

Take home examination: Family Economics, MA Yoram Weiss, April 2008

Consider a set of men and women on a given age group, say 25 – 35, who can potentially marry each other. This set may be finite or infinite. Let men and women be characterized by their age. Denote the male age by t_m and female age by t_f . Ages of men and women are distributed uniformly on some interval $[\tau, T]$. This distribution can be discrete or continuous. As people become older, their attributes *change in a systematic way*:

the wage of man m as a function of his age is

$$y_t = \alpha t_m$$

the wage of woman f as a function of her age is

$$z_t = \beta t_f$$

The fertility of a woman at age τ is q_τ , and then fertility drops gradually with age in such a way that

$$q_t = q_\tau - \gamma(t_f - \tau)$$

where

$$q_T = q_\tau - \gamma(T - \tau) > 1.$$

Thus, every married woman can have at least one child.

Single persons consume their incomes at each age (so their utility equals to their income). Married couples, combine their incomes to consume a *private* good that can be divided between the partners. Upon marriage the partners produce q children according to the wife's fertility at the time of marriage.

For simplicity, assume that couples consume their income each period (no borrowing or lending) and that all children are produced at once at the time of marriage. Also assume that following birth, husband and wife work full time.

Preferences of both men and women are given by

$$u_i = q c_i$$

where q is the number of children and c_i is the private good that spouse i consumes.

Question 1.

1. Prove that utility is transferable within couples.
2. Find an expression for the sum of utilities as a function of the ages of the two spouses and define the surplus of the marriage.
3. Show that there is negative interaction in the ages of the two spouses, explain why.
4. Show that the surplus is positive and increases with the age of the husband but need not be monotone in the age of the wife. Explain.
5. Provide sufficient condition that will ensure that the surplus rises with the wife's age.

Question 2.

Consider, first, a finite set of agents, with discrete age distributions. Let the participants in the marriage market consist of four men and five women. The ages of men are (29, 32, 33, 35) and the ages of women are (25, 27, 29, 32, 33).

1. Find the matrix of surpluses for the following parameters: $\alpha = 2$, $\beta = 1.5$, $\gamma = 0.1$, $\tau = 10$, first for $q_\tau = 4$, and next for $q_\tau = 15$. Identify the stable assignments in the two cases and explain the differences.
2. Use appendix A in chapter 9 in order to find the upper and lower bounds of the surplus share of each participant in the two cases.
3. How would the results change if, because of social norms, all couples divide their surplus *equally* between husband and wife?

Question 3.

Consider, now, a continuum of men and women whose ages are distributed uniformly over the real interval [25, 35]. Assume that there are more women than men, so the sex ratio r (measure of women relative to the measure of men) is larger than 1. Also assume that the surplus increases in the ages of both husband and wife.

1. Use chapter 8 in order to find the matching functions.
2. How old is the youngest married woman? Show that if $r = 1$ the oldest men aged 35 marry women aged 25, but if $r = 2$, then they marry women aged 30. Explain.
3. What are the end conditions? (clue: they are for the oldest men, and the youngest married women).
4. Use the end conditions and equation (20) from chapter 8 in order to compute the sharing rule.