The Economics of the Family

Chapter 8: Sharing the gains from marriage

Martin Browning
Department of Economics, Oxford University

Pierre-André Chiappori
Department of Economics, Columbia University

Yoram Weiss
Department of Economics, Tel Aviv University

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1 Introduction

In this chapter, we discuss in more detail the determination of the division of the marital surplus and how it responds to market conditions. If each couple is considered in isolation, then, in principle, any efficient outcome is possible, and one has to use bargaining arguments to determine the allocation. On the contrary, the stability of the assignments restricts the possible divisions because of the ability to replace one spouse by another one. The options for such substitution depend on the distributions of the marital relevant attributes in the populations of the men and women to be matched. In the present chapter, we precisely ask how the marriage market influences the outcome in the ideal, frictionless case discussed previously. Although the division within marriage is not always fully determined, some qualitative properties of the division can be derived from information on the joint distribution of male and female characteristics together with a specification of the household production function.

As before, we discuss separately the cases of discrete and continuous distributions. The general intuition goes as follows. In the discrete case, competition puts bounds on individual shares but does not completely determine them; this is because on the marriage market, each potential spouse has only a finite number of 'competitors', none of which is a perfect substitute - so some elements of 'bilateral monopoly' persist. In the continuous case, however, competition between potential spouses tends to be perfect, leading to an exact determination of the 'prices' - i.e., in our case, budget shares. In addition to the standard case of transferable utility, we also consider the more general case in which the rate of exchange of the spouses' utilities varies along the Pareto frontier. We provide detailed examples that illustrate how changes of the distributions of incomes or tastes of men and women can affect the division of resources within couples. We conclude with a discussion of recent developments in estimating equilibrium models of the marriage market, including the gains from marriage and the division of these gains.

The major insight obtained from the equilibrium analysis is that the sharing of the gains from marriage depends not only on the incomes or preferences of spouses in a given match but also and perhaps mainly on the overall distributions of incomes and preferences in society as a whole. Thus, a redistribution of income via a tax reform can influence the shares of the gains from marriage even if the incomes in particular couple are unaffected. Similarly a legal reform or a technological innovation that makes it easier to prevent pregnancy can influence the division of resources within married couples who chose to have children. In either case, the general equilibrium effects arise from competition with potential spouses outside the given marriage. Obviously, our assumptions regarding the agents' ability to transfer resources within marriage (and to a lesser extent the absence of frictions) are crucial for such indirect effects. It is therefore a challenging research agenda to find how important are these considerations in practice.

2 Determination of shares with a finite number of agents[1]

We start with matching between finite male and female populations. As explained in the previous chapter, while the matching pattern (who marries whom) and the associated surplus is generally unique, the allocation of surplus between spouses is not. Typically, there exists, within each couple, a continuum of allocations of welfare that are compatible with the equilibrium conditions. That does *not* mean, however, that the allocation is fully arbitrary. In fact, equilibrium imposes strict bounds on these allocations. Depending on the context, these bounds may be quite large, allowing for considerable leeway in the distribution of surplus, or quite tight, in which case the allocation is practically pinned down, up to minor adjustments, by the equilibrium conditions. We present in this section a general description of these bounds.

2.1 The two men - two women case

As an introduction, let us consider a model with only two persons of each gender discussed in Chapter 7. Assume for instance that $z_{12} + z_{21} \geq z_{11} + z_{22}$, implying that the stable match is 'off-diagonal' (man i marries woman $j \neq i$,

[1] Ellana Melnik participated in the derivation of the results of this section.

with $i, j \in \{1,2\}$). Then all pairs $(v_1,\, v_2)$ satisfying the inequalities:

$$\begin{aligned} z_{12} - z_{11} &\geq v_2 - v_1 \geq z_{22} - z_{21}, \\ z_{21} &\geq v_1 \geq 0, \\ z_{12} &\geq v_2 \geq 0, \end{aligned} \tag{8}$$

yield imputations v_1, v_2, $u_1 = z_{12} - v_{2,}$, $u_2 = z_{21} - v_1$ that support the stable assignment along the opposite diagonal. The shaded area in Figure 1 describes all the pairs that satisfy the constraints required for stability expressed in condition (8). The figure is drawn for the special case in which woman 2 is more productive than woman 1 in all marriages ($z_{22} > z_{21}, z_{12} > z_{11}$) and symmetry holds, $z_{12} = z_{21}$, implying that man 2 is also more productive than man 1 in all marriages. The main feature here is that the difference $v_2 - v_1$ is bounded between the marginal contributions of replacing woman 1 by woman 2 as spouses of man 1 and man 2. Woman 2 who is matched with man 1 cannot receive in that marriage more than $z_{12} - z_{11} + v_1$, because then her husband would gain from replacing her by woman 1. She would not accept less than $v_1 + z_{22} - z_{21}$, because then she can replace her husband by man 2 offering him to replace his present wife. The assumption that $z_{12} - z_{11} > z_{22} - z_{21}$ implies that man 1 can afford this demand of woman 2, and will therefore "win" her. In this fashion, the marriage market "prices" the different attributes of the two women. Symmetric analysis applies if we would replace (v_1, v_2) with (u_1, u_2).

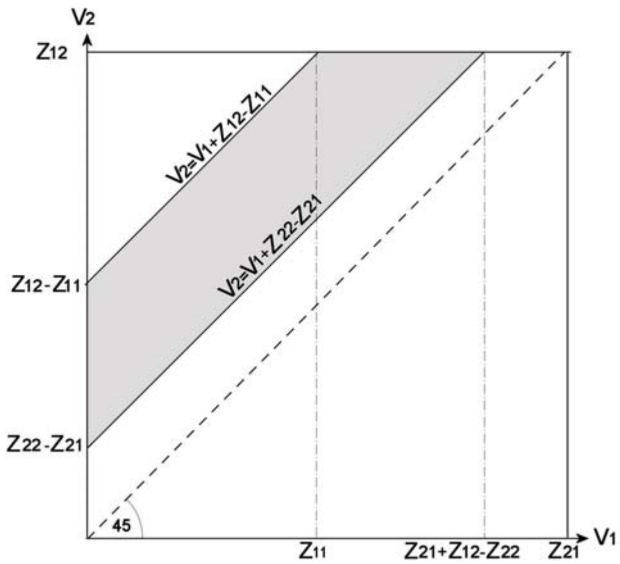

Figure 8.1: Imputations when $z_{12} + z_{21} > z_{22} + z_{11}$

Similarly, if $z_{12}+z_{21} \leq z_{11}+z_{22}$, implying that the stable match is 'diagonal', then all pairs (v_1, v_2) satisfying the inequalities

$$\begin{aligned} z_{22}-z_{21} &\geq v_2-v_1 \geq z_{12}-z_{11}, \\ z_{11} &\geq v_1 \geq 0, \\ z_{22} &\geq v_2 \geq 0, \end{aligned} \tag{8'}$$

yield imputations v_1, v_2, $u_1 = z_{11} - v_1$, $u_2 = z_{22} - v_2$ that support the stable assignment along the diagonal. The shaded area in Figure 2 describes all the pairs that satisfy the constraints required for stability expressed in condition (8'). Again the difference $v_2 - v_1$ is bounded between the marginal contributions of replacing woman 1 by woman 2 as spouses of man 1 and man 2. Because we assume that woman 2 is more attractive than woman 1, she gets a larger part of the surplus in both cases and her share in the surplus is always positive. Woman 1 who is less desirable may get no surplus at all. If she is married to man 1 who is less attractive, she may get the entire surplus. However, if she is married to man 2, he always receives a positive share and she never receives the entire surplus.

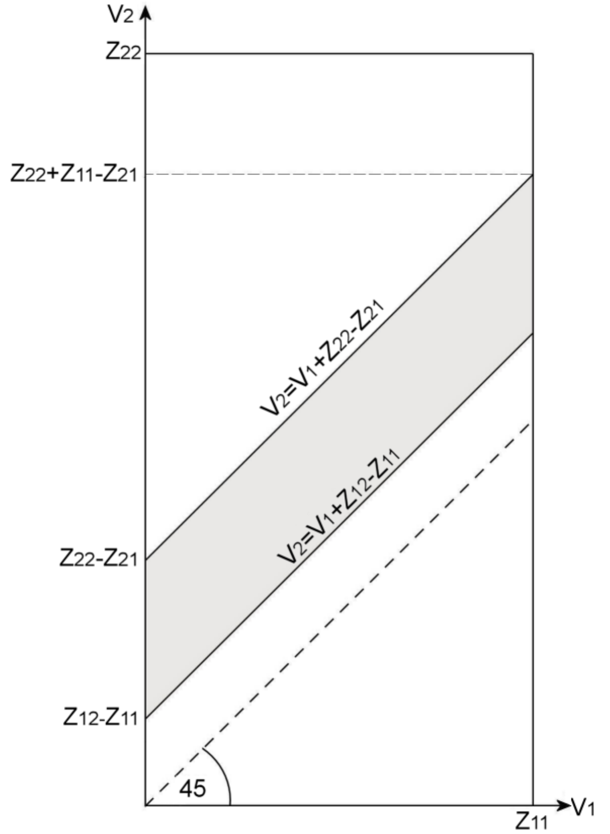

Figure 8.2: Imputations when $z_{12} + z_{21} < z_{22} + z_{11}$

The indeterminacy of prices in the marriage market reflects the fact that the "objects traded" are indivisible and have no close substitutes. Therefore, agents may obtain in the stable assignment utility levels that are strictly higher than they would in alternative marriages. When this is true for all alternative marriages it is possible to slightly shift utility between the partners of each marriage, and still maintain all the inequalities of the dual problem, without any effect on the allocation. An interesting feature, noted by Shapley and Shubik (1972) that in the core (i.e., of the set of imputations that support a stable assignment) "the fortunes of all players of the same type rise and fall together". This is seen by the upward tendency of the shaded areas in figures 7.1 and 7.2. In particular, there is a polar division of the surplus that is best for *all* men, and also a polar division that is best for *all* women.

As an illustration, let us come back to Example 7.4 in the previous Chapter. Specifically, consider the table presenting three imputations, denoted by a, b and c; for commodity, the Table is reproduced below. Note that these imputations are arranged in such a manner that the reservation utility of all men rise and those of all women decline.

Women				Men			
	a	b	c		a	b	c
v_1	2	2	1	u_1	3	4	5
v_2	5	4	3	u_2	5	5.5	6
v_3	1	0.5	0	u_3	0	0	1

In each of these three imputations, individuals who are married to each other receive their reservation utility, which together exhaust the marital input, Thus $v_2 + u_1 = z_{12} = 8$, $u_2 + v_3 = z_{23} = 6$ and $v_1 + u_3 = z_{31} = 2$. For marriages that do not form, the sum of the reservation utilities exceeds or equals the potential marital output. For instance, man 2 and woman 2 are not married to each other, and therefore $v_2+u_2 \geq z_{22} = 9$. This requirement is strict for imputations a and b and holds as equality for imputation c. Similarly, because man 1 and woman 1 are not married to each other, we must have $v_1 + u_1 \geq z_{11} = 5$. This holds as strict inequality for imputations b and c, and as equality for imputation a. The significance of the equalities is that they indicate the bounds within which it is possible to change prices without any affect on the assignment. Hence, imputation c is the best for men and the worst for women and imputation a is the best for women and the worst for men.

2.2 Bounds on levels

The previous results can readily be extended to a more general setting. We now consider N men and M women and assume that the assignment variables a_{ij} are all either zeros or ones. Start with the dual problem:

$$\min_{u_N, v_M} \sum_{i=1}^{N} u_i + \sum_{j=1}^{M} v_j \tag{P1}$$

subject to

$$u_i + v_j \geq z_{ij} \text{ for } i = 1, 2...N, j = 1, 2...M$$

Denote the solution for individual utilities (or individual 'prices') by $(\hat{u}_i, \hat{v}_j)$. From the results on duality (Gale, 1960, chapter 5) we know that the solution to this problem yields the same value as the solution to the primal problem. That is

$$\sum_{i=1}^{N} \hat{u}_i + \sum_{j=1}^{M} \hat{v}_j = \sum_{a_{i,j} \in \hat{S}} a_{ij} z_{ij},$$

where $\hat{s}$ denotes the assignment that solves the primal.

Now compare $P1$ to the dual problem when man N is eliminated, $P2$:

$$\underset{u_M, v_N}{Min} \{ \sum_{i=1}^{N-1} u_i + \sum_{j=1}^{M} v_j \} \tag{P2}$$

subject to

$$u_i + v_j \geq z_{ij} \text{ for } i = 1, 2...N-1, j = 1, 2...M$$

Denote the solution for prices by $(\bar{u}_i, \bar{v}_j)$. Again we know that the solution to this problem yields the same value as the solution to the primal problem. That is

$$\sum_{i=1}^{N-1} \bar{u}_i + \sum_{j=1}^{M} \bar{v}_i = \sum_{a_{i,j} \in \bar{S}} a_{ij} z_{ij}.$$

where $\bar{s}$ denotes the assignment that solves the primal associated with $P2$.

Notice that the values $(\hat{u}_i, \hat{v}_j)$ chosen in the dual problem $P1$ are *feasible* in the dual problem $P1$. It follows that the minimum attained satisfies

$$\sum_{i=1}^{N-1} \bar{u}_i + \sum_{j=1}^{M} \bar{v}_j \leq \sum_{i=1}^{N-1} \hat{u}_i + \sum_{j=1}^{M} \hat{v}_j \ ,$$

or

$$\sum_{a_{i,j} \in \bar{S}} a_{ij} z_{ij} \leq \sum_{a_{i,j} \in \hat{S}} a_{ij} z_{ij} - \hat{u}_N \ ,$$

implying that

$$\hat{u}_N \leq \sum_{a_{i,j} \in \hat{S}} a_{ij} z_{ij} - \sum_{a_{i,j} \in \bar{S}} a_{ij} z_{ij} \ .$$

That is, the upper bound on the utility that man N can get is his *marginal contribution* to the value of the primal program (i.e., the difference between the maximand with him and without him). Note that to calculate this upper bound we must know the assignments in *both* cases, when N is excludes and N is included. This is easily done if we assume positive or negative assortative mating. For instance, with positive assortative mating and $M > N$,

$$\sum_{a_{i,j} \in \hat{S}} a_{ij} z_{ij} - \sum_{a_{i,j} \in \bar{S}} a_{ij} z_{ij} = \sum_{i=1}^{N} z_{i,M-N+i} - \sum_{i=1}^{N-1} z_{i,M-(N-1)+i} \ . \tag{1}$$

Similar arguments apply for any man and any woman. Using the bounds for men and women who are married to each other at the stable assignment we can put bounds on the possible divisions of the gains from marriage of the husband and wife in each couple. Thus the husband's share in the couple MN is bounded by

$$z_{N,M} - \left(\sum_{i=1}^{N} z_{i,M-N+i} - \sum_{i=1}^{N} z_{i,M-1-N+i}\right) \leq \hat{u}_N \leq \sum_{i=1}^{N} z_{i,M-N+i} - \sum_{i=1}^{N-1} z_{i,M-(N-1)+i},$$

or

$$\sum_{i=1}^{N} z_{i,M-1-N+i} - \sum_{i=1}^{N-1} z_{i,M-N+i} \leq \hat{u}_N \leq \sum_{i=1}^{N} z_{i,M-N+i} - \sum_{i=1}^{N-1} z_{i,M-(N-1)+i} \ . \tag{2}$$

2.3 Bounds on Differences

With positive or negative assortative mating we can also put bounds on the *change* in utilities as one moves along the assignment profile. Let there be a positive assortative mating (i.e., the matrix z_{ij} is super modular) and suppose that $M > N$. Then man N is married to woman M, and woman $M-1$ is married to man $N-1$. At a stable assignment

$$\begin{aligned} u_N + v_M &= z_{N,M} \\ u_{N-1} + v_{M-1} &= z_{N-1,M-1} \\ u_N + v_{M-1} &\geq z_{N,M-1} \\ u_{N-1} + v_M &\geq z_{N-1,M} \end{aligned}$$

Eliminating u_N and u_{N-1} and substituting into the inequalities we get

$$\begin{aligned} z_{N,M} - v_M + v_{M-1} &\geq z_{N,M-1} \\ z_{N-1,M-1} - v_{M-1} + v_M &\geq z_{N-1,M} \end{aligned}$$

Hence,

$$z_{N,M} - z_{N,M-1} \geq v_M - v_{M-1} \geq z_{N-1,M} - z_{N-1,M-1}$$

and we get the upper and lower bounds on $v_M - v_{M-1}$. Now we also know that woman $M-2$ and man $N-2$ marry each other. Using the fact that $M-1$ and $N-1$ also marry each other we get by the same argument that

$$z_{N-1,M-1} - z_{N-1,M-2} \geq v_{M-1} - v_{M-2} \geq z_{N-2,M-1} - z_{N-2,M-2} \qquad (3)$$

and so on all the way to the lowest married couple. Because we assume more women than men, $M > N$, woman $M - N + 1$ will marry man 1. For this particular couple, we have

$$\begin{aligned} u_1 + v_{M-N+1} &= z_{1,M-N+1} \\ u_1 + v_{M-N} &\geq z_{1,M-N} \\ v_{M-N} &= 0 \end{aligned}$$

The boundary condition is therefore

$$z_{1,M-N+1} - z_{1,M-N} \geq v_{M-N+1} \ .$$

We see that along the stable assignment the prices must form an *increasing* sequence. This is a consequence of complementarity.

When we set the bounds on couple N, M in (3), we referred only to couple $N-1, M-1$. However, there are $M-1$ stability constraints, one for each woman that man N is not married to:

$$\begin{aligned} u_N &\geq z_{N,M-1} - v_{M-1} \\ u_N &\geq z_{N,M-2} - v_{M-2} \ldots \end{aligned}$$

and also $N-1$ stability constraints for woman M regarding for each man that she is not married to. We now show that the most binding constraint from all these constraints is the one that holds when man N (woman M) with woman $M-1$ (man $N-1$). That is,

$$z_{N,M-1} - v_{M-1} \geq z_{N,M-2} - v_{M-2} \geq \cdots \geq z_{N,M-N+1} - v_{M-N+1} \geq z_{N,M-N}.$$

As a first step, we show that, for man N, a marriage with woman $M-1'$s puts a more binding constraint than a marriage with woman $M-2'$s . That is,

$$z_{N,M-1} - v_{M-1} \geq z_{N,M-2} - v_{M-2}. \tag{4}$$

Equation 4 can be rewritten as :

$$z_{N,M-1} - (z_{N-1,M-1} - u_{N-1}) \geq z_{N,M-2} - v_{M-2},$$

or

$$u_{N-1} + v_{M-2} \geq z_{N,M-2} - z_{N,M-1} + z_{N-1,M-1}.$$

We know that in the stable assignment $u_{N-1} + v_{M-2} \geq z_{N-1,M-2}$, so it is enough to show that $z_{N-1,M-2} \geq z_{N,M-2} - z_{N,M-1} + z_{N-1,M-1}$. But, this follows directly from the assumption that z_{ij} is super modular. Therefore, the lower bound woman $M-1$ imposes is higher than the lower bound woman $M-2$ imposes on man N. By the same arguments, we now generally show that woman $M-k's$ constraint is more binding than woman $M-k-1's$ constraint. Now we have,

$$\begin{aligned} z_{N,M-k} - v_{M-k} &\geq z_{N,M-k-1} - v_{M-k-1}, \\ z_{N,M-k} - (z_{N-k,M-k} - u_{N-k}) &\geq z_{N,M-k-1} - v_{M-k-1}, \\ u_{N-k} + v_{M-k-1} &\geq z_{N,M-k-1} - z_{N,M-k} + z_{N-k,M-k}. \end{aligned}$$

Again, we know that in a stable assignment, $u_{N-k} + v_{M-k-1} \geq z_{N-k,M-k-1}$, so it is enough to show that $z_{N-k,M-k-1} \geq z_{N,M-k-1} - z_{N,M-k} + z_{N-k,M-k}$, which follows from the super modularity assumption that requires

$$z_{N-k,M-k-1} + z_{N,M-k} \geq z_{N,M-k-1} + z_{N-k,M-k}.$$

Therefore we can say that the lower bound woman $M-k$ imposes is higher than the lower bound woman $M-k-1$ imposes on man N, and finally conclude that the highest lower bound on man $N\prime s$ share is imposed by woman $M-1$. In a very similar way it can be shown that the highest lower bound on woman $M\prime s$ share is imposed by man $N-1$.

3 The continuous case

We now consider a continuous distribution of agents, in which equilibrium conditions typically pin down the intrahousehold allocation of welfare. The difference between the continuous case and the discrete case analyzed in the previous section is that, with a continuum of agents and continuous distributions, each agent has a very close substitute. In this case, the upper and lower bounds in 2 and 3, respectively, approach each other and in the limit coincide.

3.1 Basic results

The setting, here, is a slight generalization of the one considered in subsection 3.2 of Chapter 7. There exists a continuum of men, whose incomes x are distributed on $[0, 1]$ according to some distribution F, and a continuum of women, whose incomes y are distributed on $[0, 1]$ according to some distribution G. The measure of all men in the population is normalized to 1, and the measure of women is denoted by r. Also, we still consider a transferable utility (TU) framework. In addition, the 'marital output' is now the sum of two components: an economic output, which is a function $h(x, y)$ of individual incomes, and a fixed non monetary gain from marriage, denoted θ, which is perceived by the spouses in addition to the economic benefits. As before, h is assumed to be supermodular.

An *allocation rule* specifies the shares of the wife and husband in every marriage. If $r > 1$ and all men are married, we can index the marriage by the husband's income x. The marital output is then $h(x, \psi(x)) + \theta$ and the marital shares are $u(x)$ for the husband and $v(\psi(x))$ for the wife. If $r < 1$ and all women are married, we can index the marriage by the wife's income y. The marital output is then $h(\phi(y), y) + \theta$ and the marital shares are $u(\phi(y))$ for the husband and $v(y)$ for the wife.

As discussed before, the allocation rule that supports a stable assignment must be such that the implied utilities of the partners satisfy

$$u(x) + v(y) \geq h(x, y) + \theta \quad \forall x, y, \tag{17}$$

with equality if the partners are married to each other and inequality if they are not.[2] The utility levels $v(x)$ and $u(y)$ that satisfy (17) can be interpreted as the demand prices that men with income x and women with income y require to participate in *any* marriage. Marriages that form are consistent with the demands of both partners and exhaust family resources. Marriages that do not form are those in which resources are insufficient to satisfy the demands of *both* partners.

In particular, (17) implies that

$$u(x) = \theta + \max_y (h(x, y) - v(y)), \tag{18}$$

and

$$v(y) = \theta + \max_x (h(x, y) - u(x)).$$

[2] Note that by deducting $h(y, 0) + h(0, z)$ from both sides of equation (17), it can be written, equivalently, in terms of the surplus that the marriage generates, relative to remaining single. Also, because the values of remaining single are independent of the assignment, the condition for stable assignment can be formulated as maximization of the aggregate surplus.

That is, each partner gets the spouse that maximizes his/her "profit" from the partnership, taking into account the reservation utility (the 'price') of any potential spouse. The first order conditions for the maximizations in (18) give:

$$\begin{aligned} v'(y) &= h_y(\phi(y), y), \\ u'(x) &= h_x(x, \psi(x)). \end{aligned} \tag{19}$$

These equations have an important implication - namely that, as we move across matched couples, the welfare of each partner changes according to the *marginal* contribution of his/her *own* income to the *marital* output, irrespective of the potential impact on the partner whom one marries. The reason for this result is that, with a continuum of agents, there are no rents in the marriage market, because everyone receives roughly what he\she would obtain in the best next alternative.[3] Therefore, a change in marital status as a consequence of a marginal change in income has negligible impact on welfare, and the only gain that one receives is the marginal contribution of one's own trait. Although the change of spouse provides no additional utility, the spouse that one has influences the marginal gain from an increase in own traits, reflecting the interactions between the traits in the production of marital output.

Another important condition that needs to be satisfied in a stable assignment is that, if there are unmarried men, the poorest married man (whose income is denoted x_0) cannot get any surplus from marriage. Similarly, if there are unmarried women, the poorest married woman (whose income is denoted y_0) cannot get any surplus from marriage. Otherwise, the unmarried men or women who are slightly less rich could bid away the marginal match. This condition exploits the assumption that there is a continuum of agents. Hence, if $r < 1$ then $u(x_0) = h(x_0, 0)$ and $v(b) = \theta$. Conversely, if $r > 1$ then $v(y_0) = h(0, y_0)$ and $u(a) = \theta$. If $r = 1$, then any allocation of the gains in the least attractive match with $x = y = 0$ that satisfies $u(a) + v(b) = \theta$ is possible.

This initial disparity between the two spouses is modified as they move up the assignment profile. The main features that influence the evolution of utility differences within couples are the *local scarcity* of males and females at different levels of incomes and the strength of the interaction in traits. Assuming, for instance, that $r > 1$ and all men are married then marriages can be indexed by the husband's income. As one moves across all married couples, the utility of the husbands rises at the rate $\frac{du(x)}{dx} = h_x(x, \psi(x))$, while the utility of their assigned wives rises at the rate $\frac{dv(y)}{dy}\frac{dy}{dx} = h_y(x, \psi(x))\psi'(x)$. In this case, if men are everywhere locally scarce (i.e., $\psi'(x) < 1$), then the utility of the husband rises faster than the utility of the wife. Conversely, if there are less women than men ($r < 1$) and women are everywhere locally scarce (i.e., $\phi'(y) < 1$), the utility of the wife rises faster than the utility of the husband. Intuitively, an overall scarcity of men benefits men at the top of the income distribution

[3] The absence of rents must be distinguished from the positive surplus that the marriage creates. A positive surplus, $h(y, z) + \theta > h(y, 0) + h(0, z)$, simply means that there are positive gains from marriage, relative to the situation in which both partners become single, but this is rarely the best next alternative.

to a larger extent because these men are desired by all women; by the same token, an overall scarcity of women benefits the women at the top of the income distribution to a larger extent, because these women are desired by all men.

Integrating the expressions in (20) and using the boundary conditions described above, one can obtain a *unique* allocation rule, provided that $r \neq 1$. Basically, one first finds the allocation in the least attractive match, in which the minority type has no income, using the no rent condition. Then, the division in better marriages is determined sequentially, using the condition that along the stable matching profile each partner receives his\her marginal contribution to the marital output. The key remark is that the allocation rule is fully determined by the sex ratio r and the respective income distributions of the two sexes. The incomes of the partners in a particular marriage have no direct impact on the shares of the two partners, because the matching is endogenously determined by the requirements of stable matching.

Technically, therefore, for $r > 1$:

$$\begin{aligned} v(y) &= h(0, y_0) + \int_{y_0}^{y} h_y(\phi(t), t)\, dt, \qquad (20a) \\ u(x) &= \theta + \int_0^x h_x(s, \psi(s))\, ds, \\ y_0 &= \Psi(1 - 1/r), \end{aligned}$$

while for $r < 1$,

$$\begin{aligned} v(y) &= \theta + \int_0^y h_y(\phi(t), t)\, dt, \qquad (20b) \\ u(x) &= h(x_0, 0) + \int_{x_0}^{x} h_x(s, \psi(s))\, ds, \\ x_0 &= \Phi(1 - r). \end{aligned}$$

Finally for $r = 1$,

$$\begin{aligned} v(y) &= k + \int_0^y h_y(t, \phi(t)) dt, \qquad (20c) \\ u(x) &= k' + \int_0^x h_x(\psi(s), s) ds, \\ k + k' &= \theta. \end{aligned}$$

The first terms in the RHS of equations 20a to 20c are the utilities of the partners in the match of the lowest quality and the integrals describe the accumulated marginal changes, as we move up the stable assignment profile to marriages with higher incomes. Because of the interaction in traits, the change in the marital contribution depends on the income of the spouse that one gets. Note that marginal increases in x_0 or y_0 have *no* effect on $u(x)$ or $v(y)$, respectively, because the marginal persons with these incomes are just indifferent between marrying and remaining single.

Observe that, in marriages that involve individuals from the bottom of the male and female income distributions, members of the larger sex group have higher income. Thus, if $r > 1$ and all men are married, the men in the lowest quality matches have almost no income, while their wives have strictly positive income. The wife receives her utility as single $h(0, y_0)$ and the husband receives the remaining marital output. Conversely, if $r < 1$ and all women are married, the women in the lowest quality matches have almost no income, while their husbands will have positive income. The husband receives his utility as single $h(x_0, 0)$ and the wife obtains the rest. If $r = 1$, the allocation in the lowest quality match is indeterminate and, consequently, there is a whole set of possible sharing rules that differ by a constant of integration.

3.2 A tractable specification

Let us now slightly generalize our previous approach by assuming that male incomes x are distributed on a support $[a, A]$ according to some distribution F and female incomes y are distributed on a support $[b, B]$ according to some distribution G; the assumption of different supports for men and women is useful for empirical applications. We introduce now a simplifying assumption, namely that the output function h depends only on total family income. That is,

$$h(y, x) = H(y + x), \tag{5}$$

with $H(0) = 0$. This assumption makes sense in our transferable utility setting, since under TU a couple behaves as a single decision maker. Note that basically all examples of intrahousehold allocation with TU given in Chapter 3 satisfy this property.

Under this assumption, $h_y(y, x) = h_x(y, x) = H'(y + x)$, and assortative matching requires $h_{xy}(y, x) = H''(y + x) > 0$, so that H is increasing and convex. As above, we let $\psi(x)$ (resp. $\phi(y)$) denote the income of Mr.x's (Mrs y's) spouse. Finally, we maintain the convention that a single person with income s $(= x, y)$ achieves a utility level $H(s)$.

We are interested in how changes in the sex ratio and the distributions of income of the two sexes affect the allocation rule that is associated with a stable matching. In this analysis, we shall distinguish between two issues: (i) the shape of the allocation rule in a cross section of marriages - that is, how do the shares vary as we move up the assignment profile to couples with higher incomes, and (ii) changes of the allocation rule as parameters of the marriage market, such as the sex ratio or the male and female income distribution, change.

3.2.1 Allocation of marital output: general properties

We start by analyzing the properties of the allocation of marital output between spouses, as described by equations (20a, b, c) of Chapter 7. In the lowest quality matches, the partner that belongs to the majority group has higher income than the minimum of the corresponding income distribution, but, because of competition with lower income singles, receives no rent, and has the same income as a single. In contrast, the partner that belongs to the minority group receives a rent that equals to the total surplus generated, because there are no lower income singles to compete with. These properties exactly define the allocation of welfare between the spouses.

Under assumption (5), equation (19) in Chapter 7 becomes:

$$\frac{du(x)}{dx} = H'(x+\psi(x)), \tag{6}$$

and

$$\frac{dv(y)}{dy} = H'(\phi(y)+y) \tag{7}$$

therefore, for any married couple $(x, \psi(x)) = (\phi(y), y)$,

$$\frac{du(x)}{dx} = \frac{dv(y)}{dy}.$$

In words: the return, in terms of intrahousehold allocation of marital output, of an additional dollar of income is the same for males and females. This symmetry between genders, however, is *not* maintained when moving from one couple to another, because in general the change in husband's income between the couples does not equal to the change in the wife's income - reflecting the local scarcity of the respective genders, as discussed in Chapter 8.

Integrating (6) and (7), and assuming for instance that $r > 1$, we have:

$$u(x) = H(a+y_0) - H(y_0) + \theta + \int_a^x H'(s+\psi(s))\,ds \tag{8}$$

$$v(y) = H(y_0) + \int_{y_0}^y H'(\phi(t)+t)\,dt \tag{9}$$

Again, since women are assumed to be on the long side of the market, the poorest married woman, with income y_0, must be indifferent between marriage and singlehood; all the surplus generated by her marriage, namely $H(a+y_0) - H(y_0) - H(a) + \theta$, goes to the husband, generating a utility $H(a+y_0) - H(y_0) + \theta$. Moving up along the income distributions, the allocation evolves as described by (6) and (7).

The case $r < 1$ is similar and gives:

$$u(x) = H(x_0) + \int_{x_0}^x H'(s+\psi(s))\,ds \tag{10}$$

$$v(y) = H(x_0+b) - H(x_0) + \theta + \int_b^y H'(\phi(t)+t)\,dt \tag{11}$$

3.2.2 Linear shifts of distributions

We now introduce an additional assumption that considerably simplifies the analysis. Specifically, we assume that (i) there are as many men as women

$(r = 1)$, and (ii) that men's income distribution is a linear upward shift (LS) of the income distribution of women. That is,

$$F(t) = G(\alpha t - \beta) \quad \text{for all } t \tag{12}$$

for some $\alpha < 1, \beta > 0$.

put here figure 1

This condition is satisfied, for instance, if the income distributions of both men and women are lognormally distributed, with parameters (μ_M, σ_M) for males and (μ_F, σ_F) for females and $\sigma_M = \sigma_F$ or, alternatively if the two income distributions are uniform and the support of the male distribution is $[a, A]$ while the support of the female distribution is $[b, B]$. Then, $b = \alpha a - \beta$ and $B = \alpha A - \beta$. An illustration is provided in Figure 1. The linear shift property implies that, under assortative matching and with populations of equal size, a man with income x is paired with a woman with income $y = \alpha x - \beta$. With the previous notations, therefore, $\phi(y) = (y + \beta)/\alpha$ and $\psi(x) = \alpha x - \beta$. Equations (6) and (7) then become

$$\frac{du(x)}{dx} = H'((\alpha + 1)x - \beta), \tag{13}$$

and

$$\frac{dv(y)}{dy} = H'(((\alpha + 1)y + \beta)/\alpha), \tag{14}$$

yielding upon integration :

$$v(y) = K + \frac{\alpha}{1 + \alpha} H(\phi(y) + y) \tag{15}$$

and

$$u(x) = K' + \frac{1}{1 + \alpha} H(x + \psi(x)) \tag{16}$$

where

$$K + K' = \theta.$$

In words, the marriage between Mr. x and Mrs. $y = \psi(x)$ generates a marital output $\theta + H(x + \psi(x))$, which is divided linearly between the spouses. The non monetary part, θ, is distributed between them (he receives K, she receives K') in a way that is not determined by the equilibrium conditions (this is the standard indeterminacy when $r = 1$) but must be the same for all couples (note that K or K' may be negative). Regarding the economic output, however, the allocation rule is particularly simple; he receives some constant share $\alpha/(1 + \alpha)$ of it, and she gets the remaining $1/(1 + \alpha)$.

3.3 Comparative Statics

We now turn to examine the impact of changes in the sex ratio and income distribution.

3.3.1 Increasing the proportion of women

We begin by noting an important feature of the model, namely that *if all marriages yield a strictly positive surplus then the allocation rule has a discontinuity at $r = 1$.* Indeed, examining the expressions in (8) and (9) we see that if r approaches 1 from above we get in the limit

$$u(x) = H(a+b) - H(b) + \theta + \int_a^x H'(s + \psi(s))\, ds \qquad (17)$$

$$v(y) = H(b) + \int_b^y H'(\phi(t) + t)\, dt \qquad (18)$$

while if r approaches 1 from below we get in the limit,

$$u(x) = H(a) + \int_a^x H'(s + \psi(s))\, ds \qquad (19)$$

$$v(y) = H(a+b) - H(a) + \theta + \int_b^y H'(\phi(t) + t)\, dt \qquad (20)$$

The marital *surplus* generated by the marriage of lowest income couple, here (a, b), is equal to $H(a+b) - H(a) - H(b) + \theta$. When the two sexes are almost equal in number, a small change in the sex ratio shifts all the surplus to one of the partners in the lowest quality match, the one whose sex is in the minority, and this discontinuity is then transmitted up the matching profile to all participants in the marriage market.[4] This knife-edge property is an undesirable property of the simple model without friction. One can get rid of it either by assuming no rents for couples at the bottom of the distribution, or by limiting our attention to marginal changes in the ranges $r > 1$ or $r < 1$ which do not reverse the sign of these inequalities.

Consider, now, a *marginal* increase in the proportion of women r that maintains either $r > 1$ or $r < 1$ and assume that the shape of of income distributions of both men and women remain unchanged. From the matching rule $1 - F(x) = r(1 - G(y))$, we see that, as a consequence of such change, any married man with a given income x will now be matched with a woman with a higher y and each woman with a given y, is now matched to a man with a lower x. That is, the matching function $\psi(x)$ shifts upwards and the matching function $\phi(y)$ moves downwards. As we move along a stable assignment profile, the utility of all married men grows with their own income at a higher rate, because $h_x(x, \psi(x)) = H'(x + \psi(x))$ is higher for all x and the utility of all married women grows at slower rate because $h_y(\phi(y), y) = H'(\phi(y) + y)$ is lower for all y. It then follows from (6) and (7) that the utility of all married men rises and the utility of all married women declines; those who remain single

[4] The result that g is the only source of gain from marriage for couples at the bottom of the income distribution reflects the assumptions that $h(0,0) = 0$ and that there is a positive density of the income distribution at zero. In general, participants at the bottom of the income distribution have a positive income, so that the lowest quality match may create a monetary surplus, because of the positive interaction of traits.

are unaffected. Assuming, for instance, that $r > 1$, we have:

$$\begin{aligned}\phi(y,r) &= \Phi\left[1 - r(1 - G(y))\right] \\ \psi(x,r) &= \Psi\left[1 - \frac{1}{r}(1 - F(x))\right] \\ y_0 &= \Psi(1 - 1/r),\end{aligned}$$

and

$$\begin{aligned}\frac{\partial\phi(y,r)}{\partial r} &= -(1 - G(y)\,\Phi'\left[1 - r(1 - G(y))\right] < 0 \\ \frac{\partial\psi(x,r)}{\partial r} &= \frac{1}{r^2}(1 - F(x))\,\Psi'\left[1 - \frac{1}{r}(1 - F(x))\right] > 0 \\ \frac{\partial y_0}{\partial r} &= \frac{1}{r^2}\Psi'(1 - 1/r) > 0.\end{aligned}$$

Differentiating (8) and (9) with respect to r therefore gives:

$$\begin{aligned}\frac{\partial u(x)}{\partial r} &= \left(H'(a + y_0) - H'(y_0)\right)\frac{\partial y_0}{\partial r} + \int_a^x H''(s + \psi(s))\,\frac{\partial\psi(s,r)}{\partial r}ds > 0 \\ \frac{\partial v(y)}{\partial r} &= \left(H'(y_0) - H'(y_0 + a)\right)\frac{\partial y_0}{\partial r} + \int_{y_0}^y H''(\phi(t) + t)\,\frac{\partial\phi(t,r)}{\partial r}dt < 0\end{aligned}$$

The case $r < 1$ is similar and left to the reader.

We conclude:

A marginal increase in the proportion of women to men in the marriage market, improves (or leaves unchanged) the welfare of all men and reduces (or leaves unchanged) the welfare of all women; the impact is stronger for higher income households.

This property has been established empirically by several authors. For instance, Chiappori, Fortin and Lacroix (2002), using a collective model of labor supply, find that, other things equal, a one percentage point increase in the sex ratio (defined as the ratio of men to women in the relevant marriage market) induces husbands to transfer some 2,000 dollars (1988) of income to their spouse.

3.3.2 Shifting female income upward

Recalling our assumption that men have the higher income in the sense that their distribution dominates in the first degree the income distribution of women, i.e., $F(t) < G(t)$ for $t \in (0,1)$, we now consider a *first degree upward shift* in the distribution of female income, holding the male distribution constant. That is, the proportion of females with incomes exceeding y rises for all y, so that women become more similar to men in terms of their income, as we observe in practice. Such an upward first order shift in the distribution of female income affects the matching functions in exactly the same way as a marginal increase in the female/male sex ratio. Thus, if all men maintain their income, they all become better off. Similarly, any woman who would maintain her income would become worse off. This remark should however be interpreted with care, because it is obviously impossible for *all* women to maintain their income: when the distribution of female incomes shifts to the right, some (and possibly all) females must have higher income. In particular, those women who maintain their relative rank (quantile) in the distribution will maintain their position in the competition for men, and will be matched with a husband with the same income as before. Such women will be better off, as a consequence of the increase in their own income.

As a special case, consider the linear shift case described above; to keep things simple, assume moreover that $\beta = 0$. Suppose, now, that the income of every woman is inflated by some common factor $k > 1$ and consider a married couple with initial incomes (x, y). After the shift, the partners remain married but the wife's income is boosted to ky while the husband's income remains equal to x. If u_k and v_k denote the new individual utilities, we have from (16) and (15):

$$\begin{aligned} v_k &= K + \frac{k\alpha}{k\alpha + 1} H(ky + x) \quad \text{and} \\ u_k &= K' + \frac{1}{k\alpha + 1} H(ky + x) \end{aligned} \tag{21}$$

Differentiating in k around $k = 1$ gives:

$$\begin{aligned} \frac{\partial v_k}{\partial k} &= \frac{\alpha}{(\alpha+1)^2} H(y+x) + \frac{\alpha y}{\alpha+1} H'(y+x) \quad \text{and} \\ \frac{\partial u_k}{\partial k} &= -\frac{\alpha}{(\alpha+1)^2} H(y+x) + \frac{y}{\alpha+1} H'(y+x) \end{aligned} \tag{22}$$

One can readily check that both changes are positive (for the second one, it stems from the convexity of H). We conclude that the shift has two impacts. First, the increase in total income generates some additional surplus (the term in $yH'(y + x)$), which is shared between spouse in proportion of their respective incomes (i.e. 1 and α). In addition, a redistribution is triggered by the shift. Specifically, since the wife's share of total income is increased, so is her consumption; the husband therefore transfers to his wife an amount equal to a fraction $\alpha/(\alpha + 1)^2$ of total surplus. One can readily check that the transfer is proportionally larger for wealthier couples, since the ratio $H(y + x)/(y + x)$ increases with $(y + x)$ due to the convexity of H.

3.3.3 Empirical illustration

It is a priori not clear how important is income for matching and how to measure it. Actual incomes are rarely available, and wages are measured with a lot of noise and vary over the life cycle. For an empirical application, we estimate the *predicted hourly wage* of white men and women aged 25-40 in the CPS data and use these predictions as measures of the male and female incomes for this age group.[5] We then obtain the following results (see Figure 2):

1. The log normal distribution fits these predictions well.

2. The standard deviations for men's and women's predicted log wages are *similar* and both grow over time,

3. The mean predicted log wages of men are higher than for women but the discrepancy declines over time.

4 Male distributions of predicted log wage dominate in the first degree the female distributions in all years but the gap declines over time.

5. Within couples, there is high positive correlation between the predicted log wages of husbands and wives and this correlation rises over time indicating a high and increasing degree of positive assortative mating.

6. Finally, the ratio of men to women in the CPS sample of whites aged $25-40$ has dropped from 1.045 in 1976 to 0.984 in 2005.

Figure 3 shows the male and female income distributions estimated from the CPS data. As seen the cumulative distribution of male incomes is below the cumulative distribution of female in both years but the gap is lower in 2005, indicating a first degree dominance of the male distributions. For both man and women, the cumulative distributions are less steep in 2005, representing the general rise in inequality between 1976 and 2005.

We use this information, together with the assumption that the marital output is given by $h(x,y)=\frac{(x+y)^2}{4}$ (so that the marital surplus from marriage is $\frac{yx}{2}$), to calculate the predicted response of the shares in marital surplus to the observed changes in the male and female income distributions and in the sex ratio between the years 1976-2005. The use of log normal distribution and the specification of $h(x,y)$ allows us to use conditions (US) and (AI) and to calculate the shares using numerical integrations of (6) and (7). Figures 4 and 5 show the estimated shares in the marital surplus for men and women in 1976 and 2005. We see that men had a larger estimated share in 1976, while women had the larger share in 2005. Part of this reversal is due to the narrowing wage gap between men and women and part of it is due to the reduction in the female-male sex ratio over the period.

put here figures 2, 3, 4, 5

[5] These results are obtained by running regressions with every year for white men and women aged $25-40$ of wages on schooling experience and occupation, excluding self employed. We use up to 53 occupation dummies, which allows for a large variance given schooling and age and also captures a more permanent feature of wages because an occupation tend to relatively stable over the life cycle. For men and women who reported no occupation, we imputed the mode occupation in their schooling group. For men who did not work, we imputed wages conditioned on working and for women we also corrected for selection using the Heckman technique.

3.4 Taxation

Changes in the income distribution can also arise from a government intervention in the form of taxes and subsidies. For instance, we may consider a linear transfer scheme, such that the after tax (subsidy) income of a person with income s is $\kappa + (1-\tau)s$, with $\kappa > 0$ and $0 < \tau < 1$. Let us assume that the scheme is revenue neutral, so that its only impact is to redistribute income between and within couples. Therefore:

$$\int_0^1 xF(x)\,dx + r\int_0^1 yG(y)\,dy = \int_0^1 (\kappa + (1-\tau)x)F(x)\,dx + r\int_0^1 (\kappa + (1-\tau)y)G(y)\,dy \tag{23}$$

and

$$\kappa = \tau\frac{x + ry}{1+r} \tag{24}$$

when $x = \int_0^1 xF(x)\,dx$, $y = \int_0^1 yG(y)\,dy$ denote average incomes of male and females, respectively, so that $\frac{x+ry}{1+r}$ is average household income. Here, τ is the taxation rate, and κ is the lump sum subsidy funded by income taxation.

We can think of such an intervention as a change in the household production function from $h(x,y)$ to $\tilde{h}(x,y) = h(\kappa + (1-\tau)x, \kappa + (1-\tau)y)$. Such a transformation preserves the sign of the cross derivative w.r.t. the before tax incomes x and y. Therefore, the same pattern of a positive assortative mating is maintained and the matching functions $\psi(x)$ and $\phi(y)$ remain the same. However, the introduction of tax and transfer influences the gains from marriage, which depend on the *after* tax incomes of the partners, and the division of these gains. By construction, a progressive transfer-tax system raises the income of the poor and reduces the income of the rich. Due to positive assortative matching, the progressivity of the program is magnified, because an individual whose after tax income has increased (decreased) is typically assigned to a spouse whose after tax income has increased (decreased). Put differently, the intervention affects the surplus generated by marriage, holding the pre tax incomes fixed. For low income matches, the surplus increases and for high income matches it declines. In addition, the division of the surplus between husbands and wives is affected in general.

When assumption AI holds and only total family income matters, the household production function is modified from $h(x,y) = H(x+y)$ to $\tilde{h}(x+y) \equiv \tilde{H}(2\kappa + (1-\tau)(x+y))$. Assume, in addition, that male and female income distributions satisfy condition (LS) so that $\psi(x) = \alpha x - \beta$ and $\phi(y) = \frac{y+\beta}{\alpha}$. Then, for a larger female population ($r > 1$), utilities become:

$$\begin{aligned}\tilde{v}(y) &= H(\kappa + (1-\tau)y_0) + (1-\tau)\int_0^{y+\phi(y)} H'(\kappa + (1-\tau)s)\frac{\alpha ds}{\alpha+1} \quad (25)\\ \tilde{u}(x) &= \theta + (1-\tau)\int_0^{\psi(x)+x} H'(\kappa + (1-\tau)s)\frac{ds}{\alpha+1}\end{aligned}$$

with $k = \tau\frac{x+ry}{1+r}$. The impact of a change in the marginal income tax, t, on the utilities of women and men, respectively, is

$$\begin{aligned}\frac{\partial \tilde{v}\left(y\right)}{\partial t} &= \left(\frac{x+ry}{1+r}-y_0\right)H'\left(\alpha+\beta y_0\right)-\alpha D\left(x+y\right) \quad (26)\\ \frac{\partial \tilde{u}\left(x\right)}{\partial t} &= -D\left(x+y\right),\end{aligned}$$

where, Y denotes total family income and

$$D\left(Y\right)=\int_0^Y H'\left(\kappa+\left(1-\tau\right)s\right)\frac{ds}{\alpha+1}+\left(1-\tau\right)\int_0^Y\left(s-\frac{x+ry}{1+r}\right)H''\left(\kappa+\left(1-\tau\right)s\right)\frac{ds}{\alpha+1}. \quad (27)$$

The term $D\left(Y\right)$ is typically positive for richer households (who therefore lose from the introduction of the tax/benefit system) and negative for poorer ones. In this simple context, the corresponding gain or loss is allocated between husband and wife in respective proportions 1 and α. In addition, since single women are at the bottom of the female income distribution, their utility is typically increased by the tax/benefit scheme (this is the case whenever their income is below the mean). Equilibrium then requires the gain of the marginal woman (i.e., of the wealthiest single or poorest married woman) to be forwarded to *all* women in the distribution; hence the term $\left(\frac{x+ry}{1+r}-y_0\right)H'\left(\alpha+\beta y_0\right)$ in equation (26) representing the gain of the marginal woman. Note that this boost in income does not go to the poorer spouse, but to the spouse whose population is in excess supply. Should males outnumber females ($r<1$), they would receive the corresponding benefit. The precise impact of these changes is hard to evaluate in general and we therefore turn to a specific example.

3.5 An example

We now provide a simple example in which the shares can be easily calculated. In addition to (LS) we assume that incomes are uniformly distributed. We use again our example in Chapter 2 with public goods where $h(y,x)=\frac{(y+x)^2}{4}$, which satisfies (AI). For this example, men and women have the same marginal contribution to marriage, $h_x(y,x)=h_y(y,x)=\frac{y+x}{2}$. Assume that the incomes of men and women are uniformly distributed on $[0,1]$ and $[0,Z]$, respectively, where $Z\leq 1$. If $Z<1$, then the income distribution of men dominates in a first degree the income distribution of women, because

$$G(t)=\begin{cases}\frac{t}{Z} & if \quad 0\leq t\leq Z\\ 1 & if \quad Z<t\leq 1\end{cases} \quad (28)$$

exceeds $F(t)=t$, for all t in the interval $(0,1)$. We are also in the 'linear upward shift' case described above, with $\alpha=Z$ and $\beta=0$.To simplify further, we set $\theta=0$ so that the lowest quality matches generate no surplus. Therefore, there is no indeterminacy of the allocation rule when $r=1$ and no discontinuity in the allocation rule.

Under the assumed uniform distributions, the assignment functions are linear and given by

$$x = \phi(y) = 1 - r(1 - \frac{y}{Z}), \tag{29}$$

$$y = \psi(x) = \frac{Z}{r}[(r-1) + x]. \tag{30}$$

and the local scarcity of men is *constant* and given by $\frac{Z}{r}$. Under the simplifying assumption that $\theta = 0$, the shares of the husband and wife in the marital output can then be rewritten in the form

$$\begin{aligned} v(y) &= \frac{y^2}{4} + \frac{1}{2}\int_{y_0}^{y}[1 - r(1 - \frac{t}{Z})]dt, \\ u(x) &= \frac{x^2}{4} + \frac{1}{2}\int_{x_0}^{x}\frac{Z}{r}[(r-1) + s]ds, \\ y_0 &= \begin{cases} \frac{Z}{r}(r-1) & if \quad r > 1 \\ 0 & if \quad r \le 1 \end{cases}, \\ x_0 &= \begin{cases} 1 - r & if \quad r < 1 \\ 0 & if \quad r \ge 1 \end{cases}. \end{aligned} \tag{31}$$

Notice that $v(y) - \frac{y^2}{4}$ and $u(x) - \frac{x^2}{4}$ are the shares of the husband and wife in the marital *surplus*. Inspecting the integrals in (31), we see that the gender in short supply always receives a larger share of the surplus. In contrast, the shares of marital output of husbands and wives depend also on the location of the couple in the income distribution.

If there are more women than men, $r > 1$, the match with the lowest output is the one in which the husband has income $x = 0$, and the wife has income $y_0 = Z\frac{(r-1)}{r}$. His surplus and utility are at this point zero, while she receives the whole marital output $\frac{y_0^2}{4}$, which also equals her utility as single. Because men are always locally scarce, $\frac{r}{Z} > 1$, it follows from (31) that their utility must grow along the stable assignment at a faster rate than the utility of their assigned wives. It is readily seen that the husband's share is higher in matches with sufficiently high income. In particular, the best match with $x = 1$ and $y = Z$, yields an output of $\frac{(1+Z)^2}{4}$, of which the husband receives $\frac{1}{4} + \frac{Z}{2} - \frac{Z}{4r}$ and the wife receives $\frac{Z^2}{4} + \frac{Z}{4r}$, which is a smaller share.

If there are more men than women, $r < 1$, the match with the lowest output is the one in which the wife has income $y = 0$, and the husband's income is $x_0 = 1 - r$, and it is now the wife that has the lower utility. The local scarcity parameter can now be higher or lower than 1. If $\frac{r}{Z} > 1$, men are always locally scarce, and it follows from (31) that the husband will have a higher share in the output of *all* marriages. If, however, $\frac{r}{Z} < 1$ and women are always locally scarce, then the utility of women grows along the stable assignment profile at a faster rate than the utility of their assigned husbands, and they may eventually overtake them. Indeed, the wife's share in the best match is $\frac{Z}{2} + \frac{Z^2}{4} - \frac{rZ}{4}$ and the husband's share is $\frac{1}{4} + \frac{rZ}{4}$, which is smaller if r is sufficiently small.

This example illustrates clearly the impact of changes in the sex ratio r and the distribution of female income as indexed by Z, on the welfare of women and men. Recall that marginal increases in x_0 or y_0 have *no* effect on $u(x)$ or $v(y)$, respectively. Inspection of the integrands in (31), shows that $u(x)$ must increase in r and Z, while $v(y)$ must decrease in r and Z. As we noted above, the result that women are worse off when the mean income of women rises sounds surprising. However, the reason that a woman who maintains her income is worse off when Z rises is that there are more women with income above her, which means that she cannot "afford" anymore a husband with the same income as before. However, any woman who keeps her position in the income distribution, (i.e., whose income rose at the same proportion as Z) will obtain a husband with the same x as before the change. Then it can be shown that if $r > 1$, her surplus does not change, and if $r < 1$, her surplus rises.[6] In either case, her welfare must rise, reflecting the rise in her own income. This example can be easily generalized for the case in which there are positive non monetary gains, $\theta > 0$.

The example allows us to examine numerically the impact of a progressive transfer-tax system. Assume that male income is distributed uniformly on $[0, 1]$, while the female income is distributed uniformly on $[0, 75]$. Set $\theta = .025$ and $\tau = .7$. Now consider a balanced transfer scheme such that $\kappa(1+r) = (1-\tau)(x+ry)$. We discuss here two separate cases, one in which women are the majority and $r = 1.1$ and the other when women are the minority and $r = .9$. In the numerical example, $x = .5$ and $y = .375$. Thus, for a marginal tax of $\tau = .7$, the balanced budget constraint implies that $\kappa = .130$ when $r = 1.1$, and $\kappa = .132$. when $r = .9$.Figures (6), (7) and Table 1 summarize the results.

When women are in the majority, their share is usually less than half but rising in the income of their assigned husband (see Figure 6). The tax-subsidy intervention moderates this increase, because in low quality matches, the wife's share is determined by her income, and women with low income gain from the progressive system. When women are in the minority, their share in the marital output declines and the progressive tax system moderates this decline (see Figure 7) because in low quality matches, the husband's share is determined by his income, and men with low income gain from the progressive system. The difference in slopes between the two figures reflects the role of the non monetary

[6] The surplus of the husband and the surplus of the wife are readily obtained by calculating the integrals in (31). For $r \geq 1$, we obtain

$$\begin{aligned} s_h(y) &= u(y) - \frac{y^2}{4} = \frac{Zy^2}{4r} + \frac{1}{2}\frac{Z}{r}(r-1)y, \\ s_w(\psi(y)) &= \frac{\psi(y)\,y}{2} - s_h(y) = \frac{Zy^2}{4r}. \end{aligned}$$

For $r \leq 1$, we obtain

$$\begin{aligned} s_w(z) &= v(z) - \frac{z^2}{4} = \frac{rz^2}{4Z} + \frac{z(1-r)}{2}, \\ s_h(\phi(z)) &= \frac{\phi(z)\,z}{2} - s_w(z) = \frac{rz^2}{4Z}. \end{aligned}$$

gains, θ, that are captured by the men when $r > 1$ and by the women when $r < 1$. This effect weakens as one moves to high income couples where the monetary gains become increasingly important.

Table 1 provides the numerical values of the shares. Panel a describes the case with more women than men, $r = 1.1$. Then, all men marry and a proportion .9091 of the women remains single. The man with the lowest income, 0, is matched with a woman whose income is .0682, the man with the mean income, .5, is matched with a woman whose income is .4091, and the man with the highest income, 1, is matched to the woman with highest income, .75. Following the intervention; the after tax income of the man with lowest income rises to .1304, and that of his matched wife rises to .1781, the after tax income of the average man is reduced to .4804 and that of his matched wife rises to .3947, while the after tax of the wealthiest man is reduced to .8304 and that of his matched wife is reduced to .6554. Thus, the tax and transfers scheme reduces inequality both between and within couples.

Although the impact of the intervention on the couples with the average man or average woman is relatively small, some noticeable changes occur at the bottom and the top of the income distribution. At the bottom, the intervention raises the utilities of both men and women but women obtain a larger share of the total utility if $r > 1$ and a *smaller* share if $r < 1$. It seems surprising that a progressive policy that transfers resources to poor women reduces their share in the marital surplus. But when $r < 1$, poor women are married to men who are wealthier than they are, and the intervention makes these men less "useful" to their wives. At the top of the distribution, the intervention lowers substantially the utilities of both men and women but women gain relatively more than men if $r < 1$ and relatively less if $r > 1$. We see that the impact of the tax-subsidy intervention on each spouse reflects three different effects: an increase (decrease) in own income, an increase (decrease) in the spouse's income, and the increase in the incomes of the individuals who are just indifferent between marriage and singlehood. The first two effects influence the marital output that the matched partners can generate together. The third effect reflects the changes in the sharing of this output that are caused by the competition in the marriage market. In order to separate these effects, we examine the impact of the tax for couples for which the intervention does not affect *total* family income, and, therefore, marital output does not change. This comparison is shown in panels c and d of Table 1. We see that in both panels the wife gains income relative to the husband. However, when women are in the majority, the wife in such couples loses both in output and surplus terms. In contrast, the wife gains if women are in the minority. This difference can be traced to the impact of the intervention on the lowest quality matches, where the intervention causes a larger gain to the wife than to the husband when women are in the minority, $r < 1$, while the opposite is true when $r > 1$ (see panels a and b). These effects are transmitted along the matching profile to all couples in the marriage market.

The general conclusion that one can draw from these examples is that in a frictionless market, where the shares are determined jointly with the assignments, the simple intuition based on bargaining between two isolated partners fails. For instance, we see in panel c that, although family income remains fixed and the wife's share in the total income rises, she ends up with lower share of marital output. In other words, the allocation rule that determines the wife's and husband's utility in a particular marriage, reflects the traits of *all* participants in the marriage markets and, therefore, a change in the income distribution in the economy (society) at large can change the shares within specific marriages in a way that would not be directly predictable from the change in the within-household income distribution.

put here figures 6, 7

Table 1: Sharing of Marital Output and Surplus[7]

Panel a: Women are the Majority, $r = 1.1$

Hus. I.	Wife I.	Tax	Hus. U.	Wife U.	Hus. S.	Wife S.	U. Sh W/(W+H)	S. Sh W/(W+H)
0	0.0682	no	0.025	0.0012	0.025	0	0.0444	0
0.1304	0.1781	yes	0.0409	0.0079	0.0366	0	0.1625	0
0.5	0.4091	no	0.1472	0.0845	0.0847	0.0426	0.3646	0.3348
0.4804	0.4167	yes	0.1463	0.0798	0.0887	0.0364	0.353	0.2913
1	0.75	no	0.4795	0.3111	0.2295	0.1705	0.3935	0.4261
0.8304	0.6554	yes	0.3548	0.222	0.1825	0.1146	0.3849	0.3858

Panel b: Men are the Majority, $r = 0.9$

Hus. I.	Wife I.	Tax	Hus. U.	Wife U.	Hus. S.	Wife S.	U. Sh W/(W+H)	S. Sh W/(W+H)
0.1	0	no	0.0025	0.025	0	0.025	0.9091	1
0.2022	0.1322	yes	0.0102	0.0427	0	0.0384	0.807	1
0.55	0.375	no	0.1178	0.1211	0.0422	0.0859	0.5069	0.6707
0.5172	0.3947	yes	0.1084	0.1245	0.0415	0.0856	0.5347	0.6735
1	0.75	no	0.4188	0.3719	0.1688	0.2313	0.4704	0.5781
0.8322	0.6572	yes	0.2975	0.2821	0.1243	0.1741	0.4867	0.5834

Panel c: Women are the Majority, $r = 1.1$

Hus. I.	Wife I.	Tax	Hus. U.	Wife U.	Hus. S.	Wife S.	U. Sh W/(W+H)	S. Sh W/(W+H)
0.4762	0.3929	no	0.1366	0.0772	0.0799	0.0387	0.3612	0.3261
0.4637	0.4054	yes	0.139	0.0748	0.0852	0.0338	0.35	0.2837

Panel d: Men are the Majority, $r = 0.9$

Hus. I.	Wife I.	Tax	Hus. U.	Wife U.	Hus. S.	Wife S.	U. Sh W/(W+H)	S. Sh W/(W+H)
0.5264	0.3553	no	0.1072	0.1122	0.0379	0.0806	0.5115	0.6804
0.5007	0.381	yes	0.101	0.1184	0.0383	0.0821	0.5396	0.6819

[7] The income of men is uniform on [0,1] and the income of women is uniform on [0,0.75]. The gains from marriage is $g = 0.025$. The tax rate on income is $\beta = 0.7$ and the implied value of α that balances the budget is $\alpha = 0.1322$ at panel a and $\alpha = 0.1303$ at panel b.

3.6 Matching on preferences: Roe vs Wade and female empowerment

In the matching models presented so far, income is the trait on which people match. But other determinants can also be considered. In a recent paper, Chiappori and Oreffice (2007) use a matching model to analyze the impact of the legalization of abortion on power allocation within couples.[8] In their framework, people differ in their preferences towards children; the corresponding matching patterns - and the resulting allocation of resources - can be studied before and after legalization.

That the legalization of abortion should alter the balance of powers within couples is not surprising;[9] indeed, Oreffice (2007) has provided an empirical study based on the collective approach to household behavior, that confirms the 'empowerment' consequences of Roe vs. Wade. Still, the *mechanism* by which this empowerment occurs deserves some scrutiny. While it is not hard to convince oneself that *some* women (e.g., career-oriented women with little taste for family life) will gain from legalization, whether *all* women will is another matter. A strong objection is that women have heterogeneous preferences for fertility (or different attitudes toward abortion); some do not consider abortion as an option, either for religious and ethical reasons or because they do want children. Whether the legalization will benefit these women as well is not clear. From an economist's perspective, moreover, the new context will affect the matching process on the market for marriage, and in particular the way the surplus generated by marriage is shared between spouses. In principle, such 'general equilibrium' effects could annihilate or even reverse the direct impact of the reform, particularly for these women who are unlikely to derive much direct benefit from it.

3.6.1 Preferences

To investigate these issues, Chiappori and Oreffice consider a model in which a continuum of men and women derive utility from one private composite good c (the price of which is normalized to 1) and from children; Let the dummy variable k denote the presence ($k = 1$) or the absence ($k = 0$) of children in the household. Men have identical, quasi-linear preferences over consumption and children. The utility of single men only depends on their consumption; i.e., men cannot derive utility from (and do not share the costs of) out-of-wedlock children, due to the fact that they do not live in the same household. On the other hand, married men's utility is of the form $U_H(c_H, k) = c_H + u_H k$, where the parameter $u_H > 0$ is identical for all men in the economy. Women differ in their preferences toward children. Specifically, female utility functions take the quasi-linear form $U(c, k) = c + uk$. Here, each woman is characterized by

[8] The version presented here is a slightly simplified version of the original paper; in particular, we assume here that men have identical preferences, and concentrate on preference heterogeneity among women.

[9] See for instance Héritier (2002).

the individual-specific taste parameter u, which is distributed according to the density f over the interval $[0, U]$. We assume that any woman (single or married) who wants a child can have one. However, if she plans to have no children, unwanted births may still occur with some probability p, which depends on the available contraceptive technology and the legality of abortion.

The quasilinear structure of the male and female preferences implies that utility is transferable within marriage. For each spouse, the utility depends on the couple's fertility decision and on the share of composite good that he or she receives.

As before, we normalize the mass of men to be 1, and we denote by r the total mass of women on the market; here, we assume that $r > 1$, i.e. that women are on the long side of the market. Male income is denoted by Y. Women without children have income, x; however, if a woman has children, her income drops to y, with $y < x$, reflecting both the loss in her earning capacity due to childbearing and the cost of raising the child. Hence a single woman without children consumes her income x; if she decides to have a child (or if an unwanted pregnancy occurs), she also consumes her income (which has dropped to y) and receives a utility u from her child, which is independent of her marital status.

Regarding couples, we assume that $u_H < x - y$, i.e. that the gain received by the husband from having a child does not offset by itself the loss in income experienced by the wife. This assumption implies, in our framework, that the couple's decision to have a child or not will also depend on the wife's preferences. Therefore married women must agree with their husband on two issues. One is the fertility decision; i.e., they must decide whether to have kids or not, and the decision depends (in particular) on the wife's preferences towards children. The other decision relates to the distribution of resources within the household (i.e., the allocation of total income between male and female consumption of the composite good). Both decisions will be ultimately determined by the equilibrium on the market for marriage. Finally, we model the legalization of abortion (and generally the availability of some birth control technology) as an exogenous decrease in the probability p of experiencing an unwanted pregnancy.

3.6.2 Fertility decisions

We first consider the fertility decisions of singles and couples, starting with single individuals. Single men do not make decisions: they consume their income, and get a utility which equals to Y. Single women, on the other hand, will decide to have children if and only if the benefit compensates the income loss, i.e. if $u \geq x - y$, leading to a utility which equals $y + u$. In the alternative case when $u < x - y$, single the women chooses not to have a child and any pregnancy will be involuntary. As pregnancy occurs with probability p, the expected utility is $x(1 - p) + p\,(y + u)$. In what follows, the threshold $x - y$ by $\bar{u}$; women whose utility parameter is larger than or equal to $\bar{u}$ will be said to be referred to a as 'high' type.

Having assumed transferable utility context, couples maximize their marital surplus. The total benefit, for a couple, of having a child is $u_H + u$, whereas the cost is $x - y$. It follows that a married couple will plan to have a child if $u \geq x - y - u_H$ - then total utility is $Y + y + u_H + u$. The threshold $x - y - u_H$ is denoted $\underline{u}$; note that $\underline{u} < \bar{u}$. If $u < x - y - u_H$, only unwanted kids are born, leading to an expected total utility $Y + (1 - p)\,x + p\,(y + u_H + u)$. Women with taste parameter u smaller than $\underline{u}$ will be said to be of 'low' type, while those between $\underline{u}$ and $\bar{u}$ will be called an 'intermediate'. To summarize:

- women of 'high' type ($u \geq \bar{u}$) always choose to have a child
- women of 'intermediate' type ($\underline{u} < u < \bar{u}$) choose to have a child only when married
- women of 'low' type ($u \leq \underline{u}$) never choose to have a child

3.6.3 Stable match

We can now derive the properties of the stable match. The key element is provided by Figure 8, which plots the maximum utility $\Phi(u)$ a man can achieve when marrying a woman of taste u (in other words, $\Phi(u)$ denotes his utility if he was to appropriate *all* the surplus generated by marriage). The function Φ is increasing; i.e., it is always better (for the husband) to marry a wife with a larger taste coefficient u.

More precisely, women whose parameter u is greater than $\bar{u}$ (the 'high' type), and who would plan to have a child even when single, are the most 'attractive' from the male's perspective. While they differ in taste, this difference is irrelevant from a husband's viewpoint, since they require the same compensation c_H for getting married (namely, to be left with a private consumption which equals their income with a child, y). Women between $\underline{u}$ and $\bar{u}$ (the 'intermediate' type) come next in males' preferences. They plan to have a child only when married, and the minimum compensation they require is $c_I(u) = (x - u)(1 - p) + py$. This required compensation decreases with the individual utility u; hence men strictly prefer intermediate women with a higher u. Finally, women with a u smaller than $\underline{u}$ (the 'low' type) never plan to have a child. Again, these women are equivalent from a husband's perspective, since they require the same compensation for getting married, namely their consumption as single, i.e. $c_L = (1 - p)\,x + py$.

As often in matching models, the properties of the stable match crucially depend on the identity of the marginal spouse (i.e., the 'last' married woman). We denote by $u(r)$ the taste parameter of the marginal women (i.e., either the 'last' single woman or the 'first' married woman). Technically, $u(r)$ is defined by the fact that the measure of the set of women with a taste parameter larger than $u(r)$ equals the measure of men, which is 1; i.e., the value $u(r)$ solves the equation

$$r \int_{u(r)}^{U} f(t)\,dt = 1. \tag{32}$$

Competition between women in the marriage market implies that women who generate a larger surplus for their husband are a more desirable match.

Hence, whenever a women belonging to the intermediate type is married, then all women with a larger taste parameter are married as well - this is the case depicted in Figure 8. The intuition is that women with a larger preference for children have a comparative advantage: the compensation they need from their husband to accept marriage is smaller, because they value highly the prospect of having a child. In general, the identity of this marginal woman depends on the location of $u(r)$ with respect to the two thresholds $\underline{u}$ and $\bar{u}$.

An obvious property of stable matches in this context is that *all males receive the same utility*; indeed, they are assumed identical, and the absence of friction implies that any difference of welfare between males would be competed away. Since the marginal woman is indifferent between being married or single, her husband gets all the surplus generated by the relationship, namely $\Phi(u(r))$. Then all other men receive the same utility. Graphically, this corresponds to the horizontal line going through $\Phi(u(r))$ in Figure 8.

A crucial insight, at this point, is the following. Take any woman with a taste parameter u larger than $u(r)$. Then *the difference* $\Phi(u) - \Phi(u(r))$ *represents the surplus received by this woman.*[10] In Figure 8, for instance, the surplus received by any woman of 'high' type is depicted by a bold arrow.

put here figure 8

Using this geometric intuition, the characterization of the equilibrium is straightforward. Three cases should be distinguished:

- If $1/r \leq \underline{W} = \int_{\bar{u}}^{U} f(t)\,dt$, the excess supply of women is 'large', in the sense that there are less men than high type women. Then $u(r) \geq \bar{u}$, and the marginal married women belongs to the high type. Only (some of) these women are matched. Women of the same type who remain single decide to have a kid; all other women remain single and decide not to have children (although they may have one involuntarily). Regarding welfare issues, note that, in that case, married women receive no surplus from marriage; their consumption is the same as if single.

- If $\underline{W} < 1/r < \bar{W} = \int_{\underline{u}}^{U} f(t)\,dt$, as depicted in Figure 8, the marginal wife belongs to the intermediate type. All married women have a child, and consume the same amount, which is such that the marginal wife is indifferent between getting married and remaining single. All married women (but the marginal one) get a positive surplus from marriage, and high type women receive the maximal surplus.

put here figure 9

- Finally, when the excess supply of women is small enough (technically, $1/r \geq \bar{W}$), the marginal wife belongs to the low type (i.e. $u(r) \leq \underline{u}$

[10] If her husband's utility was $\Phi(u)$ he would get all the surplus generated by the marriage. Since his equilibrium utility is only $\Phi(u(r))$, the difference $\Phi(u) - \Phi(u(r))$ represents the part of the surplus appropriated by the wife.

- see Figure 9). Her fertility is the same with and without marriage - namely, no planned child. Stability requires that her consumption is also the same, and equals to $(1-p)\,x+py$. The same conclusion applies to all married, low type women. Other married women belong to the high or intermediate type, hence decide to have a child; their consumption is defined by the fact that men, who are in short supply, must be indifferent between the various potential spouses. Again, this condition generates a positive surplus for all women of high and intermediate types; high type women receive the largest surplus.

The variation in women's utility across the three types of equilibria exhibits interesting patterns. Not surprisingly, women are better off the smaller their excess supply on the market. However, when women's excess supply is either large or small, their welfare does not depend on the size of the imbalance. In the intermediate case, on the contrary, a marginal increase in the number of men continuously reduces the taste parameter $u\,(r)$ of the marginal woman, which ameliorates the welfare of all married women.

3.6.4 Changes in the birth control technology

We can now come to the main issue, namely the impact of a technological change in birth control that reduces the probability of unwanted pregnancies. A key assumption is that all women (including single) are given free access to the technology; a natural example could be the legalization of abortion that took place in the 70s.

The situation is depicted in Figure 10 (which, for expositional convenience, considers the case in which the risk of unwanted pregnancies goes to zero). The new technology decreases the maximum utility attainable by husbands of low or intermediate type women, resulting in a downwards shift of the graph of the function Φ. This leads to the following conclusions:

insert here Fig 10

Not surprisingly, women who do not want to have a child (either because they belong to the low type or because they are single) benefit from the technology, precisely because unwanted pregnancies become less likely. In the extreme situation in which unwanted pregnancies are eliminated, the monetary gain is thus $p\,(x-y-u)$ .More interesting is the fact that women who decide to have a child also benefit from the technology, although to a lesser extent than singles. The intuition is that the intrahousehold distribution of resources is driven by the marginal women; for a small or intermediate excess supply of women, the marginal women is indifferent between getting married and remaining single *without kid*. Her reservation utility is thus improved by the new technology. The nature of a matching game, however, implies that any improvement of the marginal agent's situation must be transmitted to all agents 'above' the marginal one.

In the case of an intermediate excess supply depicted in Figure 3, the benefit experienced by all married women, assuming the new technologies drives the risk of unwanted pregnancies to zero, is $p(x-y-u(r))$ (where, again, $u(r)$ denotes the taste parameter of the marginal married woman). This benefit continuously increases with the number of men M. When the excess supply is small, the gain is pu_H, still smaller than $p(x-y)$ (the gain for single women) but nevertheless positive. On the other hand, when the excess supply of women is 'large', married women do *not* benefit from the new technology, because the marginal woman does not use it. Hence the consequences of the new technology for married women's welfare are intimately related to the situation that prevails on the marriage market.

Finally, men cannot gain from the introduction of the new technology. When the excess supply of women is large, their utility is not affected. When the excess supply of women is small, so that the marginal wife does not want a child, the total welfare of the household is increased, but so is the reservation utility of the wife; the husband is left with the same consumption, but loses the benefit he would have received from an unwanted birth. The intermediate case is even more spectacular. Here, all marriages result in a child being born, so the total surplus generated by marriage is not affected by the innovation. What changes, however, is the intrahousehold allocation of the surplus. The new technology improves the reservation utility of the marginal woman, hence her share of resources increases. Stability requires this shift to be reproduced in all couples. All in all, the new technology results in a net transfer from the husband to the wife, which equals the expected gain of the marginal single woman, i.e. $p(x-y-u(r))$, without any change on the fertility of married couple (who actually do not use the new technology).

We thus conclude that in our model an improvement in the birth control technology, such as the legalization of abortion, generally increases the welfare of *all* women, including those who want a child and are not interested in the new technology. Note, however, that the mechanism generating this gain is largely indirect. The reason why even married women willing to have a child benefit from the birth control technology is that the latter, by raising the reservation utility of single women, raises the 'price' of all women on the matching market (although this logic fails to apply in situations of 'large' excess supply of women).

An interesting, although somewhat paradoxical implication is that reserving the new technology to married women (as was initially the case for the pill, at least for younger women) would actually *reverse* the empowerment effect. The option of marriage to women with a low taste for children, who are willing to accept a lower compensation from the husband for getting married and gaining access to the new technology, toughens the competition for husbands. Therefore, women of the high or intermediate type are made worse off by the introduction of the new technology. Only women with a very low taste parameter (i.e., below the lower marginal value) gain from the innovation. This comparison emphasizes the complex and partly paradoxical welfare impact of a new technology. On the one hand, its effects can go well beyond the individuals who actually use it, or even consider using it. Our model suggests that a major effect of legalizing abortion may have been a shift in the intrahousehold balance of powers and in the resulting allocation of resources, even (and perhaps especially) in couples who were not considering abortion as an option. On the other hand, the new technology benefits all married women only because it is available to singles. A technological improvement which is reserved to married women will have an impact on their fertility, partly because it changes the mechanisms governing selection into marriage. But its impact on women's welfare is largely negative, except for a small fraction of women who choose marriage as an access to the new technology.

4 Matching with general utilities

We now switch to the general framework in which we relax the assumption that utility is transferable. The tractability of the transferable utility framework comes at a cost. The most obvious drawback is that under TU, couples behave as singles; in particular, their demand function (i.e., the amount spent on each of the public or private commodities) does not depend on the Pareto weights. In other words, changes in male and female income distributions may trigger a reallocation of resources (or more precisely of *one* commodity) between spouses, but under TU, it cannot have income effects, and cannot result in, say, more being spent on children health or education. While this framework may be useful in many contexts, it is clearly too restrictive in other situations.

In this section, we explore the more general framework introduced in Chapter 7, in which utility is not (linearly) transferable. That is, although compensations between spouses are still possible, they need not take place at a constant 'exchange rate': there is no commodity the marginal utility of which is always identical for the spouses. In particular, the matching model is no longer equivalent to a linear optimization problem. The upside is that, now, any change affecting the wife's and husband competitive positions (e.g., a change in income distributions) will potentially affect *all* consumptions, including on public goods - which allows for a much richer set of conclusions. The downside is that the derivation of individual shares from the equilibrium or stability conditions is more complex. It remains feasible, however. We first present the general approach to the problem, then we concentrate on a specific and tractable example.

4.1 Recovering individual utilities: the general strategy

We use the same framework as in Chapter 7. Male income is denoted by x and female income is denoted by y; the Pareto frontier for a couple has the general form

$$u = H(x, y, v) \tag{33}$$

with $H(0, 0, v) = 0$ for all v. If a man with income x remains single, his utility is given by $H(x, 0, 0)$ and if a woman of income y remains single her utility is the solution to the equation $H(0, y, v) = 0$. By definition, $H(x, y, v)$ is decreasing in v; we assume that it is increasing in x and y, i.e. that a higher income, be it male's or female's, tends to expand the Pareto frontier. Also, we still consider a continuum of men, whose incomes x are distributed on $[0, 1]$ according to some distribution F, and a continuum of women, whose incomes y are distributed on $[0, 1]$ according to some distribution G; let r denote the measure of women. Finally, we assume that an equilibrium matching exists and that it is assortative - i.e., that the conditions derived in Chapter 7 are satisfied; let $\psi(x)$ (resp. $\phi(y)$) denote the spouse of Mr. x (of Mrs. y).

As previously, the basic remark is that stability requires:

$$u(x) = \max_y H(x, y, v(y))$$

where the maximum is actually reached for $y = \psi(x)$. First order conditions imply that

$$\frac{\partial H}{\partial y}(\phi(y), y, v(y)) + v'(y)\frac{\partial H}{\partial v}(\phi(y), y, v(y)) = 0.$$

or:

$$v'(y) = -\frac{\frac{\partial H}{\partial y}(\phi(y), y, v(y))}{\frac{\partial H}{\partial v}(\phi(y), y, v(y))}. \tag{34}$$

Again, we have a differential equation in v. It is more complex than in the TU case, because the right hand side depends on $v(y)$ in a potentially non linear way. Still, under mild regualrity conditions, such an equation defines v up to a constant, the value of which can be determined from the condition that the last married person in the 'abundant' side of the market receives no surplus from marriage.

Note, in particular, that from the assumptions made in Chapter 7, we have that:

$$v'(y) = -\frac{\frac{\partial H}{\partial y}(\phi(y), y, v(y))}{\frac{\partial H}{\partial v}(\phi(y), y, v(y))} > 0 \tag{25}$$

In words, richer people are always better off. Finally, once v has been computed, the condition

$$u = H(x, \psi(x), v(\psi(x))) \tag{26}$$

exactly defines u.

This framework has been applied by Chiappori and Reny (2007), who consider a population of heterogeneous agents with different risk aversions matching to share risks arising from identically distributed random incomes. They show that (i) a stable match always exists, (ii) it is unique, and (iii) it is negative assortative: among married couples, men with lower risk aversion match with more risk averse women and conversely.

4.2 A specific example

We now present another application due to Chiappori (2009).

4.2.1 Preferences

Males have identical preferences, represented by the Cobb-Douglas utility:

$$u_m = c_m Q \tag{19}$$

where c_m denotes his consumption of some private, Hicksian composite commodity and commodity Q is publicly consumed within the household. All women share the same preferences, characterized by some minimum level of consumption $\bar{c}$, beyond which private and public consumptions are perfect substitutes:

$$\begin{aligned} u_f(c_f) &= -\infty \text{ if } c_f < \bar{c} \\ &= c_f + Q \text{ if } c_f \geq \bar{c} \end{aligned}$$

An important feature here is that men and women have different preferences, in that private and public consumption are complements for men and perfect substitutes for women. We shall further assume that household income is always larger than $\bar{c}$; then female utilities are of the quasilinear form $c_f + Q$. In particular, any efficient solution involves $c_f = \bar{c}$, because beyond $\bar{c}$, spending a dollar on private consumption for the wife is inefficient: spent on the public good, the same dollar is as valuable for the wife and strictly better for the husband.

4.2.2 Efficient allocations

We first characterize the set of efficient allocations. An efficient couple solves the program:

$$\max c_m Q \tag{35}$$

under the constraints

$$c_m + c_f + Q = Y \tag{36}$$

$$u_f = c_f + Q \geq U \tag{37}$$

where Y denotes household total income and U is some arbitrary utility level. A first remark is that at any efficient allocation, the wife's utility U cannot fall below $(Y + \bar{c})/2$. As the wife receives the same consumption $\bar{c}$ in any efficient allocation, her utility varies only with the amount of the public good, Q. Once $\bar{c}$ has been spent, the husband's maximal utility is obtains when he receives his optimal bundle of private and public consumption, namely $Q = c_m = (Y - \bar{c})/2$; this choice generates a wife's utility of $(Y + \bar{c})/2$. If $U > (Y + \bar{c})/2$, however, providing her with U requires more resources to be spent on the public good (and less on his private consumption) than what he would choose by himself. Then the constraint (37) is binding. Therefore, the Pareto frontier is given by

$$u_m = H(Y, u_f) = (v - \bar{c})(Y - u_f), \tag{38}$$

where $u_f \geq \frac{Y+\bar{c}}{2}$. Moreover, one can readily compute the corresponding consumptions; namely, $Q = v - \bar{c}$ and $c_m = Y - v$. Figure 11 displays the Pareto frontier when total total income has been set to $Y = 5$ and the wife's minimal consumption to $\bar{c} = 1$, so that $(Y + \bar{c})/2 = 3$.

put here figure 11

Because of the public consumption, our simple model exhibits what Lundberg and Pollack call 'production dominance'; i.e., any single man and any single woman can do better by marrying. To see why, just note that a single man with income x chooses $Q = c_m = x/2$ for a utility of $x^2/4$, while a single woman with income $y > \bar{c}$ achieves a utility that equals y. Now, by marrying, they achieve an income $Y = x + y$. If $y \leq x + \bar{c}$, he can achieve $(x + (y - \bar{c}))^2/4 > x^2/4$ while she gets $\bar{c} + (x + y)/2 > y$. If, on the contrary, $y > x + \bar{c}$ then he can achieve $(y - \bar{c})\,x > x^2/4$, while she remains at y. Therefore, in a frictionless model like this one (and without non-monetary gains or costs), either all women or all men marry: singles can only be on one side of the marriage market.

4.2.3 Assortativeness

The Pareto frontier just derived has a particularly tractable form. Indeed, let us analyze the stability conditions along the lines previously described. For $v \geq \frac{Y+\bar{c}}{2}$, we get:

$$\frac{\partial H(Y,v)}{\partial Y} = v - \bar{c}, \frac{\partial H(Y,v)}{\partial v} = -(2v - (\bar{c} + Y)) \tag{39}$$

implying that

$$\frac{\partial^2 H(Y,v)}{\partial Y^2} = 0 \text{ and } \frac{\partial^2 H(Y,v)}{\partial Y \partial v} = 1 \tag{40}$$

As we have seen in Chapter 7, these conditions are sufficient for the existence of a unique stable match involving assortative matching. To simplify, we consider the linear shift case, where the matching functions are given by $\phi(y) = (y + \beta)/\alpha$ and $\psi(x) = \alpha x - \beta$.

4.2.4 Intrahousehold allocation of welfare

We now turn to the allocation of welfare within the couple. Equation (34) becomes:

$$\begin{aligned} v'(y) &= -\frac{\frac{\partial H}{\partial y}(\phi(y), y, v(y))}{\frac{\partial H}{\partial v}(\phi(y), y, v(y))} \\ &= \frac{v(y) - \bar{c}}{2v(y) - (\bar{c} + y + \phi(y))} \\ &= \frac{\alpha v(y) - \alpha\bar{c}}{2\alpha v(y) - (\alpha+1)y - (\alpha\bar{c} + \beta)}. \end{aligned} \tag{41}$$

Recovering the wife's utility requires solving this differential equation. For that purpose, we may, since v is strictly increasing, define the inverse function ω by:

$$v(y) = v \Leftrightarrow y = w(v)$$

Then equation 41 becomes:

$$\frac{1}{\omega'(v)} = \frac{\alpha v - \alpha\bar{c}}{2\alpha v - (\alpha+1)\,\omega(v) - (\alpha\bar{c}+\beta)},$$

or

$$\omega'(v) + \frac{(\alpha+1)}{\alpha v - \alpha\bar{c}}\omega(v) = \frac{2\alpha v - (\alpha\bar{c}+\beta)}{\alpha v - \alpha\bar{c}},$$

which is a standard first order, linear differential equation. The general solution is:

$$\omega(v) = K(v-\bar{c})^{-\frac{\alpha+1}{\alpha}} + \frac{2\alpha}{2\alpha+1}v - \frac{\beta+\bar{c}\alpha+2\alpha\beta}{2\alpha^2+3\alpha+1},$$

where K is an integration constant.

To find K, we consider the marginal couple in which the wife receives the lowest female income b and the husband receives the lowest male income $a = (b+\beta)/\alpha$. Then, we have

$$\omega(b) = K(b-\bar{c})^{-\frac{\alpha+1}{\alpha}} + \frac{2\alpha}{2\alpha+1}b - \frac{\beta+\bar{c}\alpha+2\alpha\beta}{2\alpha^2+3\alpha+1} = \frac{b+(b+\beta)/\alpha+\bar{c}}{2}.$$

which yields:

$$K = \left(\frac{b+(b+\beta)/\alpha+\bar{c}}{2} - \frac{2\alpha}{2\alpha+1}b + \frac{\beta+\bar{c}\alpha+2\alpha\beta}{2\alpha^2+3\alpha+1}\right)(b-\bar{c})^{\frac{\alpha+1}{\alpha}}.$$

To illustrate, suppose that $\beta = 0, \alpha = .8, a = 2, b = 1.6, \bar{c} = 1$. Then

$$K = 0.642,$$

and

$$\omega(v) = 0.615v + \frac{0.641}{(v-1)^{2.25}} - 0.171,$$

while the husband's utility is

$$\begin{aligned} u &= H(Y,v) = (v-1)(\omega(v)(1+1/.8)-v) \\ &= (v-1)\left(\left(0.615v + \frac{0.642}{(v-1)^{2.25}} - 0.171\right)(1+1/.8) - v\right). \end{aligned}$$

The function $v(y)$ (the inverse of $\omega(v)$) is represented plotted in Figure 12 by the thick line; the husband's utility is in dotted and thick. Also, the consumption of the public good $Q = v - \bar{c}$ is represented by dashed line, while the consumption of the husband, c_m, is represented by the thin line.

put here figure 12

As one moves up the assignment profile, the total income of the couples and utilities of both husband and wife rise. The consumption of the public good also rises. However, the private consumption of the wife remains constant at $\bar{c} = 1$, while the private consumption of the husband, c_m, first declines and than rises. This non-monotone pattern reflects a compensation to the wife to attract her into marriage via an increase in the public good to the extent that the husband must reduce his private consumption.

All the comparative statics exercises can be adapted to this general framework. For instance, suppose that we keep $\bar{c} = 1$ and $a = 2$ but shift the income distribution of women to the right so that $\alpha = 1$ and $b = 2$. Then,

$$K = 1.125,$$

and

$$\omega(v) = \left(1.125\,(v-1)^{-2} + \frac{2}{3}v - \frac{1}{6}\right),$$

while his utility is

$$\begin{aligned} u &= H(Y, v) = (v-1)\,(2\omega(v) - v) \\ &= (v-1)\left(2\left(1.125\,(v-1)^{-2} + \frac{2}{3}v - \frac{1}{6}\right) - v\right). \end{aligned}$$

The husband's and the wife's utilities for these two cases are displayed in Figure 13, where couples are indexed by male income (which remains invariant). For $\alpha = .8$, we represent the male utility, u by a thin line and the female utility, v, by a thick line. Dashed lines (thin for males and thick for females) represent u and v when $\alpha = 1$. We see that the shift of the female distribution to the right benefits both men and women. The gain is most pronounces among low-income men, who can after the shift marry a woman with a similar income to their own.

put here figure 13

5 Matching by Categories

The matching model and the associated allocation rules that we have discussed so far assume some idealized conditions that are not likely to hold in practice. The most common way to make the model more applicable is to introduce frictions and some bargaining over the resulting surplus. There is, however, an alternative modeling choice that goes part of the way towards reality and is based on the recognition that the researcher observes only part of the data that motivates and restricts choices. This is particularly true in marriage markets where explicit market prices do not exist and the division within families of consumption or time is rarely observed. This path has been followed by Choo and Siow (2008) and Chiappori, Salanié and Weiss (2010); the presentation given here follows the latter contribution.

To incorporate unobserved heterogeneity, we consider a case in which the researcher observes marriage patterns within *broad categories,* such as schooling level, race or occupation and observes only some of the individual attributes that distinguish individuals within these classes. That is, in addition to their observed class, individuals are characterized by some *observed* attributes such as income or age and by some idiosyncratic marriage related attributes that are observed by the agents in the marriage market but not by the researcher. We assume that the marital output that is generated by the match of man i, and woman j can be written in the form

$$\zeta_{ij} = \gamma_{I(i)J(j)} + \alpha_{iJ(j)} + \beta_{jI(i)}. \tag{25}$$

The first component $\gamma_{I(i)J(j)}$ depends on the *class* of the two partners, the second component $\alpha_{iJ(j)}$ depends on man i and the *class* of woman j and the third component depends on woman j and the *class* of man i. This specification embodies a strong simplifying assumption; the interaction between two married partners is always via their class identity. In particular we do not have a term that depends on both i and j.[11] We further assume that

$$\begin{aligned} \alpha_{iJ(j)} &= a'_{IJ}x_i + \varepsilon_{iJ(j)} \\ \beta_{jI(i)} &= b'_{JI}x_j + \varepsilon_{I(i)j} \end{aligned} \tag{26}$$

where x_i and x_j are the *observed* attributes of man i and woman j, respectively, a_{IJ} and b_{JI} are vectors of coefficients that represent the marginal contribution of each male (female) attribute to a marriage between a man of class I and woman of class J. The error terms $\varepsilon_{iJ(j)}$ represent the *unobserved* contribution of man i to a marriage with *any* woman of class J. Similarly, $\varepsilon_{I(i)j}$ represents the contribution of woman j to a marriage with any man of class I. We assume that these error terms are identically and independently distributed according to a type 1 extreme value (Gumbel) distribution.

A basic property of the matching model with transferable utility that we discussed in Chapter 7 is the existence of prices, one for each man, v_i, and one for each woman, u_j, that support a stable outcome. At these prices, the matching is individually optimal for *both* partners in each match. Thus, equilibrium implies that i is matched with j iff

$$\begin{aligned} u_i &= \xi_{ij} - v_j \geq \xi_{ik} - v_k \text{ for all } k, \text{ and } u_i \geq \xi_{i0}, \\ v_j &= \xi_{ij} - u_j \geq \xi_{kj} - u_k \text{ for all } k, \text{ and } v_j \geq \xi_{j0}. \end{aligned} \tag{27}$$

Under the special assumptions specified in (25) and (26), Chiappori, Salanié and Weiss (2010) prove the following Lemma:

Lemma 1 *For any stable matching, there exist numbers U_{IJ} and $V_{IJ}, I = 1, ..., M, J = 1, ..., N$, with the following property: for any matched couple (i, j) such that $i \in I$ and $j \in J$,*

$$\begin{aligned} u_i &= U_{IJ} + \alpha_{iJ} \\ &\textit{and} \\ v_j &= V_{IJ} + \beta_{Ij} \end{aligned} \tag{27}$$

[11] This simplifying assumption has been introduced in the context of transferable utility by Choo and Siow (2006). Dagsvik(2002) considers a more general error structure in the context of non transferable utility (e.g. an exogenous sharing rule).

where

$$U_{IJ} + V_{IJ} = \gamma_{IJ}$$

In words: the differences $u_i - \alpha_{iJ}$ and $v_j - \beta_{jI}$ only depend on the spouses' classes, not on who they are. Note, incidentally, that (27) is also valid for singles if we set $U_{I0} = \zeta_{I0}$ and $U_{0J} = \zeta_{0J}$.

The economic interpretation of this result is as follows. The contribution of women j and $j\prime$ who are in the same class J to a marriage with *all* men in class I differ by $\beta_{Ij'} - \beta_{Ij}$. If $v_{j'} - v_j > \beta_{Ij'} - \beta_{Ij}$ no men in I will marry woman $j\prime$ because she is 'too expensive' relative to woman j. Conversely, if $v_{j'} - v_j < \beta_{Ij'} - \beta_{Ij}$ no man in I will marry woman j. Hence, in an equilibrium in which both women j and $j\prime$ find a match with men of the same class I it must be the case that $v_{j'} - v_j = \beta_{Ij'} - \beta_{Ij}$.

Using the assumption of extreme value distribution, we can now write the probability (as viewed by the researcher) that man i marries a woman of a particular class (or remains single) in the familiar multinomial- logit form (see McFadden, 1984)

$$\begin{aligned}
\Pr(\ i \in I \ \text{ matched with } j \in J) &= \frac{\exp\left(U_{I(i)J} + a_{I(i)J}x_i\right)}{\sum_K \exp\left(U_{I(i)K} + a_{I(i)K}x_i\right) + \exp\left(U_{I(i)0} + a_{I0}x_i\right)}, \\
\Pr(i \text{ is single}) &= \frac{\exp\left(U_{I(i)0} + a_{I(i)0}x_i\right)}{\sum_K \exp\left(U_{I(i)K} + a_{I(i)K}x_i\right) + \exp\left(U_{I(i)0} + a_{I0}x_i\right)}
\end{aligned} \tag{42}$$

Analogous expressions hold for women. The terms $U_{I(i)K} + a_{I(i)K}x_i$ represent the systematic part (excluding the unobserved $\varepsilon_{iK(j)}$) of the share that man i receives upon marriage with a woman in class K. The spouse's personal attributes x_j and her idiosyncratic contribution $\varepsilon_{I(i)j}$ have no direct bearing on the probability of marriage, because in equilibrium they are already captured by the unknown constants U_{IK}. Similar remarks apply to the probability of marriage of women. The unknown parameters constants U_{IJ} and V_{IJ} adjust endogenously to satisfy the requirement that the choices of men and women are consistent with each other in the sense of market clearing. In principle, it is possible to calculate these coefficients directly by solving the market equilibrium (i.e. the linear programming problem) associated with a stable assignment. More interestingly, one can use data on actual marriage patterns and the observed attributes of participants in a "marriage market" to *estimate* the gains from marriage of these individuals (relative to remaining single).[12] Basically, the preferences for different types of spouses are "revealed" from the choice

[12] Because the probabilities in (42) are unaffected by a common proportionality factor, some normalization is required. A common practice is to set the utility from being single to zero for all individuals.

probabilities of individuals. Taking the simplest case without covariates, we see

$$\begin{aligned} \ln \frac{\Pr\left(i \in I \text{ is matched with } j \in \ J\right)}{\Pr\left(i \in I \text{ is single}\right)} &= U_{IJ} - U_{I0} \\ \ln \frac{\Pr\left(j \in J \text{ is matched with } i \in \ I\right)}{\Pr\left(j \in J \text{ is single}\right)} &= U_{IJ} - U_{J0}. \end{aligned} \tag{29}$$

Estimating separate multinomial logit for men and women, one can estimate the utilities for each gender in a marriage of each type. Summing the estimated utilities one can recover, for each matching of types (I, J), the systematic output of the marriage ξ_{IJ} (which, under the normalization that being single yields zero utility, equals the total surplus Z_{IJ}). The estimated matrix Z_{IJ} can then be analyzed in terms of the assortative matching that it implies. Of particular interest is whether or not this matrix is supermodular (implying positive assortative mating) or not. As noted by Choo and Siow (2006) and Siow (2009), the super modularity of Z_{IJ} is equivalent to the supermodularity of

$$\ln \frac{[\mu(I,J)]^2}{\sigma(I)\sigma(J)}$$

where $\mu(I, J)$ is the total number of type (I, J) marriages and $\sigma(I)$ and $\sigma(J)$ are the number of single men and single women, respectively. Such supermodularity requires that for all $I' > I$ and $J' > J$

$$\ln \frac{\mu(I',J')\mu(I,J)}{\mu(I,J')\mu(I',J)} > 0.$$

Siow (2009) uses census data on married couples in the US, where the husband and wife are 32-36 and 31-35 respectively. In each couple, the wife and the husband can belong to one of five possible schooling classes (less than high school, high school, some college, college and college plus). He compares the marriage patterns in the years 1970 and 2000 and finds that in each of the two years strict supermodularity fails to hold as in some of the off diagonal cells, the log odd ratio is negative. Looking at the whole matrix, one cannot conclude that there is more positive assortative matching in 2000 than in 1970, although some specific local log odds have increased over time.

Chiappori, Salanié and Weiss (2010) have estimated a more general model in which the choice of spouse can vary by age and cohort of birth. There are four classes of schooling: high school dropouts, high school graduates, some college and college graduates. It is assumed that a man i of group $I(i)$ who is looking to marry at age a a woman of type J obtains utility

$$\tilde{U}(a,J;i) = U(a,J;I(i),C_i) + \varepsilon^i_{a,J}, \tag{30}$$

where I and J are education classes of men and women, respectively, and C_i denotes the cohort (date of birth) of man i. Similarly, a woman j of group $J(j)$ who is looking to marry at age a a man of type I obtains utility

$$\tilde{V}(a,I;j) = U(a,I;J(j),C_j) + \varepsilon^j_{a,I}. \tag{31}$$

The error terms $\varepsilon^i_{a,J}$ and $\varepsilon^j_{a,I}$ have a type 1 extreme value distribution. The model was estimated on CPS June data for the years 1979 to 1997, using information on the age at marriage and the type of spouses of respondents aged 18-35. Preliminary findings show that endogamy among college graduates has increased in the US but, after 1970, it has decreased amongst high school dropouts.[13] The absence of a uniform increase in endogamy is consistent with the results reported by Siow (2009). These results differ, however, from those of Mare (2008) who reports a general trend of increased homogamy by schooling in the US from 1965 onwards.[14]

A more ambitious use of these models is to trace the time changes in the common factors U_{IJ} and V_{IJ} and relate them to changes in marriage market parameters such as sex ratios, costs of search and costs of separation as determined by the divorce laws. Finally as shown by McFadden(1984) the multinomial logit models provide a framework for welfare analysis. For instance one can estimate the expected gains from participation in the marriage market for individuals of different gender and levels of schooling. It should be noted, however, that comparisons across time and groups rely on various normalizing assumptions and often, implicitly or explicitly, cardinal scaling is invoked (see Train, 2009, ch.3 and Bately, 2008).

6 Appendix: Extreme Value distributions

We collect here some useful properties of extreme value disruptions (See Ben-Akiva and Lerman, 1985, ch. 5 and Johnson et al. 1995, ch. 22).

The type 1 extreme value distribution for the *maximal* extreme is
$F(x) = e^{-e^{-\frac{x-a}{b}}}$, $b > 0$.
$f(x) = \frac{1}{b} e^{-\frac{x-a}{b}} e^{-e^{-\frac{x-a}{b}}}$.
The moment generating function is $E(e^{tx}) = e^{at}\Gamma(1 - bt)$.
$E(x) = a + kb$, where $k = .57721 = -\Gamma'(1)$ is Euler's constant.
$V(x) = \frac{b^2\pi^2}{6}$.
mode=a.
median=$a - b\log(\log 2) =\cong a + .36611b$.

The parameter a is thus seen to be the *location* parameter and b is a parameter that controls the variance. This distribution is sometimes named after E. J. Gumbel and we shall say that $x \backsim G(a, b)$. The distribution is skewed to

[13] The endogamy index for a given year is

$$\frac{\frac{\mu(I,J)}{M}}{\frac{\sigma(I)\sigma(J)}{S}}$$

where M denotes here all marriages and S denotes all singles. As in the text, $\mu(I, J)$ is the number of marriages of type (I, J), $\sigma(I)$ is the number of single men of type I and $\sigma(I)$ is the number of single women of type J. All these quantities are simulated from the model. This endogamy index equals 1 if matching is random.

[14] The source of the difference is that Mare uses a log linear model in which the trend can affect only whether or not the partners have the same education (a dichotomous variable).

the right and the mean > median > mode. The distribution of the *minimal* extreme is obtained by reversing the sign of x and is skewed to the left.

The *standard form* $G(0,1)$ has mean k and variance $\frac{\pi^2}{6}$. To get an extreme value with zero mean we can set $a = -kb$ and use $G(-kb, b)$.

Basic properties

- If $x \backsim G(a,b)$ then $\alpha x + \beta \backsim G(\alpha a + \beta, \alpha b)$.
- If x_1 and x_2 are independent Gumbel variates such that $x_1 \backsim G(a_1, b)$ and $x_2 \backsim G(a_2, b)$ then $x^* = (x_1 - x_2)$ has a logistic distribution, i.e.,

$$F(x^*) = \frac{1}{1 + e^{\frac{1}{b}(a_2 - a_1 - x^*)}}.$$

- If $x_1, x_{2....}x_n$ are *iid* Gumbel variables with $G(a,b)$ and $v_1, v_{2,}..v_n$ are some constants then

$$Pr\{v_1 + x_1 \geq \ Max[v_2 + x_2....v_n + x_n]\} = \frac{e^{\frac{v_1}{b}}}{\sum_i e^{\frac{v_i}{b}}}.$$

- If $x_1, x_{2....}x_n$ are independent Gumbel variables with distributions $G(a_i, b)$ then

$$\max(x_1, x_2, ...x_n) \backsim G(b \ln(\sum_i e^{\frac{a_i}{b}}), b).$$

- In particular, if $x_1, x_{2....}x_n$ are *iid* Gumbel variables with $G(a,b)$ and $v_1, v_{2,}..v_n$ are some constants then

$$\begin{aligned} E\{Max[v_1 + x_1, v_2 + x_2....v_n + x_n]\} &= b \ln(\sum_i e^{\frac{v_i + a}{b}}) + kb \\ &= b \ln(\sum_i e^{\frac{v_i}{b}}) + a + kb. \end{aligned}$$

Thus, if $x_1, x_{2....}x_n$ are *iid* Gumbel variables with zero mean then

$$E\{Max[v_1 + x_1, v_2 + x_2....v_n + x_n]\} = b \ln(\sum_i e^{\frac{v_i}{b}}).$$

If we choose one alternative as a benchmark, say alternative 1, and normalize its value to zero, the expected utility relative to this alternative is fully determined by, and inversely related to the probability that the benchmark alternative is selected. The marginal impact of an increase in the value of specific alternative, j, is

$$\frac{\partial E\{Max[v_1 + x_1, v_2 + x_2....v_n + x_n]\}}{\partial v_j} = \frac{e^{\frac{v_j}{b}}}{\sum_i e^{\frac{v_i}{b}}},$$

which is the probability that alternative j will be selected, p_j.

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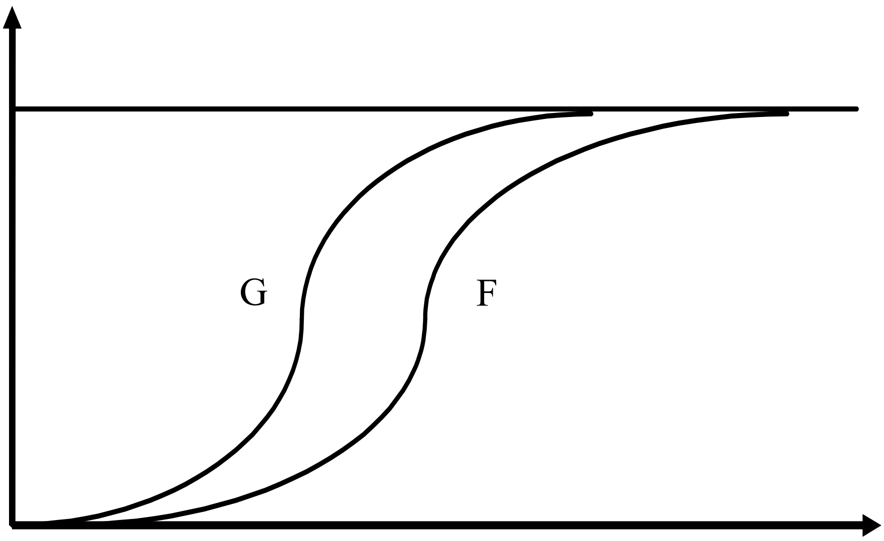

Figure 1: A linear upward shift

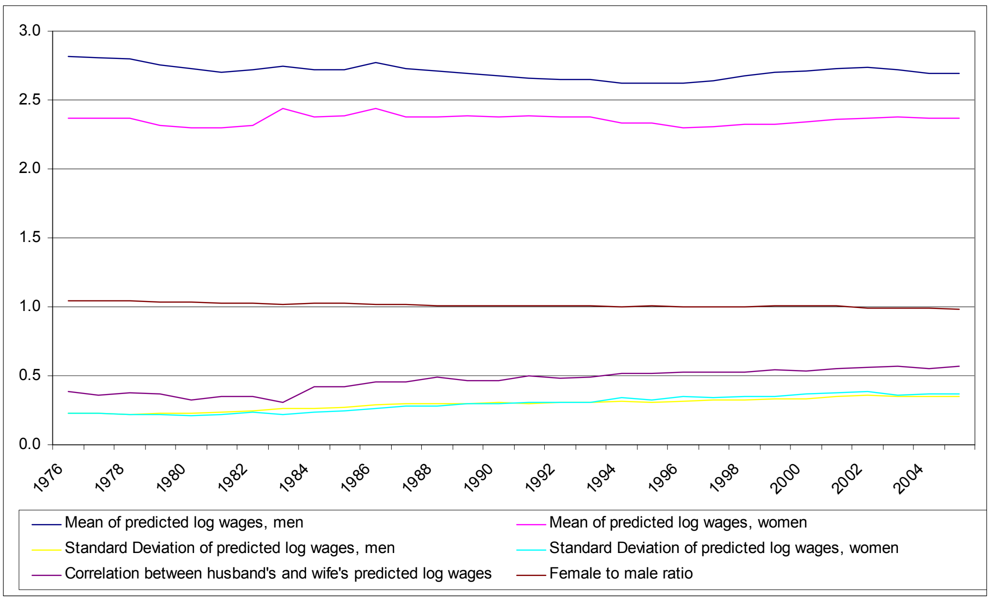

Figure 2: Parameters of the predicted log wage distribution of men and women and sex ratio, U.S. 1976-2005. Source: Current Population Surveys.

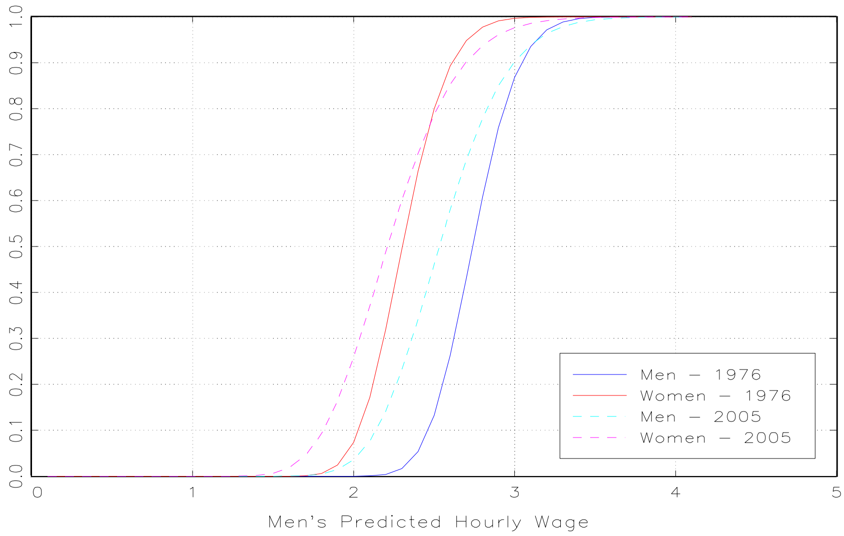

Figure 3: Cumulative distributions of predicted hourly wages of men and women

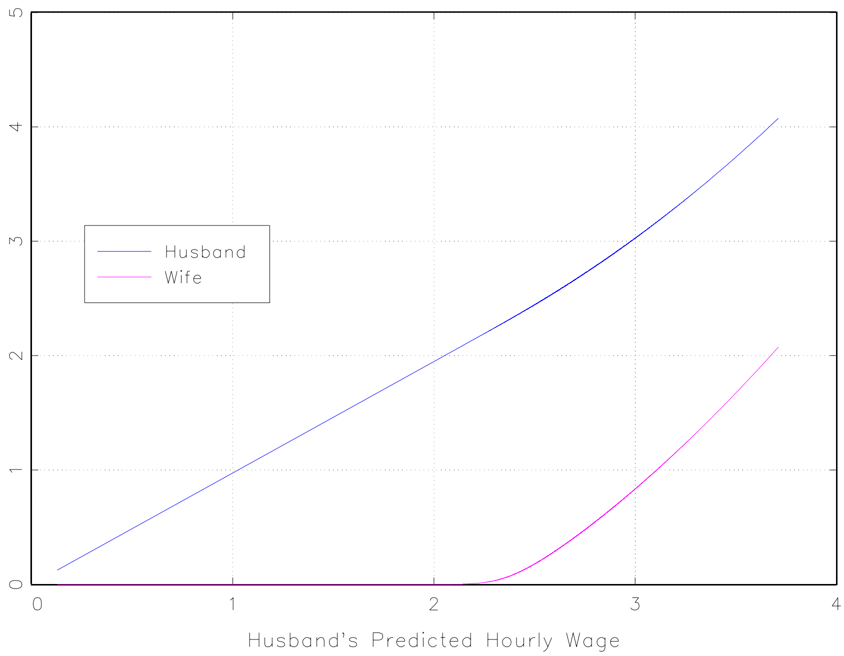

Figure 4: The surplus of married men and women in 1976, female to male ratio = 1.045

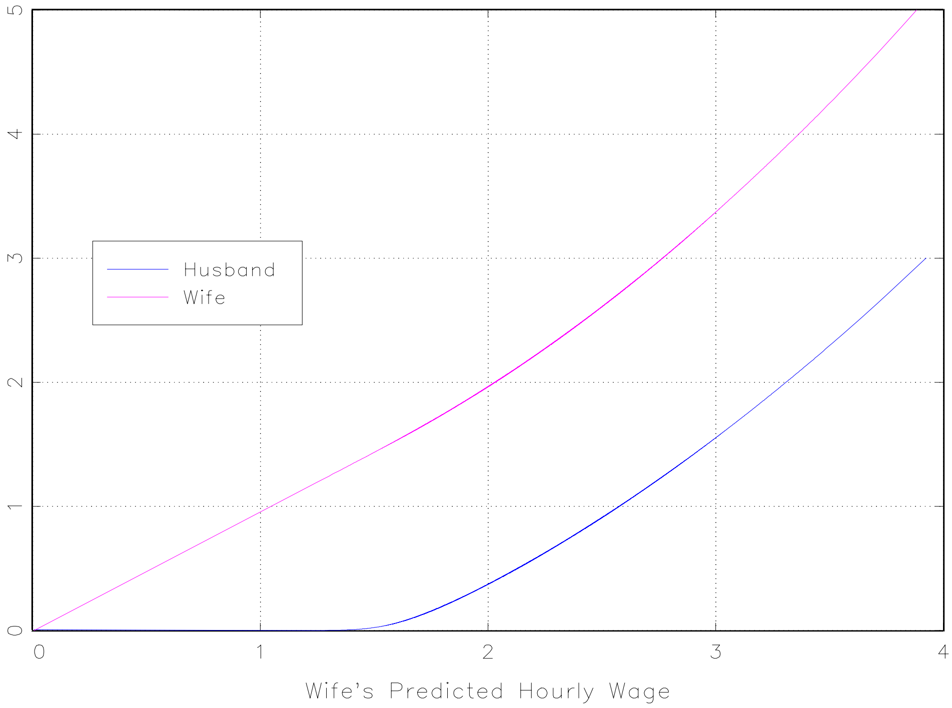

Figure 5: The surplus of married men and women in 2005, female to male ratio = 0.984

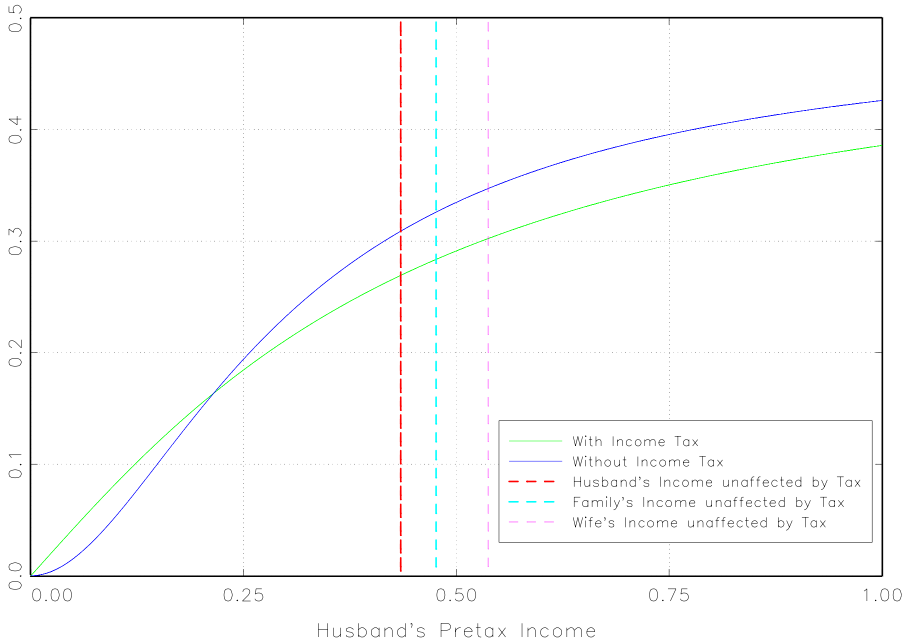

Figure 6: Share of wife's surplus relative to the sum of surpluses, women are the majority (r=1.1)

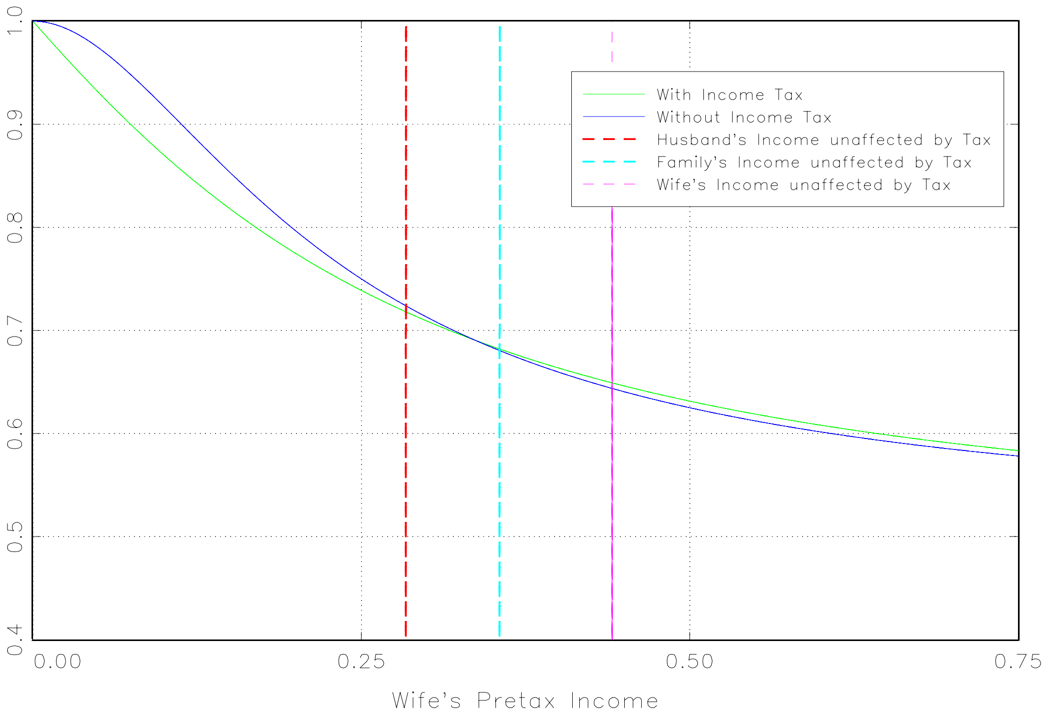

Figure 7: Share of wife's surplus relative to the joint surplus, men are the majority (r=0.9)

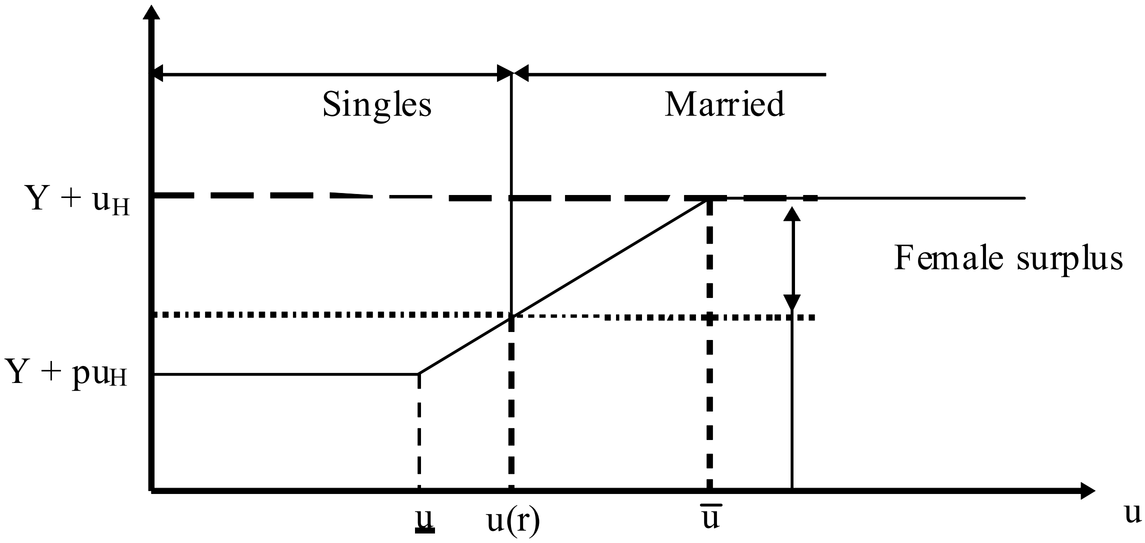

Figure 8: Maximum husband's utility as a function of the wife's taste - intermediate ESW

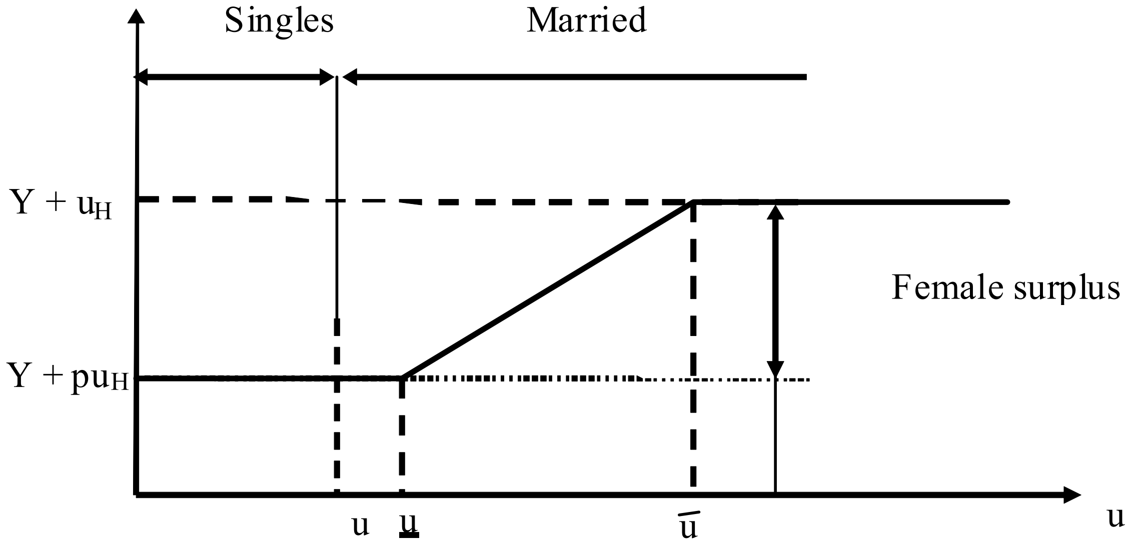

Figure 9: Maximum husband's utility as a function of the wife's taste - small ESW

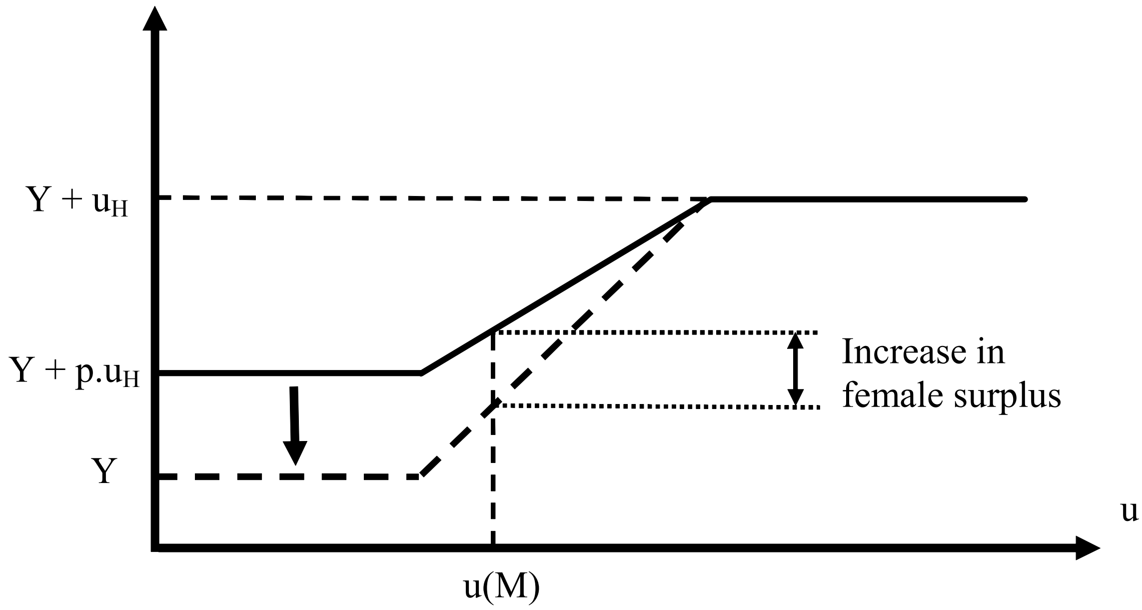

Figure 10:

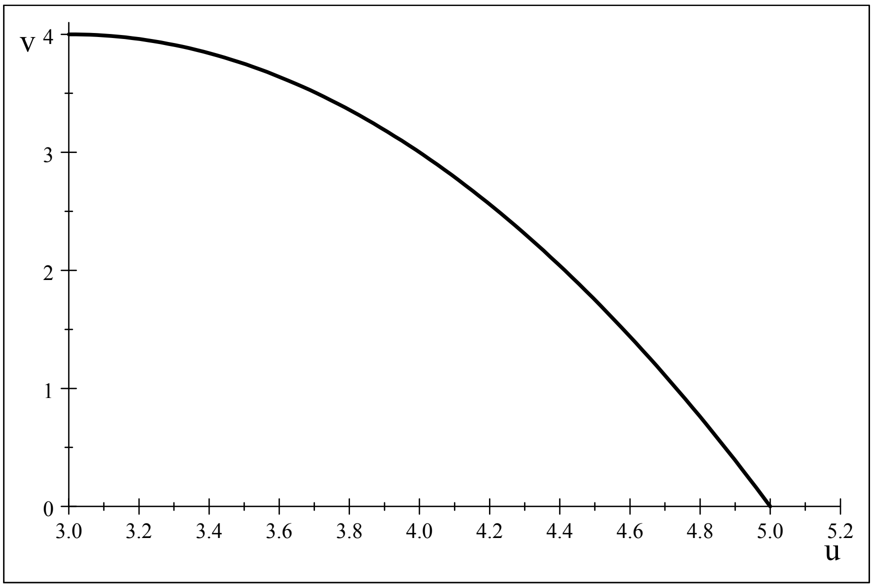

Figure 11: Pareto frontier

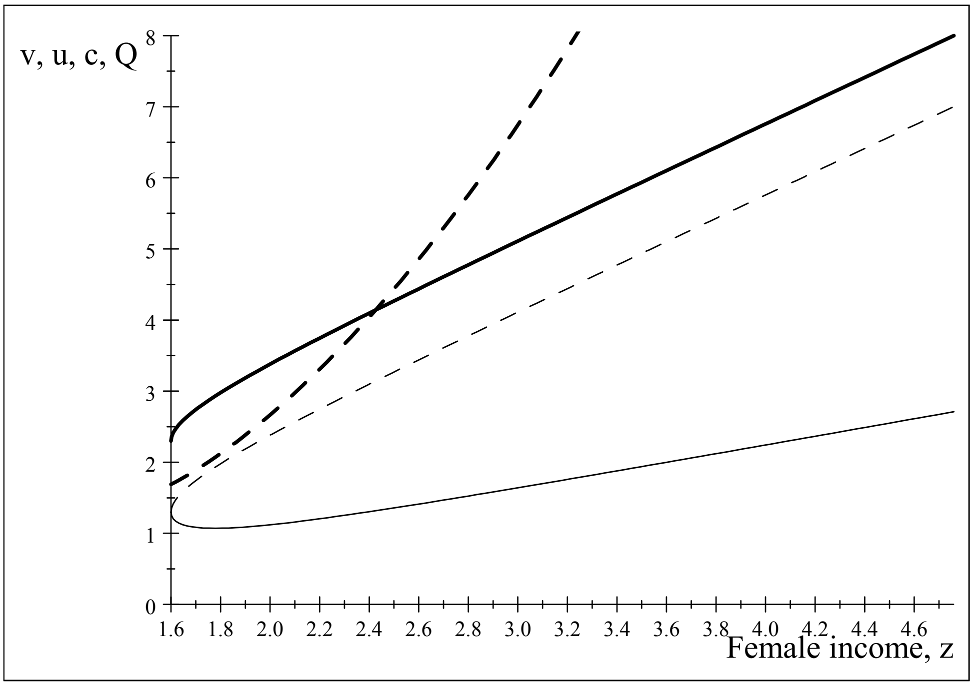

Figure 12: v -thick, u-thick-dash, c_m-thin, Q-thin-dash

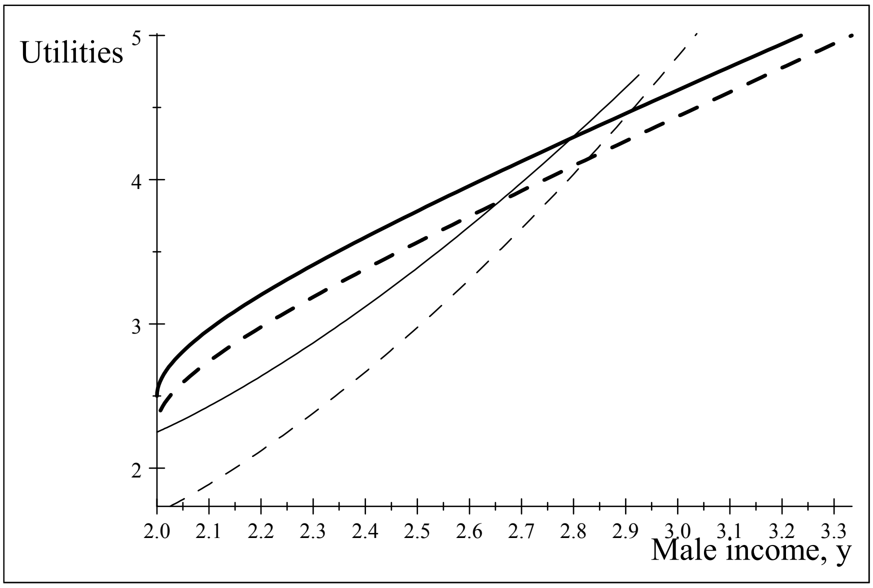

Figure 13: $\alpha = .8$: u-thin, v thick, $\alpha = 1$: u-thin-dashed, v thick-dashed