Chapter 6: Uncertainty and Dynamics in the Collective model

Martin Browning
Department of Economics, Oxford University

Pierre-André Chiappori
Department of Economics, Columbia University

Yoram Weiss
Department of Economics, Tel Aviv University

September 2010

1 Introduction

The models developed in the previous chapters were essentially static and were constructed under the (implicit) assumption of perfect certainty. As discussed in chapter 2, such a setting omits one of the most important roles of marriage - namely, helping to palliate imperfections in the insurance and credit markets by sharing various risks and more generally by transferring resources both across periods and across states of the world. Risk sharing is an important potential gain from marriage: individuals who face idiosyncratic income risk have an obvious incentive to mutually provide insurance. In practice, a risk sharing scheme involves intrahousehold transfers that alleviate the impact of shocks affecting spouses; as a result, individual consumptions within a couple may be less responsive to idiosyncratic income shocks than it would be if the persons were single. Not only are such risk-sharing mechanisms between risk averse agents welfare improving, but they allow the household to invest into higher risk/higher return activities; as such, they may also increase total (expected) income and wealth in the long run. For instance, the wife may be able to afford the risk involved in creating her own business because of the insurance implicitly offered by her husband's less risky income stream.

Another, and closely related form of consumption smoothing stems from intrafamily credit relationship: even in the absence of a perfect credit market, a spouse can consume early a fraction of her future income thanks to the resources coming from her partner. Again, intrahousehold credit may in turn enable agents to take advantage of profitable investment opportunities that would be out of the reach of a single person.

While intertemporal and risk sharing agreements play a key role in economic life in general and in marriage in particular, they also raise specific difficulties. The main issue relates to the agents' ability to credibly commit to specific future behavior. Both types of deals typically require that some agents reduce their consumption in either some future period or some possible states of the world. This ability to commit may however not be guaranteed. In some case, it is even absent (or severely limited); these are cases in which the final agreement typically fails to be fully efficient, at least in the *ex ante* sense.[1]

The theoretical analysis underpinning these issues leads to fascinating empirical questions. Again, these can be formulated in terms of testability and identifiability. When, and how, is it possible to test the assumption of perfect commitment, and more generally of *ex ante* efficiency? And to what extent is it possible to recover the underlying structure - namely individual preferences (here, aversions to risk and/or fluctuations) and the decision process (here, the Pareto weights) from observed behavior? These questions - and others - are analyzed in the present chapter.

2 Is commitment possible?

We start with a brief discussion of the commitment issue. As discussed above, credit implies repayment, and the very reason why a formal credit market may fail to be available (say, non contractible investments) may result in enforcement problems even between spouses. As the usual cliche goes, a woman will be hesitant to support her husband through medical school if she expects him to break the marriage and marry a young nurse when he finishes (this is a standard example of the hold-up problem). Similarly, risk sharing requires possibly important transfers between spouses; which enforcement devices can guarantee that these transfers will actually take place when needed is a natural question. In subsequent sections we shall consider conventional economic analyses of the commitment problem as they relate to the family. In the remainder of this section we consider possible commitment mechanisms that are specific to the family.

From a game-theoretic perspective, marriage is a typical example of repeated interactions between the same players; we know that cooperation is easier to support in such contexts.[2] This suggests that, in many case, cooperation is a

[1] A second problem is information: in general, efficient trade is much easier to implement in a context of symmetric information. Asymmetric information, however, is probably less problematic in households than in other types of relationship (say, between employers and employees or insurers and insurees), because the very nature of the relationship often implies deep mutual knowledge and improved monitoring ability.

[2] Del Boca and Flinn (2009) formulate a repeated game for time use that determines the amount of market work and housework that husbands and wives perform. Their preferred model is a cooperative model with a noncooperative breakdown point. They have a repeated game with a trigger strategy for adopting the inefficient non-cooperative outcome if the discount is too small. The value of the threshold discount factor they estimate to trigger noncooperative behavior is 0.52which implies that 94% of households behave cooperatively.

natural assumption. Still, the agents' ability to commit is probably not unbounded. Love may fade away; fidelity is not always limitless; commitment is often constrained by specific legal restrictions (for instance, agents cannot legally commit not to divorce).[3] And while the repeated interaction argument for efficiency is convincing in many contexts, it may not apply to some important decisions that are made only exceptionally; moving to a different location and different jobs is a standard example, as argued by Lundberg and Pollak (2003).

A crucial aspects of lack of commitment is that, beyond restraining efficiency in the *ex ante* sense, it may also imply *ex post* inefficiencies. The intuition is that whenever the parties realize the current agreement will be renegotiated in the future, they have strong incentives to invest now into building up their future bargaining position. Such an investment is in general inefficient from the family's viewpoint, because it uses current resources without increasing future (aggregate) income. For instance, spouses may both invest in education, although specialization would be the efficient choice, because a high reservation wage is a crucial asset for the bargaining game that will be played later.[4]

Love and all these things How can commitment be achieved when the repeated interaction argument does not hold? Many solutions can actually be observed. First, actual contracts can be (and actually are) signed between spouses. Prenuptial agreements typically specify the spouses' obligations both during marriage and in case of divorce; in particular, some provisions may directly address the hold-up problem. To come back to the previous example, a woman will less be hesitant fund her husband's training if their prenuptial agreement stipulates that she will receive, in case of divorce, a large fraction of his (future) income. Contract theory actually suggests that even if long term agreements are not feasible, efficiency can in general be reached through a sequence of shorter contracts that are regularly renegotiated (see for instance Rey and Salanié (1990).[5] Still, even though a private, premarital agreement may help alleviating the limits to commitments (say, by making divorce very expensive for one of the parties), renegotiation proofness may be an issue, especially if divorce has been made costly for both spouses; furthermore, in some countries courts are free to alter *ex post* the terms of premarital agreements. At any rate, some crucially important intrahousehold issues may hardly be contractible.

Alternative enforcement mechanisms can however be implemented. Religious or ethical factors may be important; in many faiths (and in several social groups), a person's word should never be broken. Love, affection and mutual

[3] Of course, *moral* or *religious* commitment not to divorce do exist, although they may not be globally prevalent.

[4] See Konrad and Lommerud (2000) and Brossolet (1993).

[5] In practice, prenuptial agreements are not common (although they are more frequently observed in second marriages). However, this may simply indicate that, although easily feasible, they are rarely needed, possibly because existing enforcement mechanisms (love, trust, repeated interactions) are in general sufficient. Indeed, writing an explicit contract that lists all contingencies may in fact "crowd out" the emotional bonds and diminish the role of the initial spark of blind trust that is associated with love.

respect are obviously present in most marriages, and provide powerful incentives to honoring one's pledge and keeping one's promises. Browning (2009) has recently provided a formalization of such a mechanism. The model is developed in the specific context of the location decision model of Mincer (1978) and Lundberg and Pollak (2003) but has wider application. In the location model a couple, a and b, are presented with an opportunity to increase their joint income if they move to another location. Either partner can veto the move. The problem arises when the move shifts power within the household toward one partner (partner b, say); then the other partner (a) will veto the move if she is worse off after the move. Promises by partner b are not incentive compatible since a does not have any credible punishment threat.[6] Particular commitment mechanisms may be available in this location decision model. For example, suppose there is a large indivisible choice that can be taken at the time of moving; choosing a new house is the obvious example. If this choice has a large element of irreversibility then partner b can defer to a on this choice and make the move more attractive. At some point, however, commitment devices such as this may be exhausted without persuading a that the move is worthwhile. Now assume that spouses are caring, in the usual sense that their partner's utility enter their preferences. Browning (2009) suggests that if one partner exercises too aggressively their new found bargaining power then the other partner feels betrayed and loses some regard (or love) for them. The important element is that this loss of love (by a in this case) is out of the control of the affected partner; in this sense, this is betrayal. Thus the threat is credible. In a model with mutual love, this 'punishment' is often sufficient to deter a partner from exercising their full bargaining power if the move takes place.

To formalise, consider a married couple a and b. Income, which is normalised to unity if they do not move, is divided between them so that a receives x for private consumption and b receives $1-x$. There are no public goods. Each person has the same strictly increasing, strictly concave felicity function, so that:

$$u^a = u(x), \ u^b = u(1-x) \tag{1}$$

Each person also cares for the other with individual utility functions given by:

$$\begin{aligned} W^a &= u^a + \delta^a u^b \\ &= u(x) + \delta^a u(1-x) \quad (2) \\ W^b &= \delta^b u^a + u^b \\ &= \delta^b u(x) + u(1-x) \quad (3) \end{aligned}$$

where $\delta^s \geq 0$ is person s's caring for the other person, with $\delta^a \delta^b < 1$ (see chapter 3). We assume that the caring parameters are constant and outside the control of either partner. Rather than choosing an explicit game form to choose x, we simply assume that there is some (collective) procedure that leads the household

[6] We neglect the option in which they divorce and the husband moves to the new location. Mincer (1978) explicitly considers this.

to behave as though it maximises the function:

$$W = W^A + \mu W^b$$
$$= \left(1 + \mu\delta^b\right) u(x) + (\delta^a + \mu) u(1 - x) \tag{4}$$

As discussed in chapter 4, caring modifies the Pareto weight for b to an effective value of $(\delta^a + \mu) / \left(1 + \mu\delta^b\right)$.

Now suppose there is a (moving) decision that costlessly increases household income from unity to $y > 1$. If this is the only effect then, of course, both partners would agree to move. However, we also assume that the decision increases b's Pareto weight to $\mu(1 + m)$ where $m \geq 0$. In this case there is a reservation income $y^*(m)$ such that person a will veto the move if and only if $y < y^*(m)$. In such a case there will be unrealised potential Pareto gains. Now allow that the husband can choose whether or not to exercise his new found power if they do move. If he does *not* exercise his new power then the household utility function is given by:

$$W = \left(1 + \mu\delta^b\right) u(x) + (\delta^a + \mu) u(y - x) \tag{5}$$

which obviously dominates (4). Of course, a simple statement that "I promise to set $m = 0$" has no credibility. Suppose, however, that if such a promise is made and then broken, then the wife feels betrayed. In this case her love for her husband falls from δ^a to $\delta^a(1 - \sigma)$ where $\sigma \in [0, 1]$. The fall in her caring for him is taken to be out of her control, so that a has an automatic and hence credible punishment for b choosing to take advantage of his improved position. If they move and the husband exercises his new power the household utility function is given by:

$$W = \left(1 + \mu(1 + m)\delta^b\right) u(x) + (\delta^a(1 - \sigma) + (1 + m)\mu) u(y - x) \tag{6}$$

If the husband's implicit Pareto weight is less in this case than in (5) then he will not betray his wife. In the simple case in which he does not care for her ($\delta^b = 0$) this will be the case if:

$$(\delta^a + \mu) \geq (\delta^a(1 - \sigma) + (1 + m)\mu)$$
$$\Leftrightarrow \delta^a\sigma \geq m\mu \tag{7}$$

That is, there will be a move with no betrayal if δ^a and σ are sufficiently large relative to m and μ. For example, a husband who lacks power (and hence relies on his wife's caring for resources) or has a small in increase in power (so that $m\mu$ is small) will be less likely to betray if his wife cares a lot for him (δ^a) and she feels the betrayal strongly (σ close to unity).

Psychological games A different but related analysis is provided by Dufwenberg (2002), who uses "psychological games" to discuss commitment in a family context. The basic idea, due to Geanakoplos *et al* (1989), is that the utility payoffs of married partners depend not only on their actions and the consequences in terms of income or consumption but also on the beliefs that the spouses may have on these actions and consequences. The basic assumption is that the stronger is the belief of a spouse that their partner will act in a particular manner, the more costly it is for that partner to deviate and disappoint their spouse. This consideration can be interpreted as guilt. A crucial restriction of the model is that, in equilibrium, beliefs should be consistent with the actions. Dufwenberg (2002) uses this idea in a context in which one partner (the wife) extends credit to the other spouse. For instance, the wife may work when the husband is in school, expecting to be repaid in the form of a share from the increase in family income (see Chapter 2). But such a repayment will occur only if the husband stays in the marriage, which may not be the case if he is unwilling to share the increase in his earning power with his wife and walks away from the marriage.

Specifically, consider again the two period model discussed in Chapter 2. There is no borrowing or lending and investment in schooling is lumpy. In the absence of investment in schooling, each spouse has labor income of 1 each period. There is also a possibility to acquire some education; if a person does so then their earnings are zero in the first period and 4 in the second period. We assume that preferences are such that in each period each person requires a consumption of $\frac{1}{2}$ for survival and utility is linear in consumption otherwise. This implies that without borrowing, no person alone can undertake the investment, while marriage enables the couple to finance the schooling investment of one partner. We assume that consumption in each period is divided equally between the two partners if they are together and that if they are divorced then each receives their own income. Finally, suppose that each partner receives a non monetary gain from companionship of $\theta = 0.5$ for each period they are together. The lifetime payoff if neither educate is $(2 + 2\theta) = 3$ for each of them. Since both have the same return to education, for ease of exposition we shall assume that they only consider the husband taking education.[7] If he does educate and they stay together then each receives a total of $(3 + 2\theta) = 4$ over the two periods. There is thus a potential mutual gain for both of them if the investment is undertaken and marriage continues. However, if the husband educates and then divorces, he receives a payoff of 4 in the second period and if he stays he receives only 3 $(= 2.5 + \theta)$. Thus, without commitment, he would leave in the second period[8] and the wife will then be left with a lifetime utility of 2 which is less than she would have in the absence of investment, 3. Therefore, the wife would not agree to finance her husband's education in the first period. The basic dilemma is illustrated in Figure 1, where the payoffs for the wife are at the top of each final node and the payoffs for the husband are at the bottom. The only equilibrium in this case is that the wife does not support her husband, the husband does not invest in schooling and stays in the marriage so that the

[7] The issue of what happens if the two have different returns to education is one that deserves more attention.

[8] Note that if the match quality is high enough then he will not divorce even if he educates. In the numerical example this will be the case if $\theta > 1.5$. In this case there is no need for commitment. This is analogous to the result concerning match quality and children.

family ends up in an inefficient equilibrium. However, Dufwenberg (2002) then shows that if one adds guilt as a consideration, an efficient equilibrium with consistent beliefs can exist. In particular, suppose that the husband's payoff following divorce is $4 - \gamma\tau$ where τ is the belief of the husband at the beginning of period 2 about the beliefs that his wife formed at the time of marriage, about the probability that her husband will stay in the marriage following her investment and γ is a fixed parameter. Then, if $\gamma > 2$, the husband chooses to stay in the marriage, the wife agrees to support her husband to invest and efficiency is attained. To show the existence of consistent beliefs that support this equilibrium, consider the special case in which $\gamma = 2$. Suppose that the wife actually invests, as we assume for this equilibrium. Then, she reveals to her husband that she expects to get a life time utility of at least 3 following this choice, which means that her belief, τ' about the probability that the husband would stay is such that $1 + \tau' 4 > 3$, implying $\tau' \geq \frac{1}{2}$. Knowing that, the husband's belief τ about her belief that he stays exceeds $\frac{1}{2}$. Therefore, his payoff upon leaving in the second period $4 - 2\tau$ is less or equal to his payoff if he stays, 3. Thus for any γ strictly above 2, he stays. In short, given that the wife has shown great trust in him, as indicated by her choice to support him, and given that he cares a great deal about that, as indicated by the large value of γ, the husband will feel more guilty about disappointing her and will in fact stay in the marriage, justifying his wife's initial beliefs. The husband on, his part, avoids all feelings of guilt and efficient investment will be attained. A happy marriage indeed.

Somewhat different considerations arise when we look at 'end game' situations in which the spouse has no chance to reciprocate. A sad real example of this sort is when the husband has Altzheimer's and his wife takes care of him for several (long) years, expecting no repayment from him whatsoever as he does not even know her. Here, the proper assumption appears to be that she believes that he would have done for her the same thing had the roles been reversed. Unfortunately, the consistency of such beliefs is impossible to verify. Another possibility is that she cares about him and about her children that care about him to the extent that caring for the sick husband in fact gives her satisfaction. In either case, some emotional considerations must be introduced to justify such cases of unselfish behavior in families.

The commitment issue is complex. In the end, whether agents are able to implement and enforce a sufficient level of commitment to achieve *ex ante* efficiency is an empirical issue. Our task, therefore, is to develop conceptual tools that allow a precise modeling of these problems, and empirical tests that enable us to decide whether, and to what extent, the lack of commitment is an important problem for household economics.

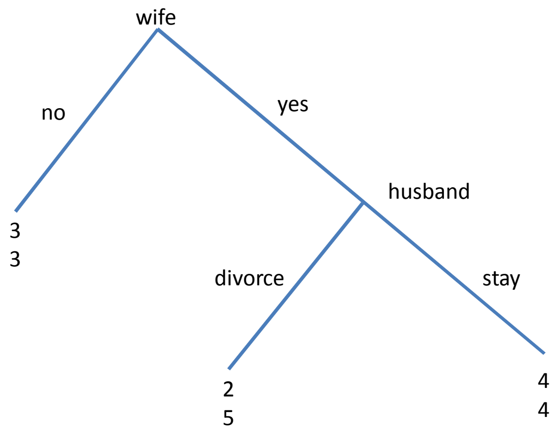

Figure 1: Game tree for investment in education

3 Modeling commitment

3.1 Full commitment

Fortunately enough, the tools developed in the previous chapters can readily be extended to modeling the commitment issues. We start from the full commitment benchmark. The formal translation is very simple: under full commitment, *Pareto weights remain constant,* either over periods or over states of the world (or both). To see why, consider for instance the risk sharing framework with two agents. Assume that there exists S states of the world, with respective probabilities $\pi_1, ..., \pi_S$ (with $\sum_s \pi_s = 1$); let y_s^a denote member a's income is state s. Similarly, let $\mathbf{p}_s$ (resp. $\mathbf{P}_s$) be the price vector for private (public) goods in state s, and $\mathbf{q}_s^a$ (resp. $\mathbf{Q}_s$) the vector of private consumption by member a (the vector of household public consumption). An allocation is *ex ante efficient* if it solves a program of the type:

$$\max_{\mathbf{Q}_s,\mathbf{q}_s^a,\mathbf{q}_s^b} \sum_s \pi_s u^a\left(\mathbf{Q}_s,\mathbf{q}_s^a,\mathbf{q}_s^b\right)$$
$$\text{subject to } \mathbf{P}'_s\mathbf{Q}_s+\mathbf{p}'_s\left(\mathbf{q}_s^a+\mathbf{q}_s^b\right)\leq\left(y_s^a+y_s^b\right) \text{ for all } s \tag{8}$$
$$\text{and } \sum_s \pi_s u^b\left(\mathbf{Q}_s,\mathbf{q}_s^a,\mathbf{q}_s^b\right)\geq \bar{u}^b$$

for some $\bar{u}^b$. As in chapter 3, if μ denotes the Lagrange multiplier of the last constraint, this program is equivalent to:

$$\max_{\mathbf{Q}_s,\mathbf{q}_s^a,\mathbf{q}_s^b} \sum_s \pi_s u^a\left(\mathbf{Q}_s,\mathbf{q}_s^a,\mathbf{q}_s^b\right)+\mu\sum_s \pi_s u^b\left(\mathbf{Q}_s,\mathbf{q}_s^a,\mathbf{q}_s^b\right)$$
$$\text{subject to } \mathbf{P}'_s\mathbf{Q}_s+\mathbf{p}'_s\left(\mathbf{q}_s^a+\mathbf{q}_s^b\right)\leq\left(y_s^a+y_s^b\right) \text{ for all } s$$

or:

$$\max_{\mathbf{Q}_s,\mathbf{q}_s^a,\mathbf{q}_s^b} \sum_s \pi_s \left[u^a\left(\mathbf{Q}_s,\mathbf{q}_s^a,\mathbf{q}_s^b\right)+\mu u^b\left(\mathbf{Q}_s,\mathbf{q}_s^a,\mathbf{q}_s^b\right)\right] \tag{9}$$
$$\text{subject to } \mathbf{P}'_s\mathbf{Q}_s+\mathbf{p}'_s\left(\mathbf{q}_s^a+\mathbf{q}_s^b\right)\leq\left(y_s^a+y_s^b\right) \text{ for all } s$$

This form shows two things. First, for any state s, the allocation contingent on the realization of this state, $\left(\mathbf{Q}_s,\mathbf{q}_s^a,\mathbf{q}_s^b\right)$, maximizes the weighted sum of utilities $u^a\left(\mathbf{Q}_s,\mathbf{q}_s^a,\mathbf{q}_s^b\right)+\mu u^b\left(\mathbf{Q}_s,\mathbf{q}_s^a,\mathbf{q}_s^b\right)$ under a resource constraint. As such, it is efficient in the *ex post* sense: there is no alternative allocation $\left(\bar{\mathbf{Q}}_s,\bar{\mathbf{q}}_s^a,\bar{\mathbf{q}}_s^b\right)$ that would improve both agents' welfare *in state s*. Secondly, the weight μ is the same across states of the world. This guarantees *ex ante* efficiency: there is no alternative allocation $\left[\left(\bar{\mathbf{Q}}_1,\bar{\mathbf{q}}_1^a,\bar{\mathbf{q}}_1^b\right),...,\left(\bar{\mathbf{Q}}_S,\bar{\mathbf{q}}_S^a,\bar{\mathbf{q}}_S^b\right)\right]$ that would improve both agents' welfare *in expected utility terms* - which is exactly the meaning of programs (8) and (9).

Finally, note that the intertemporal version of the problem obtains simply by replacing the state of the world index s by a time index t and the probability π_s of state s with a discount factor - say, δ^t.

3.2 Constraints on commitment

Limits to commitment can generally be translated into additional constraints in the previous programs. To take a simple example, assume that in each state of the world, one member - say b - has some alternative option that he cannot commit not to use. Technically, in each state s, there is some lower bound $\bar{u}_s^b$ for b's utility; here, $\bar{u}_s^b$ is simply the utility that b would derive from his fallback option. This constraint obviously reduces the couple's ability to share risk. Indeed, it may well be the case that, in some states, efficient risk sharing would require b's welfare to go below this limit. However, a contract involving such a low utility level in some states is not implementable, because it would require from b more commitment than what is actually available.

The technical translation of these ideas is straightforward. Introducing the new constraint into program (8) gives:

$$\max_{\mathbf{Q}_s,\mathbf{q}_s^a,\mathbf{q}_s^b} \sum_s \pi_s u^a\left(\mathbf{Q}_s,\mathbf{q}_s^a,\mathbf{q}_s^b\right)$$

$$\text{subject to } \mathbf{P}_s'\mathbf{Q}_s+\mathbf{p}_s'\left(\mathbf{q}_s^a+\mathbf{q}_s^b\right)\leq\left(y_s^a+y_s^b\right) \text{ for all } s, \tag{10}$$

$$\sum_s \pi_s u^b\left(\mathbf{Q}_s,\mathbf{q}_s^a,\mathbf{q}_s^b\right)\geq\bar{u}^b$$

$$\text{and } u^b\left(\mathbf{Q}_s,\mathbf{q}_s^a,\mathbf{q}_s^b\right)\geq\bar{u}_s^b \text{ for all } s \tag{11}$$

Let μ_s denote the Lagrange multiplier of constraint (C_s); the program can be rewritten as:

$$\max_{\mathbf{Q}_s,\mathbf{q}_s^a,\mathbf{q}_s^b} \sum_s \pi_s u^a\left(\mathbf{Q}_s,\mathbf{q}_s^a,\mathbf{q}_s^b\right)+\sum_s\left(\mu\pi_s+\mu_s\right)u^b\left(\mathbf{Q}_s,\mathbf{q}_s^a,\mathbf{q}_s^b\right)$$

$$\text{subject to } \mathbf{P}_s'\mathbf{Q}_s+\mathbf{p}_s'\left(\mathbf{q}_s^a+\mathbf{q}_s^b\right)\leq\left(y_s^a+y_s^b\right) \text{ for all } s$$

or equivalently:

$$\max_{\mathbf{Q}_s,\mathbf{q}_s^a,\mathbf{q}_s^b} \sum_s \pi_s\left[u^a\left(\mathbf{Q}_s,\mathbf{q}_s^a,\mathbf{q}_s^b\right)+\left(\mu+\frac{\mu_s}{\pi_s}\right)u^b\left(\mathbf{Q}_s,\mathbf{q}_s^a,\mathbf{q}_s^b\right)\right] \tag{12}$$

$$\text{subject to } \mathbf{P}_s'\mathbf{Q}_s+\mathbf{p}_s'\left(\mathbf{q}_s^a+\mathbf{q}_s^b\right)\leq\left(y_s^a+y_s^b\right) \text{ for all } s$$

Here, *ex post* efficiency still obtains: in each state s, the household maximizes the weighted sum $u^a+\left(\mu+\frac{\mu_s}{\pi_s}\right)u^b$. However, the weight is no longer constant; in any state s in which constraint (11) is binding, implying that $\mu_s>0$, b's weight is increased by μ_s/π_s. Intuitively, since b's utility cannot go below the fallback value $\bar{u}_s^b$, the constrained agreement inflates b's Pareto weight in these states by whichever amount is necessary to make b just indifferent between the contract and his fallback option. Obviously, this new contract is not efficient in the *ex ante* sense; it is only second best efficient, in the sense that no alternative contract can do better for both spouses without violating the constraints on commitment.

3.3 Endogenous Pareto weights

Finally, assume as in Basu (2006), that the fallback utility $\bar{u}_s^b$ is *endogenous*, in the sense that it is affected by some decision made by the agents. For instance, $\bar{u}_s^b$ depends on the wage b would receive on the labor market, which itself is positively related to previous labor supply (say, because of human capital accumulation via on the job training). Now, in the earlier periods b works for two different reasons. One is the usual trade-off between leisure and consumption: labor supply generates an income that can be spent on consumption goods. The second motive is the impact of current labor supply on future bargaining power; by working today, an agent can improve her fallback option tomorrow, therefore be able to attract a larger share of household resources during the renegotiation that will take place then, to the expenses of her spouse. The first motive is fully compatible with (static) efficiency; the second is not, and results in overprovision of labor with respect to the optimum level.

We can capture this idea in a simple, intertemporal version of the previous framework. Namely, consider a two-period model with two agents and two commodities, and assume for simplicity that agents are egoistic:

$$\max_{\mathbf{q}_t^a,\mathbf{q}_t^b} \sum_{t=1}^{2} \delta^{t-1} u^a\left(\mathbf{q}_t^a\right)$$

$$\text{subject to } \mathbf{p}_t'\left(\mathbf{q}_t^a+\mathbf{q}_t^b\right) \leq \left(y_t^a+y_t^b\right) \text{ for } t=1,2, \tag{13}$$

$$\sum_{t=1}^{2} \delta^{t-1} u^b\left(\mathbf{q}_t^b\right) \geq \bar{u}^b$$

$$\text{and } u^b\left(\mathbf{q}_2^b\right) \geq \bar{u}_2^b \tag{14}$$

where $\mathbf{q}_t^X = \left(q_{1,t}^X, q_{2,t}^X\right), X = a, b$; note that we assume away external financial markets by imposing a resource constraint at each period. Assume, moreover, that the fallback option $\bar{u}_2^b$ of b in period 2 is a decreasing function of $q_{1,t}^b$; a natural interpretation, suggested above, is that commodity 1 is leisure, and that supplying labor at a given period increases future potential wages, hence the person's bargaining position. Now the Lagrange multiplier of (14), denoted μ_2, is also a function of $q_{1,t}^b$. The program becomes:

$$\max_{\mathbf{q}_t^a,\mathbf{q}_t^b} \sum_{t=1}^{2} \delta^{t-1} u^a\left(\mathbf{q}_t^a\right) + \mu \sum_{t=1}^{2} \delta^{t-1} u^b\left(\mathbf{q}_t^b\right) + \mu_2\left(q_{1,t}^b\right) u^b\left(\mathbf{q}_2^a, \mathbf{q}_2^b\right)$$

$$\text{subject to } \mathbf{p}_t'\left(\mathbf{q}_t^a+\mathbf{q}_t^b\right) \leq \left(y_t^a+y_t^b\right) \text{ for } t=1,2$$

or equivalently:

$$\max_{\mathbf{q}_t^a,\mathbf{q}_t^b} \left[u^a\left(\mathbf{q}_1^a\right)+\mu u^b\left(\mathbf{q}_1^b\right)\right] + \delta\left[u^a\left(\mathbf{q}_2^a\right)+\mu u^b\left(\mathbf{q}_2^b\right)\right] + \mu_2\left(q_{1,t}^b\right) u^b\left(\mathbf{q}_2^b\right)$$

$$\text{subject to } \mathbf{p}_t'\left(\mathbf{q}_t^a+\mathbf{q}_t^b\right) \leq \left(y_t^a+y_t^b\right) \text{ for } t=1,2.$$

The first order conditions for $q_{1,1}^b$ are:

$$\mu \frac{\partial u^b\left(\mathbf{q}_1^b\right)}{\partial q_{1,1}^b} = \lambda p_{1,t} - u^b\left(\mathbf{q}_2^b\right) \frac{d\mu_2\left(q_{1,t}^b\right)}{dq_{1,t}^b}$$

which does not coincide with the standard condition for static efficiency because of the last term. Since the latter is positive, the marginal utility of leisure is above the optimum, reflecting under-consumption of leisure (or oversupply of labor). In other words, both spouses would benefit from an agreement to reduce both labor supplies while leaving Pareto weights unchanged.

4 Efficient risk sharing in a static context

4.1 The collective model under uncertainty

4.1.1 Ex ante and *ex post* efficiency

We can now discuss in a more precise way the theoretical and empirical issues linked with uncertainty and risk sharing. For that purpose, we specialize the general framework sketched above by assuming that consumptions are private, and agents have egoistic preferences. We first analyze a one-commodity model; then we consider an extension to a multi-commodity world.

We consider a model in which two risk averse agents, a and b, share income risks through specific agreements. There are N commodities and S states of the world, which realize with respective probabilities $(\pi_1, ..., \pi_S)$. Agent X $(X = a, b)$ receives in each state s some income y_s^X, and consumes a vector $c_s^X = \left(c_{s,1}^X, ..., c_{s,N}^X\right)$; let $p_s = (p_{s,1}, ..., p_{s,N})$ denote the price vector in state s. Agents are expected utility maximizers, and we assume that their respective Von Neumann-Morgenstern utilities are strictly concave, that is that agents are strictly risk averse.

The efficiency assumption can now take two forms. Ex post efficiency requires that, *in each state s of the world*, the allocation of consumption is efficient in the usual, static sense: no alternative allocation could improve both utilities at the same cost. That is, the vector $c_s = \left(c_s^a, c_s^b\right)$ solves:

$$\max u^a\left(c_s^a\right) \tag{15}$$

under the constraints:

$$\begin{aligned} u^b\left(c_s^b\right) &\geq \bar{u}_s^b \\ \sum_i p_{i,s}\left(c_{i,s}^a + c_{i,s}^b\right) &= y_s^a + y_s^b = y_s \end{aligned}$$

As before, we may denote by μ_s the Lagrange multiplier of the first constraint; then the program is equivalent to:

$$\max u^a\left(c_s^a\right) + \mu_s u^b\left(c_s^b\right)$$

under the resource constraint. The key remark is that, in this program, the Pareto weight μ_s of member b may depend on s. Ex post efficiency requires static efficiency *in each state*, but imposes no restrictions on behavior *across states.*

Ex ante efficiency requires, in addition, that the allocation of resources across states is efficient, in the sense that no state-contingent exchange can improve both agents' expected utilities. Note that, now, welfare is computed *ex ante*, in expected utility terms. Formally, the vector $c = (c_1, ..., c_S)$ is efficient if it solves a program of the type:

$$\max \sum_s \pi_s u^a\left(c_s^a\right) \tag{16}$$

under the constraints:

$$\sum_s \pi_s u^b\left(c_s^b\right) \geq \bar{u}^b \quad (17)$$

$$\sum_i p_{i,s}\left(c_{i,s}^a + c_{i,s}^b\right) = y_s^a + y_s^b = y_s,\ s = 1, ..., S \quad (18)$$

Equivalently, if μ denotes the Lagrange multiplier of the first constraint, the program is equivalent to:

$$\max \sum_s \pi_s u^a\left(c_s^a\right) + \mu \sum_s \pi_s u^b\left(c_s^b\right) = \sum_s \pi_s\left[u^a\left(c_s^a\right) + \mu u^b\left(c_s^b\right)\right]$$

under the resource constraint (18).

One can readily see that any solution to this program also solves (15) for $\mu_s = \mu$. But *ex ante* efficiency generates an additional constraint - namely, *the Pareto weight μ should be the same across states.* A consequence of this requirement is precisely that risk is shared efficiently between agents.

4.1.2 The sharing rule as a risk sharing mechanism

We now further specify the model by assuming that prices do not vary:

$$p_s = p,\ s = 1, ..., S$$

Let V^X denote the indirect utility of agent X. For any *ex post* efficient allocation, let ρ_s^X denote the total expenditure of agent X in state s:

$$\rho_s^X = \sum_i p_i c_{s,i}^X$$

Here as above, ρ^X is the *sharing rule* that governs the allocation of household resources between members. Obviously, we have that $\rho_s^a + \rho_s^b = y_s^a + y_s^b = y_s$. If we denote $\rho_s = \rho_s^a$, then $\rho_s^b = y_s - \rho_s$. Program (16) becomes:

$$W\left(y_1, ..., y_S; \mu\right) = \max_{\rho_1, ..., \rho_S} \sum_s \pi_s V^a\left(\rho_s\right) + \mu \sum_s \pi_s V^b\left(y_s - \rho_s\right) \quad (19)$$

In particular, in the absence of price fluctuations, the risk sharing problem is one-dimensional: agents transfer one 'commodity' (here dollars) across states, since they are able to trade it for others commodities on markets once the state of the world has been realized, in an *ex post* efficient manner.

4.1.3 When is a unitary representation acceptable?

The value of the previous program, $W\left(y_1, ..., y_S; \mu\right)$, describes the household's attitude towards risk. For instance, an income profile $\left(y_1, ..., y_S\right)$ is preferred over some alternative $\left(y_1', ..., y_S'\right)$ if and only if $W\left(y_1, ..., y_S; \mu\right) \geq W\left(y_1', ..., y_S'; \mu\right)$. Note, however, that preferences in general depend on the Pareto weight μ. That is, it is usually the case that profile $(y_1, ..., y_S)$ may be preferred over $(y'_1, ..., y'_S)$ for some values of μ but not for others. In that sense, W cannot be seen as a unitary household utility: the ranking over income profiles induced by W varies with the intrahousehold distribution of powers (as summarized by μ), which in turns depends on other aspects (*ex ante* distributions, individual reservation utilities,...).

A natural question is whether exceptions can be found, in which the household's preferences over income profiles would *not* depend on the member's respective powers. A simple example can convince us that, indeed, such exceptions exist. Assume, for instance, that both VNM utilities are logarithmic:

$$V^a(x) = V^b(x) = \log x$$

Then (19) can be written as:

$$\max_{\rho_1,\ldots,\rho_S} \sum_s \pi_s \log(\rho_s) + \mu \sum_s \pi_s \log(y_s - \rho_s) \tag{20}$$

First order conditions give

$$\frac{\pi_s}{\rho_s} = \frac{\mu \pi_s}{y_s - \rho_s}$$

therefore

$$\rho_s = \frac{y_s}{1+\mu}$$

Plugging into (20), we have that:

$$\begin{aligned} W(y_1, ..., y_S; \mu) &= \sum_s \pi_s \log\left(\frac{y_s}{1+\mu}\right) + \mu \sum_s \pi_s \log\left(\frac{\mu y_s}{1+\mu}\right) \\ &= \sum_s \pi_s \left[\left(\log\frac{1}{1+\mu} + \log y_s\right) + \mu\left(\log\frac{\mu}{1+\mu} + \log y_s\right)\right] \\ &= a(\mu) + 2\sum_s \pi_s \log y_s \end{aligned}$$

where

$$a(\mu) = \log\frac{1}{1+\mu} + \mu \log\frac{\mu}{1+\mu}$$

and we see that maximizing W is equivalent to maximizing $\sum_s \pi_s \log y_s$, which does not depend on μ. In other words, the household's behavior under uncertainty is equivalent to that of a *representative agent*, whose VNM utility, $V(x) = \log x$, is moreover the same as that of the individual members. Equivalently, the unitary approach - which assumes that the household behaves as if there was a single decision maker - is actually valid in that case.

How robust is this result? Under which general conditions is the unitary approach, based on a representative agent, a valid representation of household behavior under risk? Mazzocco (2004) shows that one condition is necessary and sufficient; namely, individual utilities must belong to the ISHARA class. Here, ISHARA stands for 'Identically Shaped Harmonic Absolute Risk Aversion', which imposes two properties:

- individual VNM utilities are of the harmonic absolute risk aversion (HARA) type, characterized by the fact that the index of absolute risk aversion, $-u''(x)/u'(x)$, is an harmonic function of income:

$$-\frac{u''(x)}{u'(x)} = \frac{1}{\gamma x + c}$$

  For $\gamma = 0$, we have the standard, constant absolute risk aversion (CARA). For $\gamma = 1$, we have an immediate generalization of the log form just discussed:

$$u^i(x) = \log\left(c^i + x\right)$$

  for some constants $c^i, i = a, b$. Finally, for $\gamma \neq 0$ and $\gamma \neq 1$, we have:

$$u^i(x) = \frac{\left(c^i + \gamma^i x\right)^{1-1/\gamma^i}}{1 - 1/\gamma^i}$$

  for some constants c^i and $\gamma^i, i = a, b$.

- moreover, the 'shape' coefficients γ must be equal:

$$\gamma^a = \gamma^b$$

The intuition of this result is that in the ISHARA case, the sharing rule that solves (19) is an affine function of realized income. Note that ISHARA is not simply a property of each utility independently: the second requirement imposes a compatibility restriction between them. That said, CARA utilities always belong to the ISHARA class, even if their coefficients of absolute risk aversion are different (that's because they correspond to $\gamma^a = \gamma^b = 0$). On the other hand, constant relative risk aversion (CRRA) utilities, which correspond to $c^a = c^b = 0$, are ISHARA if and only if the coefficient of relative risk aversion, equal to the shape parameter γ^i in that case, is identical for all members (it was equal to one for both spouses in our example).

4.2 Efficient risk sharing in a one-commodity world

4.2.1 Characterizing efficient risk sharing

We now characterize *ex ante* efficient allocations. We start with the case in which prices do not vary; as seen above, we can then model efficient risk sharing in a one commodity context. A sharing rule ρ shares risk efficiently if it solves a program of the form:

$$\max_{\boldsymbol{\rho}} \sum_s \pi_s \left[u^a\left(\rho\left(y_s^a, y_s^b\right)\right) + \mu u^b\left(y_s^a + y_s^b - \rho\left(y_s^a, y_s^b\right)\right)\right]$$

for some Pareto weight μ. The first order condition gives:

$$u'^a\left(\rho\left(y_s^a, y_s^b\right)\right) = \mu.u'^b\left(y_s^a + y_s^b - \rho\left(y_s^a, y_s^b\right)\right)$$

or equivalently:

$$\frac{u'^a(\rho_s)}{u'^b(y_s-\rho_s)} = \mu \text{ for each } s \tag{21}$$

where $y_s = y_s^a + y_s^b$ and $\rho_s = \rho\left(y_s^a, y_s^b\right)$.

This relationship has a striking property; namely, since μ is constant, the left hand side does not depend on the state of the world. This is a standard characterization of efficient risk sharing: *the ratio of marginal utilities of income of the agents remains constant across states of the world.*

The intuition for this property is easy to grasp. Assume there exists two states s and s' such that the equality does not hold - say:

$$\frac{u'^a(\rho_s)/d\rho}{u'^b(y_s-\rho_s)/d\rho} < \frac{u'^a(\rho_{s'})/d\rho}{u'^b(y_{s'}-\rho_{s'})/d\rho}$$

Then there exists some k such that

$$\frac{\pi_s u'^a(\rho_s)/d\rho}{\pi_{s'} u'^a(\rho_{s'})/d\rho} < k < \frac{\pi_s u'^b(y_s-\rho_s)/d\rho}{\pi_{s'} u'^b(y_{s'}-\rho_{s'})/d\rho}$$

But now, both agents can marginally improve their welfare by some additional trade. Indeed, if a pays some small amount ε to b in state s but receives $k\varepsilon$ in state s', a's welfare changes by

$$dW^a = -\pi_s u'^a(\rho_s)\,\varepsilon + \pi_{s'} u'^a(\rho_{s'})\,k\varepsilon > 0$$

while for b

$$dW^b = \pi_s u'^b(y_s-\rho_s)\,\varepsilon - \pi_{s'} u'^b(y_{s'}-\rho_{s'})\,k\varepsilon > 0$$

and both parties gain from that trade, contradicting the fact that the initial allocation was Pareto efficient.

The sharing rule ρ is thus a solution of equation (21), which can be rewritten as:

$$u'^a(\rho) = \mu u'^b(y_s-\rho) \tag{22}$$

where $\rho = \rho\left(y_s^a, y_s^b\right)$. Since the equation depends on the weight μ, there exists a continuum of efficient risk sharing rules, indexed by the parameter μ; the larger this parameter, the more favorable the rule is to member b.

As an illustration, assume that agents have Constant Absolute Risk Aversion (CARA) preferences with respective absolute risk aversions equal to α and β for a and b respectively:

$$u^a(x) = -\exp(-\alpha x)\,,\, u^b(x) = -\exp(-\beta x)$$

Then the previous equation becomes:

$$\alpha\exp\left[-\alpha\rho\left(y_s^a, y_s^b\right)\right] = \mu\beta\exp\left[-\beta\left(y_s^a + y_s^b - \rho\left(y_s^a, y_s^b\right)\right)\right]$$

which gives

$$\rho\left(y_s^a, y_s^b\right) = \frac{\beta}{\alpha+\beta}\left(y_s^a + y_s^b\right) - \frac{1}{\alpha+\beta}\log\left(\frac{\mu b}{\alpha}\right)$$

We see that CARA preferences lead to a linear sharing rule, with slope $b/(\alpha+b)$; the intercept depends on the Pareto weight μ.

Similarly, if both spouses exhibit Constant Relative Risk Aversion (CRRA) with identical relative risk aversion γ, then:

$$u^a(x) = u^b(x) = \frac{x^{1-\gamma}}{1-\gamma}$$

and the equation is:

$$\left[\rho\left(y_s^a, y_s^b\right)\right]^{-\gamma} = \mu\left[y_s^a + y_s^b - \rho\left(y_s^a, y_s^b\right)\right]^{-\gamma}$$

which gives

$$\rho\left(y_s^a, y_s^b\right) = k\left(y_s^a + y_s^b\right)$$

where

$$k = \frac{\mu^{-\frac{1}{\gamma}}}{1+\mu^{-\frac{1}{\gamma}}}$$

Therefore, with identical CRRA preferences, each spouse consumes a fixed fraction of total consumption, the fraction depending on the Pareto weight μ. Note that, in both examples, ρ only depends on the sum $y_s = y_s^a + y_s^b$, and

$$0 \leq \rho'(y_s) \leq 1$$

4.2.2 Properties of efficient sharing rules

While the previous forms are obviously specific to the CARA and CRRA cases, the two properties just mentioned are actually general.

Proposition 1 *For any efficient risk sharing agreement, the sharing rule ρ is a function of aggregate income only:*

$$\rho\left(y_s^a, y_s^b\right) = \bar{\rho}\left(y_s^a + y_s^b\right) = \bar{\rho}(y_s)$$

Moreover,

$$0 \leq \bar{\rho}' \leq 1$$

Proof. *Note, first, that the right hand side of equation (22) is increasing in ρ, while the left hand side is decreasing; therefore the solution in ρ must be unique. Now, take two pairs $\left(y_s^a, y_s^b\right)$ and $\left(\bar{y}_s^a, \bar{y}_s^b\right)$ such that $y_s^a + y_s^b = \bar{y}_s^a + \bar{y}_s^b$. Equation (22) is the same for both pairs, therefore its solution must be the same, which proves the first statement. Finally, differentiating (22) with respect to y_s gives:*

$$\frac{u''^a(\bar{\rho})}{u'^a(\bar{\rho})}\bar{\rho}' = \frac{u''^b(y_s-\bar{\rho})}{u'^b(y_s-\bar{\rho})}\left(1-\bar{\rho}'\right) \tag{23}$$

and finally:

$$\bar{\rho}'(y_s) = \frac{-\frac{u''^b(y_s-\bar{\rho})}{u'(y_s-\bar{\rho})}}{-\frac{u''^a(\bar{\rho})}{u'^a(\bar{\rho})} - \frac{u''^b(y_s-\bar{\rho})}{u'^b(y_s-\bar{\rho})}} \tag{24}$$

which belongs to the interval $[0,1]$*. Note, moreover, that* $0 < \bar{\rho}'(y_s) < 1$ *unless one of the agents is (locally) risk neutral.* ■

The first statement (23) is often called the *mutuality principle*. It states that when risk is shared efficiently, an agent's consumption is not affected by the idiosyncratic realization of her income; only shocks affecting aggregate resources (here, total income y_s) matter. It has been used to test for efficient risk sharing, although the precise test is much more complex than it may seem - we shall come back to this aspect below.

Formula (24) is quite interesting in itself. It can be rewritten as:

$$\bar{\rho}'(y_s) = \frac{-\frac{u'^a(\bar{\rho})}{u''^a(\bar{\rho})}}{-\frac{u'^a(\bar{\rho})}{u''^a(\bar{\rho})} - \frac{u'^b(y_s-\bar{\rho})}{u''^b(y_s-\bar{\rho})}} \tag{25}$$

The ratio $-\frac{u'^a(\bar{\rho})}{u''^a(\bar{\rho})}$ is called the *risk tolerance* of A; it is the inverse of A's risk aversion. Condition (25) states that the marginal risk is allocated between the agents in proportion of their respective risk tolerances. To put it differently, assume the household's total income fluctuates by one (additional) dollar. The fraction of this dollar fluctuation born by agent a is proportional to a's risk tolerance. To take an extreme case, if a was infinitely risk averse - that is, her risk tolerance was nil - then $\bar{\rho}' = 0$ and her share would remain constant: all the risk would be born by b.

It can actually be showed that the two conditions expressed by Proposition 1 are also sufficient. That is, take any sharing rule ρ satisfying them. Then one can find two utility functions u^a and u^b such that ρ shares risk efficiently between a and b.[9]

4.3 Efficient risk sharing in a multi-commodity context: an introduction

Regarding risk sharing, a multi commodity context is much more complex than the one-dimensional world just described. The key insight is that consumption decisions also depend on the relative prices of the various available commodities, and that typically these prices fluctuate as well. Surprisingly enough, sharing price risk is quite different from sharing income risk. A precise investigation would be outside the scope of the present volume; instead, we simply provide a short example.[10]

[9] The exact result is even slightly stronger; it states that for any ρ satisfying the conditions and any increasing, strictly concave utility u^A, one can find some increasing, strictly concave utility u^B such that ρ shares risk efficiently between A and B (see Chiappori, Townsend and Schulhofer-Wohl 2008 for a precise statement).

[10] The reader is referred to Chiappori, Townsend and Yamada (2008) for a precise analysis. The following example is also borrowed from this article.

Consider a two agent household, with two commodities - one labor supply and an aggregate consumption good. Assume, moreover, that agent b is risk neutral and only consumes, while agent a consumes, supplies labor and is risk averse (with respect to income risk). Formally, using Cobb-Douglas preferences:

$$U^a(c^a, l^a) = \frac{(l^a c^a)^{1-\gamma}}{1-\gamma} \text{ and } U^b(c^b) = c^b$$

with $\gamma > 1/2$. Finally, the household faces a linear budget constraint; let w_a denote 2's wages, and y (total) non labor income.

Since agent b is risk neutral, one may expect that she will bear all the risk. However, in the presence of wage fluctuations, it is *not* the case that agent a's consumption, labor supply or even utility will remain constant. Indeed, *ex ante* efficiency implies *ex post* efficiency, which in turn requires that the labor supply and consumption of a vary with his wage:

$$l^a = \frac{\rho + w_a T}{2w_a}, c^a = \frac{\rho + w_a T}{2}$$

where ρ is the sharing rule. The indirect utility of a is therefore:

$$V^a(\rho, w_a) = \frac{2^{\gamma-1}}{1-\gamma} (\rho + w_a T)^{2-2\gamma} w_a^{-(1-\gamma)}$$

while that of b is simply $V^b(y - \rho) = y - \rho$.

Now, let's see how *ex ante* efficiency restricts the sharing rule. Assume there exists S states of the world, and let $w_{a,s}, y_s$ and ρ_s denote wage, non labor income and the sharing rule in state s. Efficient risk sharing requires solving the program:

$$\max_{\rho} \sum_s \pi_s \left[V^a(\rho_s, w_{a,s}) + \mu V^b(y_s - \rho_s)\right]$$

leading to the first order condition:

$$\frac{\partial V^a(\rho_s, w_{a,s})}{\partial \rho_s} = \mu \frac{\partial V^b(y_s - \rho_s)}{\partial \rho_s}$$

In words, efficient risk sharing requires that the ratio of marginal utilities of income remains constant - a direct generalization of the previous results. Given the risk neutrality assumption for agent b, this boils down to the marginal utility of income of agent a remaining constant:

$$\frac{\partial V^a}{\partial \rho} = 2^{\gamma} (\rho + w_a T)^{1-2\gamma} w_a^{-(1-\gamma)} = K$$

which gives

$$\rho = 2K'.w_a^{\frac{1-\gamma}{1-2\gamma}} - w_a T$$

where K' is a constant depending on the respective Pareto weights. In the end:

$$l^a = K'.w_a^{\frac{-\gamma}{2\gamma-1}}, c^a = K'.w_a^{-\frac{1-\gamma}{2\gamma-1}}$$

and the indirect utility is of the form:

$$V^a = K''.w_a^{-\frac{1-\gamma}{2\gamma-1}}$$

for some constant K''. As expected, a is sheltered from non labor income risk by his risk sharing agreement with b. However, his consumption, labor supply and welfare fluctuate with his wage. The intuition is that that agents respond to price (or wage) variations by adjusting their demand (here labor supply) behavior in an optimal way. The maximization implicit in this process, in turn, introduces an element of convexity into the picture.[11]

4.4 Econometric issues

4.4.1 Distributions versus realizations

We now come back to the simpler, one-commodity framework. As expressed by Proposition 1, efficient risk sharing schemes satisfy the mutuality principle, which is a form of income pooling: the sharing rules depends only on total income, not on the agent's respective contributions y^a and y^b *per se*. This result may sound surprising; after all, income pooling is a standard implication of the unitary setting which is typically not valid in the collective framework; moreover, it is regularly rejected empirically.

The answer to this apparent puzzle is the crucial distinction between the (*ex post*) *realization* and the (*ex ante*) *distribution* of income shocks. When risk is shared efficiently, income realizations are pooled: my consumption should not suffer from my own bad luck, insofar as it does not affect aggregate resources. On the other hand, there exists a continuum of efficient allocations of resources, indexed by some Pareto weight; different weights correspond to different (contingent) consumptions. The Pareto weight, in turn, depends on the *ex ante* situations of the agents; for instance, if a has a much larger *expected* income, one can expect that his Pareto weight will be larger than b's, resulting in a higher level of consumption. In other words, the pooling property does not apply to expected incomes, and in general to any feature (variance, skewness,...) of the probability distributions of individual income streams. The main intuition of the collective model is therefore maintained: power (as summarized by Pareto weights) matters for behavior - the nuance being that under efficient risk sharing it is the distribution of income, instead of its realization, that (may) affect individual powers.

In practice, however, this raises a difficult econometric issue. Testing for efficient risk sharing requires checking whether observed behavior satisfies the

[11] Generally, the ability of risk neutral agents to adjust actions *after* the state is observed induces a "risk loving" ingredient, whereby higher price variation is preferred, and which may counterweight the agent's risk aversion.

mutuality principle, that is pooling of income realization. However, by the previous argument, this requires being able to distinguish between *ex post* realizations and *ex ante* distributions. On cross-sectional data, this is impossible.

It follows that cross-sectional tests of efficient risk sharing are plagued with misspecification problems. For instance, some (naive) tests of efficient risk sharing that can be found in the literature rely on a simple idea: since individual consumption should not respond to idiosyncratic income shocks (but only to aggregate ones), one may, on cross sectional data, regress individual consumption (or more specifically marginal utility of individual consumption) on (i) indicators of aggregate shocks (for example, aggregate income or consumption), and (ii) individual incomes. According to this logic, a statistically significant impact of individual income on individual consumption, controlling for aggregate shocks, should indicate inefficient risk sharing.

Unfortunately, the previous argument suggests that in the presence of heterogeneous income processes, a test of this type is just incorrect. To get an intuitive grasp of the problem, assume that two agents a and b share risk efficiently. However, the *ex ante* distributions of their respective incomes are very different. a's income is almost constant; on the contrary, b may be hit by a strong, negative income shock. In practice, one may expect that this asymmetry will be reflected in the respective Pareto weights; since b desperately needs insurance against the negative shock, he will be willing to accept a lower weight, resulting in lower expected consumption than a, as a compensation for the coverage provided by a.

Consider, now, a large economy consisting of many independent clones of a and b; assume for simplicity that, by the law of large numbers, aggregate resources do not vary. By the mutuality principle, efficient risk sharing implies that individual consumptions should be constant as well; and since a agents have more weight, their consumption will always be larger than that of b agents. Assume now than an econometrician analyzes a cross section of this economy. The econometrician will observe two features. One is that some agents (the 'unlucky' b's) have a very low income. Secondly, these agents also exhibit lower consumption levels than the average of the others (since they consume as much as the lucky b's but less than all the a's). Technically, any cross sectional regression will find a positive and significant correlation between individual incomes and consumptions, which seems to reject efficient risk sharing - despite the fact that the mutuality principle is in fact perfectly satisfied, and risk sharing is actually fully efficient. The key remark, here, is that the rejection is spurious and due to a misspecification of the model. Technically, income is found to matter only because income *realizations* capture (or are proxies for) specific features of income *distributions* that influence Pareto weights.

4.4.2 A simple solution

We now discuss a specific way of solving the problem. It relies on the availability of (short) panel data, and on two additional assumptions. One is that agent's preferences exhibit Constant Relative Risk Aversion (CRRA), a functional form that is standard in this literature. In practice:

$$u^a(x) = \frac{x^{1-\alpha}}{1-\alpha}, u^b(x) = \frac{x^{1-\beta}}{1-\beta}$$

The second, much stronger assumption is that risk aversion is identical across agents, implying $\alpha = \beta$ in the previous form.

Under these assumptions, the efficiency condition (22) becomes:

$$\rho^{-\alpha} = \mu (y - \rho)^{-\alpha} \tag{26}$$

leading to

$$\rho(y, \mu) = \frac{\mu^{-\frac{1}{\alpha}}}{1 + \mu^{-\frac{1}{\alpha}}} y \quad \text{and} \quad y - \rho(y, \mu) = \frac{1}{1 + \mu^{-\frac{1}{\alpha}}} y$$

In words, the sharing rule is a linear function of income, the coefficient depending on the Pareto weights. Taking logs:

$$\begin{aligned}
\log c^a &= \log \rho = \log \left(\frac{\mu^{-\frac{1}{\alpha}}}{1 + \mu^{-\frac{1}{\alpha}}} \right) + \log y, \text{ and} \\
\log c^b &= \log \left(\frac{1}{1 + \mu^{-\frac{1}{\alpha}}} \right) + \log y
\end{aligned}$$

Assume, now, that agents are observed for at least two periods. We can compute the difference between log consumptions in two successive periods, and thus eliminate the Pareto weights; we get:

$$\Delta \log c^a = \Delta \log c^b = \Delta \log y$$

In words, a given variation, in percentage, of aggregate income should generate equal percentage variations in all individual consumptions.[12]

Of course, this simplicity comes at a cost - namely, the assumption that individuals have identical preferences: one can readily check that with different risk aversion, the sharing rule is not linear, and differencing log consumptions does not eliminate Pareto weights. Assuming homogeneous risk aversions is difficult for two reasons. First, all empirical studies suggest that the cross sectional variance of risk aversion in the population is huge. Second, even if we assume that agents match to share risk (so that a sample of people belonging to the same risk sharing group is not representative of the general population), theory[13] suggests that the matching should actually be *negative assortative* (that is, more risk averse agents should be matched with less risk averse ones) - so

[12] This prediction is easy to test even on short panels - see for instance Altonji *et al* (1992) and Duflo and Udry (2004); incidentally, it is usually rejected. See Mazzocco and Saini (2006) for a precise discussion.

[13] See, for instance Chiappori and Reny (2007).

that heterogeneity should be, if anything, *larger* within risk sharing groups than in the general population.[14]

Finally, can we test for efficient risk sharing without this assumption? The answer is yes; such a test is developed for instance in Chiappori, Townsend and Schulhofer-Wohl (2008) and in Chiappori, Townsend and Yamada (2008). However, it requires long panels - since one must be able to disentangle the respective impacts of income distributions and realizations.

5 Intertemporal Behavior

5.1 The unitary approach: Euler equations at the household level

We now extend the model to take into account the dynamics of the relationships under consideration. Throughout this section, we assume that preferences are time separable and of the expected utility type. The first contributions extending the collective model to an intertemporal setting are due to Mazzocco (2004, 2007); our presentation follows his approach. Throughout this section, the household consists of two egoistic agents who live for T periods. In each period $t \in \{1, ..., T\}$, let y_t^i denote the income of member i.

We start with the case of a unique commodity which is privately consumed; c_t^i denotes member i's consumption at date t and p_t is the corresponding price. The household can save by using a risk-free asset; let s_t denotes the net level of (aggregate) savings at date t, and R_t its gross return. Note that, in general, y_t^i, s_t and c_t^i are random variables

We start with the standard representation of household dynamics, based on a unitary framework. Assume, therefore, that there exists a utility function $u\left(c^a, c^b\right)$ representing the household's preferences. The program describing dynamic choices is:

$$\max E_0 \left(\sum_t \beta^t u\left(c_t^a, c_t^b\right) \right)$$

under the constraint

$$p_t\left(c_t^a + c_t^b\right) + s_t = y_t^a + y_t^b + R_t s_{t-1}, \ \ t = 0, ..., T$$

Here, E_0 denotes the expectation taken at date 0, and β is the household's discount factor. Note that if borrowing is excluded, we must add the constraint $s_t \geq 0$.

[14] An alternative test relies on the assumption that agents have CARA preferences. Then, as seen above, the sharing rule is an affine function, in which only the intercept depends on Pareto weights (the slope is determined by respective risk tolerances). It follows that variations in *levels* of individual consumptions are proportional to variations in total income, the coefficient being independent of Pareto weights. The very nice feature of this solution, adopted for instance by Townsend (1994), is that it is compatible with any level of heterogeneity in risk aversion. Its main drawback is that the CARA assumption is largely counterfactual; empirical evidence suggests that *absolute* risk aversion decreases with wealth.

Using a standard result by Hicks, we can define household utility as a function of total household consumption; technically, the function U is defined by:

$$U(c) = \max \left\{ u\left(c^a, c^b\right) \text{ such that } c^a + c^b = c \right\}$$

and the program becomes:

$$\max E_0 \left(\sum_t \beta^t U(c_t) \right)$$

under the constraint

$$p_t c_t + s_t = y_t^a + y_t^b + R_t s_{t-1}$$

The first order conditions give the well-known Euler equations:

$$\frac{U'(c_t)}{p_t} = \beta E_t \left[\frac{U'(c_{t+1})}{p_{t+1}} R_{t+1} \right] \tag{27}$$

In words, the marginal utility of each dollar consumed today equals, in expectation, β times the marginal utility of R_{t+1} dollars consumed tomorrow; one cannot therefore increase utility by marginally altering the savings.

In practice, many articles test the empirical validity of these household Euler equations using general samples, including both couples and singles (see Browning and Lusardi 1995 for an early survey); most of the time, the conditions are rejected. Interestingly, however, Mazzocco (2004) estimates the same standard household Euler equations *separately* for couples and for singles. Using the CEX and the Panel Study of Income Dynamics (PSID), he finds that the conditions are rejected for couples, but not for singles. This seems to suggest that the rejection obtained in most articles may not be due to technical issues (for example, non separability of labor supply), but more fundamentally to a misrepresentation of household decision processes.

5.2 Collective Euler equations under *ex ante* efficiency

5.2.1 Household consumption

We now consider a collective version of the model. Keeping for the moment the single commodity assumption, we now assume that agents have their own preferences and discount factors. The Pareto program is therefore:

$$\max (1-\mu) E_0 \left(\sum_t \left(\beta^a\right)^t u^a\left(c_t^a\right) \right) + \mu E_0 \left(\sum_t \left(\beta^b\right)^t u^b\left(c_t^b\right) \right)$$

under the same constraints as above. First order conditions give:

$$\begin{aligned} \frac{u'^a\left(c_t^a\right)}{p_t} &= \beta^a E_t \left[\frac{u'^a\left(c_{t+1}^a\right)}{p_{t+1}} R_{t+1} \right] \\ \frac{u'^b\left(c_t^b\right)}{p_t} &= \beta^b E_t \left[\frac{u'^b\left(c_{t+1}^b\right)}{p_{t+1}} R_{t+1} \right] \end{aligned} \tag{28}$$

which are the *individual* Euler equations. In addition, individual consumptions at each period must be such that:

$$\frac{(\beta^a)^t}{\left(\beta^b\right)^t}\frac{u'^a\left(c_t^a\right)}{u'^b\left(c_t^b\right)}=\frac{\mu}{1-\mu} \tag{29}$$

The right hand side does not depend on t: the ratio of discounted marginal utilities of income of the two spouses must be constant through time. This implies, in particular, that

$$\frac{u'^a\left(c_t^a\right)}{u'^b\left(c_t^b\right)}=\frac{\mu}{1-\mu}\frac{\left(\beta^b\right)^t}{(\beta^a)^t}$$

If, for instance, a is more patient than b, in the sense that $\beta^a > \beta^b$, then the ratio u'^a/u'^b declines with time, because a postpones a larger fraction of her consumption than b.

An important remark is that if individual consumptions satisfy (28), then typically the aggregate consumption process $c_t = c_t^a+c_t^b$ does *not* satisfy an individual Euler equation like (27), except in one particular case, namely ISHARA utilities and identical discount factors. For instance, assume, following Mazzocco (2004), that individuals have utilities of the CRRA form:

$$u^X(c)=\frac{c^{1-\gamma^X}}{1-\gamma^X},\quad X=a,b$$

and that, moreover, $\beta^a=\beta^b=\beta$. Then (28) becomes:

$$\begin{aligned} c_t^a &= \left\{\beta p_t E_t\left[\frac{R_{t+1}}{p_{t+1}}\left(c_{t+1}^a\right)^{-\gamma^a}\right]\right\}^{-1/\gamma^a} \\ c_t^b &= \left\{\beta p_t E_t\left[\frac{R_{t+1}}{p_{t+1}}\left(c_{t+1}^b\right)^{-\gamma^b}\right]\right\}^{-1/\gamma^b} \end{aligned} \tag{30}$$

If $\gamma^a=\gamma^b$ (the ISHARA case), one can readily see that the ratio c_{t+1}^a/c_{t+1}^b is constant across states of the world; therefore

$$c_{t+1}^a=kc_{t+1},\ c_{t+1}^b=(1-k)\,c_{t+1}$$

for some constant k. It follows that:

$$\begin{aligned} c_t^a &= \left\{\beta p_t E_t\left[\frac{R_{t+1}}{p_{t+1}}\left(kc_{t+1}\right)^{-\gamma}\right]\right\}^{-1/\gamma} \\ &= k\left\{\beta p_t E_t\left[\frac{R_{t+1}}{p_{t+1}}\left(c_{t+1}\right)^{-\gamma}\right]\right\}^{-1/\gamma} \end{aligned} \tag{31}$$

and by the same token

$$c_t^b = (1-k)\left\{\beta p_t E_t\left[\frac{R_{t+1}}{p_{t+1}}\left(c_{t+1}\right)^{-\gamma}\right]\right\}^{-1/\gamma}$$

so that finally:

$$c_t = c_t^a + c_t^b = \left\{\beta p_t E_t\left[\frac{R_{t+1}}{p_{t+1}}\left(c_{t+1}\right)^{-\gamma}\right]\right\}^{-1/\gamma} \tag{32}$$

and aggregate consumption satisfies an individual Euler equation: the household behaves as a single.

However, in the (general) case $\gamma^a \neq \gamma^b$, Mazzocco shows that this result no longer holds, and household aggregate consumption does not satisfy a Euler equation even though each individual consumption does. In particular, testing the Euler conditions on aggregate household consumption should lead to a rejection even when all the necessary assumptions (efficiency, no credit constraints, ...) are fulfilled.

5.2.2 Individual consumption and labor supply

The previous, negative result is not really surprising: it simply stresses once more than groups, in general, do not behave as single individuals. What then? Well, if individual consumptions are observable, conditions (28) and (29) are readily testable using the standard approach. Most of the time, however, only aggregate consumption is observed. Then a less restrictive framework is needed. In particular, one may relax the single commodity assumption. Take, for instance, a standard model of labor supply, in which each agent consumes two commodities, namely leisure and a consumption good. The collective model suggests that individual consumptions can be recovered (up to additive constants - see chapters 4 and 5). Then tests of the Euler equation family can be performed.

As an illustration, Mazzocco (2007) studies a dynamic version of the collective model introduced in chapter 4. The individual Euler equations become, with obvious notations:

$$\begin{aligned}
\frac{\partial u^i\left(c_t^i, l_t^i\right)/\partial c}{p_t} &= \beta^i E_t\left[\frac{\partial u^i\left(c_{t+1}^i, l_{t+1}^i\right)/\partial c}{p_{t+1}} R_{t+1}\right] \\
\frac{\partial u^i\left(c_t^i, l_t^i\right)/\partial l}{w_t^i} &= \beta^i E_t\left[\frac{\partial u^i\left(c_{t+1}^i, l_{t+1}^i\right)/\partial l}{w_{t+1}^i} R_{t+1}\right]
\end{aligned} \tag{33}$$

for $i = a, b$. In particular, since individual labor supplies are observable, these equations can be estimated.

5.3 The *ex ante* inefficiency case

What, now, if the commitment assumption is not valid? We have seen above that this case has a simple, technical translation in the collective framework -

namely, the Pareto weights are not constant. A first remark, due to Mazzocco (2007), is that even in the ISHARA case, aggregate consumption no longer satisfies the martingale property (32). Indeed, let μ_t denote the Pareto weight of b in period t, and assume for the moment that μ_t does not depend on the agent's previous consumption decisions. We first have that

$$c_t^a + c_t^b = \left\{ \beta p_t E_t \left[\frac{R_{t+1}}{p_{t+1}} \left(c_{t+1}^a \right)^{-\gamma^a} \right] \right\}^{-1/\gamma^a} + \left\{ \beta p_t E_t \left[\frac{R_{t+1}}{p_{t+1}} \left(c_{t+1}^b \right)^{-\gamma^b} \right] \right\}^{-1/\gamma^b} \tag{34}$$

Moreover,

$$\frac{u'^a \left(c_t^a \right)}{u'^b \left(c_t^b \right)} = \frac{\mu_t}{1 - \mu_t} \left(\frac{\beta^b}{\beta^a} \right)^t \tag{35}$$

for all t, which for ISHARA ($\gamma^a = \gamma^b = \gamma$) preferences becomes

$$\left(\frac{c_t^a}{c_t^b} \right)^{-\gamma} = \frac{\mu_t}{1 - \mu_t} \left(\frac{\beta^b}{\beta^a} \right)^t$$

If μ_t is not constant, neither is the ratio c_t^a / c_t^b. A result by Hardy, Littlewood and Polya (1952) implies that whenever the ratio x/y is not constant, then for all probability distributions on x and y:

$$\left\{ E_t \left[(x+y)^{-\gamma} \right] \right\}^{-1/\gamma} > \left\{ E_t \left[x^{-\gamma} \right] \right\}^{-1/\gamma} + \left\{ E_t \left[y^{-\gamma} \right] \right\}^{-1/\gamma}$$

which directly implies that:

$$\left\{ \beta p_t E_t \left[\frac{R_{t+1}}{p_{t+1}} \left(c_{t+1} \right)^{-\gamma} \right] \right\}^{-1/\gamma} > c_t$$

In words, the (marginal utility of) aggregate consumption now follows a *supermartingale.*

Regarding now *individual* consumptions, one can readily check that equations (28) become:

$$\begin{aligned} (1 - \mu_t) \frac{u'^a \left(c_t^a \right)}{p_t} &= \left(1 - \mu_{t+1} \right) \beta^a E_t \left[\frac{u'^a \left(c_{t+1}^a \right)}{p_{t+1}} R_{t+1} \right] \\ \mu_t \frac{u'^b \left(c_t^b \right)}{p_t} &= \mu_{t+1} \beta^b E_t \left[\frac{u'^b \left(c_{t+1}^b \right)}{p_{t+1}} R_{t+1} \right] \end{aligned} \tag{36}$$

or equivalently:

$$\begin{aligned} E_t \left[\frac{u'^a \left(c_{t+1}^a \right)}{u'^a \left(c_t^a \right)} \frac{p_t R_{t+1}}{p_{t+1}} \right] &= \frac{1}{\beta^a} \frac{1 - \mu_t}{1 - \mu_{t+1}} \\ E_t \left[\frac{u'^b \left(c_{t+1}^b \right)}{u'^b \left(c_t^b \right)} \frac{p_t R_{t+1}}{p_{t+1}} \right] &= \frac{1}{\beta^b} \frac{\mu_t}{\mu_{t+1}} \end{aligned} \tag{37}$$

In words: under full commitment, the left hand side expressions should be constant, while they may vary in the general case. A first implication, therefore, is that whenever individual consumptions are observable, then the commitment assumption is testable. Moreover, we know that (35) holds for each t. These relations imply that μ_t is identifiable from the data. That is, if Pareto weights vary, it is possible to identify their variations, which can help characterizing the type of additional constraint that hampers full commitment.

Finally, individual consumptions are not observed in general, but individual labor supplies typically are; the same tests can therefore be performed using labor supplies as indicated above. Again, the reader is referred to Mazzocco (2007) for precise statements and empirical implementations. In particular, Mazzocco finds that both the unitary and the collective model with commitment are rejected, whereas the collective model without commitment is not. This finding suggests that while static efficiency may be expected to hold in general, dynamic (*ex ante*) efficiency may be more problematic.

5.4 Conclusion

The previous results suggest several conclusions. One is that the collective approach provides a simple generalization of the standard, 'unitary' approach to dynamic household behavior. Empirically, this generalization seems to work significantly better than the unitary framework. For instance, a well-known result in the consumption literature is that household Euler equations display excess sensitivity to income shocks. The two main explanations are the existence of borrowing constraints and non-separability between consumption and leisure. However, the findings in Mazzocco (2007) indicate that cross-sectional and longitudinal variations in relative decision power explain a significant part of the excess sensitivity of consumption growth to income shocks. Such variations, besides being interesting per se, are therefore crucial to understanding the dynamics of household consumption. A second conclusion is that the commitment issue is a crucial dimension of this dynamics; a couple in which agents can credibly commit on the long run will exhibit behavioral patterns that are highly specific. Thirdly, it is possible to develop models that, in their most general form, can capture both the 'collective' dimensions of household relationships and the limits affecting the spouse's ability to commit. The unitary model *and* the full efficiency version of the collective approach are nested within this general framework, and can be tested against it.

6 Divorce

6.1 The basic model

Among the limits affecting the spouses' ability, an obvious one is the possibility of divorce. Although divorce is, in many respects, an ancient institution, it is now more widespread than ever, at least in Western countries. Chiappori, Iyigun and Weiss (2008) indicate for instance that in 2001, among American women then in their 50s, no less than 39% had divorced at least once (and 26% had married at least twice); the numbers for men are slightly higher (respectively 41% and 31%). Similar patterns can be observed in Europe (see chapter 1). Moreover, in most developed countries unilateral divorce has been adopted as the legal norm. This implies that any spouse may divorce if (s)he will. In practice, therefore, divorce introduces a constraint on intertemporal allocations within the couple; that is, at any period, spouses must receive each within marriage at least as much as they would get if they were divorced.

Clearly, modeling divorce - and more generally household formation and dissolution - is an important aspect of family economics. For that purpose, a unitary representation is probably not the best tool, because it is essential to distinguish *individual* utilities within the couple. If each spouse is characterized, both before and after marriage, by a single utility, while the couple itself is represented by a third utility with little or no link with the previous ones, modeling divorce (or marriage for that matter) becomes very difficult and largely *ad hoc* . Even if the couple's preferences are closely related to individual utilities, for instance through a welfare function a la Samuelson, one would like to investigate the impact of external conditions (such as wages, the tax-benefit system or the situation on the marriage market) on the decision process leading to divorce; again, embedding the analysis within the black box of a unitary setting does not help clarifying these issues.

In what follows, we show how the collective approach provides a useful framework for modeling household formation and dissolution. Two ingredients are crucial for this task. One is the presence of *economic gains* from marriage. A typical example is the presence of public goods, as we have extensively discussed in the previous chapters. Alternative sources of marital gains include risk sharing or intertemporal consumption smoothing, along the lines sketched in the previous sections. At any rate, we must first recognize that forming a couple is often efficient from the pure economic perspective.

A second ingredient is the existence of *non-pecuniary benefits* to marriage. These 'benefits' can be interpreted in various ways: they may represent love, companionship, or other aspects. The key feature, in any case, is that these benefits are match-specific (in that sense, they are an indicator of the 'quality' of the match under consideration) and they cannot be exactly predicted *ex ante*; on the contrary, we shall assume that they are revealed with some lag (and may in general be different for the two spouses). The basic mechanism is that a poor realization of the non pecuniary benefits may trigger divorce, either because agents hope to remarry (and, so to speak, 'take a new draw' from the distribution of match quality), or because the match is so unsatisfactory that the spouses would be better off as singles, even at the cost of forgoing the economic gains from marriage. The existence of a trade-off between the economic surplus generated by marriage and the poor realization of non economic benefits plays a central role in most models of divorce.

More specifically, we shall consider a collective framework in which couples may consume both private and public goods, and marriage generates a non-pecuniary benefit. In principle, this benefit can enter individual utilities in an arbitrary manner. In what follows, however, we concentrate on a particular and especially tractable version of the model, initially due to Weiss and Willis (1993, 1997), in which the non monetary gain is additive; that is, the utility of each spouse is of the form

$$U^i = u^i\left(q^i, Q\right) + \theta^i, \quad i = a, b$$

where $q^i = \left(q_1^i, ..., q_n^i\right)$ is the vector of private consumption of agent i, $Q = (Q_1, ..., Q_N)$ is the vector of household public consumption, and θ^i is the non monetary gain of i. In particular, while the total utility does depend on the non monetary components θ^i, the marginal rates of substitution between consumption goods does not, which simplifies the analysis.

For any couple, the pair $\left(\theta^a, \theta^b\right)$ of match qualities is drawn from a given distribution Φ. In general, any correlation between θ^a and θ^b is possible. Some models introduce an additional restriction by assuming that the quality of the match is the same for both spouses - that is, $\theta^a = \theta^b$.

To keep things simple, we present the model in a two periods framework. In period one, agents marry and consume. At the end of the period, the quality of the match is revealed, and agents decide whether to remain married or split. If they do not divorce, they consume during the second period, and in addition enjoy the same non monetary gain as before. If they split, we assume for the moment that they remain single for the rest of the period, and that they privately consume the (previously) public goods.[15] The prices of the commodities will not play a role in what follows; we may, for simplicity, normalize them to unity.

Finally, let y^a and y^b denote the agents' respective *initial* incomes, which they receive at the beginning of each period; and to simplify, we assume no savings and borrowing. In case of divorce, the couple's total income, $y^a + y^b$, is split between the ex-spouses. The rule governing this division leads to an allocation in which a receives some $D^a\left(y^a, y^b\right)$ and b receives $D^b\left(y^a, y^b\right) = y^a + y^b - D^a\left(y^a, y^b\right)$. For instance, if incomes are considered to be private property of each spouse, then $D^i\left(y^a, y^b\right) = y^i, i = a, b$, whereas an equal distribution rule would lead to $D^a\left(y^a, y^b\right) = D^b\left(y^a, y^b\right) = \left(y^a + y^b\right)/2$. A natural interpretation is that the rule $D = \left(D^a, D^b\right)$ is exogenous and imposed by law; however, while an agent cannot be forced to transfer to the ex-spouse more than the legal amount D, he may freely elect to do so, and will in some cases (see next subsection). An alternative approach considers divorce contracts as endogenous, for instance in a risk sharing perspective.[16]

We may now analyze the couple's divorce decision. First, the second period utility of agent i if divorced is simply $V^i\left(D^i\left(y^a, y^b\right)\right)$ (where, as before, V^i is agent i's indirect utility). If, on the other hand, the spouses remain married,

[15] Some commodities may remain public even after divorce; children expenditures are a typical example. For a detailed investigation, see Chiappori et al. (2007).

[16] See for instance Chiappori and Weiss (2009).

then they choose some efficient allocation; as usual, their consumption plan therefore solves a program of the type:

$$\max u^a\left(q^a, Q\right) + \theta^a$$

under the constraints:

$$\begin{aligned} \sum_j \left(q_j^a + q_j^b\right) + \sum_k Q_k &= y^a + y^b \\ u^b\left(q^b, Q\right) + \theta^b &\geq \bar{u}^b \end{aligned}$$

where $\bar{u}^b$ is a constant. Let $\left(\bar{q}^a, \bar{q}^b, \bar{Q}\right)$ denote the solution to this program, and $\bar{u}^a = u^a\left(q^a, Q\right) + \theta^a$ the corresponding utility for a. Note that both are functions of $\bar{u}^b$; we note therefore $\bar{u}^a\left(\bar{u}^b\right)$. Let $\mathcal{P}_M$ denote the Pareto set if married, that is the set of utilities $\left(u^a, u^b\right)$ such that $u^a \leq \bar{u}^a\left(u^b\right)$; in words, any pair of utilities in $\mathcal{P}_M$ can be reached by the couple if they remain married.

Then we are in one of the following two situations:

- either the reservation point $\left(V^a\left(D^a\left(y^a, y^b\right)\right), V^b\left(D^b\left(y^a, y^b\right)\right)\right)$, representing the pair of individual utilities reachable through divorce, belongs to the Pareto set if married, $\mathcal{P}_M$. Then there exists a second period distribution of income which is preferred over divorce by *both* spouses. The efficiency assumption implies that this opportunity will be taken, and the model predicts that the marriage will continue.
- or, alternatively, $\left(V^a\left(D^a\left(y^a, y^b\right)\right), V^b\left(D^b\left(y^a, y^b\right)\right)\right)$ is outside $\mathcal{P}_M$. Then the marriage cannot continue, because any second period allocation of resources the spouses may choose will be such that one spouse at least would be better off as a single; therefore, divorce must follow.

The model thus provides a precise description of the divorce decision; namely, divorce takes place whenever it is the efficient decision under the constraint that agents cannot receive less than their reservation utility $V^i\left(D^i\left(y^a, y^b\right)\right), i = a, b$.

Some remarks are in order at this point. First, the argument presented above assumes that divorce is unilateral, in the sense that each partner is free to terminate the marriage and obtain divorce, even if the spouse does not agree. An alternative setting requires mutual consent - that is, divorce cannot occur unless *both* spouses agree. An old question of family economics is whether a shift from mutual consent to unilateral has an impact on divorce rates; we shall consider that question in the next subsection.

Secondly, the fact that spouses *may* disagree about divorce - that is, a spouse may ask for divorce against the partner's will - does not imply that they *will*. In the setting just presented, a partner who would consider divorce may sometimes be 'bribed back' into marriage by her spouse, through an adequate redistribution of income. Only when such a redistribution cannot take place, because the cost to the remaining partner would exceed the benefits of remaining married, will divorce occur. In that sense, there is not disagreement about divorce in this model; simply, divorce sometimes comes out as the best solution available.

A third remark is that, ultimately, divorce is triggered by the realization of the match quality parameters $\left(\theta^a, \theta^b\right)$. Large values of the θs inflate the Pareto frontier, making it more likely to contain the divorce threat point; conversely, poor realizations contract it, and divorce becomes probable. Formally, it is easy to check that the divorce decision is monotonic in the θs, in the sense that if a couple remains married for some realization $\left(\bar{\theta}^a, \bar{\theta}^b\right)$, then they also do for any $\left(\theta^a, \theta^b\right)$ such that $\theta^i \geq \bar{\theta}^i$, $i = a, b$; and conversely, if they divorce for some $\left(\bar{\theta}^a, \bar{\theta}^b\right)$, so do they for any $\left(\theta^a, \theta^b\right)$ such that $\theta^i \leq \bar{\theta}^i$, $i = a, b$. In general, there exists a divorce frontier, namely a decreasing function ϕ such that the coupe divorce if and only if $\theta^a < \phi\left(\theta^b\right)$. Note, however, that for a 'neutral' realization $\theta^a = \theta^b = 0$, the couples always remains married, because of the marital gains arising from the presence of public consumption; negative shocks are required for a marriage to end.

Finally, how is the model modified when divorced agents are allowed to remarry? The basic principle remains valid - that is, agents (efficiently) divorce if no point within the Pareto frontier if married can provide both agents with the same expected utility as if single. The latter value is however more difficult to compute, because it now includes the probability of finding a new mate multiplied by the utility the ex spouse will get in their new marriage. In other words, one need to predict which particular allocation of resources and welfare will prevail in newly formed couples - a task that requires a more complete investigation of the equilibrium forces governing the (re)marriage market. We shall come back to this issue in the second part of the book.

6.2 Divorce under transferable utility and the Becker-Coase theorem[17]

6.2.1 The TU framework

We now further investigate the divorce model under an additional assumption - namely, that utility is transferable between spouses, both during and after marriage. Technically, we first assume that preferences of married individuals are of the *generalized quasi-linear* (*GQL*) form (see Bergstrom, 1989).

$$u_m^i\left(q^i, Q\right) = F(Q)\, q_1^i + G_m^i\left(Q, q_{-1}^i\right) + \theta^i,\ i = a, b \tag{38}$$

where $q_{-1}^i = \left(q_2^i, ...q_n^i\right)$. Here, the functions F and $G_m^i,\ i = a, b$, are positive, increasing, concave functions such that $F(0) = 1$ and $G_m^i(0) = 0$.

Secondly, we assume that preferences if single take the *strictly quasi-linear* form:

$$u_s^i\left(q^i, Q\right) = q_i^1 + G_s^i\left(Q, q_{-1}^i\right),\ i = a, b, \tag{39}$$

[17] The material presented in this subsection is borrowed from Chiappori, Iyigun and Weiss (2007).

where again the G_s^i, $i = a, b$, are increasing concave functions, with $G_s^i(0) = 0$. Because of quasi-linearity, the optimal consumptions of public goods and private goods other than good 1 are given by the conditions:

$$\frac{\partial G_s^i\left(Q, q_{-1}^i\right)}{\partial Q^j} = 1, \quad 1 \leq j \leq N \qquad \text{and} \qquad \frac{\partial G_s^i\left(Q, q_{-1}^i\right)}{\partial q_{ki}^i} = 1, \quad 2 \leq k \leq n \; .$$

Neither these conditions nor the optimal levels of all private and public consumptions (except for good 1) depend on income. Let these optimal levels be denoted $\left(\bar{Q}, \bar{q}_{-1}^i\right)$. To simplify notations, we choose units such that $G_s^i\left(\bar{Q}, \bar{q}_{-1}^i\right) = \sum_{j=1}^N \bar{Q}_j + \sum_{k=2}^n \bar{q}_k^i$, $i = a, b$. Then, the indirect utility of a single person equals his or her income.

Now, consider a man with income y^a married with a woman with income y^b. There is a unique efficient level for the consumption of each of the public goods and each of the private goods 2 to n. Moreover, these levels depend only on the total income of the partners, $y = y^a + y^b$. If we define

$$\eta(y) = \max_{\left(Q, q_{-1}^a, q_{-1}^b\right)} \left\{ \begin{array}{c} F(Q)\left[y - \sum_{j=1}^N Q_j + \sum_{k=2}^n \left(q_k^a + q_k^b\right)\right] \\ +G_m^a\left(Q, q_{-1}^a\right) + G_m^b\left(Q, q_{-1}^b\right) \end{array} \right\}$$

then the Pareto frontier is given by

$$u_m^a + u_m^b = \eta(y) + \theta^a + \theta^b, \tag{40}$$

Here, u_m^a and u_m^b are the attainable utility levels that can be implemented by the allocations of the private good q_1 between the two spouses, given the efficient consumption levels of all other goods. The Pareto frontier is a straight line with slope -1: utility is transferable between spouses (see chapter 3). Assuming, as is standard, that the optimal public consumptions are such that $F(Q)$ is increasing in Q, we see that $\eta(y)$ is *increasing and convex* in y.[18] Moreover, $\eta(0) = 0$ and $\eta'(0) = F(0) = 1$. Since η is convex, this implies that $\eta(y) > y$ and $\eta'(y) > 1$ for all $y > 0$.

Finally, if divorce takes place, the post-divorce utility of agent i is $V_s^i\left(D^i\left(y^a, y^b\right)\right) = D^i\left(y^a, y^b\right)$. In particular, we see that

$$V_s^a + V_s^b = D^a\left(y^a, y^b\right) + D^b\left(y^a, y^b\right) = y^a + y^b = y \tag{41}$$

In this framework, the divorce decision takes a particularly simple form. Indeed, agents divorce if and only if the point $\left(V_s^a, V_s^b\right)$ is outside the Pareto set when married. Given (40), this occurs when the sum $V_s^a + V_s^b$ is larger than $\eta(y) + \theta^a + \theta^b$. Using (41), we conclude that divorce takes place whenever:

$$\eta(y) + \theta^a + \theta^b < y$$

[18] By the envelope theorem, the derivative $\eta'(y)$ is equal to $F(Q)$. Therefore, η is increasing in y and, if $F(Q)$ is increasing in y as well, then η is convex. Note that a sufficient (but by no means necessary) condition is that public consumptions are all normal.

or equivalently:

$$\theta^a + \theta^b < y - \eta(y) \tag{42}$$

Condition (42) has a simple, probabilistic translation; namely, the probability that a couple with total income y divorces is simply

$$P = \Pr\left(\theta^a + \theta^b < y - \eta(y)\right) = \Phi(y - \eta(y))$$

where Φ is the cumulative distribution function of $\theta^a + \theta^b$. As expected, the threshold $\bar{\theta} = y - \eta(y)$ is negative and decreases with income: wealthier couples are less likely to divorce, because they receive larger economic gains from marriage. Note also that the divorce decision only depends on the realization of the sum $\theta^a + \theta^b$: under transferable utility, a poor realization of θ for one spouse can always be compensated by a transfer from the partner.

6.2.2 The Becker-Coase theorem

This result has several consequences. One is that *the divorce decision does not depend on the law governing post divorce income allocation*; indeed, condition (42) above is independent of the rule D.

Moreover, let us compare the two dominant legal systems governing divorce, namely unilateral divorce and mutual consent. One can readily see that in both cases, agents divorce if and only if condition (42) is satisfied. The result is obvious under unilateral divorce, because condition (42) implies that no intrahousehold resource allocation can provide both agents with at least as much as their utility if single. The case of mutual consent is slightly more complex, because even when condition (42) is satisfied, the post divorce allocation D may be such that one member, say a, strictly loses from divorce (of course, (42) then requires that her spouse, b, strictly gains). But then b may bribe a into divorcing by offering a post divorce allocation that is more favorable to a than D^a. Of course, the price for b is that he will receive less than D^b. But condition (42) precisely state that this is still better for b than remaining married.

We can therefore conclude that *the laws governing divorce have no impact on divorce probability.* This neutrality result, initially established by Becker in a slightly less general framework, is in fact a natural consequence of the well-known Coase (1960) theorem, stating that the allocation of the surplus stemming from a decision has no impact on the decision taken. This does *not* mean that divorce laws are irrelevant, but simply that they only influence the distribution of welfare between the spouses, both in marriage and after divorce - not the divorce decision itself.[19]

The corresponding intuition is easy to grasp from Figure 2. Under transferable utility, both the Pareto frontier when married and the Pareto frontier when divorced are straight line with slope -1. Therefore, they cannot intersect; one Pareto set must be included within the other. The optimal divorce decision simply picks up the larger Pareto set. What legal dispositions can do is vary the

[19] A recent attempt to test this theoretical prediction is Wolfers (2006).

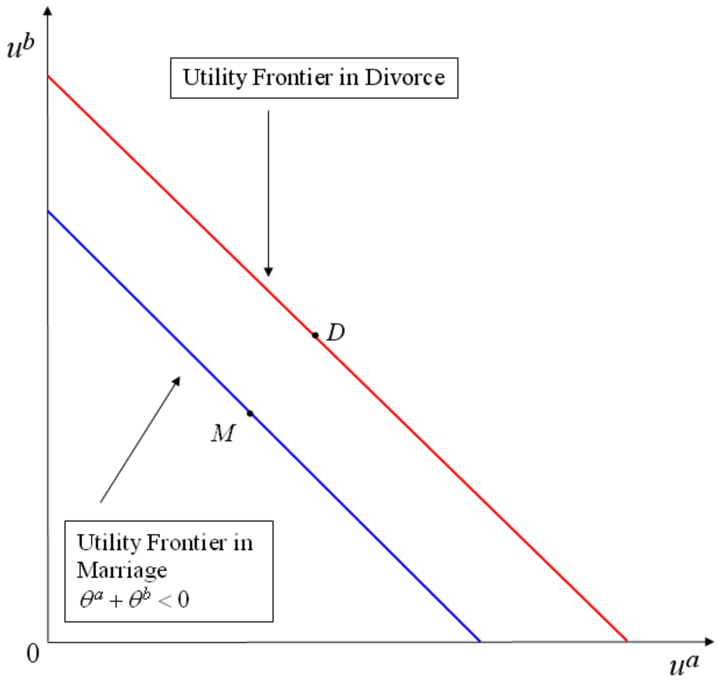

Figure 2: Pareto frontiers in marriage and divorce, no public goods

post divorce allocation along the post divorce Pareto frontier. But if the latter is located within the Pareto set when married, there always exist a particular redistribution of marital surplus that will make both spouses better off than divorce; if, conversely, it is located outside, then whatever the planned allocation of resources within the couple, it is always possible to redistribute income after divorce in such a way that both agents prefer separation.

Finally, it is important to understand the assumptions that are needed for the Becker-Coase theorem to hold. Chiappori, Iyigun and Weiss (from now on CIW) show that there are three. One is that utility is transferable within marriage (which, in our setting, justifies the GQL form taken for utilities when married). A second requirement is that utility be transferable after divorce; here, we have therefore assumed quasi linear preferences for singles. Finally, the slopes of the two Pareto frontiers (before and after divorce) must be equal. While these requirements are indeed satisfied in the example just given, they are in fact quite unlikely to hold in reality. For instance, the assumption of quasilinear preferences if single is totally *ad hoc* . Assume, on the contrary, that preferences if single have the same general form as when married - that is, that:

$$u_s^i\left(q^i, Q\right) = F_s^i\left(Q\right) q_1^i + G_s^i\left(Q, q_{-1}^i\right) + \theta^i,\ i = a, b$$

The question, now, is whether commodity Q, which was publicly consumed when the couple was married, remains public after divorce. In many cases, it does not; for instance, housing typically stops being jointly consumed after the separation. Then the second requirement is not satisfied in general. In other situations, the commodity remains public, in the sense that it still enters both ex-spouse's utilities; this is the case for children consumption, for instance. However, the utility adults derive from children's well being may well change after divorce, especially for the parent who does not have full custody. Technically, the F_s^i function is now different between spouses, which violates either the second or the third requirement. All in all, CIW argue that, in general, these requirements are unlikely to be fulfilled - therefore that the Becker-Coase result is unlikely to hold.

An important implication is that the claim, frequently encountered in the literature, that the Becker-Coase theorem is a consequence of the efficiency assumption is incorrect. Whenever any of the CIW requirements is violated, the neutrality result does not hold true. Then the general model developed in the previous subsection, which only assumes efficient behavior (including for divorce decisions), remains valid; but one can find situations in which couples would split under unilateral divorce but not under mutual consent - and also, more surprisingly, cases in which this intuition is reversed, in the sense that divorce occurs under mutual consent but not under unilateral divorce. Figure 3, borrowed from Clark (1999) and Chiappori, Iyigun and Weiss (2007), illustrates the latter case. With mutual consent, each partner has a "property right" on the allocation within marriage, represented by M. This point is contained in the divorce frontier and both partners can be made better of by renegotiating the divorce settlement and leaving the marriage. In contrast, with unilateral

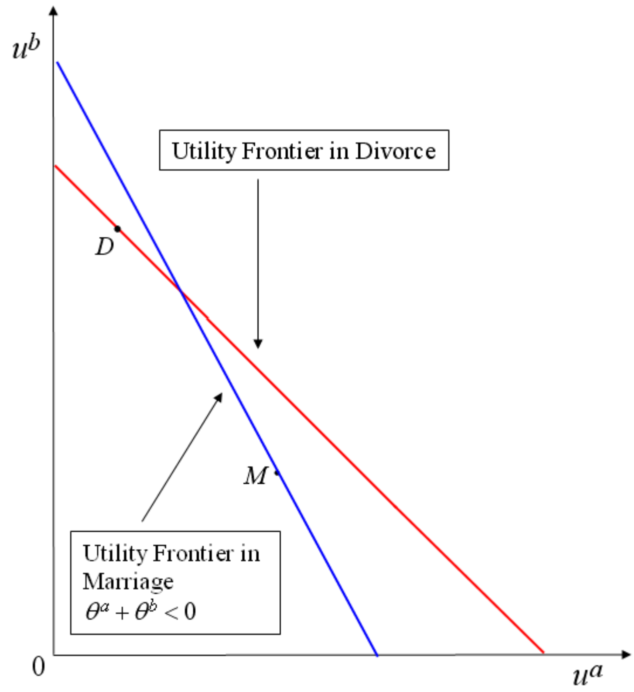

Figure 3: Divorce without transferable utility

divorce partners have property rights on their divorce allocation, represented by D. This point, however, is contained within the marriage frontier and the partners can find an allocation within marriage that will sustain the marriage.

6.3 Divorce and investment in children: a non transferable utility example

Endogenous divorce raises some particular contracting issues that do not arise when divorce is exogenous. This is particularly true when we take into account marriage specific investments, such as children - who are (at least partially) 'specific' in the sense that the welfare derived by the parents from the presence of children is often reduced upon divorce (that is, parents suffer a 'capital loss' upon divorce). This kind of problem usually motivates post divorce transfers in the form of child support that will be discussed at length in Chapter 11. Here we wish to examine the role of these post divorce transfers on the investment in children when they are young. To highlight their role, we shall now discuss an extreme case in which such transfers are not possible *within marriage,* because all goods that a couple consumes are public; therefore post divorce transfers are the only feasible transfers between the spouses.

Agents live two periods. Marriage takes place at the beginning of the first period and each marriage produces one child. Caring for the child requires an investment of time by both parents in the first period and the outcome (child quality) is enjoyed in the second period. The household production function for child quality is

$$Q = \sqrt{(1 + t^a)(1 + t^b)} \tag{43}$$

where t^a and t^b are are the proportions of available time spent on child care by a and b, respectively. The time constrains are

$$\begin{aligned} 0 &\leq t^a \leq 1 \\ 0 &\leq t^b \leq 1 \end{aligned} \tag{44}$$

The opportunity cost of the time spent with children in the first period is market work. In the second period there is no need to spend time on children and both spouses work full time. However the wage in the second period of life depends on the amount of market work in the first period. We normalize the first period wage of a to 1 and assume that $w^b < 1$. We further assume that the second period wages are directly proportional to the first period labor supply - that is, they are equal to $\gamma(1-t^a)$ and $\gamma w^b(1-t^b)$ for a and b respectively, where $\gamma > 1$. Effectively, this means that incomes in the two period are proportional, which simplifies the analysis considerably.

The utility that parents derive from the child (or child quality) depends on whether or not the parents live together. If the parents stay married, their utility from quality is $\alpha \ln Q$, but if the parents separate, their utility from child quality is reduced to $(1-\delta)\alpha \ln Q$, where $0 < \delta < 1$. The utility parents depends on the child quality, on their consumption of goods q and if married, the quality of their match, θ, that is revealed only after one period of marriage.

If the partners are married, the utility of both partners is

$$u_m = \ln q + \alpha \ln Q + \theta \tag{45}$$

Divorce may occur if the realized value (revealed at the beginning of the second period) is sufficiently low. Following divorce, the utilities of the former spouses are

$$u_d = \ln q_d^i + (1 - \delta)\alpha \ln Q, \;\; i = a, b \tag{46}$$

where q_d^i denotes the post divorce consumption of the two spouses. Note that we assume here that when a couple is married *all* good are public. The only way to influence the division of the gains from marriage is through transfer in the aftermath of divorce. As we shall show, such transfers can influence the investment in children during marriage and probability of divorce.

As in the previous subsection, we continue to assume no borrowing or lending. Then,

$$\begin{aligned} q_1 &= w^a(1 - t^a) + w^b(1 - t^b) \\ q_2 &= \gamma w^a(1 - t^a) + \gamma w^b(1 - t^b) = \gamma q_1 \end{aligned} \tag{47}$$

where q_1 denotes the joint consumption in the first period, while q_2 is the joint consumption if the partners remain married or the sum of their private consumptions if they separate. Thus, the allocation of time in the first period determines the consumption available to the parents each period as well as the quality of the child that they enjoy in the second period. The only issue then is how is this allocation determined.

A necessary condition for an efficient allocation of time is that the cost of producing child quality, in terms of the foregone earnings of the couple during the two periods of life, should be minimized. In the this example, these costs are

$$C(Q) = (1+\gamma)(w^a t^a + w^b t^b) \tag{48}$$

and cost minimization takes a simple form. In particular if there is an interior solution and both partners contribute time to the child[20] then we must have

$$w^b(1+t^b) = 1 + t^a \tag{49}$$

Whether or not an interior solution arrises, efficiency requires that the low wage person, b, should contribute more time to the child and the question is if and how such *unequal* contribution can be implemented. The answer depends on the contracting options that the couple have. We shall assume here that the partners can always commit, at the time of marriage, on some post divorce allocation of resources, provided that it falls within some legal bounds. The justification for this assumption is that the event of separation and the resources available upon separation can be verified so that contracts contingent on these variables can be enforced by law. Denoting by β the share received by the low wage person, b, the post divorce consumption levels are

$$\begin{aligned} q_d^a &= (1-\beta)[\gamma w^a(1-t^a) + \gamma w^b(1-t^b)] \\ q_d^b &= \beta[\gamma w^a(1-t^a) + \gamma w^b(1-t^b)]. \end{aligned} \tag{50}$$

It is more difficult, however, to verify the time allocation and in particular time spent on children, and we shall allow for the possibility that partners cannot commit at the time of marriage on how much time they will spend with the child.

Following the realization of θ at the beginning of the second period, and given the predetermined quality of children and divorce contract, marriage will continue if

$$\alpha \ln Q + \ln q_2 + \theta \geq (1-\delta)\alpha \ln Q + \max\{(\alpha \ln q_d^a, \alpha \ln q_d^b) \tag{51}$$

and dissolve otherwise. This rule holds because, by assumption, utility is not transferable within marriage and each partner is free to walk away from the marriage. Clearly, the person who can attain higher consumption outside marriage will trigger the divorce.

[20] The efficiency requirements include regions in which only one person contributes. These regions depend on the desire for children relative the wages of the two spouse. If $\alpha < 1$ the mother will work only at home and the father only in the market. To allow for an interior solution, we assume that $2 > \alpha > 1$. Then for $\alpha w^b > 1$, both partners work part time at home and part time in the market.

Examining equation (42), we see that if the wife receives a higher share of family resources upon divorce, $\beta > \frac{1}{2}$, she will trigger the divorce and, divorce occurs if

$$\theta < -\delta\alpha \ln Q + a \ln \beta)$$

If the husband obtains the larger share, $\beta < \frac{1}{2}$, he will trigger the divorce an divorce occurs if

$$\theta < -\delta \ln \alpha Q + \alpha \ln(1-\beta)$$

Finally, with equal sharing divorce occurs if

$$\theta < -\delta\alpha \ln Q - \alpha \ln 2$$

The probability of divorce is, therefore,

$$\text{Prob(divorce)} = \begin{cases} F(-\delta\alpha \ln Q + \ln(1-\beta)) & \text{if} \quad \beta \leq \frac{1}{2} \\ F(-\delta\alpha \ln Q - \ln 2) & \text{if} \quad \beta = \frac{1}{2} \\ F(-\delta\alpha \ln Q + \ln \beta) & \text{if} \quad \beta \geq \frac{1}{2} \end{cases} \tag{52}$$

where $F(.)$ is the cumulative distribution of θ. We assume that this distribution is symmetric with zero mean. We see that a high child quality, Q, and high loss of child quality upon divorce, δ, generate higher gains from continued marriage and reduce the probability of divorce. A negative shock to θ is required to initiate a divorce, because of the cost associated with reduced child quality, represented here by the term $\delta\alpha \ln Q$, and loss of the utility gains from joint consumption, which depends on the allocation of resources upon divorce ($\ln(1-\beta)$) if $\beta \leq \frac{1}{2}$ or $\ln \beta$ if $\beta \geq \frac{1}{2}$).

At this point we can already make three observations:

- An increase in child quality reduces the probability of divorce.
- For a given child quality, Q, the lowest probability of divorce is attained when $\beta = \frac{1}{2}$.
- For $\beta \neq \frac{1}{2}$, divorce is inefficient in the sense that the spouse who triggers the divorce does not internalize the reduced welfare of the spouse who is left behind and would rather stay married for at least some range of θ's below the trigger. Note that the contrast to the results in the previous section, where divorce was efficient and the probability;of divorce was independent of the division of income in the aftermath of divorce. The Becker-Coase theorem does not hold when transfers within marriage are not feasible.

We now turn to the determination of the investment in children in the first period. We first consider the benchmark case of equal sharing, with $\beta = \frac{1}{2}$. Defining the trigger value for divorce as

$$\theta^* = -\delta\alpha \ln Q - \ln 2, \tag{53}$$

the expected utility of each of the two partners is then

$$E(u) = \ln q_1 + (1 - F(\theta^*)[\alpha \ln Q + \ln q_2] + \int_{\theta^*}^{\infty} \theta f(\theta) d\theta \quad (54)$$

$$+F(\theta^*[(1-\delta)\alpha \ln Q + \ln \frac{q_2}{2}]$$

$$= \ln q_1 + \ln q_2 + \alpha \ln Q + \int_{\theta^*}^{\infty} \theta f(\theta) d\theta + F(\theta^*)\theta^*$$

Maximizing $E(u)$ with respect to t^a and t^b, respectively, we obtain the first order conditions for an interior solution

$$\frac{1}{q_1} + \frac{\gamma}{q_2} = [1 - \delta F(\theta^*] \frac{\alpha}{2(1+t^a)} \quad (46a)$$

$$\frac{w^b}{q_1} + \frac{w^b \gamma}{q_2} = [1 - \delta F(\theta^*)] \frac{\alpha}{2(1+t^b)} \quad (46b)$$

The interpretation of these two conditions is transparent. For each spouse, the couple equates the expected marginal gain in terms of child quality, associated with an increase in the time investment, to the marginal costs in terms of forgone consumption of the parents in the two periods. The two conditions together imply condition (0) which means that, under equal division, efficiency is maintained. Importantly, there is no need for the partners to commit on the time spent with the child because the Nash equilibrium that arises under non cooperation satisfies exactly the *same* conditions. That is, in equilibrium, each spouse, including the low wage person who is called upon to supply more hours, would do it from selfish reasons, provided that the other spouse supplies the efficient quantity of time.

The situation is quite different if the partners choose ex-ante an unequal division but cannot commit on the allocation of time. For concreteness, consider the case that in which the low wage person, b, is the wife and she receives a lower share of family resources, $\beta < \frac{1}{2}$. Now each spouse will maximize his\her *different* payoff functions. Let the new trigger function be

$$\hat{\theta} = -\delta \alpha \ln Q + \ln(1 - \beta). \quad (55)$$

Then, the choice of t^a as a function t^b is determined by the maximization with respect to t^a of

$$E(u^a) = \ln q_1 + \ln q_2 + \alpha \ln Q + \int_{\hat{\theta}}^{\infty} \theta f(\theta) d\theta + F(\hat{\theta})(\hat{\theta}), \quad (56)$$

with the first order condition

$$\frac{1}{q_1} + \frac{\gamma}{q_2} = [1 - \delta F(\hat{\theta}] \frac{\alpha}{2(1+t^a)}. \quad (57)$$

Similarly, the choice of t^b as a function t^a is determined by the maximization with respect to t^b of

$$E(u^b) = \ln q_1 + \ln q_2 + \alpha \ln Q + \int_{\hat{\theta}}^{\infty} \theta f(\theta) d\theta + F(\hat{\theta})\hat{\theta} + F(\hat{\theta})[\ln \beta - \ln(1-\beta], \quad (58)$$

with the first order condition

$$\frac{w^b}{q_1} + \frac{\gamma w^b}{q_2} = [1 - \delta F(\hat{\theta}) + f(\hat{\theta}) \ln \frac{\beta}{1-\beta}] \frac{\alpha}{2(1+t^b)}. \quad (59)$$

We see that the expected marginal reward from exerting effort is smaller to the wife (note that for $\beta < \frac{1}{2}$, $\ln \frac{\beta}{1-\beta} < 0$). The wife takes into account her lower consumption, and thus higher marginal utility from consumption, following divorce. She responds by shifting additional time in the first period into work so that her future wage will be higher. This defensive investment in market work by the wife causes an inefficient time allocation. Examining conditions (49) and (50), we see that the requirement for cost minimization is *not* satisfied.

When partners cannot commit on the allocation of time, commitments made at the time of marriage should adjust. One may assume that the husband has a higher bargaining power at the time of marriage, because of his higher wage and thus higher consumption as single. However, it makes sense for the husband to give up some of his power, which will raise the "pie" available during marriage that he and his the wife enjoy equally.

Returning now to the case of equal division and efficient allocation of time, we can provide some further analysis of the investment decision. Using the efficiency conditions (and constant returns to scale) we have that, in an interior solution,

$$Q = \sqrt{w^b}(1 + t^b). \quad (60)$$

We also have that

$$\begin{aligned} q_1 &= 1 + w^b - t^a - w^b t^b \quad (61) \\ &= 2(1 + w^b) - 2\sqrt{w^b} Q \end{aligned}$$

We can, therefore, rewrite condition (46b) in the form

$$\frac{2\sqrt{w^b}}{1 + w^b - \sqrt{w^b} Q} = [1 - \delta F(-\delta \alpha \ln Q - \ln 2] \frac{\alpha}{Q}. \quad (62)$$

Condition (54) then determines the desired child quality and we can then use the efficiency conditions to trace back the implied allocation of time. The left hand side of (54) represents the marginal disutility (associated with lost consumption) and unambiguously rises with Q. However, the right hand side of (54), which represents the expected marginal utility from having children in the second period, involves two conflicting effects: A higher level of child quality reduces the marginal utility from children and also reduces the probability of divorce. Therefore, the marginal expected utility can either rise or fall and the outcome depends on the shape of the *hazard* associated with the distribution of quality match $F(\theta)$. Specifically,

$$\frac{d}{dQ}\alpha[\frac{1-\delta F(-\delta\ln Q-\ln 2)}{Q}]=\alpha[\frac{(\delta^2 f(-\delta\ln Q-\ln 2)-(1-\delta F(-\delta\ln Q-\ln 2))}{Q^2}]$$

which is negative if.

$$\delta^2\frac{f(-\delta\ln Q-\ln 2)}{1-F(-\delta\ln Q-\ln 2)}<1.$$

This condition is satisfied, for instance, for the normal distribution if $\sigma\geq 1$, because then the hazard is an increasing function and its value at zero is $\sqrt{\frac{2}{\pi}\frac{1}{\sigma}}$.

Assuming that the expected marginal utility from children declines with Q, it is easy to see that the investment in children is reduced in response to increasing risk, represented here by a mean preserving increase in the spread of the *shocks* to match quality. However, the investment in children may also rise in order to stabilize the marriage. In either case, the "efficient" family responds to such change in circumstances in a way which is optimal for *both* spouses, without having equality in action. What is required, of course, is for both partners to have equal interest in the outcome and, given that all goods in marriage are public, such harmony can be achieved by a binding commitments of an equal division of resources upon divorce. These considerations can go part of the way in explaining the prevalence of equal divisions following divorce. A binding commitment is required, because expost, after divorce has occurred and the investments have been made, there is no incentive for further transfers. However at the time of marriage, a spouse with a higher income may be willing to commit on a post divorce transfer in order to induce the seemingly less powerful spouse to invest in children.

References

[1] Altonji, Joseph G., Hayashi, Fumio and Laurence J. Kotlikoff, 'Is the Extended Family Altruistically Linked? Direct Tests Using Micro Data', **American Economic Review,** 82 (1992), 1177-1198.

[2] Basu, Kaushik, 'Gender and Say: A Model of Household Behaviour With Endogenously Determined Balance of Power', **Economic Journal**, 116 (2006), 558-580.

[3] Becker, Gary, **Treatise on the family, (**Cambridge Mass: Harvard University Press, 1991).

[4] Brossolet, C., 'Fondement de la division du travail dans les modéles économiques du ménage', Paris, Editions Arguments, (1992).

[5] Bourguignon, Francois, Browning, Martin, Chiappori, Pierre-André and Valérie Lechene, 'Intra Household Allocation of Consumption: A Model and Some Evidence from French Data', **Annales d'Economie et de Statistique,** 29 (1993), 137-156.

[6] Browning, Martin, 'Love Betrayel and Commitment', University of Oxford, (2009).

[7] Browning, Martin and Annamaria Lusardi, 'Household Saving: Micro Theories and Micro Facts', **Journal of Economic Literature,** 34 (1996), 1797-1855.

[8] Bergstrom, Theodore C., 'A Fresh Look at the Rotten Kid Theorem–and Other Household Mysteries', **Journal of Political Economy,** 97 (1989), 1138-1159.

[9] Borch, Karl, 'The Safety Loading of Reinsurance Premiums', **Skandinavisk Aktuarietidskrift**, 43 (1960), 163–184.

[10] Chiappori, Pierre-André, 'Characterizing the Properties of the Generalized Quasi-linear Utilities', Columbia University, (2007).

[11] Chiappori, Pierre-André and Philip J. Reny, 'Matching to Share risk', Columbia University, (2005).

[12] Chiappori, Pierre-André, Townsend, Robert and Hiro Yamada, 'Sharing Wage Risk', Columbia University, (2008).

[13] Chiappori, Pierre-André and Yoram Weiss, 'Marriage Contracts and Divorce: an Equilibrium Analysis', Tel Aviv University, (2009).

[14] Chiappori, Pierre-André and Yoram Weiss, 'Divorce, Remarriage, and Welfare: A General Equilibrium Approach', **Journal of the European Economic Association,** 4 (2006), 415-426.

[15] Chiappori, Pierre-André and Yoram Weiss, 'Divorce, Remarriage, and Child Support', **Journal of Labor Economics,** 25 (2007), 37-74.

[16] Chiappori, Pierre-André, Iyigun, Murat, and Yoram Weiss, 'Investment in Schooling and the Marriage Market', **American Economic Review**, 99 (2009), 1689-1713.

[17] Chiappori, Pierre-André, Iyigun, Murat, and Yoram Weiss, 'Public Goods, Transferable Utility and Divorce Laws', IZA Discussion Paper No. 2646, (2007).

[18] Chiappori, Pierre-André, Iyigun, Murat, and Yoram Weiss, 'Spousal Matching, Marriage Contracts and Property Division in Divorce', Tel Aviv University, (2006).

[19] Clark, Simon, 'Law, Property, and Marital Dissolution', **The Economic Journal,** 109 (1999), C41-C54.

[20] Coase, Ronald H., 'The Problem of Social Cost', **Journal of Law and Economics,** 3 (1960), 1-44.

[21] Del Boca, Daniela and Christopher Flinn, 'Endogeneous Household Interaction', CHILD Working Papers wp08_09, CHILD - Centre for Household, Income, Labour and Demographic economics - ITALY, (2009).

[22] Duflo, Esther and Christopher Udry, 'Intrahousehold Resource Allocation in Cote d'Ivoire: Social Norms, Separate Accounts and Consumption Choices', NBER Working Paper No. 10498, (2004).

[23] Dufwenberg, Martin, 'Marital investments, time consistency and emotions', **Journal of Economic Behavior & Organization,** 48 (2002), 57-69.

[24] Geanakoplos, John, Pearce, David and Ennio Stacchetti, 'Psychological games and sequential rationality', **Games and Economic Behavior,** 1 (1989), 60-79.

[25] Hardy, G. H., Littlewood, J. E., and G. Polya, **Inequalities,** (Cambridge Mathematical Library, 1952).

[26] Konrad, Kai A. and Kjell Erik Lommerud, 'The bargaining family revisited', **Canadian Journal of Economics,** 33 (2000), 471-487.

[27] Lundberg, Shelly and Robert A. Pollak, 'Separate Spheres Bargaining and the Marriage Market', **Journal of Political Economy,** 101 (1993), 988-1010.

[28] Lundberg, Shelly and Robert A. Pollak, 'Efficiency in Marriage', **Review of Economics of the Household,** 1 (2003), 153-167.

[29] Mazzocco, Maurizio, 'Saving, Risk Sharing, and Preferences for Risk', **American Economic Review,** 94 (2004), 1169-1182.

[30] Mazzocco, Maurizio and Shiv Saini, 'Testing Efficient Risk Sharing with Heterogeneous Risk Preferences: Semi-parametric Tests with an Application to Village Economies', 2006 Meeting Papers 108, Society for Economic Dynamics, (2006).

[31] Mazzocco, Maurizio, 'Household Intertemporal Behaviour: A Collective Characterization and a Test of Commitment', **Review of Economic Studies,** 74 (2007), 857-895.

[32] Mincer, Jacob, 'Family Migration Decisions', **Journal of Political Economy,** 86 (1978), 749-773.

[33] Rey, Patrick, and Bernard Salanie, 'Long-term, Short-term and Renegotiation: On the Value of Commitment in Contracting', **Econometrica,** 58 (1990), 597-619.

[34] Townsend, Robert M, 'Risk and Insurance in Village India', **Econometrica,** 62 (1994), 539-591.

[35] Weiss, Yoram and Robert J. Willis, 'Transfers among Divorced Couples: Evidence and Interpretation', **Journal of Labor Economics,** 11 (1993), 629-679.

[36] Weiss, Yoram and Robert J. Willis, 'Match Quality, New Information, and Marital Dissolution', **Journal of Labor Economics,** 15 (1997), S293-S329.

[37] Wolfers, Justin, 'Did Unilateral Divorce Laws Raise Divorce Rates? A Reconciliation and New Results', **American Economic Review,** 96 (2006), 1802-1820.