

Take home examination in Family Economics

Receive Monday July 4 2006, 12.00

Return Monday July 11 2006, 12.00

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Sources:

1. Shapley, L. and M. Shubik (1972), "The Assignment Game I: The Core, " *International Journal of Game Theory* , 1, 11-130.
2. B-C-W chapters 8,12.
3. Chiappori, P.A. M. Iyigun and Yoram Weiss (2006), "Investment in Schooling and the Marriage Market".

Part 1. Consider two individuals, man i and woman j with preferences given by

$$u_i = c_i q + \theta_i \quad (1)$$

$$u_j = c_j q + \theta_j, \quad (2)$$

where c is a private good, q is a public good that can be shared if the two people marry but is private if they remain single and θ is the emotional gain from being married (relative to remaining single). The public good is produced within the household according to a household production function

$$q = e + \gamma t, \quad (3)$$

where e denotes purchased market goods, t is total time spent working at home by the two spouses and γ is an efficiency parameter which equals zero if the two individuals are single but is positive if they marry and have children. Time not spent at home is spent at work and the wages are denoted by w_i and w_j . The time constraints are

$$0 \leq t_i \leq 1 \quad (4)$$

$$0 \leq t_j \leq 1$$

If the partners marry, the family budget constraint is

$$e + c_i + c_j = w_i(1 - t_i) + w_j(1 - t_j). \quad (5)$$

If they remain single, they have separate budget constraints

$$e_i + c_i = w_i(1 - t_i),$$

$$e_j + c_j = w_j(1 - t_j).$$

1. Find the optimal allocation of resources if the partners marry and if they remain single.

2. Draw the Pareto frontier upon marriage and indicate on the same graphs the utilities if the partners remain single.
3. Calculate the surplus generated by a marriage between i and j .
4. How would the surplus change if *only* the mother spends time on the child and the required time is τ where $0 < \tau < 1$.

Part 2. Consider two equally large populations of men and women to be matched. Individuals live for two periods. Each person can choose whether to acquire schooling or not and whether and whom to marry. Investment takes place in the first period of life and marriage in the second period. Investment in schooling is lumpy and takes one period so that a person who invests in schooling works only in the second period, while a person who does not invest works in both periods. To simplify, we assume no credit markets. All individuals of the same schooling earn the same wage rate and we denote the wage of an educated persons by w_2 and the wage of an uneducated persons by w_1 where $w_2 > w_1$.

The schooling level (class) of man i by $I(i)$ where $I(i) = 1$ if i is uneducated and $I(i) = 2$ if he is educated. Similarly, we denote the class of woman j by $J(j)$ where $J(j) = 1$ if j is uneducated and $J(j) = 2$ if she is educated. The surplus generated by a marriage of man i and woman j is

$$s_{ij} = z_{I(i)J(j)} + \theta_i + \theta_j, \quad (6)$$

where θ_i and θ_j represent the non-economic gains of man i and woman j from their marriage and $z_{I(i)J(j)}$ is the material surplus that the marriage generates. In addition to the returns in the market or marriage, investment in schooling is associated with idiosyncratic costs (benefits) denoted by μ_i for men and μ_j for women. The idiosyncratic preference parameters are assumed to be independent of each other and across individuals. We denote the distributions of θ and μ by $F(\theta)$ and $G(\mu)$ and assume that these distributions are symmetric around their zero means.

1. Suppose that all married couples have no children, $\gamma = 0$. Find the surplus matrix and the equilibrium shares.
2. How will the result changes if all married couples have children, $\gamma > 0$ and $\tau > 0$ and the couples act efficiently? Explain.
3. How will the result change if all married couples have children, $\gamma > 0$ and $\tau > 0$ and the mother alone takes care of the childrent? Explain.