased.[7]

By definition, a capital value is the expected present value of an anticipated benefit stream. Equivalently, the imputed interest income on the capital value is equal to current income plus (minus) any anticipated capital gain (loss). In an existing match, the total current income associated with a match is its realized quality, x, by definition. The match faces the prospect of dissolution at an exogenous expected frequency δ, and the total capital loss to the pair is its surplus value because both return to the single status. Finally, when a separation occurs because one or the other is met by an alternative partner who can offer a match of greater value, neither a loss nor a gain in the original pair's total capital value occurs. The sum of the pair's capital value is the same before and after the dissolution whether the rival for the desired partner bids up to the willingness of the other partner to pay or compensates the deserted partner for his or her share of the value of the original match. Consequently, the capital value of an existing match of quality x satisfies

$$rW(x) = x - \delta S(x).^{8} \tag{9}$$

[7] Becker et al. (1977) and Mortensen (1978) point out this social efficiency property of the auction scheme and of the compensation scheme, respectively. However, neither scheme is efficient when realistic complications are added. Both parties to the original match have an incentive to exaggerate its quality when the rival does not know the truth. This information asymmetry causes a problem of implementing the compensation scheme that is not present in the case of an auction. But, when an endogenous search effort on the part of matched individuals is allowed, matches turn over too frequently under the auction scheme because it induces an incentive to search, for the purpose of increasing one's share of the surplus of the existing match rather than of finding a more valuable one. This incentive is not present under the compensation scheme.

[8] Formally, the pair receives the joint-benefit flow x until the match terminates for exogenous reasons. In this event, which occurs at future uncertain date T, both be-

The analogous relationship for the expected present value of the future income stream associated with the unmatched state is more complicated. A single female generates meetings with males at frequency λ_m. However, the male met at random is single with probability equal to the fraction of all males who are single, denoted as u_m, and is matched with probability $1 - u_m$. In addition, the single female is met by some single male with frequency equal to the aggregate rate at which single males meet females, divided by the number of females. Hence, if n_f and n_m equal the total number of female and male individuals, respectively, this meeting rate is equal to $\lambda_m u_m n_m / n_f = \lambda_m u$ when the numbers of the two types are equal.

If the potential partner met is single, then the capital gain associated with the meeting is zero if the match is not acceptable and is equal to the individual's share of the surplus capital value of the match otherwise. When we let Y denote the uncertain quality of the match that is distributed according to the cumulative distribution function $F(y)$, the uncertain capital gain associated with meeting another single of the opposite type is $\theta \max[S(Y), 0]$ if female, and $(1 - \theta)\max[S(Y), 0]$ if male.

If the potential partner met is matched, then the capital gain associated with a meeting depends on both the quality of the match denoted as Y and the uncertain quality of the existing match involving the potential partner. Let Z denote the latter and let the $G(z)$ represent its cumulative distribution function. In a meeting generated by a single, a new partnership forms, and the single obtains a capital gain equal to the difference in the surplus capital values if and only if this difference is positive. In other words, the uncertain capital gain associated with meeting a matched individual is $\max[S(Y) - S(X), 0]$.

Let b_i, $i = m$ or f, denote the benefit flow when single. The imputed interest income on the capital value associated with the single state is equal to b_i plus the sum of the products of each of the three different meeting rates and the expected capital gain associated with each type of meeting. The expected present value of the future income stream antici-

come single. Since the present value of the sum of their future benefit flows when single is $V_f + V_m$ by definition and T is an exponential random variable with mean $1/\delta$ by assumption, the expected present value of their joint future benefits when matched is

$$
\begin{aligned}
W(x) &= E\left[\int_0^T be^{-rt}dt + e^{-rT}(V_f + V_m)\right] \\
&= \int_0^\infty \left[(b/r)(1 - e^{-rT}) + e^{-rT}(V_f + V_m)\right]\delta e^{-rT}dT \\
&= \frac{b}{r + \delta} + \frac{\delta}{r + \delta}(V_f + V_m),
\end{aligned}
$$

which is equivalent to eq. (9).

pated by a single male can be analogously defined. In the case of an equal number of males and females, that is, $n_f = n_m$, these arguments imply

$$rV_f = b + (\lambda_f + \lambda_m)u\theta\int \max[S(y), 0]dF(y) + \lambda_f(1 - u)\int\int \max[S(y) - S(z), 0]dF(y)dG(z) \tag{10a}$$

in the case of the single female, and

$$rV_m = b_m + (\lambda_m + \lambda_f)u(1 - \theta)\int \max[S(y), 0]dF(y) + \lambda_m(1 - u)\int\int \max[S(y) - S(z), 0)]dF(y)dG(z) \tag{10b}$$

in the case of a single male, where $u = u_f = u_m$ is the common fraction of unmatched males and females.

To derive the endogenously determined surplus capital value associated with any match, add the respective sides of equations (10a) and (10b), and to obtain

$$r(V_f + V_m) = b + \lambda u\int \max[S(y), 0]dF(y) + \lambda(1 - u)\int\int \max[S(y) - S(z), 0]dF(y)dG(z), \tag{11}$$

where

$$b = b_f + b_m \tag{12a}$$

is the sum of the pair's benefit flows as singles and

$$\lambda = \lambda_m + \lambda_f \tag{12b}$$

is the sum of the average rates at which the two generate meetings as singles. The right side of equation (11) is the joint value in flow terms to a pair of continued search once they meet.

By subtracting respective sides of equation (11) from equation (9) and then rearranging terms using the identity equation (7), one obtains the following expression for the endogenous surplus capital value associated with any given match of quality:

$$(r + \delta)S(x) = x - b - \lambda u\int \max[S(y), 0]dF(y) - \lambda(1 - u)\int\int \max[S(y) - S(z), 0]dF(y)dG(z). \tag{13}$$

It is a simple matter to verify that a unique surplus capital value function satisfies this equation and that the function is continuous and increasing in x. Specifically, the solution is of the form

$$S(x) = (x - x^*)/(r + \delta), \tag{14}$$

where x^* is the *reservation quality,* the smallest quality that yields a nonnegative surplus. In other words, a match of quality x is acceptable if and only if $x \geq x^*$. By substituting the right side of (14) for the function everywhere that it appears in (13), one finds that the reservation quality is the unique solution to the following equation:

$$x^* = b + [\lambda/(r + \delta)][u\int \max(y - x^*, 0)dF(y) + (1 - u)\int\int \max(y - z, 0)dF(y)dG(z)]. \quad (15)$$

In other words, the reservation quality is equal to the sum of the joint benefits enjoyed as singles plus the expected present value of joint future gains associated with continued search by both parties.

To establish that endogenous match turnover is a possibility, one needs to show that conditions exist under which the reservation quality, x^*, is less than the largest possible quality available, which is the smallest solution to $F(x^+) = 1$. The solution to equation (15) is greater than or equal to b in general and is less than the largest value of match quality if and only if b is less than the largest possible quality; that is $b < x^+$. Of course, if this condition is not satisfied, there is no incentive for match formation in the first place. Therefore, a match that is acceptable to two singles is not stable relative to a match of higher quality that might become available to one of the partners in the future. Endogenous separation occurs in the model because it does not pay to wait for the perfect match, that which is of quality x^+.

The observant reader will have noted that the fraction of single agents, u, and the fraction of matched pairs of quality x or less, $G(x)$, are neither exogenous nor stationary in the model. Because the aggregate rate at which singles meet is the sum of the rate at which single individuals meet single members of the opposite sex, which is $(\lambda_m + \lambda_f)u^2 n$, where n is the total number of individuals of each sex, and because a meeting results in the formation of a match if and only if realized match quality is acceptable, which occurs in each case with probability equal to $Pr(X \geq x^*) = [1 - F(x^*)]$, new matches form at an aggregate rate equal to the product $(\lambda_m + \lambda_f)u^2\text{n}[1 - F(x^*)]$ on average. The average aggregate rate of separation is the product of the exogenous separation hazard δ and the number of matched pairs, which is $(1 - u)n$, because endogenous turnover does not affect the aggregate number of pairs. Of course, the time rate of change in the number of matched pairs is the difference between these two flows, and, consequently, the fraction matched converges over time to a steady-state value defined by the equality of the two flows. Specifically, the steady-state fraction of singles of either type is the positive root of the following quadratic equation:

$$\lambda u^2[1 - F(x^*)] - \delta(1 - u) = 0. \quad (16a)$$

One can show that the positive root is a number greater than zero and less than unity, which tends to zero as the exogenous separation rate δ tends to zero. Furthermore, the solution u^* is an increasing function of the reservation quality because an increase in x^* decreases the probability that a meeting of two single individuals of opposite types will form a match.

The steady-state distribution of match quality can be derived in a similar manner. Since the number of matches of quality x or less, $(1 - u)G(x)$, is a stock, its steady-state value is that which equates the flows in and out. The rate at which new matches of this quality form is the product of the rate at which singles meet and the probability that realized match quality is both acceptable and less than or equal to x, which is $\lambda u^2 n[F(x) - F(x^*)]$. The rate at which matches of quality x or less separate is the sum of the exogenous rate, which is $\delta[1 - u]G(x)n$, and the endogenous rate. The endogenous separation rate is the product of the rate at which singles meet matched individuals of the opposite sex whose existing match is of quality x or less, $\lambda u[1 - u]G(x)n$, and the probability that the realized quality of the alternative exceeds x, which is $[1 - F(x)]$. Equating the inflow and outflow, one obtains

$$G(x) = \frac{\lambda u^2[F(x) - F(x^*)]}{\delta(1 - u) + \lambda u(1 - u)[1 - F(x)]} = \frac{u[F(x) - F(x^*)]}{1 - F(x) + u[F(x) - F(x^*)]}, \quad (16b)$$

where the second equality follows by substitution from (16a).

An equilibrium steady-state solution to the model is any reservation quality and a steady-state fraction of unmatched agents and a distribution of match qualities that simultaneously satisfy equation (15) and the two equations of (16). Obviously, in a labor-market context, the fraction of unmatched agents is the unemployment rate. Diamond (1981, 1982), Mortensen (1982*a*, 1982*b*), and Pissarides (1984, 1985), among others, have studied the equilibrium steady-state properties of models of this type as a means of gaining insight into how matching processes that operate in the labor market determine the unemployment rate.[9]

Diamond (1981) argues that the equilibrium reservation quality is too low because two singles who meet do not take account of the effect of their acceptance decision on their own future availability as single potential partners for others. The presence of this overlooked external effect can be seen by noting that a larger fraction of unmatched agents benefits the typical pair of searching singles by virtue of equation (11). Specifi-

[9] See Hosios (1986) for a more comprehensive review of this literature than that provided here as well as a commendable attempt to develop a general framework within which to pose the question of the social efficiency of the steady-state unemployment rate.

cally, the sum of the imputed income that they can expect, the right side of (11), increases with u because the expected capital gain attributable to a random meeting of any two singles, $\int \max[S(y), 0]dF(y)$, exceeds the capital gain attributable to a meeting of a single and a matched individual, $\int \max[S(y) - S(z), 0]dF(y)dG(z)$.[10] Consequently, given a higher reservation quality, a larger steady-state fraction of singles would be available by virtue of equation (16a), and the greater availability would benefit all singles by increasing the probability of meeting another single. Hence, if a singles convention were held, the participants would all have an ex-ante incentive to agree to legislate a higher reservation quality than that resulting from individually rational behavior, given the actions of others. Diamond shows that the reservation quality that maximizes average "aggregate" wealth exceeds the steady-state reservation quality for this reason.[11]

In a labor-market context, the steady-state unemployment rate is too low by implication. As a remedy, Diamond suggests a subsidy to unemployment like that implicit in the payment of unemployment insurance benefits. Since the unemployment rate is equivalent to the job-vacancy rate in this model, paying employers to keep jobs open has the same effect. In a marriage-market context, a subsidy would take the form of a bribe paid in every period to remain single. Such a benefit or bribe increases the value b in equation (15) and, consequently, increases the equilibrium-reservation quality and fraction of unmatched agents.

Although the equilibrium-reservation quality may be too low by virtue of this argument, the implication that the steady-state unemployment rate or fraction of unmatched agents is also too small is not robust, as Mortensen (1982*a*, 1982*b*) and Pissarides (1984, 1985) show by generalizing the model to allow for endogenous search effort. This can be done by supposing that the frequencies at which single individuals generate meetings are determined by the search effort or intensity levels chosen by them. If one

[10] As it stands, this argument is incomplete because the steady-state distribution of existing match quality, $G(z)$, also depends on u by virtue of (16b). However, because the distribution of quality worsens as u increases, in the sense that the fraction of quality x or less increases for all x, this secondary effect reinforces the primary one pointed out in the text.

[11] Although Diamond's model differs from that discussed here, his is formally equivalent when divorce is not allowed. In this case, only singles meet for the purpose of match formation, and the aggregate meeting rate is the convex quadratic function $f(u) = \lambda u^2$. Generally, one can show that the reservation quality is too low (high) if $f'(u) >(<) f(u)/u$ in this version of the model. Consequently, the conclusion is sensitive to the functional form of the relationship between the meeting rate and fraction of unmatched agents, known in the literature as the "meeting technology." Specifically, the equilibrium is efficient if and only if the meeting technology is linear, i.e., $f(u) = uf'(u)$. See Hosios (1986) for a proof.

lets $c_f(\lambda)$ and $c_m(\lambda)$ represent the cost of meeting other agents at frequency λ for the typical female and male, respectively, then the frequency that maximizes the expected present value of an individual single's future income, given the frequency chosen by all other single agents, solves the problem defined on the right side of

$$rV_f = \max_{\lambda}\{b_f + (\lambda + \lambda_m)u \int \max[\theta S(y), 0]dF(y) + \lambda(1 - u)\int\int \max[S(y) - S(z), 0]dF(y)dG(z) - c_f(\lambda)\} \tag{17a}$$

in the case of a female and

$$rV_m = \max_{\lambda}\{b_m + (\lambda_f + \lambda)u\int \max[(1 - \theta)S(y), 0]dF(y) + \lambda(1 - u)\int\int \max[S(y) - S(z), 0]dF(y)dG(z) - c_m(\lambda)\} \tag{17b}$$

in the case of a male.

Given the reservation quality and the unemployment rate, the elements of the pair (λ_f, λ_m) that simultaneously solve these two problems, which can be characterized as the noncooperative solution to a strategic game of search-effort choice, are too small. Specifically, because no single individual takes account of the fact that a greater search effort on his part benefits singles of the other sex by decreasing the time required to find an acceptable match, that is, the right side of (17a) increases with the meeting frequency chosen by the males while the right side of (17b) increases with the meeting frequency chosen by females, all single agents could anticipate a larger expected discounted stream of future income if all were to search more intensively.

Finally, because the steady-state unemployment rate decreases with the search intensities of all the singles, the presence of this external effect can imply that the steady-state unemployment rate or fraction of singles is too large rather than too small, when we hold reservation quality constant. If so, then this externality justifies subsidizing the search effort either by making social investments that reduce the cost of finding potential partners or by paying people directly to search more intensively. In a labor-market context, examples of facilitating investments include the establishment of a federal computerized job and unemployed-worker data bank that could facilitate the matching process, a proposal that has been around for a long time. A state-sponsored computerized dating service is the analogue for the marriage market.[12]

[12] One way to view the problem is that each individual takes account only of his share of the surplus capital value of any future match that might result as a consequence of a meeting generated by that individual's search effort, while the social gain attributable

Job-worker matching models that incorporate the fact that future returns accruing to a specific match are uncertain initially and are only realized through experience include those developed by Johnson (1978), Jovanovic (1979*a,* 1979*b,* 1984), and Viscusi (1979). Originally, these models were motivated by the observed instability of employment relationships among younger workers. The explanation offered by the models is that new entrants to the labor market are "job shopping." Specifically, the typical new worker knows neither his unique abilities nor what job would make the best use of them. Finding a "fit," a match of these abilities with a job that uses them, is a trial and error process that takes time. That frequent job changes by new entrants are a reflection of this process is the job-shopping hypothesis.

A simple specification of the essential features of the job-shopping hypothesis follows: First, assume that no information about the quality of a specific match is observed initially, although the expected quality of any random match, defined as

$$\mu = \int x dF(x), \tag{18}$$

is known to both parties. Second, suppose that, after some period of experience, which for the purpose of illustration is assumed to be exponentially distributed with hazard rate η, the true quality of a specific match is revealed. The idea is that some period of experience is required to learn true quality. In this specification, the expected length of the required period is $1/\eta$. When the true value is realized, the matched pair decide to separate or not. If a separation occurs, then both partners become single. Of course, even if no separation occurs at this point in time, a separation may occur subsequently if one or the other partner finds a better alternative.

One is tempted to identify the time period required to learn about the true value of a match with formal probation periods found in many labor markets and with "dating" or "living together" in the context of marriage. Although both institutional arrangements seem to recognize the partners' needs to learn about one another before the formation of a "permanent" relationship, the learning process usually continues beyond the date at which the relationship is consummated.

In any case, the formal structure imposed above distinguishes between two matched states that I shall call "tenured" and "untenured." A match

to the formation of a new match is the entire surplus. In Mortensen (1982*a,* 1982*b*), I show that the socially efficient search-effort levels are chosen by all agents if the agent responsible for generating each meeting receives the entire surplus capital value of any acceptable match, at least when the meeting technology is linear.

is tenured only after the truth is known. In order to modify the relationships in the model in a manner that recognizes this distinction, let W_0 represent the total capital value of the untenured state and $W_1(x)$ the total capital value of a tenured match known to be of quality x. The decision to separate or not is based on the surplus value of a tenured match of revealed quality, which is given by

$$S_1(x) \equiv W_1(x) - V_f - V_m. \tag{19}$$

Provided that the pair can agree that the female member receives some positive share of the surplus, θ, the match continues if the realized surplus capital value is positive. However, if the quality turns out to be less than the reservation quality, denoted as x^* and defined by

$$S_1(x^*) = 0, \tag{20}$$

then the pair separates because both obtain larger expected streams of future income as singles. In other words, the match has proven to be a mistake.

If one of the partners to an acceptable match is met subsequently by an unmatched agent, the match dissolves if the alternative yields a higher surplus capital value, for the reasons already described. This event occurs if the quality of the existing tenured match, x, is less than a second reservation quality, denoted as x^{**} and defined as that which equates the surplus capital value of tenured with that of an untenured match; that is,

$$S_1(x^{**}) = S_0, \tag{21}$$

where

$$S_0 = W_0 - V_f - V_m. \tag{22}$$

By specifying the appropriate analogues of equation (9) for both W_0 and $W_1(x)$ and of the equations of (10) for V_f and V_m, one can solve the model for both notions of surplus capital value and the two endogenously determined reservation qualities. For our purposes, it is sufficient to note that

$$x^{**} > x^* \text{ if } S_0 > 0, \tag{23}$$

by virtue of equations (20) and (21). In other words, if untenured matches offer a positive capital surplus, which they must in order to motivate search by single individuals, then acceptable low-quality tenured matches exist, those for which $x^* < x < x^{**}$, that can dissolve subsequently. Finally, because it can be shown that

$$x^{**} < x^+, \tag{24}$$

tenured matches of quality equal to or larger than x^{**} but less than x^+ are permanent because no untenured match offers a higher initial value; that is, $W_1(x) > W_0$ for all $x > x^{**}$ by virtue of equations (19)–(22).

Developing realistic generalizations of these simple models of matching under conditions of uncertainty and costly search continues to be a fruitful research program. One issue of classic theoretical importance is the relationship between the equilibrium solution to this type of model and the concept of a stable match structure introduced in the pure theory of matching. Gale (1986) has shown that a related search-theoretic model of sequential bilateral exchange in the traditional setting of a finite number of divisible commodities implies that each trader receives his or her competitive allocation, those determined by prices that equate supply and demand, in the limit as either the meeting rate tends toward infinity or the rate of time preference tends toward zero. This result suggests the following general conjecture: the set of equilibrium match structures converges to the set of stable match structures as either the meeting rate, λ, tends toward infinity or the sum of the rate of time preference and the rate of exogenous turnover, $r + \delta$, tends toward zero. Indeed, the conjecture is true for the simple models sketched here. If the conjecture were also true more generally, then the set of stable match structures can be justifiably regarded as the states that realistic matching processes seek, at least to a first-order approximation.

EMPIRICAL IMPLICATIONS

Theories of matching suggest two related rational explanations for job-worker separation: (1) Acceptable but unstable employment relationships form as a consequence of search costs even if there is no uncertainty about the quality of a specific match. (2) Given initial uncertainty, mistakes occur because the true quality of a match is fully revealed to the parties involved only through subsequent experience. For both reasons, the typical young worker should be expected to switch employers more frequently during the process of finding a job that fits the worker's talents, while the typical older worker, having found a good fit, can be expected to continue with the current employer more or less indefinitely. To the extent that earnings are positively correlated with the "quality" of a match, the typical worker's labor income should increase more rapidly during the initial "job shopping" process. Empirical support for this characterization of the typical worker's employment and earning experience over the life cycle is strong.

For example, in a recent study of a representative sample of individual earnings and employment records, the Longitudinal Employer-Employee

Data Survey (LEEDS) file, Topel and Ward (1985) report the following startling facts: Among the young men who entered the labor market sometime between 1957 and 1973, the average number of different jobs held during the first 10 years of employment experience was 5.5. Only one in 20 new entrants worked for the same employer throughout the first 10-year period. The average completed duration of a first job was only 1.5 years. Yet, after two years with the same employer, the relationship could be expected to last for another five. These figures reflect the following facts about separation propensities: Given no prior work experience, the fraction of job-worker matches that dissolve in a three-month period falls from 37% in the first quarter of a relationship to 7.5% in the eighth. After nine to 16 years experience in one or more jobs, the corresponding figures are 26% and 5.5%.

These observations reflect the typical pattern of adjustment of new workers to the labor market—an initial period of frequent movement among employers followed by a few employment relationships of relatively long duration. The fact that young labor-market entrants (five years of experience or less) enjoy triple the annual earnings growth of prime-age workers (see Mincer and Jovanovic 1982) is consistent with a matching story. Although the large positive association between the level of individual earnings and both years of work experience and current job tenure, when we hold experience constant, has been well established for some time, the realization that matching theories can account for these findings is much more recent. Traditionally, these results have been attributed to the accumulation of human capital, general and specific.

According to received human-capital theory, the quality of a match improves with age because the worker acquires skills as a consequence of learning and training on the job. Earnings growth associated with increments to general work experience is attributed to the acquisition of general ability, while that associated with additional tenure with a specific employer is attributed to improvement in productivity on the current job only.

The simple fact that good matches last longer does not provide an alternative explanation for the observed association between earnings and both experience and tenure, as Topel (1986) has recently noticed. If jobs were pure search goods in the sense that a worker's job-specific productivity is fully known when a job commences, then the theory of matching alone implies only a positive experience effect on earnings. Similarly, only a positive tenure effect is implied if no prior information exists about a worker's job-specific productivity. Sample selection induced by rational mobility decisions can account for both effects only if jobs are partly "search" and partly "experience" goods. The following example illustrates why.

Suppose that each worker lives two periods and assume that all workers are identical in the sense that their productivities are random draws from the same distribution. In the pure-search-good version of the model, each worker receives an offer equal his known job-specific productivity at the beginning of each of the two periods. Let $\mu + \epsilon_0$ and $\mu + \epsilon_1$ represent this sequence of random offers, where ϵ_0 and ϵ_1 are identically and independently distributed with zero mean. Assume that in the first period, each worker accepts the offer. Hence, the average earnings of all workers in the first period equal $w_0 = \mu + E(\epsilon_0) = \mu$. In the second period, some workers will choose to reject and others will choose to accept the alternative offer. Movement is optimal if and only if $\mu + \epsilon_1 > \mu + \epsilon_0$. Hence, the average second-period earnings of movers and stayers are, respectively, $w_m = \mu + E(\epsilon_1 | \epsilon_1 > \epsilon_0)$ and $w_s = \mu + E(\epsilon_0 | \epsilon_0 > \epsilon_1)$. Because movers in the second period have one more period of experience than movers in the first and the same tenure, the implied return to experience is $w_m - w_0 = E(\epsilon_1 | \epsilon_1 > \epsilon_0) > E(\epsilon_1) = 0$ as a consequence of the sample selection induced by their decisions to move. Because in the second period stayers have the same experience as movers but one more period of tenure, the return to tenure is $w_s - w_m = E(\epsilon_0 | \epsilon_0 > \epsilon_1) = E(\epsilon_1 | \epsilon_1 > \epsilon_0) = 0$ since ϵ_0 and ϵ_1 are identically and independently distributed (i.i.d.); that is, there is no tenure effect.

Topel (1986) also points out that the measured tenure effect is strictly smaller than any true job-specific productivity increment if either the true increment is positive or a positive cost of moving is present. This implication is a consequence of the fact that the wage offer required to induce movement must be strictly larger than that received in the first period in either of these cases. Hence, OLS estimates of the contribution of tenure to earnings may underestimate the causal effect of job-specific training on earnings in the presence of the sample selection induced by the pure-search-good version of the model.

In the pure-experience-good version of the model, only the worker's expected productivity in all jobs is known initially. Let this quantity, μ, equal the wage offered on any job in the first period. Assume that worker's job-specific productivity, $\mu + \nu$, is known after one period and is equal to the second period wage, where $E(\nu) = 0$. Again, workers are identical in the sense that μ is the same and each lives two periods. Since the only alternative wage in the second period of life is μ, the wage paid during the first period in any job, a worker stays if and only if $\mu + \nu > \mu$. Hence, $w_0 = \mu$, $w_m = \mu$, and $w_s = \mu + E(\nu | \nu > 0)$, which imply no experience increment, that is, $w_m - w_0 = 0$, and a positive tenure increment, that is, $w_s - w_m = E(\nu | \nu > 0) > E(\nu) = 0$.

Finally, by "mixing" these two specifications, namely, by letting the first-period wage on any job equal $\mu + \epsilon$ and the second-period wage equal $\mu + \epsilon + \nu$, where ϵ and ν are independent and each has zero mean, rational mobility decisions imply the following:

$$w_0 = \mu + E(\epsilon_0) = \mu$$

$$w_s = \mu + E(\epsilon_0 + \nu | \epsilon_0 + \nu > \epsilon_1)$$

$$w_m = \mu + E(\epsilon_1 | \epsilon_1 > \epsilon_0 + \nu).$$

Consequently, both effects are positive; that is,

$$w_m - w_0 = E(\epsilon_1 | \epsilon_1 > \epsilon_0 + \nu) > 0$$

because $E(\epsilon_1) > 0$, and

$$w_s - w_m = E(\epsilon_0 + \nu | \epsilon_0 + \nu > \epsilon_1) - E(\epsilon_1 | \epsilon_1 > \epsilon_0 + \nu) > 0$$

because $\epsilon_0 + \nu$ is a mean preserving spread of ϵ_1, given that ϵ_0 and ϵ_1 are i.i.d. and ν has zero mean.

Recently, several different studies have attempted to ascertain the importance of the matching process as an explanation of wage growth over the life cycle by using methods that correct for the kinds of selection bias implied by optimal mobility decisions. The authors include Altonji and Shakotko (1987), Marshall and Zarkin (1987), Topel (1986), and Mincer (1986). Although both data and statistical methods differ across studies, the conclusions are quite similar. Specifically, earnings gains associated with work experience are due in large measure, but not exclusively, to mobility. Little remains of any causal tenure effect on earnings once selection effects of the type implied by the pure-"experience"-good version of the matching model are taken into account, although results may be biased in favor of the reported conclusion to the extent that the pure-"search"-good version of the model is the more appropriate, for the reasons discussed by Topel (1986).

SUMMARY

The existence of stable match structures suggests that job separation and divorce are not caused by the inability of a voluntary matching process to find partnership structures that are impervious to competition. Nevertheless, unstable structures can form when matching requires time, is costly, and takes place under conditions of uncertainty both because it is not rational to wait indefinitely for the perfect partner and because experience is required to discover the value of a specific partnership. Recent empirical evidence on the reasons for job turnover and wage growth over the life cycle support this view of the job-worker matching process.

The theoretical analogy between job-worker matching and the sorting process implicit in marriage suggests that search costs and match-specific learning may be important as explanations of divorce as well. Indeed, Becker, Landes, and Michael (1977) raise this question in the context of the empirical observation that longer marriages, like employment relationships, are less likely to separate. Does divorce risk fall primarily because those matches that survive are the good ones? Unfortunately, in the case of marriage, no indicator of match quality seems to exist that would permit one to obtain an answer using the techniques applied in the case of worker-employer relationships. At this point in time, one can only speculate.

REFERENCES

Altonji, Joseph, and Robert Shakotko. 1987. "Do Wages Rise with Job Seniority?" *Review of Economic Studies* 54:437–59.

Becker, Gary. 1973. "A Theory of Marriage: Part 1." *Journal of Political Economy* 81:813–46.

Becker, Gary, Elisabeth M. Landes, and Robert T. Michael. 1977. "An Economic Analysis of Marital Instability." *Journal of Political Economy* 85 (6): 1141–88.

Crawford, Vincent P., and Elsie Marie Knoer. 1981. "Job Matching with Heterogeneous Firms and Workers." *Econometrica* 49 (5): 437–50.

Diamond, Peter. 1981. "Mobility Costs, Frictional Unemployment, and Efficiency." *Journal of Political Economy* 89:789–812.

Diamond, Peter. 1982. "Wage Determination and Efficiency in Search Equilibrium." *Review of Economic Studies* 49:217–27.

Diamond, Peter, and Eric Maskin. 1979. "An Equilibrium Analysis of Search and Breach of Contract." *Bell Journal of Economics* 10:282–316.

Dreze, J. H., and J. Greenberg. 1980. "Hedonic Coalitions: Optimality and Stability." *Econometrica* 48 (4): 987–1003.

Gale, David, and Lloyd Shapley. 1962. "College Admission and the Stability of Marriage." *American Mathematical Monthly* 69:9–15.

Gale, Douglas. 1986. "Bargaining and Competition, Parts I and II." *Econometrica* 54:785–818.

Hosios, Arthur J. 1986. "On the Efficiency of Matching and Related Models of Search and Unemployment." Working paper. University of Toronto, Department of Economics.

Johnson, William. 1978. "A Theory of Job Shopping." *Quarterly Journal of Economics* 92:261–77.

Jovanovic, Boyan. 1979*a*. "Firm-specific Capital and Turnover." *Journal of Political Economy* 87:972–90.

———. 1979*b*. "Job Matching and the Theory of Turnover." *Journal of Political Economy* 87:1246–60.

———. 1984. "Matching, Turnover, and Unemployment." *Journal of Political Economy* 92:108–22.

Kelso, Alexander S., and Vincent P. Crawford. 1982. "Job Matching, Coalition Formation, and Gross Substitutes." *Econometrica* 50 (6): 1483–1504.

Marshall, Robert C., and Gary A. Zarkin. 1987. "The Effects of Job Tenure on Wage Offers." *Journal of Labor Economics* 5 (3): 301–24.

Mincer, Jacob. 1986. "Wage Changes in Job Changes." Pp. 171–98 in *Research in Labor Economics,* Pt. A. Edited by R. Ehrenberg. Greenwich, Conn.: JAI.

Mincer, Jacob, and Boyan Jovanovic. 1982. "Labor Mobility and Wages." Pp. 21–63 in *Studies in Labor Markets,* edited by S. Rosen. Chicago: University of Chicago Press.

Mortensen, Dale T. 1978. "Specific Capital and Labor Turnover." *Bell Journal of Economics* 9:572–86.

———. 1982*a*. "Property Rights and Efficiency in Mating, Racing and Related Games." *American Economic Review* 72 (5): 968–79.

———. 1982*b*. "The Matching Process as a Noncooperative Bargaining Game." Pp. 233–58 in *The Economics of Information and Uncertainty,* edited by J. J. McCall. Chicago: University of Chicago Press.

Pissarides, Chris A. 1984. "Search Intensity, Advertising and Efficiency." *Journal of Labor Economics* 2:128–43.

———. 1985. "Taxes, Subsidies and Equilibrium Unemployment." *Review of Economic Studies* 52:121–35.

Roth, Alvin E. 1984*a*. Stability and Polarization of Interests in Job Matching." *Econometrica* 52 (1): 47–57.

———. 1984*b*. "The Evolution of the Labor Market for Medical Interns and Residents: A Case Study in Game Theory." *Journal of Political Economy* 92 (6): 991–1016.

Rubinstein, Ariel, and Asher Wolinsky. 1985. "Equilibrium in a Market with Sequential Bargaining." *Econometrica* 53:1133–50.

Shapley, Lloyd S., and Martin Shubik. 1972. "The Assignment Game I: The Core." *International Journal of Game Theory* 1:111–30.

Topel, Robert. 1986. "Job Mobility, Search, and Earnings Growth: A Reinterpretation of Human Capital Earnings Functions." Pp. 199–233 in *Research in Labor Economics,* edited by R. Ehrenberg. Greenwich, Conn.: JAI.

Topel, Robert, and Michael Ward. 1985. "Job Mobility and the Careers of Young Men." Chicago: University of Chicago, NORC.

Viscusi, W. Kip. 1979. "Job Hazards and Worker Quit Rates: An Analysis of Adaptive Worker Behavior." *International Economic Review* 20:29–58.