# Take home examination: Family Economics, MA Yoram Weiss, July 2007

In all questions below, we consider models with frictions, where meetings follow a Poisson process with a fixed arrival rate $\lambda$. That is, in a short time interval $h$ the probability of a meeting is $\lambda h + o(h)$ and the probability of no meeting is $1 - \lambda h + o(h)$. The time between successive meetings is a random $\tau$ variable with an exponential distribution

$$prob(\tau \leq t) = 1 - e^{-\lambda t}. \tag{1}$$

The arrival rate $\lambda$ is determined by market forces and cannot be influenced by search effort. But partners can decide upon which marriages to form. Agents are risk neutral. There is a fixed interest rate $r$ and equal number of men and women, $N$, where $N$ is "large".

**Question 1**. Consider a case with *transferable utility* and suppose that all men and women are *identical.* Upon meeting, a couple draws a random draw of match quality $x$, from a given distribution $F(x)$, where $x$ denotes the *flow* of *joint* benefits per unit of time during marriage. The flow of benefits from being single is normalized to zero.

There is no voluntary divorce, however, marriages break exogenously and duration of marriage is a random variable $l$ with an exponential distribution

$$prob(l \leq t) = 1 - e^{-\delta t}. \tag{2}$$

Denote the values of being single for men and women by $V_m$ and $V_f$, respectively and let $z(x)$ be the *expected* joint gains of being married.

1. Give an explicit expression for $z(x)$ and define the surplus of the marriage.
2. Suppose that each spouse receives in marriage his value of being single plus some fixed positive share $\theta$ of the marital surplus, where $\theta_m + \theta_f = 1$ What is the reservation policy of each man and woman?
3. Show that there is some *common* value of $x$, $x^*$, such that above it both partners wish to marry and below it no one wants to marry. Explain.
4. Find an equation that determines $x^*$ uniquely and show that $x^*$ rises with $\lambda$ and declines in $r$ and $\delta$. Explain
5. Find the steady state proportion of married (and divorced individuals) and examine how it reacts to changes in parameters.
6. Can the model explain the increase in marital turn over in the marriage market that we observe? Explain
7. Do changes in $\theta_m$ or $\theta_f$ influence the proportion married? Do they affect the marital surplus? Explain.

**Question 2**. Consider now a case of *non transferable utility* and heterogenous populations of men and women. Suppose that men are characterized by their incomes, $y_m$ for men and $y_f$ for women. A single person consumes his own income and when a couple marries, both partners consume the entire family income $y_m + y_f$ (consumption is a local public good). Assume that life is infinite and marriages last for ever.

1.Characterize the Nash solution and the resulting assignment.
2. What would be the stable assignment without frictions?
3. Explain how and why the solutions differ with and without frictions.

**Question 3**. Consider, finally, a case of *non transferable utility* with only two income *groups* for each sex. Half of the men have income $y_m^h = 3$ and half have $y_m^l = 2$. Half of the women have income $y_f^h = 1$ and half have $y_f^l = 0$. As in question 2, there is no divorce but assume now that agents have random life that is exponentially distributed with parameter $\delta$ and that the same proportion $\delta$ of the population is born and dies at any point in time, without affecting the income distributions. Assume that every born agent is initially single. For simplicity, assume that the death of spouses is perfectly correlated so that there are no widows or widowers.

1. Show that the discount rate now becomes $r\prime = r + \delta$. Explain.
2. Find the marriage patterns that arise in a Nash equilibrium.
3, Discuss the qualitative effects of $r$ and $\lambda$ and $\delta$.
4. What is the impact of the assumption that, on average, men are richer than women?
5. Find the steady state proportions of married (and singles) among men and women of each type.

**Sources.**
Burdett, Kenneth and Melvyn G. Coles, 'Long-Term Partnership Formation: Marriage and Employment', **The Economic Journal,** 109 (1999), 307-334.
Mortensen, Dale T., 'Matching: Finding a partner for life or otherwise', **American Journal of Sociology**, 94 (1998, supplement), s215-s240.
Browning-Chiappori-Weiss Chapter 8.