Take home examination in Family Economics

Receive Friday June 28 2002, 12.00
Return Sunday July 7 2002, 12.00

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Sources:

1. Becker, Gary S. (1991), *A Treatise on the Family*, Cambridge and London: Harvard University Press (Chapter 8) .
2. Bergstrom, Theodore C. (1989) "A Fresh Look at the Rotten Kid Theorem–and Other Household Mysteries", *Journal of Political Economy*, 97(5), p. 1138-59.
3. Chiappori, Pierre-Andre', Werning, Ivan (2002), "Comment on Rotten Kids, Purity and Perfection". *Journal of Political Economy*, 110, p. 475-480.
4. Cornes, Richard C. and Silva, Emilson C. D. "Rotten Kids, Purity, and Perfection.", *Journal of Political Economy.* 107 (October 1999): 1034-40.

Problem 1. Imagine a two stage game played by an altruistic patriarch or head with utility $W[u_0, ..., u_n]$ and n kids with utilities $u_i(x_i)$. Each kid can choose an action a_i. This action enters as input into a family "production function"

$$I = f(a_1, ...a_n),$$

which generates a single good as output, later to be split and consumed by all kids. In addition every kid has some exogenous income y_i that can also be taxed and redistributed by the head.

In the first stage each kid i chooses an action a_i in order to solve

$$\max_{\{a_i\}} u_i(x_i)$$

$$s.t. \;\; x_i = t_i(a_i, ..., a_n) + y_i - a_i,$$

where t_i is the transfer that he receives from the head, and $t_i(a_i, ..., a_n)$ is the reaction function of the patriarch from the second stage. We may interpret $t_i + y_i$ as consumption and a_i as effort. Effort cannot be controlled by the head directly.

In the second stage the head chooses transfers t_i to maximize his utility subject to a resource constraint that is determined by the actions of the kids in the first stage. That is

$$\max_{\{t_1,...,t_n\}} W(u_1, ..., u_n)$$

$$s.t \;\; \sum(t_i + y_i) = f(a_1^*, ...a_n^*) + \sum y_i,$$

where the actions $(a_1^*, ...a_n^*)$ are given. We focus on the subgame perfect equilibrium of this game, which means that the father cannot commit to a scheme whereby the payment to i is conditioned on his effort.

1. Show that the result of this mechanism is efficient (provided that some plausible restrictions are imposed on $W(u_1, ..., u_n)$) and that income pooling holds (in the sense that the outcome depends only on the sum of the $y_i^{\prime}$s).

2. Re-examine efficiency and income pooling for the case

$$\begin{aligned} u_i &= x_i(1-a_i),\ i=1,2, \\ x_i &= t_i(a_1,a_2)+y_i,\ i=1,2, \\ W(u_1,u_2) &= \sqrt{u_1}+\sqrt{u_2}, \\ f(a_1,a_2) &= a_1+a_2. \end{aligned}$$

Explain the difference in the results in cases 1 and 2.

Problem 2. Consider a household consisting of a patriarch or head endowed with some income Y, and two kids 1 and 2. There are two goods, one private x and one public Q; both prices are set to 1. Let $U^i\left(x^i, Q\right)$ denote kid i's utility, as a function of his private consumption x^i and the level Q of public goods. Also, let $W\left(U^1, U^2\right)$ denote the patriarch's welfare index. For simplicity, assume that W is symmetric: $W\left(U^1, U^2\right) = W\left(U^2, U^1\right)$. The players play a two stage game. At stage one, each kid decides on some amount q^i he will contribute to the public good; then the level of public goods is $Q = q^1 + q^2$. At stage two, taking Q as given, the patriarch uses the remaining income $Y - Q$ to buy private good and redistributes it across children, in order to maximize W. The goal of the problem is to see whether the resulting allocation of public good is efficient and optimal from the patriarch's viewpoint.

1. Write the program that characterizes efficient allocations of the public good. What are the corresponding first order conditions?

2. We now solve the game, beginning with stage 2. Taking Q as given, write the program of the patriarch. Write its first order conditions.

3. Will the optimal distribution $\left(x^1, x^2\right)$ chosen by the patriarch depend on the individual contributions $\left(q^1, q^2\right)$ or only on the total quantity of public good $Q = q^1 + q^2$

4. At stage 1, the kids take the redistribution rule above as given; i.e., they understand that the head will respond to any choice $\left(q^1, q^2\right)$ by choosing the optimal redistribution of the private commodity defined in 3. The kids play simultaneously and non cooperatively. Write each kid's program.

5. Assuming kid i's program has an interior solution, write the first order condition.

6. Assume both programs have an interior solution. Show that the resulting allocation of private and public goods is efficient and optimal from the patriarch's viewpoint.

Problem 3. It is now assumed that preferences are given by

$$U^i(x^i, Q) = \log(x^i) + \alpha_i f(Q),$$

(where f is a concave, strictly increasing function) and the patriarch's welfare function is

$$W(U^1, U^2) = \lambda^1 U^1 + \lambda^2 U^2.$$

Compute the outcome of the second-stage game as a function of Q

Write the first order conditions of the kid's programs. Are they compatible?

What is the role of interior solution in the efficiency result obtained in problem 2.

Problem 4. Consider a couple where each member i allocates his (her) time between market work ℓ^i (at some wage w_i), domestic work t^i and leisure $1 - \ell^i - t^i$. Labor income plus some fixed initial wealth y are used to buy consumption goods $x_1, ...x_n$ on the market, at given prices $p_1, ...p_n$. These goods and domestic work are used as inputs for the domestic production function; the latter produces a private good z, according to the technology

$$z = f\left(x_1, ...x_n, t^1, t^2\right).$$

Finally, z is allocated within the couple; member i receives z^i (with $z^1 + z^2 = z$).

Member 1 (the rotten kid) is egoistic, his utility function is of the form :

$$u^1(z^1, 1 - \ell^1 - t^1) = z^1 + v^1(1 - \ell^1 - t^1).$$

Member 2 (the patriarch) is altruistic, her utility function is of the form :

$$W\left[u^1, u^2\right],$$

where

$$u^2(z^2, 1 - \ell^2 - t^2) = z^2 + v^2(1 - \ell^2 - t^2).$$

The goal of the problem is to check whether the rotten kid theorem applies.

1. Show that all Pareto efficient allocations require the same level of labor supply, domestic work and market purchases. Write the corresponding program.
2. For any given (possibly suboptimal) $x_1, ...x_n, t^1, t^2, \ell^1, \ell^2$, what is the form of the Pareto set in the $\left(u^1, u^2\right)$ plane.
3. Does the rotten kid theorem apply?

Problem 5. Based on your answers, and the readings, comment on the role of the Rotten Kid Theorem in justifying the *unitary* model in a family context. Compare it to the role of assumption of transferable utility.