The Economics of the Family

Chapter 7: Matching on the Marriage Market: Theory

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1 Introduction

Individuals in society have many potential partners. This situation creates competition over the potential gains from marriage. In modern societies, explicit price mechanisms are not observed. Nevertheless, the assignment of partners and the sharing of the gains from marriage can be analyzed within a market framework. The main insight of this approach is that the decision to form and maintain a particular union depends on the whole range of opportunities and not only on the merits of the specific match. However, the absence of explicit prices raises important informational issues. There are two main issues distinguishing the approaches used in the matching literature. The first issue concerns the information structure and the second relates to the extent of transferability of resources among agents with different attributes. Specifically, models based on frictionless matching assume that perfect and costless information about potential matches is available to all participants; the resulting choices exclusively reflect the interaction of individual preferences. Such models may belong to several classes, depending on whether or not compensating transfers are allowed to take place between individuals and, if so, at what 'exchange rate'. Still, they all rely on a specific equilibrium concept, namely stability. Formally, we say that an assignment is *stable* if:

(i) There is no married person who would rather be single.

(ii) There are no two (married or unmarried) persons who prefer to form a new union.

The interest in stable marriage assignments arises from the presumption that in a frictionless world, a marriage structure which fails to satisfy (i) and (ii) either will not form or will not survive.

Models based on frictionless matching are studied in the next three Sections. An alternative approach emphasizes the role of frictions in the matching process; in these models, based on search theory, information is limited and it takes time to find a suitable match. The corresponding framework will be discussed in the last Section.

2 Stable Matching without transfers: the Gale-Shapley Algorithm

We begin our analysis of the marriage market assuming that there are no frictions - i.e., that each man and woman knows the potential gains from marrying any potential mate. Marriage can be viewed as a voluntary matching of males and females, allowing for the possibility of staying single. We consider here only monogamic marriages so that each person can have at most one spouse of the opposite sex. These assignments can be presented by matrices with 0/1 entries depending upon whether or not male i is married to female j. Since we consider only monogamic marriages, there is at most one none zero entry in each column and row. An illustration of such a representation with 4 men and 3 women, where man 1 is married to woman 3, man 2 is married to woman 1, man 3 is single and man 4 is married to woman 2 is shown in Example 7.1:

Example 7.1

	1	*2*	*3*
1	0	0	1
2	1	0	0
3	0	0	0
4	0	1	0

We first study matching when agents cannot make transfers between each other. We thus assume that a marriage generates an outcome for each partner that is fully determined by the individual traits of the partners; this outcome cannot be modified by one partner compensating the other for his or her deficient traits. Although somewhat extreme, this assumption captures situations where, because of public goods and social norms that regulate within family allocations, there is limited scope for transfers, so that the success of a marriage mainly depends on the attributes of the partners. However, an undesired marriage can be avoided or replaced by a better one. Although there is no scope for trade within marriage, there is margin for trade across couples.

Let there be a given number of men, M, and a given number of women, N. We designate a particular man by i and a particular woman by j. Assume that each man has a preference ranking over all women and each woman has a preference ordering over all men. Such preferences can be represented by a $M \times N$ bi-matrix with a pair of utility payoffs, (u_{ij}, v_{ij}) in each cell. For a given j, the entries v_{ij} describe the preference ordering of woman j over all feasible males, $i = 1, 2...M$. Similarly, for a given i, the entries u_{ij} describe the preference ordering of man i over all feasible women $j = 1, 2...N$. We may incorporate the ranking of the single state by adding a column and a row to the matrix, denoting the utility levels of single men and women by u_{i0} and v_{0j}, respectively. The preferences of men and women are datum for the analysis. However, the representations of these preferences by the utility payoffs are only unique up to monotone transformations. An illustration of such a representation with 4 men and 3 women is shown in Example 7.2:

Example 7.2

	1	*2*	*3*	*0*
1	u_{11}, v_{11}	u_{12}, v_{12}	u_{13}, v_{13}	u_{10}
2	u_{21}, v_{21}	u_{22}, v_{22}	u_{23}, v_{23}	u_{20}
3	u_{31}, v_{31}	u_{32}, v_{32}	u_{33}, v_{33}	u_{30}
4	u_{41}, v_{41}	u_{42}, v_{42}	u_{43}, v_{43}	u_{40}
0	v_{01}	v_{02}	v_{03}	

Gale and Shapley (1962) were the first to demonstrate that a stable matching always exists, and suggested an algorithm which generates a stable outcome. For simplicity, we assume here that all rankings are strict. To begin, let each man propose marriage to his most favored woman. A woman rejects any offer which is worse than the single state, and if she gets more than one offer she rejects all the dominated offers; the non rejected proposal is put on hold ('engagement'). In the second round, each man who is not currently engaged proposes to the woman that he prefers most among those women who have not rejected him. Women will reject all dominated offers, including the ones on hold. The process stops when no male is rejected. Convergence is ensured by the requirement that no woman is approached more than once by the same man; since the number of men and women is finite, this requirement implies that the process will stop in finite time. The process must yield a stable assignment because women can hold all previous offers. So if there is some pair not married to each other it is only because either the man did not propose (implying that he found a better mate or preferred staying single) or that he did and was rejected (implying that the potential wife had found a better mate or preferred staying single).

The stable assignment that is realized in the way just described need not be unique. For instance, a different stable assignment may be obtained if women make the offers and men can reject or store them. Comparing these stable assignments, it can be shown that if all men and women have strict preferences, the stable matching obtained when men (women) make the proposal is weakly preferred by *all* men (women). This remarkable result shows that social norms of courting can have a large impact on matching patterns (see Roth and Sotomayor 1990, ch. 2).

As an example, let there be 3 men and 3 women and consider the matrix of utility payoff in Example 7.3: (setting the value of being single of all agents to zero)

Example 7.3

		Women		
		1	2	3
Men	1	3,2	2,6	1,1
	2	4,3	7,2	2,4
	3	1,1	2,1	0,0

Note that, in this case, there is divergence of preferences among men; man 1 ranks woman 1 above women 2 and 3, while men 2 and 3 both put woman 2 at the top of their ranking. Similarly, there is divergence of preferences among women; man 1 is the most attractive match for woman 2, while women 1 and 3 both consider man 2 as the best match. There is also a lack of reciprocity; man 1 would rather marry woman 1 but, alas, she would rather marry man 2. As a consequence there are two possible stable assignments, depending on whether men or women move first.

If men move first, man 1 proposes to woman 1, and men 2 and 3 both propose to woman 2, who rejects man 3, but keeps man 2. In the second round, man 3 proposes to woman 1 who rejects him. In the last round, man 3 proposes to woman 3 and is not rejected so that the procedure ends up with the outcome emphasized in bold letters in the matrix below:

		Women		
		1	2	3
Men	1	**3,2**	2,6	1,1
	2	4,3	**7,2**	2,4
	3	1,1	2,1	**0,0**

One can check directly that this assignment is stable. Men 1 and 2 obtain their best option and do not wish to change spouse, while man 3 cannot find a better match who is willing to marry him.

Now, if women move first, woman 2 proposes to man 1 and women 1 and 3 both propose to man 2, who rejects woman 3, but keeps woman 1. In the second round, woman 3 proposes to man 1 who rejects her. In the last round, woman 3 proposes to man 3 and is not rejected so that the procedure ends up with the outcome emphasized in bold letters in the matrix below:

		Women		
		1	2	3
Men	1	3,2	**2,6**	1,1
	2	**4,3**	7,2	2,4
	3	1,1	2,1	**0,0**

Again, one can check directly that this assignment is stable. Women 1 and 2 obtain their best option and do not wish to change spouse, while woman 3 cannot find a better match who is willing to marry her. It is seen that the first assignment, in which men move first is better for all men (except for man 3 who is indifferent) and the second assignment, in which women move first is better for all women (except for woman 3 who is indifferent).

A very special case arises if women and men can be ranked by a single male trait x and a single female trait y. This assumption introduces a strong commonality in preferences, whereby all men agree on the ranking of all women and vice versa. Specifically, let us rank males and females by their marital endowment (i.e., $x_{i+1} > x_i$ and $y_{j+1} > y_j$), and let us assume that there exists a "household output function" $h(x_i, y_j)$ that specifies the marital output as a function of the attributes of the two partners.[1] This output is then consumed jointly as a public good, or shared between the partners in some rigid fashion (equally for instance) in all marriages. A natural question is: Who marries whom? Would a stable assignment associate a male with a high marital endowment to a female with high marital endowment (what is called *positive assortative mating*)? Or, to the contrary, will a highly endowed male be matched with a low endowment female (*negative assortative mating*)? The answer obviously depends on the properties of the function $h(x, y)$. It is easy to show that if $h(x, y)$ is strictly increasing in both traits, the unique stable assignment is one with perfect positive assortative mating. To see that, suppose that men propose first. In the first round, all men will propose to the woman with the highest female attribute and she will reject all offers but the one from the best man. In the second round, all remaining men will propose to the second best woman and she will reject all but the second best man and so on. The situation when women propose first is identical. Symmetrically, if the male and female traits have opposing effects on output, the unique stable assignment is one with perfect negative assortative mating.

In addition to the identification of stable assignments, one can use the Gale-Shapley algorithm to obtain simple comparative static results. Allowing for unequal numbers of men and women, it can be shown that a change in the sex ratio has the anticipated effect. An increase in the number of women increases the welfare of men and harms some women. The same result holds in many to one assignment.

3 Stable Matching with transferable utilities: the Becker-Shapley-Shubik model

3.1 The basic framework

The properties of the previous model heavily depend on the assumption that transfers are impossible, so that a person cannot 'compensate' a potential part-

[1] This "household output function" should be distinguished from the standard household production function described in the previous sections, which take the attributes of the spouses as fixed. Here we are interested in a reduced form that depends only on attributes after all relevant activities have been chosen so as to achieve intrahousehold efficiency.

ner for marrying him or her despite some negative traits. In practice, this assumption is hard to maintain. Whenever some commodities are privately consumed, a spouse can reduce his\her private consumption to the partner's benefit, which de facto implements a compensation. We now consider the opposite polar case in which not only transfers are feasible, but there is a medium of exchange that allows partners to transfer resources between them *at a fixed rate of exchange*; that is, we assume that utilities are transferable (see Chapter 3).

Instead of introducing two matrices $u = (u_{ij})$ and $v = (v_{ij})$ as in the case of non-transferable utility, we now consider a unique output matrix with entries ζ_{ij} which specifies the total output of each marriage. Given the assumption of transferable utility, this total output can be divided between the two partners. We denote the utility payoff of the husband by u_{ij} and the utility payoff of the wife by v_{ij}. Thus, by definition, if i and j form a match we have

$$u_{ij} + v_{ij} = \zeta_{ij} \tag{1}$$

As in the previous section with no transfers, we are interested only in *stable* matching. The question is: for a given matrix $\zeta = \left(\zeta_{ij}\right)$, which are the stable assignments, and what are the corresponding allocations of output (or *imputations*) within each marriage. Note that the question is, in a sense, more difficult than in the case with no transfers, since the distribution of output between members is now endogenous and has to be determined in equilibrium. Still, it is relatively easy to apply the criteria for stability in the case of transferable utility. Specifically, one can show that *a stable assignment must maximize total output over all possible assignments*. It is this simple and powerful result that makes the assumption of transferable utility attractive in matching models.

3.1.1 Two examples

To understand this result, consider first the simplest possible case. Let there be two people of each sex. Assuming that marriage dominates the single state (i.e. if any two individuals remain unattached they can gain by forming a union), there are two possible assignments: Man 1 marries woman 1 and man 2 marries woman 2, or man 1 is married to woman 2 and man 2 is married to woman 1. In testing for stability we treat the potential marital outputs ζ_{ij} as given and the divisions u_{ij} and v_{ij} as variables. Suppose, now, that the assignment in which man 1 marries woman 2 and man 2 marries woman 1 (the off diagonal assignment) is stable. Then, the following inequalities must hold

$$u_{12} + v_{21} \geq \zeta_{11} \tag{2a}$$

$$u_{21} + v_{12} \geq \zeta_{22} \tag{2b}$$

If the first inequality fails to hold then male 1 and female 1, who are currently not married to each other, can form a union with a division of utilities which will improve upon their current situations, defined by u_{12} and v_{21}. If the second inequality does not hold then man 2 and woman 2, who are presently not married to each other, can form a union and divide utilities so as to improve over the current values u_{21} and v_{12}. From equation 1 we have $\zeta_{12} = u_{12} + v_{12}$ and $\zeta_{21} = u_{21} + v_{21}$ so that equation (2a) can be rewritten as

$$\zeta_{12} - v_{12} + \zeta_{21} - u_{21} \geq \zeta_{11}. \tag{2a'}$$

Adding conditions (2a') and (2b) we obtain

$$\zeta_{12} + \zeta_{21} \geq \zeta_{11} + \zeta_{22} \tag{3}$$

By a similar argument, an assignment along the main diagonal will be stable only if (3) is reversed. Condition (3) is not only necessary but also sufficient for stability of the off diagonal assignment. For if it is satisfied we can find values of u and v such that (2a) and (2b) hold. Such imputations support the stability of the assignment since it is then impossible for both partners to gain from reassignment. More generally, for an arbitrary number of men and women, if there are any two alternative assignments, such that under one assignment the aggregate output is lower then that assignment cannot be stable, because for any division of the marital outputs there must be at least two individuals in different marriages who can gain by forming a new union.

To illustrate the implications of the transferable utility assumption and the implied maximization of *aggregate* marital output, let us consider a second example. There are 3 men and 3 women and consider the matrix of marital outputs below:

Example 7.4

		Women		
		1	2	3
Men	1	5	8	2
	2	7	9	6
	3	2	3	0

Notice that the entries in this matrix are just the sums of the two terms in Example 7.3 discussed above. In this regard, non transferable utility can be thought of as a special case of transferable utility, where the division of the output in each marriage is predetermined and cannot be modified by transfers between spouses. For instance, if each partner receives half of the marital output in *any* potential marriage, the Gale Shapley algorithm yields the unique stable outcome, which is on the diagonal of this matrix. In contrast, with transferable utility, the unique assignment that maximizes aggregate marital output is *not* on the diagonal as indicated by the bold numbers in the matrix below. This assignment yields aggregate output of 16, compared with an aggregate output of 14 on the diagonal.

	Women			
		1	2	3
Men	1	5	**8**	2
	2	7	9	**6**
	3	**2**	3	0

Though all men would obtain the highest marital output with woman 2, and all women would obtain the highest output with man 2 (implying that ζ_{22} is the largest entry in the marital output matrix 7.4), the best man and the best woman are *not* married to each other. With transfers, the assignment on the diagonal is no longer stable, because if couple $1,1$ and couple $2,2$ exchange partners, there is an aggregate gain of 1 unit of the transferable good. Then man 1 can, despite his lower contribution to the marital output, bid away the best woman by offering her a larger amount of private consumption and still be better off than in the initial match with woman 1. Similarly, woman 1 can bid away the best man by offering him a larger share of private consumption and still be better off than in the initial match with man 1. The higher *aggregate* output achievable when man 2 and woman 2 are *not* married to each other implies that, for any division of the marital output of 9 that these partners can obtain together, at least one of the partners can be made better off in an alternative marriage.

3.1.2 Stable matching with a finite number of agents

Let us now consider the general assignment problem with M males and N females. Let ζ_{ij} denote the total output of a marriage between male i and female j, and let ζ_{i0} (resp. ζ_{0j}) be the utility that person i (resp. person j) receives as single (with $\zeta_{00} = 0$ by notational convention). Then the difference $z_{ij} = \zeta_{ij} - \zeta_{i0} - \zeta_{0j}$ is the marital *surplus* that male i and female j generate by marrying each other.

We define assignment indicators, a_{ij}, such that $a_{ij} = 1$ if and only if i is married with j and $a_{ij} = 0$ otherwise. We also define $a_{i0} = 1$ if and only if i is single, and similarly $a_{0j} = 1$ if and only if j is single. Then, following Gale (1960, chapters 1 and 5) and Shapley and Shubik (1972), we may describe the stable assignment as a solution to an integer linear programming problem:

$$\max_{a_{ij}} \sum_{i=0}^{M} \sum_{j=0}^{N} a_{ij}\zeta_{ij} \tag{4}$$

subject to

$$\sum_{j=0}^{N} a_{ij} = 1, \; i = 1, 2., .M, \tag{5i}$$

$$\sum_{i=0}^{M} a_{ij} = 1, \; j = 1, 2., .N. \tag{5j}$$

A first remark is that since $a_{0j} = 1 - \sum_{i=1}^{M} a_{ij}$ and $a_{i0} = 1 - \sum_{j=1}^{N} a_{ij}$ the program can be rewritten as

$$\max_{a_{ij}} \sum_{i=1}^{M} \sum_{j=1}^{N} a_{ij} \left(\zeta_{ij} - \zeta_{i0} - \zeta_{0j}\right) + C = \max_{a_{ij}} \sum_{i=1}^{M} \sum_{j=1}^{N} a_{ij} z_{ij} + C \tag{4'}$$

subject to

$$\begin{aligned} \sum_{j=1}^{N} a_{ij} &\leq 1, \ i = 1, 2., .M, &\text{(5i')} \\ \sum_{i=1}^{M} a_{ij} &\leq 1, . \ j = 1, 2., .N, &\text{(5j')} \end{aligned}$$

where $C = \sum_{i=1}^{M} \zeta_{i0} + \sum_{j=1}^{N} \zeta_{0j}$ is the aggregate utility of singles. Therefore, the maximization of aggregate marital output over all possible assignments is equivalent to the maximization of aggregate surplus and, without loss of generality, we can normalize the individual utilities by setting $\zeta_{i0} = \zeta_{0j} = 0$ for all i and j.

Secondly, one can actually assume that in the problem above, the a_{ij} can be real numbers in the $(M-1)$-dimensional simplex (instead of constraining them to be integers). Intuitively, a_{ij} can then be interpreted as the probability that Mr. i marries Mrs. j. Note, however, that given the linearity of the structure, the solution of this generalized problem is anyway attained with all a_{ij} being either zero or one.

The basic remark, at that point, is that the program thus defined is a standard, linear programming problem; i.e., we want to find a vector (a_{ij}) that maximizes the linear objective (4) (or (4')) subject to the linear constraints (5i) and (5j) (resp. (5i') and (5j')). We can therefore use the standard tools of linear programming - specifically, duality theory. Associated with the maximization of aggregate surplus which determines the assignment is a dual cost minimization problem that determines the set of possible divisions of the surplus. Specifically, one can define a dual variable u_i for each constraint (5i') and a dual variable v_j for each constraint (5j'); the dual program is then:

$$\min_{u_i, v_j} \left(\sum_{i=1}^{M} u_i + \sum_{j=1}^{N} v_j\right) \tag{6}$$

subject to

$$\begin{aligned} u_i + v_j &\geq z_{ij}, \ i \in \{1, ..., M\}, \ j \in \{1, ..., N\} &\text{(7)} \\ u_i &\geq 0, \ v_j \geq 0. \end{aligned}$$

The optimal values of u_i and v_j can be interpreted as shadow prices of the constraints in the original maximization problem (the primal). Thus, $u_i + v_j =$

z_{ij} if a marriage is formed and $u_i + v_j \geq z_{ij}$ otherwise.[2] This result is referred in the literature as the complementarity slackness condition, see for instance Gale (1978). It has a very simple interpretation. Any man i is a resource that be can allocated to any woman, but only one woman, in society. Similarly woman j is a resource that can be allocated to any man in society, but only one man. The shadow price of each constraint in 5' describes the social cost of moving a particular man (woman) from the pool of singles, where he (she) is a potential match for others. The sum of these costs $u_i + v_j$ is the social cost of matching man i and woman j and z_{ij} is the social gain. Thus if $u_i + v_j > z_{ij}$, the costs exceed the gains and the particular marriage would not form. However, if a marriage is formed then $u_i + v_j = z_{ij}$ and each person's share in the resulting surplus equals the opportunity costs of the two spouses in alternative matches.

The crucial implication of all this is that the shadow price u_i is simply the share of the surplus that Mr.i will receive at the stable match (and similarly for v_j); consequently, conditions (7) are nothing else than the stability conditions, stating that if i and j are not matched at the stable match, then it must be the case that the surplus they would generate if matched together (i.e. z_{ij}) is not sufficient to increase *both* utilities above their current level!

These results have a nice interpretation in terms of decentralization of the stable match. Indeed, a stable assignment can be supported (implemented) by a reservation utility vector, whereby male i enters the market with a reservation utility u_i and is selected by the woman that gains the highest surplus $z_{ij} - u_i$ from marrying him. Similarly, woman j enters with a reservation utility v_j and is selected by the man who has the highest gain $z_{ij} - v_j$ from marrying her. In equilibrium, each agent receives a share in marital surplus that equals his\her reservation utility. In a sense, u_i and v_j can be thought of as the 'price' that must be paid to marry Mr. i or Mrs. j; each agent maximizes his/her welfare taking as given this 'price' vector.

It is important to note that the informational requirements for implementing a stable assignment with transferable utility is quite different than for the Gale-Shapley no transfer case. For the latter, we only require that each person can rank the members of the opposite sex. With transferable utility, the planner needs to know the surplus values of all possible matches and each agent should know the share of the surplus that he\she can receive with any potential spouse.

In general, there is a whole set of values for u_i , v_j that support a stable assignment. For instance, the stable assignment for example 7.4 can be supported by the three (of many) imputations denoted by a, b and c listed below.

Women				Men			
	a	b	c		a	b	c
v_1	2	2	1	u_1	3	4	5
v_2	5	4	3	u_2	5	5.5	6
v_3	1	0.5	0	u_3	0	0	1

[2] Conversely, a_{ij} can be seen as the dual variable for constraint (7). In particular, if $a_{ij} > 0$, then the constraint must be binding, implying that $u_i + v_j = z_{ij}$.

		Women		
		1	2	3
Men	1	5	**8**	2
	2	7	9	**6**
	3	**2**	3	0

The reader can readily check that each of these imputations supports a stable match. That is, $u_i + v_j \geq z_{ij}$, with equality if a marriage forms and inequality otherwise. Note that as we move from a to b to c, the share of the husband in the marital surplus rises (or does no change) while the share of the wife declines (or does not change) in *all* of the three stable marriages. The next chapter provides an extensive discussion on the division of the surplus in a stable assignment.

3.1.3 Extension: continuum of agents

Finally, although the previous argument is presented in a finite setting, it is fully general, and applies to continuous models as well. From a general perspective, we only need that the set of men and the set of women, denoted X and Y, be complete, separable metric spaces equipped with Borel probability measures F and G; note that no restriction is imposed on the dimension of these spaces (it may even be infinite). There exists a surplus function $h(x, y)$ which is only assumed to be upper semicontinuous. The problem can be stated as follows. Can one find a measure Φ on $X \times Y$ such that:

- The marginals of Φ on X and Y are F and G, respectively.
- The measure Φ solves $\max_{\Phi} \int_{X \times Y} h(x, y)\, d\Phi(x, y)$, where the max is taken over the set of measures satisfying the previous conditions.

A complete analysis of this problem is outside the scope of this book; the reader is referred to Chiappori, McCann and Neishem (2010) or Ekeland (2010) for recent presentations. Let us just mention that the existence of a stable match obtains in general; this comes from the fact that the linear optimization problem does have a solution under very general assumptions.

3.2 Assortative mating

3.2.1 The basic result

Suppose, as above, that each male is endowed with a single characteristic, x, and each female is endowed with a single characteristic, y, which positively affects the family's output. When can we expect the stable assignments to exhibit either positive or negative assortative mating? Again, the answer is quite different from the no transfer case. It follows in the present case from the observation that a stable assignment must maximize the aggregate marital output (or surplus) over all possible assignments.

Specifically, let, as above,

$$\zeta_{ij} = h(x_i, y_j) \tag{9}$$

be the household output function that specifies the marital output as a function of the attributes of the two partners. We say that a function $h(x_i, y_j)$ is *super modular* if $x' > x$ and $y\,' > y$ always imply that

$$h(x', y') + h(x, y) \geq h(x', y) + h(x, y'), \tag{10}$$

and it is *sub modular* if inequality (10) is always reversed. This definition captures the idea of complementarity and substitution as usually understood. Rewriting (10) in the form

$$h(x', y') - h(x', y) \geq h(x, y') - h(x, y), \tag{10'}$$

we see that the requirement is that the contribution to marital output of a given increase in the female attribute rises with the level at which the male trait is held fixed. By a similar rearrangement, the impact of a given increase in the male's attribute rises in the female's attribute. Note also that if h is twice differentiable then h is super (sub) modular if the second cross derivative h_{yx} is always positive (negative).[3] The condition that h_{yx} is monotonic is sometimes called the *single crossing* or the *Spence-Mirrlees* condition; indeed, a similar condition is crucial in contract theory, signalling models (a la Spence) and optimal taxation (a la Mirrlees).

The basic result is that complementarity (substitution) in traits must lead to a positive (negative) assortative mating; otherwise aggregate output is not maximized. Assuming that $h(x, y)$ is increasing in x and y, we obtain that in the case of positive assortative mating, the best man marries the best woman, and if there are more women than men the women with low female quality remain single.[4] If there is negative assortative mating, the best man marries the worst woman among the married women but if there are more women than men, it

[3] Indeed, for any given (m, f) define

$$H(m', f') = h(m', f') + h(m, f) - h(m', f) - h(m, f').$$

Then

$$H_{m'}(m', f') = h_{m'}(m', f') - h_{m'}(m', f)$$

which is positive for $f\prime > f$ if $h_{mf} \geq 0$ (since h_m is then increasing in f). Similarly,

$$H_{f'}(m', f') = h_{f'}(m', f') - h_{f'}(m, f') \geq 0$$

for $m\prime > m$ if $h_{mf} \geq 0$. Hence H is increasing in its arguments, and $H(m, f) = 0$; we conclude that $H(m', f') \geq 0$ whenever $m\prime > m$ and $f\prime > f$ and $h_{mf} \geq 0$.

[4] Let $\bar{m}$ and $\bar{f}$ denote the endowments of the "best" man and woman and suppose that they are *not* married to each other and instead man $\bar{m}$ marries some woman whose female attribute is $f' < \bar{f}$ and woman $\bar{f}$ marries some man whose attribute is $m' < \bar{m}$. Then stability

is the women with the lower female attributes who remain single. [5] In other words, who marries whom depends on second order derivatives of $h(x, y)$ but who remains single depends on the first order derivatives of $h(x, y)$. If there is no interaction in traits and the marginal contribution of each agent is the same in all marriages, any assignment is (weakly) stable and it does not matter who marries whom, because whichever way we arrange the marriages the aggregate output of all marriages remains the same.

We may explain these results intuitively by referring again to the basic idea of a stable assignment. Complementarity (substitution) implies that males with high x will be willing to pay more (less) for the female attribute. Thus, if x stands for money and y stands for beauty, the wealthy men will be matched with the pretty women if and only if their (marginal) willingness to pay for beauty is higher. If there is negative interaction between money and beauty, the most wealthy man will *not* marry the most pretty woman, because whichever way they divide their gains from marriage, either he is bid away by a less pretty woman or she is bid away by a poorer man.

This result is in a sharp contrast to the no transferable case, where monotonicity in traits is sufficient to determine the outcome.[6] The consequence is that

of these matches requires the existence of divisions such that

$$\begin{aligned} u_{\bar{m},f\prime} + v_{m',\bar{f}} &\geq h(\bar{m},\bar{f}), \\ u_{m',\bar{f}} + v_{\bar{m},f} &\geq h(m',f'), \\ u_{\bar{m},f\prime} + v_{\bar{m},f\prime} &= h(\bar{m},f\prime), \\ u_{m',\bar{f}} + v_{m',\bar{f}} &= h(m',\bar{f}), \end{aligned}$$

which implies that

$$h(\bar{m},f\prime) + h(m',\bar{f}) \geq h(m',f') + h(\bar{m},\bar{f})$$

and contradicts (strict) super modularity. Thus complementarity implies that the best man must marry the best woman. Eliminating this couple, and restricting attention to the next best pair, we see that it must marry too and so on.

[5] Suppose that are more women than men. Then there must be some woman f_m such that all woman with lesser quality are single. Otherwise, there must be a married woman with a lower quality than some single woman, which under monotonicity implies that aggregate output is not maximized. Now man $\bar{m}$ and f_m must marry each other. If they are not married to each other and, instead, man $\bar{m}$ marries some woman whose female attribute is $f' > f_m$ and woman f_m marries some man whose attribute is $m' < \bar{m}$, then stability of these matches requires the existence of divisions such that

$$\begin{aligned} u_{\bar{m},f\prime} + v_{m',f_m} &\geq h(\bar{m},f_m), \\ u_{m',f_m} + v_{\bar{m},f} &\geq h(m',f'), \\ u_{\bar{m},f'} + v_{\bar{m},f\prime} &= h(\bar{m},f\prime), \\ u_{m'f_m} + v_{m',f_m} &= h(m',f_m), \end{aligned}$$

which implies that

$$h(\bar{m},f\prime) + h(m',f_m) \geq h(m',f') + h(\bar{m},f_m)$$

and contradicts (strict) sub modularity. Eliminating this couple, and restricting attention to the next best pair, i.e. the second best man among men and the second worst woman among all married women must marry too, and so on.

[6] However, monotonicity may fail to hold when super modularity holds. A potentially important case is when preferences are single peaked in the attribute of the spouse. In such cases, we can have assortative mating in the sense that married partners have similar traits,

assortative (negative or positive) mating is more prevalent in the absence of transfers, because it is impossible for agents with less desirable traits to compensate their spouses through a larger share of the marital output (see Becker, 1991, ch. 4 and Becker and Murphy, ch. 12). The sad message for the econometrician is that, based on the same information, namely the household production function, one can get very different outcomes depending on the ability to compensate within households, a feature that we usually cannot directly observe.

Finally, the impact of traits on the value of being single does not affect these considerations, because the welfare of each person as single depends only on his own traits. Therefore, in the aggregate, the output that individuals obtain as singles is independent of the assignment. Although the value of being single does matter to the question who marries, it does not affects who marries whom, in equilibrium.

3.2.2 Examples

In many models, the surplus function takes a specific form. Namely, the two traits x and y can often be interpreted as the spouses' respective incomes. Following the collective approach described in the previous Chapters, we may assume that a couple consisting of a husband with income x and a wife with income y will make Pareto efficient decisions; then it behaves as if it was maximizing a weighted sum of individual utilities, subject to a budget constraint. The important remark is that the constraint only depends on the sum of individual incomes. Than the Pareto frontier - or in our specific case the value of the surplus function $h(x,y)$ which defines it - only depends on the sum $(x+y)$[7]; i.e.:

$$h(x,y) = \bar{h}(x+y)$$

The various properties described above take a particular form in this context. For instance, the second cross derivative h_{xy} is here equal to the second derivative $\bar{h}''$. It follows that we have assortative matching if $\bar{h}$ is convex, and negative assortative matching if $\bar{h}$ is concave. The interpretation is as above: a convex $\bar{h}$ means that an additional dollar in income is more profitable for wealthier people - meaning that wealthier husbands are willing to bid more aggressively for a rich wife than their poorer competitors. Conversely, if $\bar{h}$ is concave then the marginal dollar has more value for poorer husbands, who will outbid the richer ones.

In models of this type, the TU assumption tends actually to generate convex output functions, hence assortative matching. To see why, consider a simple model of transferable utility in the presence of public good. Preferences take

but individuals with extreme traits may fail to marry. The interested reader may consider the case in which the marital surplus is given by $g-(m-f)^2$.

[7] Of course, while the Pareto set only depends on total income, the *location* of the point ultimately chosen on the Pareto frontier depends on individual incomes - or more specifically on the location of each spouse's income within the corresponding income distribution. These issues will be analyzed in the next Chapter.

the form

$$u_i = c_i g(q) + f_i(q), \tag{2}$$

where c and q denote private and public consumption, respectively. The Pareto frontier is then

$$u_a + u_b = h(Y) = \max_q [(Y - q)g(q) + f(q)],$$

where $f(q) = f_a(q) + f_b(q)$ and $Y = x + y$. By the envelope theorem, $h'(Y) = g(q)$ and therefore,

$$h''(Y) = g'(q)\frac{dq}{dz} = \frac{-(g'(q))^2}{(y-q)g''(q) - 2g'(q) + f''(q)} > 0.$$

The denominator must be negative for the second order conditions for a maximum to hold. Hence, if there is an interior solution for q, the household production function is *convex* in family income, Y, implying that the two incomes x and y must be complements.

As an illustration, recall examples 2.1 and 2.2 from Chapter 2. In example 2.1, the spouses pool their (fixed) incomes and share a public good. Individual preferences where of the form $u_i = c_i.q$, compatible with (2). If we now rank men and women by their incomes we have a situation in which the household production function is $h(x, y) = \frac{(x+y)^2}{4}$. This is a convex function of total income; there is a positive interaction everywhere, leading to assortative sorting.

In contrast, for example 2.2, in which division of labor has led to marital output given by $\max(w_i, w_j)$ - which is *not* a function of total income. Here, we obtain negative assortative mating. This holds because a high wage person is more useful to a low wage person, as indicated by the submodularity of $h(x, y) = \max(x, y)$.[8] For instance, if man i has wage i and woman j has wage j, the output matrix for the 3 by 3 case is:

Example 7.5

		Women		
		1	2	3
Men	1	1	*2*	**3**
	2	2	**2**	*3*
	3	**3**	3	3

[8] For all $m\prime \geq m$ and $f\prime \geq f$, we have $\max(m', f') + \max(m, f) \leq \max(m', f) + \max(m, f')$ .Going over the six possible orderings of four numbers $m'm, f', f'$ satisfying $m\prime \geq m$ and $f\prime \geq f$, we see that

$$\begin{array}{lcl}
m' \geq m \geq f' \geq f & \Rightarrow & m' + m \leq m' + m, \\
m' \geq f' \geq m \geq f & \Rightarrow & m' + m \leq m' +, f', \\
m' \geq f' \geq f \geq m & \Rightarrow & m' + f \leq m' + f', \\
f' \geq f \geq m' \geq m & \Rightarrow & f' + f \leq f + f', \\
f' \geq m' \geq f \geq m & \Rightarrow & f' + f \leq m' +, f', \\
f' \geq m' \geq m \geq f & \Rightarrow & f' + m \leq m' + f'.
\end{array}$$

implying three stable assignments; the opposite diagonal (in bold), one close to it in which couples (1,3) and (2,2) exchange partners (emphasized), and a symmetric one in which couples (3,1) and (2,2) exchange partners. The assignment also depends on the location of the wage distribution for each gender. As an extreme case, let the worst woman have a higher wage than the best man. Then in all marriages the female wage determines the outcome and all assignments are equally good.

Note, finally, that in the absence of any interaction, we have $h(x, y) = x + y$; this describes a situation where the two spouses simply pool their incomes and consume only private goods. Since the output is a linear function of both incomes, any assignment of men to women is stable. It is interesting that although the assignment is completely indeterminate, the set of imputations shrinks substantially and is given by

$$\begin{aligned} v_i &= x_i + p, \\ u_j &= y_j - p, \end{aligned} \tag{11}$$

for some fixed p. Thus, in the absence of interaction in traits, the *same* transfer p occurs in *all* marriages and we may interpret it as a common bride price or dowry, depending on whether p is positive or negative in equilibrium.[9] As we shall show in the next Chapter, if there is interaction in traits, this single price is replaced by an intrahousehold allocation rule that depends on the attributes of both partners.

3.3 Matching with a continuum of agents

The discussion above shows that a crucial feature of the problem is the interaction in the traits that the two partners bring into marriage. We shall for the time being focus here on situations where income is the only marital trait and individual incomes are complement in the household output function - i.e., $h(x, y)$ is super modular, or $h_{xy}(x, y) > 0$. Moreover, we assume here that there

[9] Consider any two couples, (i, j) and (r, s), in a stable assignment. Then, using the duality results,

$$\begin{aligned} u_i + v_j &= m_i + f_j \\ u_r + v_s &= m_r + f_s, \end{aligned}$$

because the imputations for married couples exhaust the marital output. Also, because couples (i, s) and (r, j) are not married to each other

$$\begin{aligned} u_i + v_s &\geq m_i + f_s \\ u_r + v_j &\geq m_r + f_j \end{aligned}$$

But none of these inequalities can be strict, because their sum must equal to the sum of the equalities above. It then follows that in *all* marriages on *any* stable assignment

$$\begin{aligned} u_i - u_r &= m_i - m_r \\ v_j - v_s &= f_j - f_s, \end{aligned}$$

which is equivalent to (11).

exists a *continuum* of men, whose incomes x are distributed on $[0,1]$ according to some distribution F, and a continuum of women, whose incomes y are distributed on $[0,1]$ according to some distribution G. The measure of all men in the population is normalized to 1, and the measure of women is denoted by r. We allow different income distributions for men and women.

The assumed positive interaction implies a positive assortative matching. Therefore, if a man with income x is married to a woman with income y, then the set of men with incomes above x must have the same measure as the set of women with incomes above y. Thus, for all x and y in the set of married couples,

$$1 - F(x) = r(1 - G(y)). \tag{13}$$

Hence,

$$x = \Phi[1 - r(1 - G(y))] \overline{=} \phi(y), \tag{14}$$

where $\Phi = F^{-1}$, or equivalently,

$$y = \Psi\left[1 - \frac{1}{r}(1 - F(x))\right] \overline{=} \psi(x), \tag{15}$$

where $\Psi = G^{-1}$ and $\psi = \phi^{-1}$.

If $r = 1$, the assignment matches men and women of the same quntile in their respective income distributions. Condition (13) modifies this rule when the male and female populations are of unequal size. The sex ratio r and the differences in the male and female income distributions determine the husband's and wife's incomes for each pair that marries. All men and women are married if there is an equal measure of men and women, $r = 1$. All women are married if there is scarcity of women, $r < 1$, implying that men with income x less than $x_0 = \Phi(1 - r)$ remain single. All men are married if there is scarcity of men, $r > 1$, implying that women with income y less than $y_0 = \Psi(1 - 1/r)$ remain single. If $r > 1$, then the function $y = \psi(x)$ determines the income of the wife for each man with income x in the interval $[0,1]$. Similarly, if $r < 1$, then the function $x = \phi(y)$ determines the husband's income of each woman with income y in the interval $[0,1]$. We shall refer to these functions as the *matching functions* and to the resulting assignment as the *assignment profile.*

In Figure 1 we show the matching function $\psi(x)$ for the case in which x is distributed uniformly on $[0,1]$, y is distributed uniformly on $[0,\sigma]$, $\sigma < 1$ and $r > 1$. Applying (13) and solving

$$1 - x = r(1 - \frac{y}{\sigma}),$$

we obtain

$$\psi(x) = \frac{\sigma}{r}(r - 1 + x).$$

We see that women with incomes y such that $y \leq y_0 = \frac{\sigma}{r}(r-1)$ remain single. Women with incomes in the range $[y_0, y'] = [\frac{\sigma}{r}(r-1, \frac{\sigma}{r}(r-1+x')]$ marry men with incomes in the range $[0, x\prime]$. Finally, women with incomes in the range

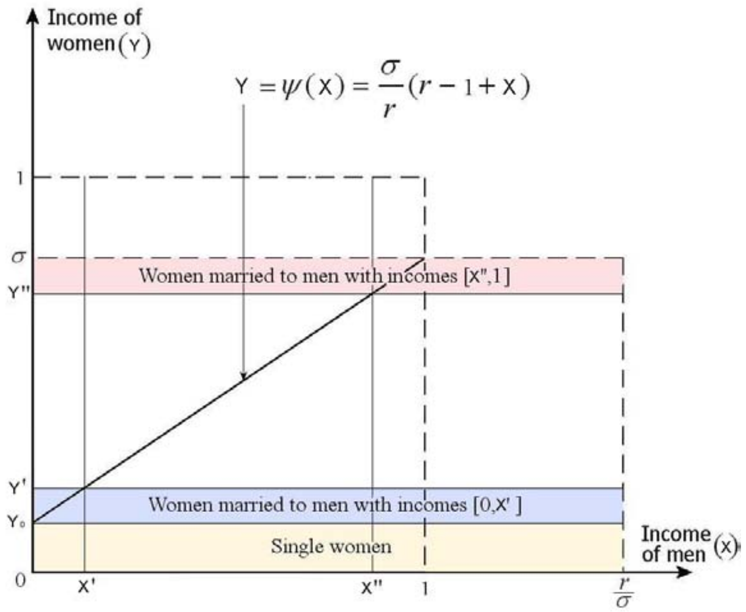

Figure 1: Positive Assortative Mating

$[y'', \sigma] = [\frac{\sigma}{r}(r-1+x''), \sigma]$ marry men with incomes in the range $[x'', 1]$. Thus women with higher incomes marry men with higher incomes. Note the equality in the measures of women and men in these intervals, as indicated by the areas of the corresponding rectangulars. For instance the rectangular with base $x\prime$ and height 1 has the same area as the rectangular with base $\frac{r}{\sigma}$ and height $\frac{\sigma}{r}(r-1+x') - \frac{\sigma}{r}(r-1)$. Such equality of measures must hold throughout the assignment profile.

The slope of each matching function is related to the *local scarcity* of men relative to women. Men are locally scarce if there are more women than men at the assigned incomes $(\phi(y), y) = (x, \psi(x))$. Or, equivalently, if an increase in the husband's income is associated with a smaller increase in the income of the matched wife. That is,

$$\begin{aligned} \frac{dx}{dy} &= \phi'(y) = r\frac{g(y)}{f(\phi(y))} > 1, \qquad (16) \\ \frac{dy}{dx} &= \psi'(x) = \frac{1}{r}\frac{f(x)}{g(\psi(x))} < 1. \end{aligned}$$

Men are locally abundant if these inequalities are reversed.

3.4 Multidimensional matching

The previous discussion explicitly refers to a one-dimensional framework. Assortative matching is harder to define when several dimensions (or several traits) are involved; moreover, conditions like supermodularity or single-crossing do not have an obvious extension to a multidimensional setting. Still, they can be generalized; again, the reader is referred to Chiappori, McCann and Neishem (2010) or Ekeland (2010) for recent presentations. The main insights can briefly be described as follows. Assume that X and Y are finite dimensional. Then:

1. The Spence-Mirrlees condition generalizes as follows: if $\partial^x h\,(x_0, y)$ denotes the *superdifferential* of h in x at (x_0, y), then for almost all x_0, $\partial^x h\,(x_0, y_1)$ is disjoint from $\partial^x s(x_0, y_2)$ for all $y_1 \neq y_2$ in Y. This is the 'twisted buyer' condition in Chiappori, McCann and Neishem 2010.

2. If the 'twisted buyer condition is satisfied, then the optimal match is unique; in addition, it is *pure*, in the sense that the support of the optimal measure Φ is born by the graph of some function $y = \phi\,(x)$; i.e., for any x there exists exactly one y such that x is matched with y with probability one.

3. There exists a relaxation of the 'twisted buyer' condition (called the 'semi-twist') that guarantees uniqueness but not purity.

The notion of 'superdifferential' generalizes the standard idea of a linear tangent subspace to non differentiable functions. If h is differentiable, as is the case is most economic applications, then $\partial^x h\,(x_0, y)$ is simply the linear tangent (in x) subspace to h at (x_0, y), and the condition states that for almost all x_0, there exists a one to one correspondence between y and $\partial^x h\,(x_0, y)$. Note that if X and Y are one-dimensional, then $\partial^x h\,(x_0, y)$ is fully defined by the partial $\partial h/\partial x\,(x_0, y)$, and the condition simply requires that $\partial h/\partial x\,(x_0, y)$ be strictly monotonic in y - i.e., the sign of $\partial^2 h/\partial x \partial y$ be constant, the standard single-crossing condition, Similarly, if X and Y are one-dimensional, then purity imposes a one-to-one matching relationship between x and y; if this matching is continuous, it has to be monotonic, i.e. matching must be either positive or negative assortative (in that sense, purity is a generalization of assortativeness to multi-dimensional settings).

In general, purity rules out situations in which a subset of agents (with a positive measure) randomize between several, equivalent matches. Such situations may be frequent in practice; in particular, Chiappori, McCann and Neishem (2010) show that they are likely to occur when agents are located on an Hotelling-type circle. Finally, only recently have empirical models of multidimensional matching been developed; the main reference, here, is Galichon and Salanié (2009).

4 Matching with general utilities

In the previous two sections, the matching process is studied in specific and somewhat extreme settings: either transfers cannot take place at all, or they can be made at a constant exchange rate (so that reducing a member's utility by one 'unit' increases the spouse's utility by one unit as well). We now consider the general case, in which although transfers are feasible, there is no commodity that allows the partners to transfer utilities at a fixed rate of exchange. Then the utility frontier is no longer linear and it is impossible to summarize the marital output from a match by a single number. In this more general framework, stability is defined in the same manner as before, that is, an assignment is stable if no pair who is currently not married can marry and choose an allocation of family resources that yields a result which is better for both of them than under the existing assignment and associated payoffs. Observe that the assignment and payoffs are simultaneously restricted by this definition. However, it is no longer true that aggregate marital output must be maximized - actually, such an 'aggregate output' is not even defined in that case. Mathematically, the matching model is *no longer* equivalent to an optimization problem.

Still, it is in principle possible to simultaneously solve for the stable assignment and the associated distribution(s) of surplus. The interested reader is referred to Roth and Sotomayer (1990, ch. 6), Crawford (1991), Chiappori and Reny (2006) and Legros and Newman (2007). To give a quick idea of how the general problem can be approached with a continuum of agents, let us assume, as above, that each agent is characterized by one trait, and let's assume that this trait is income (assumed to be exogenous). Male income is denoted by x and female income is denoted by y. We no longer assume transferable utility; hence the Pareto frontier for a couple has the general form

$$u = H(x, y, v) \tag{3}$$

with $H(0, 0, v) = 0$ for all v.

As above, if a man with income x remains single, his utility is given by $H(x, 0, 0)$ and if a woman of income y remains single her utility is the solution to the equation $H(0, y, v) = 0$. By definition, $H(x, y, v)$ is decreasing in v; we assume that it is increasing in x and y, i.e. that a higher income, be it male's or female's, tends to expand the Pareto frontier. Also, we still consider a continuum of men, whose incomes x are distributed on $[0, 1]$ according to some distribution F, and a continuum of women, whose incomes y are distributed on $[0, 1]$ according to some distribution G; let r denote the measure of women.

Finally, let us *assume* for the moment that an equilibrium matching exists and that it is assortative. Existence can be proved under mild conditions using a variant of the Gale-Shapley algorithm; see Crawford (1991), Chiappori and Reny (2006). Regarding assortativeness, necessary conditions will be derived below. Under assortative matching, the 'matching functions' ϕ and ψ are defined exactly as above (eq. 13 to 15).

Let $u(x)$ (resp. $v(y)$) denote the utility level reached by Mr. x (Mrs. y) at the stable assignment. Then it must be the case that

$$u(x) \geq H(x, y, v(y))$$

for all y, with an equality for $y = \psi(x)$. As above, this equation simply translates stability: if it was violated for some x and y, a marriage between these two persons would allow increasing *both* utilities. Hence:

$$u(x) = \max_y H(x, y, v(y))$$

and we know that the maximum is actually reached for $y = \psi(x)$. First order conditions imply that

$$\frac{\partial H}{\partial y}(\phi(y), y, v(y)) + v'(y)\frac{\partial H}{\partial v}(\phi(y), y, v(y)) = 0.$$

while *second* order conditions for maximization are

$$\frac{\partial}{\partial y}\left(\frac{\partial H}{\partial y}(\phi(y), y, v(y)) + v'(y)\frac{\partial H}{\partial v}(\phi(y), y, v(y))\right) \leq 0 \quad \forall y.$$

This expression may be quite difficult to exploit. Fortunately, it can be simplified using a standard trick. The first order condition can be written as:

$$F(y, \phi(y)) = 0 \quad \forall y$$

where

$$F(y, x) = \frac{\partial H}{\partial y}(x, y, v(y)) + v'(y)\frac{\partial H}{\partial v}(x, y, v(y)). \tag{4}$$

Differentiating:

$$\frac{\partial F}{\partial y} + \frac{\partial F}{\partial x}\phi'(y) = 0 \quad \forall y,$$

which implies that

$$\frac{\partial F}{\partial y} \leq 0 \text{ if and only if } \frac{\partial F}{\partial x}\phi'(y) \geq 0.$$

The second order conditions can hence be written as:

$$\left(\frac{\partial^2 H}{\partial x \partial y}(\phi(y), y, v(y)) + v'(y)\frac{\partial^2 H}{\partial x \partial v}(\phi(y), y, v(y))\right)\phi'(y) \geq 0 \quad \forall y.$$

Here, assortative matching is equivalent to $\phi'(y) \geq 0$; this holds if

$$\frac{\partial^2 H}{\partial x \partial y}(\phi(y), y, v(y)) + v'(y)\frac{\partial^2 H}{\partial x \partial v}(\phi(y), y, v(y)) \geq 0 \quad \forall y.$$

Since $v'(y) \geq 0$, a sufficient (although not necessary) condition is that

$$\frac{\partial^2 H}{\partial x \partial y}(\phi(y), y, v(y)) \geq 0 \text{ and } \frac{\partial^2 H}{\partial x \partial v}(\phi(y), y, v(y)) \geq 0.$$

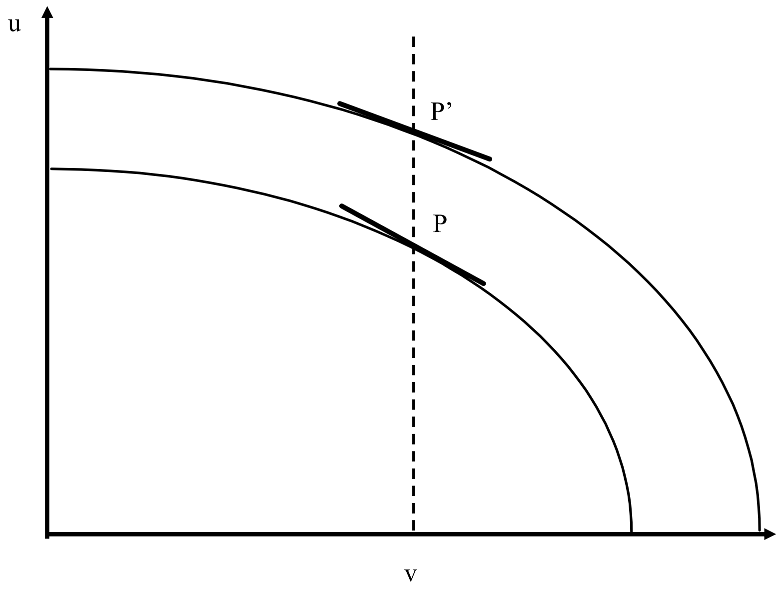

Figure 2: The slope condition

One can readily see how this generalizes the transferable utility case. Indeed, TU implies that $H(x, y, v(y)) = h(x, y) - v(y)$. Then $\frac{\partial^2 H}{\partial x \partial v} = 0$ and the condition boils down to the standard requirement that $\frac{\partial^2 H}{\partial x \partial y} = \frac{\partial^2 h}{\partial x \partial y} \geq 0$. General utilities introduces the additional requirement that the cross derivative $\frac{\partial^2 H}{\partial x \partial v}$ should also be positive (or at least 'not too negative'). For instance, a homothetic expansion of the Pareto set will typically satisfy this requirement. Geometrically, take some point on the Pareto frontier, corresponding to some female utility v, and increase x - which, by assumption, expands the Pareto set, hence shifts the frontier to the North East (see Figure 2). The condition then means that at the point corresponding to the same value v on the new frontier, the slope is less steep than at the initial point.

The intuition is that whether matching is assortative depends not only on the way total surplus changes with individual traits (namely, the usual idea that the marginal contribution of the husband's income increases with the wife's income, a property that is captured by the condition $\frac{\partial^2 H}{\partial x \partial y} \geq 0$), but also on how the 'compensation technology' works at various income levels. With general utilities, while the technology for transferring *income* remains obviously linear, the cost (in terms of husband's utility) of transferring *utility* to the wife varies with incomes. The second condition implies that, keeping the wife's utility level fixed, a larger income alleviates the cost (in terms of husband's utility) of providing an additional unit of utility to the wife. Then wealthy males have a double motivation for bidding aggressively for wealthy women: they benefit more from winning, and their 'bidding costs' are lower. They will thus systematically win. Note, however, that when the two partials $\frac{\partial^2 H}{\partial x \partial y}$ and $\frac{\partial^2 H}{\partial x \partial v}$ have opposite signs, the two aspects - benefits from winning and cost of bidding - vary with income in opposite directions. Assume, for instance, that $\frac{\partial^2 H}{\partial x \partial y} \geq 0$ but $\frac{\partial^2 H}{\partial x \partial v} \leq 0$. Then the outcome is uncertain because while wealthy males still value wealthy females more than poor males do, they are handicapped by their superior cost of bidding.

5 Search

We now turn to the alternative approach that stresses that in real life the matching process is characterized by scarcity of information about potential matches. The participants in the process must therefore spend time and money to locate their best options, and the set of potential partners they actually meet is partially random. The realized distribution of matches and the division of the gains from each marriage are therefore determined in an equilibrium which is influenced by the costs of search and the search policies of other participants.

5.1 The basic framework

The main ingredients of the search model are as follows. There is a random process which creates meetings between members of society of the opposite sex. When a meeting occurs the partners compare their characteristics and evaluate their potential gains from marriage. Each partner anticipates his share in the joint marital output. If the gains for both partners from forming the union exceed their expected gain from continued search then these partners marry. Otherwise, they depart and wait for the next meeting to occur (see Mortensen, 1988).

We assume that meetings occur according to a Poisson process. That is, the waiting times between successive meetings are i.i.d exponential variables with mean $1/\lambda$. Within a short period h, there is a probability of a meeting given by $\lambda h + o(h)$ and a probability of no meeting given by $1 - \lambda h + o(h)$, where, $o(h)/h$ converges to zero as h approaches zero. The arrival rate λ is influenced by the actions of the participants in the marriage market. Specifically, imagine an equal number of identical males and females, say N, searching for a mate. Let s_i denote the "search intensity" (i.e. number of meetings per period) initiated by a particular male. If all females search at the same intensity s_f , they will generate Ns_f contacts per period distributed randomly across all males. In this case, the probability that male i will make a contact with some female, during a short interval, h, is $(s_f + s_i)h$. If all males search at a rate s_m and all females at a rate s_f then the rate of meetings between agents of opposite sex is

$$\lambda = s_m + s_f. \tag{31}$$

The key aspect in (31) is that activities on both sides of the market determine the occurrence of meetings. A limitation of the linear meeting technology is that the number of searchers, N, has no effect on the arrival rate λ. Each participant who searches actively and initiates meetings must bear a monetary search cost given by $c_i(s)$, $i = m, f$, where we allow the costs of search to differ by sex. The total and the marginal costs of search increase as search intensity increases. When a meeting occurs the marital output (quality of match) that the partners can generate together is a random variable, z, drawn from some fixed distribution, $F(z)$. Having observed z, the couple decides whether to marry or not. With transferable utility, the decision to marry is based on the total output that can be generated by the couple within marriage relative to the expected total output if search continues. Hence, a marriage occurs if and only if

$$z \geq v_m + v_f, \tag{32}$$

where, v_m and v_f denote the value of continued search for the male and female partners, respectively. These values depend, in equilibrium, on the search intensity that will be chosen if the marriage does not take place. Specifically, for $i = m, f$,

$$rv_i = \underset{s}{Max}\{(s + s_j) \int_{v_m+v_f}^{\infty} (w_i(z) - v_i)dF(z) - c_i(s)\}, \tag{33}$$

where r is the instantaneous interest rate and $w_i(z)$ denote the shares of the gains of marital output that male and female partners expect.[10]

By definition,

$$w_m(z) + w_f(z) = z. \tag{34}$$

Equation (33) states that the value of being an unattached player arises from the option to sample from offers which arrive at a rate $s + s_f$ and are accepted only if (32) holds. Each accepted offer yields a surplus of $w_i(z) - v_i$ for partner i. Integration over all acceptable offers yields expected gain from search. Since each participant controls his own intensity of search, he will choose the level of

[10] This equation can be derived by using the following standard argument. Let h be a short time interval. Then, the Bellman equation for dynamic programming is

$$\begin{aligned} v_i &= \underset{s}{Max} e^{-rh}\{\lambda h(s + s_j)[p(z \geq v_m + v_f)(E_z(Max(v_i, w_i(z)|z \geq v_m + v_f)] \\ &\quad +[1 - \lambda h(s + s_j)]v_i\} + o(h). \end{aligned}$$

Note that, due to the stationarity of the Poisson process and the infinite horizon, v_i and and $w_i(z)$ do not depend on time. Approximating $e^{-rh} \simeq 1 - rh$, cancelling terms that do not depend on h and rearranging, we obtain

$$\begin{aligned} \underset{s}{Max}\{\lambda h(s + s_j)[p(z &\geq v_m + v_f)E_z(Max(v_i, w_i(z)|z \geq v_m + v_f) - v_i] \\ +[1 - \lambda h(s + s_j)]v_i - rh^2\lambda(s + s_j)[(p(z &\geq v_m + v_f)E_z(Max(v_i, w_i(z)|z \geq v_m + v_f) - v_i]\} \\ +o(h) &= rhv_i. \end{aligned}$$

Dividing both sides of this equation by h, we obtain (33) as the limit when h approach zero.

s that maximizes his value in the unattached state. Therefore, with identical individuals in each gender,

$$\int_{v_m+v_f}^{\infty} (w_i(z) - v_i)dF(z) = c_i'(s),\ i = m, f. \tag{35}$$

The marginal benefits from search, the L.H.S of (35), depend on the share that a person of type i expects in prospective marriages. As $w_i(z)$ rises, holding z constant, he or she searches more intensely. Hence, the equilibrium outcome depends on the allocation rules that are adopted. The literature examined two types of allocation rules. One class of allocation rules relies on Nash's axioms and stipulates

$$w_i(z) = v_i + \gamma_i(z - v_m - v_f), \tag{36}$$

where, $\gamma_i \geq 0$ and $\gamma_m + \gamma_f = 1$, $i = m, f$. The parameter γ_i allows for asymmetry in the bilateral bargaining between the sexes due to preferences or social norms. The crucial aspect of this assumption, however, is that outside options, reflected in the market determined values of v_m and v_f, influence the shares within marriage.

Wolinsky (1987) points out that a threat to walk out on a potentially profitable partnership is not credible. Rather than walking away, the partners exchange offers. When an offer is rejected, the partners search for an outside opportunity that would provide more than the expected gains from an agreement within the current marriage. Hence, during the bargaining process the search intensity of each partner is determined by

$$\int_{y}^{\infty} (w_i(x) - w_i(y))dF(x) = c_i'(s),\ i = m, f, \tag{37}$$

where, y is the quality of the current marriage and $w_i(y)$ is the expected share in the current marriage if an agreement is reached. Since $y \geq v_m + v_f$ and $w_i(y) \geq v_i$, a person who searches for better alternatives during a bargaining process will search less intensely and can expect lower gains than an unattached person. The threat of each partner is now influenced by two factors: The value of his outside opportunities (i.e., the value of being single), which enters only through the possibility that the other partner will get a better offer and leave; The value of continued search during the bargaining process, including the option of leaving when an outside offer (whose value exceeds the value of potential agreement) arrives. Therefore, the threat points, v_i , in (36) must be replaced by a weighted average of the value of remaining without a partner and the value of continued search during the bargaining (the weights are the probabilities of these events). Given these modified threat points, the parameters γ_i that determine the shares depend on the respective discount rates of the two partners and the probabilities of their exit from the bargaining process. The logic behind this type of formula, due to Rubinstein (1982), is that each person must be indifferent between accepting the current offer of his partner or rejecting it, searching for a better offer and, if none is received, return to make a counter offer that the partner will accept.

Given a specification of the share formulae, one can solve for the equilibrium levels of search intensities and the values of being unattached. For instance, if the shares are determined by (36) and γ_i is known, then equations (35) and (36) determine unique values for s_m, s_f, v_m, and v_f. Because of the linear meeting technology, these equilibrium values are independent of the number of searchers. Observe that although the share formulae depend on institutional considerations, the actual share of marital output that each partner receives depends on market forces and is determined endogenously in equilibrium.

We can close the model by solving for the equilibrium number of unattached participants relative to the population. Suppose that each period a new flow of unattached persons is added to the population and the same flow of married individuls exit. To maintain a steady state, this flow must equal the flow of new attachments that are formed from the current stock of unattached. The rate of transition into marriage is given by the product of the meeting rate λ and the acceptance rate $1 - F(z_0)$, where z_0 is the reservation quality of match. Using (31) and (32), we obtain

$$u(s_m + s_f)(1 - F(v_m + v_f)) = e \tag{38}$$

where, u is the endogenous, steady state, rate of non-attachment and e is the exogenous constant rate of entry and exit.

The meeting technology considered thus far has the unsatisfactory feature that attached persons "do not participate in the game". A possible extension is to allow matched persons to consider offers from chance meetings initiated by the unattached, while maintaining the assumption that married people do not search. In this case divorce becomes an additional option. If an unattached person finds a married person who belongs to a marriage of quality z and together they can form a marriage of quality y then a divorce will be triggered if $y > z$. The search strategies will now depend on the relative numbers of attached and unattached persons. Specifically, (33) is replaced by

$$\begin{aligned} rv_i \quad = \quad & \underset{s}{Max}\{u(s + s_j) \int_{v_m+v_f}^{\infty} (w_i(z) - v_i)dF(z) + \\ & (1-u)s \int_{v_m+v_f}^{\infty} \int_{y}^{\infty} (w_i(z) - w_i(y) - v_i)dG(z)dF(y) - c_i(s)\}, \end{aligned} \tag{39}$$

where, $G(z)$ is the distribution of quality of matched couples.[11] Observe that the expected returns from meeting an attached person are lower than those of

[11] The second term in equation (39) is derived from the following argument. Suppose i is a male and he meets a married woman who together with her current husband has marital output y. Together with i, the marital output would be z, where $z \geq y$. The threat point of this woman in the bargaining with man i is what she would receive from her current husband

meeting with an unmarried one. Therefore, the higher is the aggregate rate of non-attachment the higher are the private returns for search.

Assuming that partners are ex-ante identical, the search models outlined above do not address the question who shall marry whom. Instead, they shift attention to the fact that in the process of searching for a mate there is always a segment of the population which remains unmatched, not because they prefer the single state but because matching takes time. A natural follow up to this observation is the question whether or not there is "too much" search. Clearly, the mere existence of waiting time for marriage does not imply inefficiency since time is used productively to find superior matches. However, the informational structure causes externalities which may lead to inefficiency. One type of externality arises because in deciding on search intensity participants ignore the higher chance for meetings that others enjoy. This suggests that search is deficient. However, in the extended model which allows for divorce there is an additional externality operating in the opposite direction. When two unattached individuals reject a match opportunity with $z < v_m + v_f$, they ignore the benefits that arise to other couples from a higher non attachment rate. Thus, as in a related literature on unemployment, it is not possible to determine whether there is too much or too little non attachment.

An important aspect of equation (39) is the two way feedback between individual decisions and market outcomes. The larger is the proportion of the unattached the more profitable is search and each unattached person will be more choosy, further increasing the number of unattached. As emphasized by Diamond (1982) such reinforcing feedbacks can lead to multiplicity of equilibria. For instance, the higher is the aggregate divorce rate the more likely it is that each couple will divorce. Therefore, some societies can be locked into an equilibrium with a low aggregate divorce rate while others will settle on a high divorce rate. There are some additional features which characterize search for a mate and can be incorporated into the analysis. First, as noted by Mortensen (1988), the quality of marriage is revealed only gradually. Moreover, each partner may have private information which is useful for predicting the future match quality (see Bergstrom-Bagnoli, 1993). Second, as noted by Oppenheimer (1988), the offer distribution of potential matches varies systematically with age, as the number and quality of available matches change and the information about a person's suitability for marriage sharpens. Finally, meetings are not really completely random. Unattached individuals select jobs, schools and leisure activities so as to affect the chances of meeting a qualified person of the opposite sex (see Goldin, 2006).

5.2 Search and Assortative Mating

Models of search add realism to the assignment model, because they provide an explicit description of the sorting process that happens in real time. Following

when she threatens to leave him, which is $y - v_f$. Thus, the total surplus of the new marriage is $z - (y - v_m) - v_m$. Hence, following bargaining, man i will receive in the new marriage $v_m + \gamma_m(z - y) = v_m + w_i(z) - w_i(y)$.See Mortnesen (1988).

Burdett and Coles (1999), consider the following model with non transferable utility whereby if man m marries woman f, he gets f and she gets m. Assume a continuum of men, whose traits m are distributed on $[0, \bar{m}]$ according to some distribution F, and a continuum of women, whose traits f are distributed on $\left[0, \bar{f}\right]$ according to some distribution G. To bring in the frictions, assume that men and women meet according to a Poisson process with parameter λ. Upon meeting, each partner decides whether to accept the match or to continue the search. Marriage occurs only if both partners accept each other. A match that is formed cannot be broken. To ensure the stationary of the decision problem, we assume a fixed and equal number of infinitely lived men and women.

Each man chooses an acceptance policy that determines which women to accept. Similarly, each woman chooses an acceptance policy that determines which men to accept. These policies are characterized by reservation values, R, such that all potential partners with a trait exceeding R are accepted and all others are rejected. The reservation value that each person chooses depends on his\her trait. In particular, agents at the top of the distribution of each gender can be choosier because they know that they will be accepted by most people on the other side of the market and hence continued search is more valuable for them. Formally,

$$\begin{aligned} R_m &= b_m + \frac{\lambda \mu_m}{r} \int_{R_m}^{\bar{f}} (f - R_m) dG_m(f), \qquad (40) \\ R_f &= b_f + \frac{\lambda \mu_f}{r} \int_{R_f}^{\bar{m}} (m - R_f) dF_f(m), \end{aligned}$$

where, the flow of benefits as single, b, the proportion of meetings that end in marriage, μ, and the distribution of "offers" if marriage occurs, all depend on the trait of the person as indicated by the m and f subscripts. The common discount factor, r, represents the costs of waiting.

In equilibrium, the reservation values of all agents must be a best response against each other, yielding a (stationary) Nash equilibrium. The equilibrium that emerges is an approximation of the perfect positive assortative mating that would be reached without frictions. Using the Gale-Shapley algorithm to identify the stable outcome, we recall that, in the absence of frictions, this model generates a positive assortative mating. Thus, if men move first, all men will propose to the best woman and she will keep only the best man and reject all others. All rejected men will propose to the second best woman and she will accept the best of these and reject all others and so on. This outcome will also emerge here if the cost of waiting is low or frictions are not important, because λ is high. However, if frictions are relevant and waiting is costly, agents will compromise. In particular, the "best" woman and the "best" man will adopt the policies

$$R_{\bar{m}} = b_{\bar{m}} + \frac{\lambda}{r} \int_{R_{\bar{m}}}^{\bar{f}} (f - R_{\bar{m}}) dG_m(f), \tag{40'}$$
$$R_{\bar{f}} = b_{\bar{f}} + \frac{\lambda}{r} \int_{R_{\bar{f}}}^{\bar{m}} (m - R_{\bar{f}}) dF_f(m).$$

Thus, the best man accepts some women who are inferior to the best woman and the best woman accepts some men who are inferior to the best man, because one bird at hand is better than two birds on the tree.

The assumption that the rankings of men and women are based on a single trait, introduces a strong commonality in preferences, whereby all men agree on the ranking of all women and vice versa. Because all individuals of the opposite sex accept the best woman and all women accept the best man, μ is set to 1 in equation (40') and the distribution of offers equals the distribution of types in the population. Moreover, if the best man accepts all women with f in the range $[R_{\bar{m}}, \bar{f}]$ then all men who are inferior in quality will also accept such women. But this means that all women in the range $[R_{\bar{m}}, \bar{f}]$ are sure that all men accept them and therefore will have the *same* reservation value, $R_{\bar{f}}$, which in turn implies that all men in the range $[R_{\bar{f}}, \bar{m}]$ will have the same reservation value, $R_{\bar{m}}$. These considerations lead to a *class structure* with a finite number of distinct classes in which individuals marry each other. Having identified the upper class we can then examine the considerations of the top man and woman in the rest of the population. These individuals will face $\mu < 1$ and a *truncated* distribution of offers that, in principle, can be calculated to yield the reservation values for these two types and all other individuals in their group, forming the second class. Proceeding in this manner to the bottom, it is possible to determine all classes.

With frictions, there is still a tendency to positive (negative) assortative mating based on the interactions in traits. If the traits are complements, individuals of either sex with a higher endowment will adopt a more selective reservation policy and will be matched, on the average, with a highly endowed person of the opposite sex. However, with sufficient friction, it is also possible to have negative assortative mating under complementarity. The reason for this result is that, because of the low frequency of meetings and costs of waiting, agents in a search market tend to compromise. Therefore, males with low m, expect some women with high f to accept them, and if the gain from such a match is large enough, they will reject all women with low f and wait until a high f woman arrives.

The class structure result reflects the strong assumption that the utility that each partner obtains from the marriage depends only on the trait of the *other* spouse, so that there is no interaction in the household production function between the traits of the two spouses. In general, there will be some mingling of low and high income individuals, but the pattern of a positive assortative mating is sustained, provided that the complementarity in traits is large enough to motivate continued search for the "right" spouse. Smith (2006) provides a (symmetric) generalization of the problem where if man m marries woman f he receives the utility payoff $v = \pi(m, f)$ and she receives the utility payoff $u = \pi(f, m)$. It is assumed that this function is increasing in its second argument, $\pi_2(x, y) > 0$, so that all men prefer a woman with a higher f and all women prefer a man with a higher m, but individuals can differ in the intensity of their ordering.[12] He then shows that a sufficient condition for positive assortative mating, in the sense of a higher likelihood that a rich person will have a rich spouse, is that $log[\pi(m, f)]$ be super modular. That is, $m > m\prime$ and $f > f\prime$ imply that

$$\pi(m, f)\pi(m', f') > \pi(m, f')\pi(m', f). \tag{41}$$

The reason for such a condition is that one needs sufficiently strong complementarity to prevent the high types from accepting low types, due to impatience.

Surprisingly, the assumption of transferable utility loses some of its edge in the presence of frictions. In particular, it is no longer true that the assignment is determined by the maximization of the aggregate marital output of all potential marriages. To see why, consider the following output matrix:

Example 7.6

		Women		
		1	2	3
Men	1	4	1	0
	2	1	0	1
	3	0	1	4

where aggregate output is maximized on the main diagonal. With frictions, this assignment is in general *not* stable, because man 2 and woman 2 will prefer continued search to marriage that yield, 0, even if the value of being single is 0. The reason is that they can marry other men and women with whom they can obtain 1, who might be willing to marry them if the arrival rate of offers is low or the cost of waiting is high.

Generally speaking, the nature of the assignment problem changes, because of the need to consider the cost of time spent in search, as well as the benefits from matching. An additional complication, relative to the case of non-transferable utility, is the presence of rents. As we have seen, when meetings are random, and agents adopt reservation polices for accepted matches, the realized match will generally exceed the outside options of the married partner so that the rules for dividing the rents enter into the analysis. As a consequence, one generally needs stronger conditions to guarantee assortative matching. Shimer and Smith (2000) provide an analysis of the degree of complementarity that must hold to guarantee positive assortative mating if rents are divided equally in all marriages. Positive assortative mating, in the sense that a high m male is

[12] Intensity is a meaningful concept because given the risky environment agents are endowed with a Von Neumann Morgenstern utility function that is unique up to a linear transformation.

more likely to match with a high f female (on the average) requires, in addition to the supermodularity of $h(m, f)$, the supermpdularity of the logs of its partial derivatives and the log of the cross derivative $h_{mf}(m, f)$. This means that the simple predictions of the frictionless model carry over only under restrictive assumptions. For instance, $h(m, f) = \frac{(m+f)^2}{4}$, which, as we have shown, arises naturally in the presence of public goods, does *not* satisfy these requirements.[13]

6 Bargaining In Marriage (BIM)

As we have just seen, search models with random and intermittent meetings provide a natural framework to deal with rents and bargaining over rents in the marriage market. However, if marriage specific capital, such as children, is generated during marriage, then rents and bargaining can arise even without uncertainty and frictions. As is well known from models of specific human capital (see Becker (1993 ch. 3)) the accumulation of capital that is useful only in a particular relation makes the division of the gain from marriage partially insulated from competition. There is, therefore, a scope for bargaining over such rents.

It has been recently pointed out by Lundberg and Pollak (2009) that if the division resulting from bargaining in marriage is fully anticipated prior to marriage and if, in addition, binding contracts cannot be made at marriage, then the assignment into marriage must be based on the Gale Shapley algorithm. Specifically, Lundberg and Pollak contrast their 'BIM' (Bargaining In Marriage) framework with the standard, 'BAMM' (Binding Agreements on the Marriage Market) model, which is one of the possible foundation of the Becker-Shapley-Shubik construct. In a BIM world, any promise I may make before marriage can (and therefore will) be reneged upon minutes after the ceremony; there is just no way spouses can commit beforehand on their future behavior. Moreover, 'upfront' payments, whereby an individual transfers some money, commodities or property rights to the potential spouse *conditional on marriage*, are also excluded. Then the intrahousehold allocation of welfare will be decided after marriage, irrespective of the commitment made before. Marriage decision will therefore take the outcome of this yet-to-come decision process as given, and we are back in a non transferable utility setting in which each partner's share of the surplus is fixed and cannot be altered by transfers decided ex ante.

This result is an outcome of the assumed inability to credibly bid a person prior to marriage either by payments up-front or by short term commitments. This argument raises some important modeling issues about the working of the marriage market. A first remark is that it is not clear why premarital contracting is assumed away. Historically, contracts specifying what one brings into

[13] Specifically, the partial derivatives $\frac{m+f}{2}$ are not log super modular because $m > m\prime$ and $f > f\prime$ imply that

$$(m + f)(m' + f') < (m + f')(m' + f).$$

marriage and what the husband and wife take away upon divorce were universal (see Anderson, 2007). In modern societies prenuptial contracts still exist, although they are less prevalent. One possibility is that formal contracting and the associated enumeration of contingencies would "crowd out" the emotional trust on which the partners rely. This argument, however, has somewhat ambiguous implications, because the mere existence of such emotional trust seems to imply the existence of at least some minimum level of 'emotional commitment' - an idea that has been formalized by Browning (2009). Another important issue is verification. Typically it is difficult for the courts to verify the division of consumption or work within families. It must however be emphasized that commitment on intrahousehold allocation is *not* needed to implement a BAMM solution. Any transfer that (i) is decided ex ante, i.e. before marriage, and (ii) can be used to alter the spouse's respective bargaining positions after marriage, can do the trick. For instance, if the husband can, at (or just before) marriage, sign a legally enforceable contract specifying the transfers that would occur in case of separation, then we are back to a BAMM framework: I can now 'bid' my wife by offering her a very advantageous contract, because *even if we do not ultimately divorce,* the additional bargaining power provided to her by the ex ante contract will allow her to get a larger share of household resources - and is therefore equivalent to an ex post cash transfer. An even more striking example is the 'payment for marriage' situation, in which the husband can transfer a predetermined amount to his wife upon marriage (say, by offering her an expensive ring, or putting the couple's residence under her name, or even writing a check). Again, the size of the transfer can be used in the bidding process, and the relevant concept is again BAMM. Conversely, the BIM framework basically requires that no ex ante contract can ever be signed, and no conditional payment can ever be made.

A second concern is that even if we accept the total absence of commitment, Gale-Shapley still needs not be the relevant equilibrium concept. To see why, consider the extreme situation in which marriage can be done and undone at very low cost. Then at any moment of marital life, each spouse has a many close substitutes on the market, and the intrahousehold allocation will typically reflect this fact. Although, technically, this is not a BAMM situation (no binding agreement can be signed by assumption), the relevant concept is still the TU model a la Becker-Shapley-Shubik, because each spouse receives exactly her/his reservation value and the latter is fully determined by market equilibrium forces (at least when the number of potential spouses is 'large enough'). In other words, even in the extreme no transfer/no commitment case, the BIM framework applies only insofar as marriage decision can only be reversed at some cost, and only within the limits defined by this cost.

It is clear, in practice, that entry into marriage is a major decision that can be reversed only at some cost. However, as in any modeling choice, "realism" of the assumptions is not the only concern. Equally important is to have a tractable model that allows one to predict the marriage market outcomes under *varying* conditions. In this regard, the presence of transaction costs is quite problematic. To see this, consider again our example 7.3. Suppose that a new woman, 4, unexpectedly enters a marriage market that has been in one of the two equilibria discussed in section 1.1. Let the new payoffs matrix be as below:

Example 7.3a

		Women			
		1	2	3	4
Men	1	3,2	2,6	1,1	2,1
	2	4,3	7,2	2,4	5,4
	3	1,1	2,1	0,0	.5,.5

By assumption, woman 4 is preferred to woman 3 by all men and one would expect that in the new assignment woman 3 will become single. Suppose, however, that all existing couples bear a transaction cost of .75. Then it is easy to see that if the original equilibrium was the one in which men moved first, no man will marry woman 4 and she will remain single. In contrast, if the original equilibrium was the one in which women moved first then man 2 will take woman 4 and his ex-wife (woman 1) will first propose to man 1 who will reject her and then to man 3 who will accept her, so that woman 3 will become single. Thus, in general, it is impossible to predict what would happen when a new player enters the market, without knowing the bargaining outcomes in all marriages, the potential bargaining outcome that the entrant will have with all potential existing partners and the relational capital accumulated in all existing marriages. Such information is never available to the observer. In contrast, the Becker-Shapley-Shubik framework can predict the outcome very easily, using only information about the place of the new woman in the income distribution of women and the form of the household production function that specifies the within couple interaction between men and women of different attributes.

Given the different implications of alternative models of the marriage market, it seems prudent to consider several alternatives, depending on the application. In subsequent chapters we shall apply search models to analyze marriage and divorce when match quality is uncertain, and we shall apply the standard assignment model to discuss the determination of the division of gains from marriage when men and women differ in their attributes.

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