The Economics of the Family

Chapter 11:

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1 Introduction

The purpose of this chapter is to examine in more detail the role of children in marriage and divorce.[1] In particular, we wish to discuss the determination of expenditures on children and their welfare under various living arrangements, with and without the intervention of the courts. There is a growing concern that the higher turnover in the marriage market causes more children to live with single mothers or step parents,[2] which may be harmful to the children.[3] Part of the problem is that, following separation, fathers are less willing

[1]This chapter extends the results reported in Chiappori and Weiss (2007) to include both time and money as inputs to the child welfare.

[2]In the US, year 2005, 68 percent of children less than 18 years old lived with two parents (including step parents), 23 percent lived only with their mother and 5 percent lived only with their father, the rest lived in households with neither parent present.

[3]There is substantial evidence that children of divorced parents do not perform as well as comparable children in intact families. See Argys et al.(1998), Lamb et al.(1999), Hetherington and Stanley-Hagan

to transfer resources to the custodial mothers (i.e., their ex-wives). A major objective of our analysis is to explain how transfers between separated parents are determined and how they vary with marriage market conditions.[4]

Separation may entail an inefficient level of expenditures on children for several reasons: 1) If the parents remarry, the presence of a new spouse who cares less about step children reduces the incentives to spend on children from previous marriages. 2) If the parents remain single then, in addition to the loss of the gains from joint consumption, the custodial parents may determine child expenditures without regard to the interest of their ex-spouse. 3) Parents that live apart from their children can contribute less time and goods to their children and may derive less satisfaction from them. These problems are amplified if the partners differ in income and cannot share custody to overcome the indivisibility of children. The custodial parent is usually the mother who has some comparative advantage in caring for children but has lower income. The father has often limited access to the child and low incentive to provide for him. The outcome is that the level of child expenditures following separation is generally below the level that would be attained in an intact family, reducing the welfare of the children and possibly the welfare of their parents.

An important consequence of having children is that they create ex-post wage differences between men and women. The basic reason for such differences is biological in nature. The mother is the one who gives birth as she is more capable of taking care of the child at least initially. As noted by Becker (1993) this initial difference may have large economic consequences. When the mother takes care of the child, her future earning capacity erodes. Then, because of the reduced earning capacity of the mother and her inherent advantage in child care, a pattern of specialization arises, whereby the father

(1999), Gruber (2004) and Stafford and Yeung (2005). Such empirical evidence should be interpreted with some care, for two reasons. First, dysfunctional families are more likely to generate both divorce *and* poor child performance. Bj́orklund and Sundstrom (2006) argue that inferior performances of divorced children can largely be attributed to selection effects. Second, even if divorce causes poor performance at the individual level, the impact of the *aggregate* divorce rate on the welfare of children is a different issue. As shown by Piketty (2003) the increase in the divorce rate in France has *reduced* the gap in school performance between children of divorced parents and children from intact families.

[4] See Weiss and Willis (1985, 1993), Del-Boca (2003), Case et al (2003) and Chiappori and Weiss (2007).

works more in the market and the mother works more at home.[5] This pattern is most pronounced if the couple remains married and can coordinate activities. Following separation, however, the allocation of time may change, and a custodial mother may spend less time on her child if she remarries, because a foster father cares less about the child than a natural father.

The ex-post asymmetry between parents has strong implications for the divorce decision and the incentive to produce children. Because men maintain or increase their earning capacity during marriage, they have higher expected gains from divorce. Under divorce at will, they will initiate the divorce, at some situations in which the mother would like to maintain the marriage. If transfers within marriage are limited due to a large component of public consumption, separations will be inefficient, implying that the gains from having children are smaller to the mother than to the father. Because the production of children requires both parents, the mother may avoid birth in some situations in which the husband would like to have a child. The consequence is an then inefficient production of children.

To overcome these problems, the partners have an incentive to sign *binding* contracts that will determine some transfers between the spouses. The purpose of the transfers is to induce an efficient level of child expenditures following divorce and to guarantee efficient separation and child production by restoring the symmetry between the parents. It is generally not possible to obtain such a first best outcome, because of some important limitations on transfers. First, transfers within marriage can only partially compensate for *common* factors that affect both partners, such as the failure of the marriage. If the partners separate then transfers can compensate for differences in the gains and costs from divorce, but these transfers are limited too. In particular, it is not possible to condition the transfer on the allocation within a household which is usually not observed by a third party.

Legal intervention is required to enforce binding contracts. In practice, enforcement of alimony and child support contracts is imperfect. This is not simply a matter of lack of resources or determination on the part of the legal authorities. There is a basic conflict

[5] See also Chichilnisky (2005) and Albanesi and Olivetti (2009).

between private needs and social needs that results from the externalities that prevail in the marriage market. One issue is that parents and child interests may conflict, even if parents care about their children. For instance, a mother may choose to remarry even if the child under her custody is harmed, because she gains more than the child from the presence of a new spouse. Another issue is the impact of the divorce and fertility decisions of a given couple on the prospects for remarriage and the gains from remarriage of others. In marriage markets with frictions, competition does not force a couple to internalize the impact on potential mates, because meetings are to a large extent random and rents prevail. Therefore, a contract that a couple is willing to sign is not necessarily optimal from a social point of view. A related issue is that contracts that couples are willing to sign may at the time of marriage, before the quality of match is observed, may be inefficient ex-post after divorce has occurred and the impact of the contract on the divorce and fertility decisions is not relevant any more. In this case, the partners may wish to renegotiate, thereby creating a lower level of welfare for both of them from an ex-ante point of view.

The benefits from having children depend on the contracts that the parents employ to regulate these decisions and on the prospects of remarriage that are determined in the marriage market. Consequently, the incentives to produce children depend not only on the risk of divorce, triggered by changing circumstances in a specific household, such as falling out of love, but also on the general situation in the marriage market. The larger is the proportion of couples that divorce, the better are the remarriage prospects. In the absence of children, or with children but adequate transfers, this would increase the probability of divorce. However, with children, remarriage may have a negative effect on the child because the new husband of the custodial mother may be less interested in his welfare. We may refer to this problem as the "Cinderella effect" (see Case et al., 1999). This effect reduces the incentive of the non custodial father to support the child, because part of the transfer is "eaten" by the new husband. In addition, non custodial parents who are committed to their custodial ex-spouse are less attractive as potential mates for remarriage. Thus, the larger is the proportion of such individuals among the divorcees, the less likely it is that a particular couple will divorce, and the more likely it is that each couple will have children. In this chapter, we use a simple model to illustrate the interactions among these considerations in a general equilibrium framework and highlight the potential consequences for parents and children.

2 The Model

We consider here a given cohort with equal number of men and women. Individuals live for two periods and can be married or single in each of these periods. A household consists of one or two adults and possibly one child. We treat fertility as a choice variable and each couple decides on whether or not it should have a child in the first period. If a married couple with a child divorces, the mother becomes the sole custodian and . We assume that childless men and women are identical and both earn the same wage w_h. However, if a couple has a child then, because the mother is the one who gives birth, her second period wage drops to a lower level, w_l.

2.1 The technology and preferences

The household pools the incomes of its members and allocates it to buy an adult good a and a child good c. Each parent has one unit of time which can be allocated between market work and child care. Let h_m and h_f denote the time spent by mother and father in market work, respectively. Then, the amount of time they spend at home is $t_j = 1 - h_j$, where, $0 \leq h_j \leq 1$ for $j = m, f$.

The household production function is

$$q = \alpha a + t + g(c), \tag{1}$$

where

$$t = \beta t_f + \gamma t_m. \tag{2}$$

The output q is interpreted as the child's utility or "quality". The parameter α describes the marginal effect of the adult good, a, on the child's quality, the parameters β and γ represent the productivities of the father and mother, respectively, in household work and t is total time spent on the child, measured in efficiency units. The function $g(c)$ is assumed to be rising in c and concave, with $g(0) = 0$. The linearity in t is assumed to allow corner solutions whereby family members *specialize* either in household work or market work. To determine the pattern of specialization under different household structures, we assume

$$\begin{aligned} \gamma &> w_l(1+\alpha), \\ \beta &< w_h(1+\alpha), \end{aligned} \tag{3}$$

where, w_l is the wage of the mother and w_h is the wage of the father. That is, the mother is more productive at home, while the father is more productive in the market. This may hold either because the mother has an absolute advantage in home production $\gamma > \beta$ or that she has absolute disadvantage in market work, $w_l < w_h$, because of the erosion in her wage due to her withdrawal from the labor force during child birth.

The adult good a is shared by all members of a household. The marginal utility of each adult from the adult good is set to 1, while the marginal utility of the child is set to the constant α that is smaller than 1. In contrast, the child good, c, is consumed only by the child. However, indirectly, child consumption matters to the parents of the child, who care about its welfare. The utility of a child is defined to be identical to its quality, q, and the utility of each parent is defined as the sum $a + q$. Thus, both parents care about their joint child, wherever the child lives. In this sense, child quality is a collective good for the natural parents.[6]

Taken together, these assumptions impose a quasi-linear structure that implies that the time and specific goods spent on the child do not depend on the household's total income, if a positive amount of the adult good is consumed. In that case, any additional income is spent only on the adult good. Income effects on the child are present, however, if the adult good is not consumed.

Married couples also enjoy a match specific "love" factor which we denote by θ. This factor is random and not known at the time of marriage. The quality of match θ is

[6] A parent that lives apart from the child may enjoy it to a lesser degree, and we may set the parent's utility to $a+\delta q$ if the parent and child live in a separate households. The parameter δ may be interpreted as a discount factor that captures the idea that "far from sight is far from heart".

revealed at the end of each period.[7] We assume that θ is independent across couples and is distributed with some known distribution $F(\theta)$ that is symmetric and has a mean of zero.[8] Individuals, who marry at the beginning of the first period, observe θ at the end of the first period and can then decide whether or not to break the marriage and look for a new match. If marriage continues it will have the same θ. If a new marriage is formed its θ will be a random draw from $F(\theta)$.

A negative shock to θ can cause dissolution of the marriage. Following divorce, the parents may remain single or remarry, so that the child may live in a household that consists of one or two adults. Household structure affects both the technology and the household decision making. We assume that a parent can spend time on a child only if they live in the same household, but may spend money on the child even if they live apart. If both parents live together with their child in an intact family, all household goods are public and there is no conflict as to how much should be spent on the child. However, if the family breaks the parents may have conflicting interests, because the costs of caring for the child good will not be the same when they do not share the adult good. In addition, if the custodial parent remarries, then child quality is influenced by the foster parent who may care less about the child than his natural parents.

2.2 The legal framework

We consider a modern society in which individuals can marry or divorce at will. However, the partners can sign binding contracts, enforced by law, that specify the custody arrangement and child support payments following divorce or remarriage. Such contracts may be signed at the time of divorce or at the time of marriage. An interim contract signed at the time of divorce takes the presence of children and the separation as given, and it's main objective is to influence the expenditures on children under the different household structures that may arise if each parent remarries or remains single. An ex-ante contract,

[7] In contrast to chapter 10, we simplify here by eliminating the premarital signal of the quality of the match.

[8] The zero mean assumption implies that in the second period agents marry only "for money". In the first period, however, the average married couple enjoys a positive non monetary gain, because the option of divorce eliminates some of the downward risk. The model can be easily generalized to the case with $\bar{\theta} > 0$.

signed at the time of marriage, aims to influence the fertility and separation decisions as well. We discuss here simple and familiar contracts in which the mother obtains custody and the father commits, at the time of divorce, to pay the mother a fixed amount that is not contingent on whether one or both of the parents remarry. Such binding contracts are in fact enforced by law.

Except for the enforcement of arrangement that the partners may reach, the law may also intervene by setting standards within which the partners can operate. Custody is most often given to the mother on the ground that she can take better care of the child, while the father obtains visitation rights.[9] Often, custody assignment is associated with some amount of child support that is mandated by law. The guiding principle is that the custody assignment and the mandated payments should minimize the harm to the child. Such legal constraints may affect the agreements that partners would reach when bargaining in the "shadow of the Law" (see Mnookin and Kornhauser, 1977).

2.3 The meeting's technology

As in chapter 10, we assume that, each period, a person meets a *random* draw from the population of the opposite sex in the same age group. If this person is already married then such a meeting is "wasted" and no new marriage is formed. This feature creates "increasing returns" in meetings (see Diamond and Maskin, 1979), whereby it is more likely to meet a single person if there are more singles around.[10]

Only if two singles meet, they can form a new household. We denote by p the proportion of singles (divorcees) that one meets in the second period of life. Because the expected quality of match, θ, which is revealed with a lag, equals zero and the material gains from marriage are positive, every meeting of two divorcees results in a marriage,

[9] See Cancian and Meyer (1998). However, the share of joint physical custody has increased over time. Halla (2009) examines the impact of state differences in this trend and concludes that the option of joint custody has raised the incentives of men to marry, with little impact on divorce.

[10] Lauman et al. (1994, Table 6.1) report that about half of the marriages arise from meeting in school, work, and private party and only 12 percent originate in specialized channels such as social clubs or bars. The establishment of more focused channels, where singles meet only singles, is costly and they will be created only if the "size of the market" is large enough. Also, as noted by Mortensen (1988), the search intensity of the unattached decrease with the proportion of attached people in the population. The reason is that attached individuals are less likely to respond to an offer, which lowers the return for search (see Chapter 7)..

so that p is also the probability of remarriage. A new element in this chapter is that, in the second period, one can meet individuals that differ in their attractiveness as partners for remarriage, because of the presence of children, lack of income or commitments to previous spouses. However, the consequences of having a child may differ for men and women. For women it implies a lower wage. For men it may imply commitments to the ex-wife. In either case, a person with a child is a less attractive match.

3 Household structure and child care

We begin our analysis with the allocation of resources by couples with one child. This allocation depends on whether the partners live with their child or are separated.

3.1 Intact family

If both parents live with their child in an intact family, the utility of husband and wife is the same for all allocations and the two spouses will agree to maximize their common utility subject to the household budget constraint

$$a + c = w_l h_m + w_h h_f. \tag{4}$$

Because the mother is assumed to have the comparative advantage in home production, the father spends all his discretionary time in the market, $h_f = 1$, while the mother will spend all her discretionary time at home. To verify the optimality of this outcome, note that, due to assumption (3), an increase the father's work in the market raises the utility of *both* parents by $w_h(1+\alpha) - \beta$, while an increase in the mother's work at home raises their common utility by $\gamma - w_l(1+\alpha)$.

Given this specialization pattern, the amount spent on the child is determined by equalizing the marginal utilities from a and c, that is,

$$1 + \alpha = g'(c). \tag{5}$$

We denote the unique solution to (5) by c^* and assume that $c^* < w_h$ so that a positive amount is spent on the adult good a.

The utilities of the three family members in an intact family are

$$\begin{aligned} u_c &= g(c^*) + \gamma + \alpha(w_h - c^*) \\ u_m &= u_f = g(c^*) + \gamma + (1+\alpha)(w_h - c^*). \end{aligned} \tag{6}$$

We shall denote the above common utility of father and mother in an intact family (with a child) by u^*.

3.2 Separation, custody and voluntary transfers

If the parents separate, one of the parents receives custody over the child. Only the custodial parent can contribute household time to the child, but both parents can participate in the child expenditures. The non custodial parent continues to care about the child and may wish to transfer resources to it voluntarily. Transfers can be earmarked, in the form of tuition and health care for instance, or fungible in which case the custodian treats it as regular income that can be allocated according to the custodian's preferences. Generally, an ear-marked transfer is preferred by the non custodial parent, because part of the fungible transfer is "taxed" by the custodial parent (and also the new spouse if remarriage occurs) and does not reach the child. Realistically, the father can rarely transfer money directly to the child, especially when he is young. We shall, therefore, discuss here only fungible transfers. The transfers are determined at the time of separation, prior to meeting a new partner and are binding when the new marital status of the parents is realized. Such voluntary commitments are enforced by law and typical examples are child support and alimony agreements. We shall discuss separately non contingent contracts, such as child support in which the payment usually does not depend on the marital status of the parents, and contingent contracts, such as alimony in which the payment may stop if the ex-spouse remarries.

In practice, custody is often given to the mother, based on the idea that she can or willing to take better care of the child. In terms of our model, this is rationalized by the assumed comparative advantage that the mother has in housework. However, it is possible that the child is better of with the father because of the higher adult consumption that he provides. The custody choice is, therefore, related to the transfers that occur between the divorced parents. If transfers are sufficiently high, it is possible to restore at least partially the efficient division of labor that is attained under marriage, so that the child and consequently the parents are less harmed by the divorce. For the time being, we shall assume that the mother receives custody and will address this issue again after we derive the equilibrium level of transfers.

3.2.1 Single custodial mother

If the custodial mother remains single, she will choose the amount of time that she spends at work h_m, her adult consumption a_m and the amount of child goods c so as to maximize her own utility subject to her budget constraint, taking as given the amount that the ex-husband transfers to her, s. Her utility is then defined as the solution to the program

$$\begin{aligned} u_m(s) &= \underset{a_m,c,h_m}{Max} \{(1+\alpha)a_m + \gamma(1-h_m) + g(c)\} \\ & \quad s.t. \\ a_m + c &= w_l h_m + s, \\ 0 &\leq h_m \leq 1. \end{aligned} \tag{7}$$

The mother's choices as a function of s are summarized in Figure 1 below. Because of the quasi linear structure of the problem, the solution has three distinctly different regions. For low levels of s, the mother withdraws some time from the child and works in the market part time. She then spends all her disposable income on child goods. The optimum conditions in this region are

$$\begin{aligned} w_l g'(c) &= \gamma, \\ c &= w_l h_m + s. \end{aligned} \tag{8}$$

Thus, the mother spends a *fixed* amount of money, $\hat{c}$ on child goods and works in the market the minimal amount of time required to achieve this target. As s rises, the mother reduces her market work until it reaches zero, spending more time on child care.

For high levels of s, the mother does not work in the market and allocates her disposable income between the child and adult goods. The optimum conditions in this region are

$$\begin{aligned} g'(c) &= 1+\alpha, \\ c+a_m &= s. \end{aligned} \tag{9}$$

That is, the mother will spend a *fixed* amount of money, c^*, on the child and adjust her adult consumption according to the level of s.

For intermediate values of s, satisfying

$$\frac{\gamma}{w_l} > g'(s) > 1+\alpha, \tag{10}$$

the mother will not work and will not consume adult goods, so that all her income and free time are devoted to the child.

This pattern of behavior reflects our assumption that the mother has comparative advantage in child care $\gamma > w_l(1+\alpha)$ which is seen to imply that, for the mother, the child comes first and she spends resources on herself only when she is sufficiently wealthy. The utility of the child is then

$$q(s) = \begin{cases} g(\hat{c}) + \gamma(1 - \frac{\hat{c}-s}{w_l}) & if \quad s \leq \hat{c}, \\ g(s) + \gamma & if \quad \hat{c} < s < c^*, \\ g(c^*) + \gamma + \alpha(s - c^*) & if \quad s \geq c^*, \end{cases} \tag{11}$$

and the utility of the mother is

$$u_m(s) = \begin{cases} g(\hat{c}) + \gamma(1 - \frac{\hat{c}-s}{w_l}) & if \quad s \leq \hat{c}, \\ g(s) + \gamma & if \quad \hat{c} < s < c^*, \\ g(c^*) + \gamma + (1+\alpha)(s - c^*) & if \quad s \geq c^*. \end{cases} \tag{12}$$

3.2.2 Remarried custodial mother

If the custodial mother remarries, she may spend less time and money on the child, because her *new* husband receives little or no benefits from such spending. This is in contrast to the case of an intact family where, by assumption, *both* parents benefit equally from the time and money spent on the child. To sharpen our results, we assume that the new husband derives no utility at all from the step child and depends only on the adult good, a, that the remarried couple purchases.[11] The mother, however, cares about both the adult good, a, and the child good, c. All this means that the child good is a private good for the wife in the new household. Because of the potential conflict of interests, bargaining is required to determine the amount spent on the adult good in the new household.[12] Assuming that the bargaining outcome is efficient, it must be on the Pareto frontier within the new household. Let $y_h = w_h - s'$ denote the income that the new husband brings into marriage, net of his obligations to his ex-wife, $s\prime$.[13] Then, the Pareto utility frontier is given by

$$u_m = \begin{cases} (1+\alpha)a + \gamma + g(y_h + s - a) & if \quad y_h < a < y_h + s - \hat{c} \\ (1+\alpha)a + \frac{\gamma}{w_l}(w_l + y_h + s - a - \hat{c}) + g(\hat{c}) & if \quad y_h + s - \hat{c} \leq a \leq y_h + w_l + s - \hat{c} \\ (1+\alpha)a + g(y_h + w_l + s - a) & if \quad y_h + w_l + s - \hat{c} < a < y_h + w_l + s \end{cases} . \tag{13}$$

and described in Figure 2, which is drawn for the case in which the transfer from the father, s, exceeds the efficient level of expenditure on the child good, c^*. For levels of a close to y_h, the mother spends all her time on the child and the implied expenditure on the child good c exceeds c^*. In this case, *both* spouses want to reduce c and increase a, although the child may be hurt from such a substitution. As a is raised sufficiently so that c reaches c^* the newly formed couple enters the region of *conflict.* Any further increase in a and the associated decrease in c, benefits the new husband but reduces the mother's (and the child's) utility. Initially, the mother continues to spends all her time

[11]The new husband's utility also depends on the utility of his child from the previous marriage, which is taken as given in the bargaining of the remarried couple.

[12]Akashi-Ronquest (2009) reports lower child investments following remarriage (compared with intact families) and that an increase in the hourly wage of a biological mother significantly improves her child investment when her husband is a stepfather of the child, while there is no such effect for mothers living with the biological father of the child. The author interprets these findings as bargaining on child quality in step families.

[13]Since we shall examine only symmetrical equilibria, there is no loss of generality in assuming that all other fathers make the same payments.

on the child but when a reaches $y_h + s - \hat{c}$, she starts to work part time and continues to do so until a reaches $y_h + w_l + s - \hat{c}$. In this segment, the Pareto frontier is *linear* because the child good is held at a fixed level, $c = \hat{c}$, and any increase in a is achieved by an increase in h_m which raises the father's utility by $w_l dh_m$ and reduces the mother's utility by $((1+\alpha)w_l - \gamma)dh_m$. At high levels of a, exceeding $y_h + w_l + s - \hat{c}$, the mother works full time in the market and as a rises, the amount of child good is reduced until it reaches zero and the new husband obtains all the household resources, $y_h + w_l + s$.

To proceed with the analysis, one must determine how the conflict between the spouses is resolved and which particular point on the Pareto frontier is selected. For simplicity, we assume that the new husband obtains *all* the surplus from remarriage, so that the point on the Pareto frontier is selected so as to make the mother *indifferent* between remarriage and remaining single. This allows us to illustrate the general equilibrium issues in a relatively simple manner. The reader may interpret the model as a worst case scenario from the point of view of the mother and child.[14] The efficient level of adult consumption is then defined as the solution of the following maximization program

$$\begin{aligned} a(s, y_h) &= \underset{a,h_m,c}{Max}\ a \\ &\quad s.t. \\ a + c &= w_l h_m + y_h + s, \\ (1+\alpha)a + \gamma(1-h_m) + g(c) &\geq u_m(s), \\ 0 &\leq h_m \leq 1. \end{aligned} \tag{14}$$

For remarriage to take place, it must be the case that the solution of this program satisfies $a(s, y_h) \geq y_h$, otherwise the new husband would be better off as single.

[14] We could use instead a symmetric Nash-Bargaining solution to determine the bargaining outcome (see Chiappori and Weiss, 2007). The Nash axioms imply that the bargaining outcome must maximize the product of the gains from remarriage, relative to remaining single, of the two partners. This model yields similar qualitative results, because the mother is assumed to have lower income and therefore her options outside marriage are worse than those of men. The magnitudes of the welfare loss of the child and mother would, of course, be smaller if the mother gets a larger share of the gains from marriage.

The first order conditions for the efficient allocation within the new household are

$$\begin{aligned} -1-\lambda(g'(c)-(1+\alpha)) &= 0, \\ w_l-\lambda((1+\alpha)w_l-\gamma) &= 0 \; if \; 0<h_m<1, \\ w_l-\lambda((1+\alpha)w_l-\gamma) &\le 0 \; if \; h_m=0, \\ w_l-\lambda((1+\alpha)w_l-\gamma) &\ge 0 \; if \; h_m=1, \end{aligned} \tag{15}$$

where λ is a Lagrange multiplier such that $\frac{1}{\lambda}$ equals the slope of the Pareto frontier in the new household (see equation (13) and Figure 2). In an interior solution with $0 < h_m < 1$ we have

$$\lambda = \frac{w_l}{(1+\alpha)w_l-\gamma} < 0, \tag{16a}$$

because a marginal increase raises the utility of the husband by w_l and reduces the utility of the wife by $(1+\alpha)w_l-\gamma$. As in the case of a single mother household, $w_l g'(c)=\gamma$ and $c=\hat{c}$ as long as the mother works part time in the market. If h_m is at the boundaries of 0 or 1, the level of child expenditures c is determined by the requirement that the utility of the mother is equated to her reservation utility $u_m(s)$ and

$$\lambda = \frac{1}{(1+\alpha)-g'(c)}. \tag{16b}$$

In this section, we shall consider only equilibria with moderate levels of transfers such that the mother spends some time in the labor market when she is remarried. This allows us to exploit the linear Pareto frontier that arises in this case, which substantially simplifies the calculations. Such interior solution requires that the net income of the new husband is sufficiently high to motivate the mother to work part time but not so large as to cause her to work full time.

If the remarried mother has no surplus then

$$a(s,y_h)+q(s,y_h)=u_m(s). \tag{17}$$

When the mother remarries, she obtains more of the adult good that she shares with her new husband but the child's utility $q(s,y_h)$ is lower. This implies that the mother's remarriage has a negative impact on the father, but he can mitigate this effect by transferring money to the mother, that is by increasing s. Specifically,

$$\begin{aligned} \frac{\partial a(s,y_h)}{\partial s} &= \lambda(u_m'(s) - \frac{\gamma}{w_l}) \geq 0, \\ \frac{\partial a(s,y_h)}{\partial y_h} &= 1-\lambda(1+\alpha) > 1, \\ \frac{\partial q(s,y_h)}{\partial s} &= u_m'(s) - \frac{\partial a(s,y_h)}{\partial s} > 0, \\ \frac{\partial q(s,y_h)}{\partial y_h} &= -\frac{\partial a(s,y_h)}{\partial y_h} \leq 0. \end{aligned} \tag{18}$$

An increase in the transfer s raises the utility that the mother would receive as single and improves her bargaining position in the newly formed household. Consequently, the remarried mother works less and spends more time with the child, which raises the utility of the child.[15] However, an increase in s also has the unintended effect of raising the new husband's utility, who "eats" part of the transfer. An increase in the net income of the new husband raises his gain from marriage $a(s,y_h) - y_h$ because the mother spends less time with the child and more time in the market. The mother is willing to do such a sacrifice of child quality because she is compensated by a higher level of adult consumption, jointly with the new husband.

The result that remarried mothers work more in the market may seem counterfactual. [16] We emphasize that market work is just one way of transferring resources from the child to the new husband and the crucial assumption is the availability of a linear transfer in some non trivial range. For instance, the mother may spend less time with the child and more time with the new husband in joint leisure activities. As long as such substitutions are available at a fixed rate of exchange, the results are the same as if the remarried mother would spend time working in the market.

[15] Note that $\frac{\partial a(s,y_h)}{\partial s} = 0$ if $s = \hat{c}$, $\frac{\partial a(s,y_h)}{\partial s} = 1$ if $s = c^*$, and $1 > \frac{\partial a(s,y_h)}{\partial s} > 0$ for $\hat{c} < s < c^*$.

[16] As seen in Chapter 1, Figure 12, the raw data suggests that divorced women work more than married women. However Seitz (1999) shows that correcting for selection and unobserved attributes, there is no significant difference in labor supply of divorced and married women, while remarried women work significantly more than married women, as our model suggests.

4 Transfers, the interim perspective

Following separation, the parents can be in four different states, depending on the new marital status of their ex spouses:

1. Both parents are single, which happens with probability $(1-p)^2$.
2. The father remains single while the mother is remarried, which happens with probability $p(1-p)$.
3. The mother remains single but the father is remarried, which happens with probability $(1-p)p$.
4. Both parents remarry, which happens with probability p^2.

Note that, by assumption, the probability of remarriage is the same for the husband and wife, and that meetings and subsequent remarriages are independent across parents.

Anticipating these contingencies, the father may be willing to commit to transfer money to the custodial mother with the intention to influence the welfare of the child, of whom he continues to care.[17] Each father makes his choice of s separately, taking the choice of others, $s\prime$ as given. These payments are made at the time of divorce, before the marital status of the ex-spouses is known. We, therefore, must use expectations in determining the optimal level of the transfer. The expected utility of the father is, therefore,

$$\begin{aligned} V_f &= (1-p)^2[w_h - s + q(s)] + (1-p)p[w_h - s + q(s, w_h - s')] \\ &\quad + p(1-p)[a(s', w_h - s) + q(s)] + p^2[a(s', w_h - s) + q(s, y - s')], \end{aligned} \tag{19}$$

and

$$\frac{\partial V_f}{\partial s} = (1-p)[q'(s) - 1] + p[\frac{\partial q}{\partial s} - \frac{\partial a}{\partial y_h}]. \tag{20}$$

[17]Another possible motive is that the father maintains an altruistic motive towards his ex-wife. In this chapter, however, we ignore this added altruistic link and confine our attention only to the case in which parents care about their children.

We first note that the father will never choose voluntarily transfer s that exceeds c^* because, in this case, the single mother would spend the marginal dollar on the adult good. The father then receives a marginal benefit of α from the transfer if the mother remarries single and $1 + a - \frac{\gamma}{\gamma-(1+\alpha)w_l}$ if she remarries. But his expected cost in terms of the adult good is higher, because a transfer of a dollar costs the father 1 dollar if he remains single and $\frac{\partial a}{\partial y_h} = \frac{\gamma}{\gamma-(1+\alpha)w_l}$ if he remarries (see equations (11) and (18)).

Under our maintained assumption that the remarried mother works part time, equation (20) can be rewritten as

$$\frac{\partial V_f}{\partial s} = \begin{cases} (1-p)(\frac{\gamma}{w_l} - 1) + p(\frac{\gamma}{w_l} - \frac{\partial a}{\partial y_h}) & if \quad 0 \leq s < \hat{c}, \\ (1-p)(g'(s) - 1) + p(g'(s) - 2\frac{\partial a}{\partial y_h}) & if \quad \hat{c} \leq s \leq c^*, \end{cases} \tag{21}$$

where $\frac{\partial a}{\partial y_h} = 1 - \lambda(1+\alpha) = \frac{\gamma}{\gamma-(1+\alpha)w_l}$.

The two branches in (21) reflect changes in the mother's behavior as a function of the transfer s that she would receive if she would have remained single. In the region $0 < s < \hat{c}$, the single mother would work part time, so that $c = \hat{c}$ and $q'(s) = \frac{\gamma}{w_l}$. The independence $\frac{\partial V_f}{\partial s}$ from s in this region implies that either the father will contribute nothing or he will voluntarily commit on a transfer of at least $\hat{c}$. Which of these two possibilities applies depends on the basic parameters of the model and the probability of remarriage. The father is certainly willing to transfer resources to his ex-wife if he and the mother remain single with high probability, because then the marginal benefit in terms of child quality, $\frac{\gamma}{w_l}$, exceeds the marginal costs in terms of the forgone consumption of the father, which is 1. The father will be more reluctant to contribute if p is large, because then the cost for him is larger, $\frac{\partial a}{\partial y_h}$. The father will contribute at all p if $\gamma > (2+\alpha)w_l$, which means that the mother is highly effective in caring for the child. If this requirement is not satisfied, there will be some critical p below which the father will give at least the amount $\hat{c}$, but above which he will give nothing. In the region $\hat{c} < s < c^*$, the mother uses the transfer to increase child consumption and because of the concavity of $g(s)$, the marginal value of the transfer $\frac{\partial V_f}{\partial s}$ declines with s. Hence, an interior solution can exist in this region.

We can now characterize the father's incentives to support the mother.

Proposition 1 *Let* $s^*(p)$ *be the* optimal *level of voluntary commitment that the father is willing to make at the time of divorce. Then,* $s^*(p) \leq c^*$*, declines in the probability of remarriage,* p*, and is independent of the income of the new husband whom the mother may remarry. For a sufficiently high comparative advantage of the mother in child care, i.e.,* $\gamma > (2+\alpha)w_l$*, the optimal transfer,* $s^*(p)$*, exceeds* $\hat{c}$*. The transfer is then set to* $s^*(p) = c^*$ *if* $p < p_1$*, where* p_1 *satisfies*

$$1 + \alpha = (1 - p_1) + 2p_1 \frac{\partial a}{\partial y_h}. \tag{22}$$

Otherwise, if $p \geq p_1$*,* $s^*(p)$ *is determined by the unique solution to*

$$g'(s) = (1 - p) + 2p \frac{\partial a}{\partial y_h}. \tag{23}$$

The optimal transfer declines with the remarriage probability for two reasons: the marginal impact of a transfer on child quality is larger (or equal) when the mother is single, and the cost of giving are higher if the father remarries.

The independence of the transfer from the new husband's income implies that although the expected utility of each husband depends on the transfers by others, the *marginal* impact of s is not and, therefore, s is independent of $s\prime$. This feature is reflected in the fact that $\frac{\partial a}{\partial y_h} = \frac{\gamma}{\gamma - (1+\alpha)w_l}$ is a constant. However, if the mother does not work when she is remarried, or works full time, the utility frontier for a remarried couple is no longer linear and the marginal impact of the transfer to the mother will depend on the net income of her new husband y_h.

4.1 Partial equilibrium

Suppose that all couples have children. Then, at given probability of remarriage, p, the equilibrium outcome is that all fathers will transfer the *same* amount $s^*(p)$ to the mother if the marriage dissolves. That is, given that other fathers choose $s\prime = s^*(p)$, each father independently chooses $s = s^*(p)$. This equilibrium requirement is trivially satisfied here, because the optimal choice of each father is (locally) independent of the choices of others. We refer to the equilibrium as partial because, as we shall see shortly, the remarriage and fertility rates must also be set at equilibrium levels.

In this partial equilibrium, the mother works part time when she is remarried but not as single. The reason for this difference is that she must compensate her new husband for the option of sharing the adult good.

The amount of time that the mother spends in market work is

$$h_m(p) = \frac{(1+\alpha)(w_h - \hat{c}) + g(\hat{c}) - g(s^*(p))}{\gamma - (1+\alpha)w_l} \tag{24}$$

and a sufficient condition for an interior solution $0 < h_m(p) < 1$ for all p is that

$$\gamma - (1+\alpha)w_l > (1+\alpha)(w_h - \hat{c}) > g(c^*) - g(\hat{c}). \tag{25}$$

Basically, the mother should have a sufficiently high comparative advantage in child care to motivate her to spend some time at home and the net income of the new husband should not be so high that the mother is driven completely into the market, contrary to her comparative advantage.[18]

We see that when p rises and *all* husbands reduce their contribution, the remarried mothers increase their hours of work and thus reduce the amount of time spent with the child. The implied adult consumption in the remarried household

$$a(p) = w_l h_m(p) + w_h - \hat{c} \tag{26}$$

rises in p but the child's utility if the mother remarries

$$q(p) = \alpha a(p) + \gamma(1 - h_m(p)) + g(\hat{c}) \tag{27}$$

declines in p, because the mother's time is more important for the child than the added adult good.

The expected utilities of the three family members, evaluated at the time of divorce,

[18] The sufficient condition (25) is much stronger than we need because, as we shall show shortly, the equilibrium remarriage rate is bounded by $\frac{1}{2}$.

are

$$\begin{aligned} V_m(p) &= g(s^*(p)) + \gamma, \\ V_c(p) &= g(s^*(p)) + \gamma - pa(p), \\ V_f(p) &= g(s^*(p)) + \gamma + (1-p)(w_h - s^*(p)). \end{aligned} \tag{28}$$

Compared with an intact family with $\theta = 0$, all three family members are worse off if the marriage breaks. The child received less child goods because the transfer from the father $s^*(p)$ is lower than c^* (except at low probability of remarriage $p < p_1$) and also less time if the mother remarries. Both the mother and the father suffer from the reduction in child quality. In addition, there is a loss of resources resulting from the inability to share consumption goods when the parents remain single. This cost is born mainly by the mother. The assumption that the mother receives no surplus implies that she pays for the adult good in terms of the child's quality, so that her utility is unaffected by remarriage but that of the child is reduced by $a(p)$. The father, on the other hand, gets the benefits from sharing $a(p)$ with the new wife and, in addition, he consumes the adult good when he is single. In fact, he consumes as single more of the adult good than he would under marriage. The outcome of this asymmetry is that the father's expected utility following separation is higher than the mother's.

The expected utility of all family members in the aftermath of divorce declines with the probability of remarriage, p. This is a surprising result, given that remarriage is voluntary. It can be traced to the fact that a higher remarriage rate does not only make it easier to remarry, which is *individually* welfare enhancing, but also affects behavior in a way that may be harmful to others. Thus, although the mother fully internalizes that the child is worse off upon remarriage, this does not stop her from remarrying if she is compensated by higher adult consumption. Nor does she take into account the negative impact of her remarriage on her ex-husband. The father's incentives to transfer money to the custodial mother decline as the probability of remarriage rises, because he anticipates that part of it will be spent on adult goods that are not as useful to the child, mainly because of a presence of a third party in the form of the new husband. As a result of this reluctance to contribute, mothers are worse off even if they remain single. Finally, each father is worse off mainly because the child is worse off when the mother remarries and he cannot fully remedy that by the use of transfers, due to the principal-agent issues that we described. This loss of control is sufficiently costly to offset the gains that the father receives when he remarries and obtains all the surplus.

5 Divorce

Having observed the realized quality of the current match, each spouse may consider whether or not to continue the marriage. A parent will agree to continue the marriage, if given the observed θ the utility in marriage exceeds his/her expected gains from divorce. Under divorce at will, the marriage breaks if

$$u^* + \theta < \max\left(V_m, V_f\right) - b, \tag{29}$$

where u^* is the common utility of the husband and wife if the marriage would continue (not incorporating the quality of the match) and b is a fixed cost associated with divorce. The fixed costs reflect the emotional, legal and relocation costs associated with the change in marital status that affects the child and parents. We assume that these costs are higher for couples with children and are shared equally by the two spouses.

The particular value of θ that triggers divorce is given by

$$\theta^*(p) = max\left(V_m(p), V_f(p)\right) - u^* - b. \tag{30}$$

The critical value θ^* is seen to equal the expected gains from divorce, relative to remaining married, evaluated at $\theta = 0$, which is the mean value of θ in the population. In other words, the couples that divorce are those with a realized quality of match that is below the unconditional expectation of the gains from divorce, *before* θ is observed. These expected gains are negative, because an intact marriage with $\theta = 0$ is better for all parties. The probability that a couple will divorce is then

$$\Pr\{\theta \leq \theta^*\} = F(\theta^*), \tag{31}$$

where $F(.)$ is the cumulative distribution of θ.

Our previous analysis implies that

Proposition 2 *If all couples have a child, then in a partial equilibrium where all fathers choose the optimal transfer $s^*(p)$, the divorce decision at any expected remarriage rate, p, is determined by the father. The critical value of θ that triggers divorce $\theta^*(p)$ is negative and declines in the probability of remarriage, p.*

Because of the ex-post asymmetry between the partners, separation may be inefficient. The father, who has strictly higher expected gain from divorce than the mother, will initiate the divorce at some θ such that the mother wants the marriage to continue and inflict on her a loss of match quality.[19]

5.0.1 Couples without children

If the parents do not have a child, the "material" utility (not including the love component θ) of *each* parent in an intact family is $u^* = 2w_h$, reflecting the assumptions of income pooling and joint consumption of the adult public good. However, a parent that remains single consumes only w_h. If *all* couples do not have a child, the symmetry between the parents is reestablished and both expect upon separation to receive

$$V_m(p) = V_f(p) = (1-p)w_h + p2w_h. \tag{32}$$

An important difference from the case with children is that the expected utility of the two parents, as evaluated at the time of divorce, *rises* with the probability of remarriage. This is simply an outcome of the option to share consumption upon remarriage, without any negative impact of divorce on child quality.

The critical value of θ that triggers divorce is now given by

$$\theta^*(p) = -(1-p)w_h \tag{33}$$

[19] We note, however, that if $\delta < 1$ so that the non custodial father suffers from the distance from his child, the father's gains from divorce decline and may be lower than the mother's.

which rises with p. That is, the higher is the probability of remarriage the more likely it is that a particular couple will divorce. This result is in sharp contrast to that in Proposition 2, illustrating the marked difference that children might have on the divorce decisions.

6 Fertility

So far, we took the number of children as given and assumed that all couples have children. We now examine the decision to have children.

We view children as an investment good that the parents produce at some cost during the first period of marriage, before the quality of match is revealed. To simplify, we assume that only one child can be produced. The costs of having a child are the forgone earnings of the mother associated with child birth and child rearing. We assume that the mother cannot work in the first period if she gives birth, so that w_h is lost in the first period. Also, because of the mother's withdrawal from the labor force, her second period wage erodes from w_h to w_l. The benefits from the child that accrue in the second period depend on the probabilities of divorce and remarriage and on the parents' ability to care for the child in the aftermath of divorce. To avoid trivial solutions, we assume that children may be a bad or good investment, depending on the circumstances. In particular, a couple that obtains the average draw $\theta = 0$ and chooses not to divorce gains from having had children. This is equivalent to saying that children are desired if divorce is not an option. However, when divorce is an option, children may be a liability if they lock the parents into bad matches.

An important feature of the analysis is that the decisions of each couple whether to divorce or to have a child depend not only on the circumstance of the couple, e.g., if it suffered a negative shock, but also on the decisions of other couples to have children and to divorce, as well as on the contracts that they sign. These decisions by other couples influence the prospects of remarriage and the quality of potential mates. To simplify our analysis, we focus here on the case in which, *in equilibrium,* all couples have children or all couples do not have children. Therefore, we only need to consider the benefits of a particular couple from having a child, conditioned on whether or not *all* other couples have children.

Given the choices of others, the expected life time utility of a parent j in a particular couple is

$$W_{j,n}(p) = u_n^0 + \int_{\theta_n^*(p)}^{\infty} (u_n^* + \theta) f(\theta) d\theta + F(\theta_n^*(p))(V_{j,n}(p) - b_n), \tag{34}$$

where, $j = f$ for the (potential) father and, $j = m$ for the (potential) mother, and n is a choice variable that equals to 1 if the couple has children and 0 otherwise. The term u_n^0 in equation (34) represents the utility of the two partners in the first period, which is $2w_h$ if the couple has no children and only w_h if a child is born, because of the mother's withdrawal from the labor force during child birth. The term u_n^* represents the parents utility if marriage continues, which is $2w_h$ if the couple has no children and u^* if there is a child. The fixed costs of separation b_n are assumed to be larger when the couple has children.[20]

The expected life time utility is higher for the partner with the higher gains from divorce who determines the divorce decision. In fact, the expected life time utility can be rewritten as

$$W_{j,n}(p) = \begin{cases} u_n^0 + u_n^* + \int_{-\infty}^{\theta_n^*(p)} (\theta_n^*(p) - \theta) f(\theta) d\theta & if \quad V_{j,n}(p) \geq V_{i,n}(p) \\ u_n^0 + u_n^* + \int_{-\infty}^{\theta_n^*(p)} (\theta_n^*(p) - \theta) f(\theta) d\theta & \\ \quad -F(\theta_n^*(p))(V_{i,n}(p) - V_{j,n}(p)) & if \quad V_{j,n}(p) < V_{i,n}(p) \end{cases} \tag{34'}$$

where the term $u_n^0 + u_n^*$ is the value of the marriage if it never breaks and the term $\int_{-\infty}^{\theta_n^*(p)} (\theta_n^*(p) - \theta) f(\theta) d\theta$ is the *option value* of breaking the marriage if it turns sour because of a bad draw of θ. The option to sample from the distribution of θ is a motivation for marriage that exists even if marriage provides no other benefits. However, this option is available only to the person with the higher gains from divorce, who determines the divorce. When the marriage breaks, an event that happens with probability $F(\theta_n^*(p))$, the spouse who does not initiate the divorce and is left behind suffers a capital loss given by $V_{i,n}(p) - V_{j,n}(p)$. The value of the option for the spouse who determines the divorce,

[20] It would be more realistic to allow discounting of future utilities in (34). However, this does not add any new conceptual issues and to economize on notation we set the discount factor to unity.

increases in the gains from divorce, $F(\theta_n^*(p)$, and also with the variability in the quality of match, because then the ability to avoid negative shocks becomes more valuable.

We define the benefit of spouse j from having a child as

$$B_j(p) = W_{j,1}(p) - W_{j,0}(p). \tag{35}$$

Because the production of children must involve both partners, a couple will have a child if and only if *both* partners agree to have a child. That is if,

$$B(p) = Min\{B_m(p), B_f(p)\} \geq 0. \tag{36}$$

Because the father determines the divorce decision, his life time utility must also be higher, $W_{f,1}(p) \geq W_{m,1}(p)$. Now imagine that a particular couple departs from the general pattern and chooses not to have a child. If the husband remains single, he will get his income w_h and if he remarries he will get $w_h + w_l h_w$, because his new wife is a custodial mother who spends some of her time on the child. The wife also gets w_h if she remains single, because she has no child, and has the same wage as her husband. However if she remarries she will get $w_h + w_h - s^*(p)$. For a sufficiently large gap between w_h and w_l, a non custodial father brings more income into the marriage than a custodial mother who generally works only part time.[21] Therefore, if all other couples have a child, the wife in a couple that chooses not to have a child expects to gain from divorce *more* than her husband, $V_{f,0}(p) < V_{m,0}(p)$. In this case, she will determine the divorce decision, and consequently, her life time utility is higher, $W_{f,1}(p) < W_{m,1}(p)$. It follows that the mother has lower benefits from having the child, $B_m(p) < B_f(p)$.

Proposition 3 *The wife determines whether or not the couple produces children.*

When the value of having a child is strictly lower for the wife, there may be some p such that she will prefer not to have a child when the husband would like to have a child. In such a case, the father may be willing to sign a binding ex-ante contract which would

[21] Recalling that $s^*(p) \leq c^*$, a sufficient condition is that $w_h > w_l + c^*$.

transfer money to the mother upon separation if this would induce her to have a child. We shall return to this issue in the concluding section.[22]

7 Equilibrium

Equilibrium requires consistency among the choices of the participants in the marriage market and realization of their expectations. The first consistency requirement is that the aggregate divorce rate coincides with the expected remarriage rate. Assuming independence of the marital shocks across couples and a large population, the proportion of couples that will choose to divorce is the same as the probability that a particular couple divorces. The decision of each couple to divorce depends on the expected remarriage rate, p. Assuming that a person can remarry only with a divorcee and that meetings are random, we require that, in equilibrium, the realized aggregate divorce rate must equal the expected remarriage rate of all agents. That is,

$$p = F(\theta^*(p)). \tag{37}$$

Because the gains from divorce for a couple with $\theta = 0$ are negative, the threshold $\theta^*(p)$ is negative and it then follows from our assumptions on $F(p)$ that any solution of (37) must be such that $p < \frac{1}{2}$.

When fertility is endogenous, we have the additional requirement that the expected gain from divorce must reflect the optimal fertility choices of the participants in the marriage market. Thus, in an equilibrium without children we must have that

$$p = F(\theta_0^*(p)), \tag{38}$$

and $B(p) < 0$. That is, the expected gains from divorce are calculated based on the assumption that all singles are childless, and given these expectations no couple wishes to

[22] We note again that if $\delta < 1$, so that the non custodial father suffers from the distance from his child, the father's benefit from having a child could be smaller than the mother's.

have a child. Similarly, in an equilibrium in which every couple has a child we must have

$$p = F(\theta_1^*(p)), \tag{39}$$

and $B(p) > 0$.

The third requirement from equilibrium is that the participants contracting choices must be optimal, given by $s^*(p)$.[23]

For any given legal environment, it is convenient to rewrite the equilibrium condition in the form

$$F^{-1}(p) = \theta^*(p). \tag{40}$$

This formulation separates the properties of the distribution of the unanticipated shocks from the properties of the trigger $\theta^*(p)$ that summarized the impact of the expected remarriage rate on the expected gains from divorce. Because $F^{-1}(p)$ rises in p, while $\theta^*(p)$ declines (rises) in p when a child is (not) present, there may be two equilibrium points: a high divorce (remarriage) without children and a low divorce (remarriage) with children.

7.1 Numerical Example 1

We now present a numerical example that illustrates some of the results. We adopt here a slightly more flexible formulation, allowing the father to suffer a utility loss when he lives separately from the child, $\delta < 1$, and allowing discounting of future utilities, $\Delta < 1$.[24]

Figure 3 shows the optimal transfers that the father promises to his ex-wife at the time of divorce as a function of the prospective remarriage rate, p, and the implied consequences

[23] These equilibrium requirements implicitly assume symmetric equilibria in which all agents behave in the same manner. Such equilibria are a natural choice given that all agents are initially identical, but other equilibrium may exist. In a more general analysis, one can incorporate also mixed equilibria such that some couples choose to have a child, some choose to remain childless and all couples are *indifferent* between having and not having a child. However, because such equilibria tend to be unstable, we are less interested in them and will not introduce the additional notation that is required to characterize them.

[24] With these modifications (19) and (34) become

$$\begin{aligned} V_f &= (1-p)^2[w_h - s + \delta q(s)] + (1-p)p[w_h - s + \delta q(s, w_h - s')] \\ &\quad + p(1-p)[a(s', w_h - s) + \delta q(s)] + p^2[a(s', w_h - s) + \delta q(s, y - s')], \end{aligned}$$

for the child when the mother remarries and remains single.[25] The optimal transfer from the father to his ex-wife, $s^*(p)$, declines with p because the marginal impact of the transfer on the child is lower when the mother remarries. As a consequence, the utility of the child when the mother remains single, $\gamma + g(s^*(p))$, declines too. The child's utility when the mother remarries, $\alpha a(p) + \gamma(1 - h_m(p)) + g(\hat{c})$, declines because a lower transfer implies that the mother works more and spends less time on the child; but the reduction in the mother's caring time has a stronger effect than the child's gains from the higher consumption of the adult good. The child's utility when the mother is single exceeds the child's utility when the mother remarries, because the mother is "paying" for her gain of adult good by reducing the utility of the child. Therefore, the child' loss from remarriage equals $a(p)$, which rises with p, as the bargaining position of the mother worsens when the father transfers less.

As a consequence of the decline in the optimal transfer, the expected utilities at the time of divorce of the child, mother and father all decline (see Figure 4). Assuming a moderate loss for the father when he lives apart from the child, $\delta = .75$, the expected utility of the father is higher than that of the mother through most of the relevant range of p. Consequently, the father determines the divorce decision if $.05 < p < .5$, while the mother determines the divorce decision if $p < .05$.[26]

In Figure 5, we plot the maximum of the husband's and wife's expected gains (losses) from divorce, including the fixed cost of divorce, for couples with and without children. These gains rise for couples without children because remarriage enhances joint consump-

and

$$W_{j,ch}(p) = u^0_{ch} + \Delta\{ \int_{\theta^*_{ch}(p)}^{\infty} (u^*_{ch} + \theta) f(\theta) d\theta + F(\theta^*_{ch}(p))(V_{j,ch}(p) - b_{ch})\},$$

respectively, where $0 < \delta$, $\Delta \leq 1$. In the figures, we set $\delta = .75$ and $\Delta = .625$.

[25] The graphs are drawn for the case in which the mother's productivity at home is $\gamma = 1.75$, the mother's wage is $w_l = .5$ and $w_h = 1$. We specify $g(c) = g_1(1 - (1 + c)^{-g_2})$, where $g_1 = g_2 = \frac{9}{4}$ and $\alpha = \frac{1}{4}$. Thus c^* satisfies $g_1 g_2 (1 + c)^{-g_2 - 1} = 1 + \alpha$, implying $c^* = .5378$ and $g(c^*) = 1.3956$, $\hat{c}$ satisfies $g^2(1 + c)^{-g_2 - 1} = \frac{\gamma}{w_l}$ implying $\hat{c} = .1203$.

[26] The difference between the father's and mother's expected gains from divorce is

$$(g(s^*(p)) + \gamma)(\delta - 1) + (1 - p)(w_h - s^*(p)),$$

which is always positive if $\delta = 1$, but can be negative for $\delta < 1$.

tion and decline for couples with children, because remarriage implies lower spending on the child that dominate the gains from joint consumption. The intersections of these curves with the inverse probability function at $p = .214$ and $p = .334$ represent potential equilibria, where the realized divorce rate equals the expected remarriage rate.[27] A higher potential equilibrium point arises when all couples do not have children because, by assumption, such couples have lower fixed cost of separation and, in addition, they do not suffer from the reduced welfare of the child when the marriage breaks. To make sure that the two intersections in Figure 5 satisfy all the requirements for equilibrium, we must further verify that, at the higher intersection with $p = .334$, no couple without a child wants to deviate and have a child when all the others do not have a child, while in the low intersection with $p = .214$, no couple with a child wants to deviate and have no child when all others have a child.

Figure 6 shows the incentives of the husband and wife to deviate and have no child when all other couples have a child and their child support is set at the optimal level $s^*(p)$. The expected life time utilities of the husband and wife when all couples have children decrease with the probability of remarriage, with the mother's life time utility being slightly lower than the father's (except for very low p, $p < .05$), reflecting the father's higher expected gain from divorce. In contrast, the life time utilities that the parents obtain upon deviating to not having a child *rise* with the probability of remarriage because of the gain from joint consumption. With this structure, a deviation would occur only at a sufficiently high probability of remarriage. Because both partners are required to produce a child, it is sufficient for a deviation to occur that *one* of the two parents refuses to have a child. We see that the wife wants to deviate only if the remarriage rate exceeds .28, while the husband wants to deviate only if the remarriage rate exceeds .36. Thus, the intersection in Figure 5 at $p = .214$ is an equilibrium with children. By a similar argument, it can be seen that the intersection in Figure 5 at $p = .334$ is an equilibrium without children, because neither the husband nor the wife wish to have a child if all others do not have a child (see Figure 7).[28]

[27] The inverse probability is drawn for the case in which the match quality, θ, is uniformly distributed over $[-d, d]$, so that $p = prob\{\theta \leq x\} = \frac{d+x}{2d}$ and $x = d(2p-1)$. In the figures, we set $d = 2$.

[28] If all other couples do not have a child, a husband would like to deviate and have a child only if

In Table 1, we provide some comparative static results. The first panel shows the benchmark parameters. The second panel shows the impact of changes in the variance of the quality of match, holding the mean constant. The inverse probability is drawn for the case in which the match quality, θ, is uniformly distributed over $[-d, d]$, so that such an increase is represented by an increase in d. The higher is d, the more likely it is that the realized match will be sufficiently low to trigger divorce. Therefore, the equilibrium divorce rate rises with d. At a low d, $d = 1.5$, the only equilibrium is the one with children and for a high d, $d = 2.5$, the only equilibrium is without children. For intermediate values of d ($d = 2.0$ and $d = 2.2$) there are two equilibria for each value of d. It is then possible that a small exogenous change i.e., a rise in d from 2 to 2.5 will cause a large change in the divorce rate, shifting the equilibrium from a divorce (remarriage) rate of $p = .214$, (with children) to .375 (without children), with a noticeable rise in the utility of both parents. This change illustrates a *social multiplier* effect where the higher willingness of each couple to divorce, as a consequence of the exogenous shock (i.e., the rise in d), increases the aggregate divorce rate, which further increases the incentives to divorce. The rise in the life time utility of the parents with d illustrates our observation that marriage has an option value, because bad outcomes to the quality of the match can be avoided through divorce. However, the child, who is a passive agent that cannot directly influence the divorce decision, suffers from the dissolution of the marriage.

The third panel of Table 1 illustrates the impact of an increase in the fixed costs of divorce in the presence of children, b_1.[29] An increase in b_1 reduces the divorce rate of couples with children and thereby reduces the expected life time utility of the parents who cannot so easily recover from bad matches. The child, of course, gains from such a change, because he is better off in an intact family and, by assumption, does not suffer from a bad quality of match. Although such a change does not directly affect the outcomes if all couples do not have children, it still may influence the equilibrium outcome through the impact on the incentives to have children. Thus if b_1 is reduced from .25 to .05 then the equilibrium without children disappears and the only equilibrium is with children.

$p < .19$, while a wife would like to have a child only if $p < .24$. In calculating the deviation, we take into account that when the couple will have the child the father will commit to pay child support according to $s^*(p)$.

[29] We assume no fixed cost of separation in the absence of children, i.e., $b_0 = 0$.

The last panel of Table 1 illustrates the impact of changes in the utility of the father, as δ rises and he suffers less from living apart from the child. Such an increase in proximity raises the utility of the father directly, but it also raises his willingness to transfer money to the custodial mother and, consequently, the child and mother gain too. Notice that for $\delta = 1$, the father would like to have a child but the mother prefers not to have a child if all other couples do not have a child. This conflict could, in principle, be resolved by ex ante contracting at the time of marriage.

8 Further issues

In this concluding section we discuss some departures from the standard contract that we analyzed and examine their implications.

8.1 The custody assignment

If the father has the custody, he will spend only the minimal amount of time on child care because, under our assumption that $w_h(1+\alpha) > \beta$, any hour spent on child care could be better used in the market. He will spend on the child good c^* and spend the remainder of his income on the adult good. This is true whether or not he receives transfers from the mother. Therefore, the mother has no incentive to transfer to the custodial father, as any additional dollar is spent on the adult good and $\alpha < 1$. If all couples choose father custody then in a remarried couple, the new wife will work in the market, because she cannot spend time on her child who lives in a different household. The expected utilities of the family members are then

$$\begin{aligned} V_m(p) &= \alpha(w_h - c^*) + g(c^*) + p\alpha w_l + w_l + p(w_h - c^*), \\ V_c(p) &= \alpha(w_h - c^*) + g(c^*) + p\alpha w_l, \\ V_f(p) &= (1+\alpha)(w_h - c^*) + g(c^*) + p(1+\alpha)w_l. \end{aligned} \tag{41}$$

We see that under father's custody, the child receives less time but consumes more of the adult and child goods. Thus, the justification for the prevalence of mother custody must rest on the assumption that, in the case of children, time is more important then money, that is γ is large relative to $\alpha(w_h - c^*)$. For a small remarriage probability, the condition $\gamma > \alpha(w_h - c^*)$ is sufficient to ensure that mother custody is better for the child. In this case, the father also prefers that the child will be with the mother, because for a small p his expense on child support, $s^*(p)$, is about the same as he would spend himself on the child, c^*, and the potential gain from the mother contribution of time exceeds the gains that the father has from sharing adult consumption with the child. However, if $\gamma < \alpha(w_h - c^*) + w_l$ then, for a small p, the mother would prefer that the father will have the custody, because this would free her to earn some extra money in the labor market. In this case, the child is a "hot potato" that each parent prefers that the *other* will take care of it. This reflects, of course, the potential for free riding that exists in the provision of public goods. Thus, γ must exceed $\alpha(w_h - c^*) + w_l$ for the two parties to agree on mother custody.[30]

An increase in the probability of remarriage p decreases the welfare of the child under the mother's custody but raises it under the father's custody. This difference is caused by the shift of the custodial mother towards market work when she remarries. The custodial father works at the same intensity whether he is married or not and the child gains from the added adult consumption when the father remarries. Therefore, for a large probability of remarriage father's custody becomes more attractive and a larger gap between γ and $\alpha(w_h - c^*)$ is required to justify mother's custody under the voluntary commitments discussed so far. A possible resolution is to mandate (and enforce) some *minimal* child support transfer from the non custodial father to the custodial mother.

8.2 Mandated and contingent contracts

The courts often consider the "accustomed standard of living" of the parties as a standard for divorce settlements. Because living alone is more costly then living together and there is always a risk of remaining single, it is *impossible* to restore the same standard of living for *all* parties. The problem is exacerbated by the principal-agent issues emphasized here

[30] For alternative models of custody assignment see Atteneder and Halla (2007) and Rasul (2006).

that imply a level of transfers that is insufficient to restore efficiency. We now discuss some alternative contracting options that may restore efficiency.

The law also singles out children as worthy of special consideration in divorce settlements. This concern is justified because, as we have seen, even if parents are altruistic and internalize the welfare of the child, the child as a passive party can be hurt by the divorce. It is then natural to apply the accustomed standard of living principle only to the child, as a constraint on the parents' contracting choices. For instance, the law may mandate a level of child support $s = c^*$. As we have shown, such a transfer would indeed induce a single mother to choose the efficient level of child care, spending all her time on child care. This, however, is not true for a remarried mother, who still may be forced to work part time to comply with the interest of her new husband. As long as the courts cannot interfere with within-household allocations (that are hard to verify) and the father cannot transfer directly to the child, because his money transfer is fungible and can be consumed by the mother and her new husband, it is hard to expect that the child interests will be maintained simply by mandating a money transfer. There is, however, one notable exception. If the mother has sufficient bargaining power to take *all* the gains from marriage, then she would solve the mirror image of problem (13) and maximize her utility subject to the constraint that the new husband is just indifferent between remaining single and remarriage. Formally, this problem is the same as problem (7) that mother solves as single and setting $s = c^*$ would indeed induce her to maintain the efficient outcome when she remarries.

This brief discussion illustrates that in search markets with rents that are subject to bargaining, it is important to specify the relative bargaining power of the parties that determines the share of the surplus that each party gets. In the Nash bargaining model this is determined by considerations such as impatience and risk aversion that, of course, need not be equal across genders. More broadly, social norms such as egalitarianism and sex roles may also affect the bargaining outcome.

Another, and potentially more fruitful, direction is to enlarge the set of contracts that the courts are willing to enforce. In principle, child support payments should depend on the marital status of both parents, because the costs and benefits of post divorce transfers depend on these states.[31] In practice, child support is not contingent on marital status but there are other payments such as alimony that are often contingent on the marital status of the mother. Because we assume that all transfers are fungible, the name attached to these payments does not really matter, but it does matter how flexible they are and to what contingences they respond.

Now imagine that a father can pay different amounts to the custodial mother depending upon whether or not she is remarried. Suppose further that the father is forced by law to pay a fixed amount of child support $s = c^*$ but can augment it by an additional payment σ that he pays the custodial mother only if she is single. Then, the efficient allocation within the remarried household is determined by

$$\underset{h_m, c}{Max}\ E(a|\sigma, \sigma') = w_l h_m + w_h - c - (1-p)\sigma' \tag{42}$$

$$s.t.$$

$$(1+\alpha)[w_l h_m + w_h - c - (1-p)\sigma'] + \gamma(1-h_m) + g(c) \geq u_m(c^* + \sigma),$$

$$0 \leq h_m \leq 1.$$

The chosen values of h_m and c depend on the transfer that the father promises the mother if she remains single, σ, and the *expected value* of the new husband's gross income $w_h - (1-p)\sigma'$. Only the expectation matters because the remarried partners are risk neutral with respect to a and because the mother's work time, h_m, and the expenditures on the child good, c, are determined before the marital status of the ex-wife of the new husband is known.

In contrast to a non contingent transfer, a transfer given to the mother *only* when she is single does not change the utility frontier of the remarried couple and therefore must *reduce* the expected utility of the new husband. This implies that with contingent payments, the father is able to attain a larger impact on the child's utility and is willing

[31] In theory, the transfers should depend on the marital status of *all* agents that participate in the marriage market. But this, of course, is highly impractical.

to contribute more to the custodial mother. In fact, by setting $\sigma = w_h - c^* - (1-p)\sigma'$ the father can eliminate *all* the gains from marriage of the new husband and restore efficiency. We then obtain the following characterization (see Appendix).

Proposition 4 *If all couples have children, then the commitment equilibrium for a given remarriage probability p, is such that: For $p < p_0$, the only symmetric equilibrium is one in which all fathers pay only the mandatory payment $s = c^*$ and $\sigma = 0$. For $p > p_1$, the only symmetric equilibrium is one in which all fathers voluntarily commits to pay their ex-wife $\sigma = \frac{w_h - c^*}{2-p}$ if she remains single. For $p_1 \geq p \geq p_0$, both types of equilibrium can arise. The equilibrium $\sigma = \frac{w_h - c^*}{2-p}$ is efficient and*

$$\begin{aligned} V_c(p) &= \gamma + g(c^*) + \alpha \frac{w_h - c^*}{2-p}, \\ V_m(p) &= V_f(p) = \gamma + g(c^*) + (1+\alpha)\frac{w_h - c^*}{2-p}. \end{aligned} \tag{43}$$

The pattern described in the proposition suggests reinforcement; one is willing to commit to his wife if others do, but not if they do not. As is well known, such positive feed backs can yield multiple equilibria. We also see that higher probability of remarriage is conducive to equilibria in which fathers are willing to commit on a payment that is conditioned on the event that the mother remains single, because such promises are carried out less often and are more likely to yield benefits.

When efficiency is restored, the child suffers only from the reduced adult consumption that is caused by the risk of remaining single. If the mother is sure to remarry, i.e., $p = 1$, then the child is as well off as in an intact family. That is, the father was practically replaced by the new husband with no harm to the child. This favorable outcome was achieved by eliminating the marital surplus completely and effectively eliminating the power of the new husband from extracting any rents which may harm the child upon remarriage. Importantly, the contingent transfer restores ex-post symmetry between the parents, which implies that divorce will also be efficient. Finally, because of the efficiency in child care, all family members benefit from a higher probability of remarriage.

These results are in sharp contrast to the case of non contingent transfers and raise the question why contingent contracts are not more prevalent. The basic problem with such contracts is that they are not attractive when the probability of remarriage is low, because then the father is very likely to bear the costs, when the mother remains single, and correspondingly unlikely to reap the benefits when she remarries. As we shall show in a subsequent section, this problem can be mitigated if the courts would also enforce contracts that are signed at the time of marriage. Before we turn to that case, however, let us illustrate the impact of contingent contracts with a numerical example.

8.3 Numerical Example 2

We now present an example with contingent contracts. The parameters are the same as in numerical example 1, except that we now set $\delta = 1$. This change is made to increase the motivation of the father to support the child and the motivation of the couple to have children, so that an equilibrium with voluntary transfers and children can be supported.

Figure 8 presents three potential equilibrium points for the divorce and remarriage rates associated with the following alternatives:

1. All couples have children and the father pays the mother a *fixed* payment c^* and, in addition, a *contingent* payment $\sigma = \frac{w_h - c^*}{2-p}$ that the mother receives only if she remains single.

2. All couples have children and the father pays the mother only the *fixed* payment c^*.

3. All couples have no children and no transfers are made upon divorce.

Case 3 is identical to the one discussed in the previous section. Case 2 is a modification of the case discussed in the previous section; the child support payment is still unconditional but fathers are forced to pay more than they would pay voluntarily. However, because part of this payment is eaten by the new husband of the remarried mother, the child and consequently the father are still harmed by remarriage, which is the same qualitative result that we had before. The main departure is case 1, where the contingent payment given to the mother raises her bargaining power to the extent that the child is not harmed from remarriage and, consequently, the gains from divorce of both parents rise with the prospects of remarriage. For the chosen parameters, this case yields an equilibrium divorce rate $p = 356$.[32]

Figures 9 and 10 describe the impacts of deviations from the transfer patterns when all parents have children. Figure 9 shows that if all parents commit on the transfer $\sigma = \frac{w_h - c^*}{2-p}$ that restores efficiency then for $p < .142$, each father, taken separately, would be better off by unilaterally deviating to $\sigma = 0$. Thus, a symmetric equilibrium in which everyone commits to $\sigma = \frac{w_h - c^*}{2-p}$ cannot exist in this range. Figure 10 shows that if all parents pay only the compulsory payment c^* and $\sigma = 0$ then, for $p > .302$, each father taken separately would be better off if he unilaterally commits to the mother to pay her *all* his disposable income, $w_h - c^*$, if she remains single. Thus an equilibrium where no one wishes to commit cannot exist in this range. Proposition 4 states that if $p_1 > p > p_0$ there may be *two* partial equilibria, one in which every father commits on a positive σ and one in which no father commits. However, for the chosen parameters, the equilibrium divorce rate associated with having children is above $p_1 = .302$, implying that the equilibrium in which every father commits at $p = .356$ is the only potential equilibrium when all parents have children. Table 2 shows that the equilibrium at $p = .356$ is indeed a full equilibrium in the sense that, with the implied child support transfers, all couples prefer to have children. For the assumed parameters, this is the only equilibrium because, if all couples have no children, there is an incentive to deviate to a situation with a child and full commitment $\sigma = w_h - c^*$. Thus, the full equilibrium is *unique*.

Comparing the results in Table 2 to the last row in Table 1, we can see the impact of different legal regimes when all parameters of the model are the same. Suppose that all couples have children. Then, if transfers are not contingent and determined optimally, the child's expected utility is 2.613. If fathers are forced to pay the mother a transfer of $s = c^*$, the child's expected utility rises to 2.811 and if contingent transfers are also enforced, the child's expected utility is 3.216, which is only slightly less than the child's utility in an intact family, 3.260. As we move across these alternatives, the transfer from each father to his ex-wife rises when the marriage breaks and, consequently, the expected

[32] An interesting point is that if the efficiency of child expenditures is restored by appropriate transfers then the gains from divorce with children can exceed the gains from divorce without children, despite the higher fixed costs of divorce associated with children. The reason is that couples without children have higher joint consumption (in terms of the adult good), which can make divorce more costly for them.

life time utility of each mother rises. The surprising result, however, is that the father is also better off and his expected utility levels are 3.456, 3.475 and 3.485, respectively. The result that a *compulsory* increase in child support above the individually optimal level, $s^*(p)$ raises the father's expected utility reflects a positive *contract externality*, whereby the commitment made by each father to his ex-wife benefits other fathers when they remarry. The second increase, associated with raising the contingent payment, σ, from 0 to $\frac{w_h - c^*}{2-p}$ benefits each father separately because of the rise in the expected utility of the child. The rise in the remarriage prospects as p rises from .344 to .356 raises the incentives of all fathers to contribute to their ex-wives. In this respect, a higher *aggregate* divorce rate can serve as a coordination device that can benefit children and raises the incentives to have children.

8.4 Transfers, the ex-ante perspective

At the interim stage, when the fertility has already been determined, the purpose of the contract is to induce the custodial mother to spend all her time on the child if she remarries. A contract that is signed at the time of marriage can also influence the divorce and the father would be willing to commit for a broader range of p. Thus, in contrast to the ex-post contract, where the husband gains from the commitment to pay the single mother only if she remarries, the ex ante contract can benefit the father even if the mother remains single. Of course, the husband is willing to pay only if the mother would not have the child in the absence of contract. It is easy to find parameters of the model such that the mother would prefer to have a child even without a contract, in which case there is no role for voluntary ex-ante commitments by the husband. However, we shall focus here on the case in which, in the absence of binding contracts, the mother does not wish to have the child but if binding contracts are enforced then the mother may prefer to have a child, depending on the expected remarriage rate and the decision of others to have children.

Suppose that all couples have children and sign an ex-ante contract, at the time of marriage, that promises the mother $\sigma = \frac{w_h - c^*}{2-p}$ if the mother remains single. Then, the gains from divorce are the same for the two partners and separations are efficient for all p and, therefore, the expected utility at the time of marriage is the same for the husband

and wife. Since under such contract both partners want the child at the same values of p, the production of children is efficient too. *Both* partners would agree to sign such a contract at the time of marriage, if their expected utility is higher than it would be in the absence of contract and no children. Finally, the ex-ante contract is renegotiation proof, because it coincides with the interim contract.

It is puzzling why ex-ante contracts that are signed at the time of marriage are not prevalent among all couples with children. The implementation of such contracts in earlier times suggests that the enforcement of ex-ante contracts is not the issue. Rather, in modern societies with free marriage, based in part on mutual attraction, the general sense is that emotional commitments are more important than legal agreements and thinking of contingencies and writing them down may "kill love". However, prenuptial contracts are often signed, at the time of marriage, by couples in which one (or both) of the partners bring into the marriage substantial property, which is more common on second marriages.

9 Conclusion

As the last two chapters illustrate, marriage markets with search frictions, in which the meeting technology displays increasing returns, may have multiple equilibria, because of the various search and contracting externalities. In chapter 10, we did not allow any contracting and, as a consequence, obtained the result that equilibria with higher turnover, that is, higher divorce and remarriage rates, provides all participants with a higher welfare. The reason is that an increase in the *aggregate* divorce rate, raises the prospects of remarriage, which makes it easier to replace bad marriages by better ones. In chapter 11, we allowed parents to transfer resources in the aftermath of divorce, based on the insight that, in the presence of children, marriage dissolution does not eliminate all ties between the partners because both parents continue to care about their child, which motivates post divorce transfers.[33] However, the impact of transfers on the marriage market and the welfare of children is quite complex, because the willingness of each parent to transfer to his\her ex-spouse depends on the transfers that potential mates for remarriages expect

[33] This is in sharp contrast to employment relationships that end in separation, in which case the ex-partners are no longer tied with each other.

from their ex-spouses. This contract externality can operate in different ways, depending on the type of contracts that are enforced by law. If only unconditional transfers are enforced, higher divorce and remarriage rates reduce the incentive to transfer money to the custodial mother, because a dollar transferred to her is less likely to reach the child than if she remarries. The consequence is that children may be worse off in high divorce equilibria. The outcome is completely reversed if the contracts environment is enriched and contingent contracts are also enforced. If the non custodial father promises the mother a payment that is contingent on her remaining single, then her bargaining position vis-a-vis her new husband is improved and the welfare of the child can be protected. Fathers have a stronger incentive to make such commitments when the remarriage rate is high, because then the payments to the custodial mothers are made relatively rarely, while the non custodial fathers are rewarded for their commitments more often. The outcome, in this case, is that equilibria with higher *aggregate* divorce (and remarriage) can be welfare enhancing. In particular, children who would suffer from the break of the marriage of their parents if it would happen in isolation, can gain from being in environment in which a higher proportion of marriages dissolve.

10 Appendix: Contingent contracts

The purpose of this appendix is to prove Proposition 4. We assume throughout a mandatory child support payment of c^* so that $s = c^* + \sigma$ and $\sigma \geq 0$. Hence, by (12), $u'_m(s) = 1 + \alpha$ and $q\prime(s) = \alpha$.

10.1 The choice of contract

The expected utility of the father is now

$$\begin{aligned} V_f &= (1-p)^2[w_h - (c^* + \sigma) + q(c^* + \sigma)] + (1-p)p[w_h - c^* + E(q|\sigma, \sigma')], \quad \text{(A1)} \\ &\quad + p(1-p)[E(a|\sigma', \sigma) + q(c^* + \sigma)], \\ &\quad + p^2[E(a|\sigma', \sigma) + E(q|\sigma, \sigma')]. \end{aligned}$$

From (42) we have, that in any household

$$\begin{aligned}
\frac{\partial E(a|\sigma,\sigma')}{\partial \sigma} &= \lambda u'_m(c^*+\sigma) = \lambda(1+\alpha) < 0, \qquad \text{(A2)}\\
\frac{\partial E(q|\sigma,\sigma')}{\partial \sigma} &= u'_m(c^*+\sigma) - \frac{\partial E(a|\sigma,\sigma')}{\partial \sigma} = (1+\alpha)(1-\lambda) > 0,\\
\frac{\partial E(a|\sigma,\sigma')}{\partial \sigma'} &= -(1-p)(1-\lambda(1+\alpha)) < 0,\\
\frac{\partial E(q|\sigma,\sigma')}{\partial \sigma'} &= -\frac{\partial E(a|\sigma,\sigma')}{\partial \sigma} > 0,
\end{aligned}$$

where λ is the Lagrange multiplier for the participation constraint of the wife. Therefore,

$$\begin{aligned}
\frac{\partial V_f}{\partial \sigma} &= (1-p)^2[-1+\alpha] + (1-p)p[(1+\alpha)(1-\lambda_m)] + \qquad \text{(A3)}\\
&\quad p(1-p)[-(1-p)(1-\lambda_f(1+\alpha))+\alpha)] + p^2[-(1-p)(1-\lambda_f(1+\alpha))\\
&\quad +(1+\alpha)(1-\lambda_m)],
\end{aligned}$$

where λ_m and λ_f denote the Lagrange multipliers if the mother or father remarry, respectively.

An interior solution for σ exists if $\frac{\partial V_f}{\partial \sigma} = 0$ and

$$\frac{\partial^2 V_f}{\partial \sigma^2} = (1+\alpha)p[-\frac{\partial \lambda_m}{\partial \sigma} + (1-p)\frac{\partial \lambda_f}{\partial \sigma}] < 0. \qquad \text{(A4)}$$

From the first order conditions to (13), if both the mother and the new wife of the father work part time then $\lambda_m = \lambda_f = \frac{w_l}{(1+\alpha)w_l-\gamma}$. Otherwise, $\lambda_j = \frac{1}{1+\alpha-g'(c)}$ for $j = m, f$. When the mother does not work, any increase in σ increases the consumption of the child and decreases λ_m. Similarly, when the father remarries and his wife does not work any increase in his commitment to his ex-wife raises the consumption of the step child and decreases λ_f. Hence, $\frac{\partial \lambda_m}{\partial \sigma} \leq 0$ and $\frac{\partial \lambda_f}{\partial \sigma} \leq 0$. But in a symmetric equilibrium all couples make the same choices and $\frac{\partial \lambda_m}{\partial \sigma} = \frac{\partial \lambda_f}{\partial \sigma}$, which would imply that $\frac{\partial^2 V_f}{\partial \sigma^2} \geq 0$. Therefore, there is *no* interior symmetric equilibrium, and the only symmetric equilibria are such that all couples must be at one of the boundaries, $\sigma = 0$ or $\sigma = w_h - c^* - (1-p)\sigma'$. Notice that the upper boundary is not determined by the budget constraint but by the requirement that the mother is just indifferent between remarriage and remaining single.

Suppose that all other couples set $\sigma' = \frac{w_h - c^*}{2-p}$. Then by setting

$$\sigma = w_h - c^* - (1-p)\sigma' = \frac{w_h - c^*}{2-p} \tag{A5}$$

the father can guarantee that the mother is just indifferent between marriage and remaining single. This is seen by noting that the mother participation constraint becomes

$$(1+\alpha)[w_l h_m + \frac{w_h - c^*}{2-p} + c^* - c] + \gamma(1-h_m) + g(c) \geq \gamma + g(c^*) + (1+\alpha)\frac{w_h - c^*}{2-p} \tag{A6}$$

But

$$\begin{aligned} & \underset{c,h_m}{Max}\{(1+\alpha)[w_l h_m + c^* - c] + \gamma(1-h_m) + g(c)\} \qquad \text{(A7)} \\ \leq\ & \underset{c,h_m}{Max}\{\gamma h_m + (1+\alpha)(c^* - c) + \gamma(1-h_m) + g(c)\} \\ =\ & \gamma + g(c^*). \end{aligned}$$

Therefore, (A6) must hold as equality.

It remains to show that $\sigma = \frac{w_h - c^*}{2-p}$ is indeed an optimal choice, given that others maintain $\sigma' = \frac{w_h - c^*}{2-p}$. For a marginally lower σ we have that $\frac{\partial V_f}{\partial \sigma}$ approaches ∞ because $\lambda_m = \frac{1}{1+\alpha - g'(c)}$ approaches $-\infty$ as c approaches c^*. For marginally higher σ we have that $\frac{\partial V_f}{\partial \sigma}$ approaches $-\infty$, because when the mother chooses not to marry the father suffers a discrete loss since he pays the mother σ with certainty. Thus, $\sigma = \frac{w_h - c^*}{2-p}$ is a *local* maximum. However, it need not be a global maximum. In particular, for a small p, it is always the case that $\frac{\partial V_f}{\partial \sigma} < 0$, because the father bears the costs with high probability and the benefits with a low probability, so that $\sigma = 0$ is also a *local* maximum. However, in contrast to the selection of $\sigma = \frac{w_h - c^*}{2-p}$, the selection of $\sigma = 0$ is a local maximum *only* if p is small. The difference arises because at low levels of σ, $c < c^*$ so that λ_m and λ_f are finite and $\frac{\partial V_f}{\partial \sigma}$ must change sign from negative to positive as p rises from zero to one.

We have noted in the text that if remarried mothers work part time then $\frac{\partial V_f}{\partial \sigma}$ is independent of σ'. However, if the mother does not work or works full time so that $\lambda_m = \frac{1}{1+\alpha - g'(c)}$, then an increase in σ' will raise c, λ_m becomes more negative and $\frac{\partial V_f}{\partial \sigma}$ rises. That is, σ and σ' are *strategic complements.*

The characterization in the text follows from the following observations: For any fixed σ', the global maximum is at $\sigma = 0$ if p is sufficiently small, say less than p_0, and at $\sigma = \frac{w_h - c^*}{2-p}$ if p is sufficiently large, say larger than p_1. Because of complementarity, one is more inclined to give if others do, and therefore p_1 must exceed p_0.

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Table 1: Impact of Change in Parameter on the Equilibrium Divorce (Remarriage) Rate and Life Time Utilities of Family Members

Part 1: Benchmark Parameters

Husband's wage	$w_h = 1$
Wife's wage	$w_l = 0.5$
Mother's productivity in child care	$\gamma = 1.75$
Father's productivity in child care	$\beta = 0$
Child's marginal utility from the adult good	$\alpha = 0.25$
Distribution of shocks	$\theta\tilde{}U[-d, d],\ d = 2$
Utility from child expenditures	$g(c) = 2.25 * (1 - \frac{1}{(1+c)^{2.25}})$
Child expenditure levels	$c^* = 0.5378,\ \hat{c} = 0.1203,\ g(c^*) = 1.4$
Proximity factor of father's utility from the child	$\delta = 0.75$
Discount factor	$\Delta = 0.625$
Fixed cost of separation without a child	$b_0 = 0$
Fixed cost of separation with a child	$b_1 = 0.35$
Father's second period utility	$\gamma + g(c^*) + (1 + \alpha)(w_h - c^*) = 3.72$
Mother's second period utility	$\gamma + g(c^*) + (1 + \alpha)(w_h - c^*) = 3.72$
Child's second period utility	$\gamma + g(c^*) + \alpha(w_h - c^*) = 3.26$

Part 2: Change in the Variability of Shock, d

	Equilibrium with children					Equilibrium without children			
	Divorce rate:	Utilities:	Husband	Wife	Child	Divorce rate:	Utilities:	Husband	Wife
$d = 1.5$	0.139	Life Time	3.345	3.340	2.784	0.250	Life Time	3.309	3.309
		Deviation	3.287	3.286			Deviation	3.329	3.333
		Second period	2.989	2.932			Second period	1.250	1.250
$d = 2$	0.214	Life Time	3.384	3.373	2.606	0.334	Life Time	3.389	3.389
		Deviation	3.351	3.352			Deviation	3.356	3.358
		Second period	2.925	2.844			Second period	1.334	1.334
$d = 2.2$	0.235	Life Time	3.403	3.390	2.556	0.353	Life Time	3.421	3.421
		Deviation	3.379	3.381			Deviation	3.373	3.374
		Second period	2.905	2.819			Second period	1.353	1.353
$d = 2.5$	0.262	Life Time	3.434	3.419	2.492	0.375	Life Time	3.470	3.470
		Deviation	3.423	3.426			Deviation	3.401	3.401
		Second period	2.879	2.788			Second period	1.375	1.375

Part 3: Change in the Fixed Cost of Separation with a Child, b1

	Equilibrium with children					Equilibrium without children			
	Divorce rate:	Utilities:	Husband	Wife	Child	Divorce rate:	Utilities:	Husband	Wife
$b_1 = 0.05$	0.274	Life Time	3.420	3.405	2.463	0.334	Life Time	3.389	3.389
		Deviation	3.357	3.360			Deviation	3.392	3.395
		Second period	2.866	2.775			Second period	1.334	1.334
$b_1 = 0.25$	0.234	Life Time	3.395	3.383	2.559	0.334	Life Time	3.389	3.389
		Deviation	3.353	3.354			Deviation	3.366	3.369
		Second period	2.906	2.820			Second period	1.334	1.334
$b_1 = 0.35$	0.214	Life Time	3.384	3.373	2.606	0.334	Life Time	3.389	3.389
		Deviation	3.351	3.352			Deviation	3.356	3.358
		Second period	2.925	2.844			Second period	1.334	1.334
$b_1 = 0.45$	0.193	Life Time	3.374	3.364	2.656	0.334	Life Time	3.389	3.389
		Deviation	3.349	3.349			Deviation	3.347	3.349
		Second period	2.944	2.868			Second period	1.334	1.334

Part 4: Change in the Proximity Factor, δ

	Equilibrium with children					Equilibrium without children			
	Divorce rate:	Utilities:	Husband	Wife	Child	Divorce rate:	Utilities:	Husband	Wife
$\delta = 0.5$	0.126	Life Time	3.347	3.349	2.454	0.334	Life Time	3.389	3.389
		Deviation	3.340	3.344			Deviation	3.324	3.331
		Second period	2.576	2.607			Second period	1.334	1.334
$\delta = 0.75$	0.214	Life Time	3.384	3.373	2.606	0.334	Life Time	3.389	3.389
		Deviation	3.351	3.352			Deviation	3.356	3.358
		Second period	2.925	2.844			Second period	1.334	1.334
$\delta = 0.85$	0.255	Life Time	3.408	3.377	2.620	0.334	Life Time	3.389	3.389
		Deviation	3.357	3.355			Deviation	3.385	3.369
		Second period	3.092	2.896			Second period	1.334	1.334
$\delta = 1$	0.321	Life Time	3.456	3.375	2.613	0.334	Life Time	3.389	3.389
		Deviation	3.367	3.361			Deviation	3.448	3.375
		Second period	3.357	2.953			Second period	1.334	1.334

Table 2: The Incentives to Deviate at Alternative Potential Equilibria

Equilibrium with children, $\sigma = \frac{w_h - c^*}{2-p}$

Divorce rate:	Life Time Utilities:	Husband	Wife	child
0.356	With a child, $\sigma = \frac{w_h - c^*}{2-p}$	3.485	3.485	3.216
	With a child, $\sigma = 0$	3.422	3.417	
	No child	3.369	3.343	

Equilibrium with children, $\sigma = 0$

Divorce rate:	Life Time Utilities:	Husband	Wife	child
0.344	With a child, $\sigma = 0$	3.475	3.410	2.811
	With a child, $\sigma = w_h - c^*$	3.477	3.540	
	No child	3.378	3.354	

Equilibrium without children

Divorce rate:	Life Time Utilities:	Husband	Wife
0.334	No child	3.389	3.389
	With a child, $\sigma = w_h - c^*$	3.453	3.540
	With a child, $\sigma = 0$	3.452	3.414

Parameters are the same as in the benchmark of Table 1, except that $\delta = 1$, instead of $\delta = 0.75$.

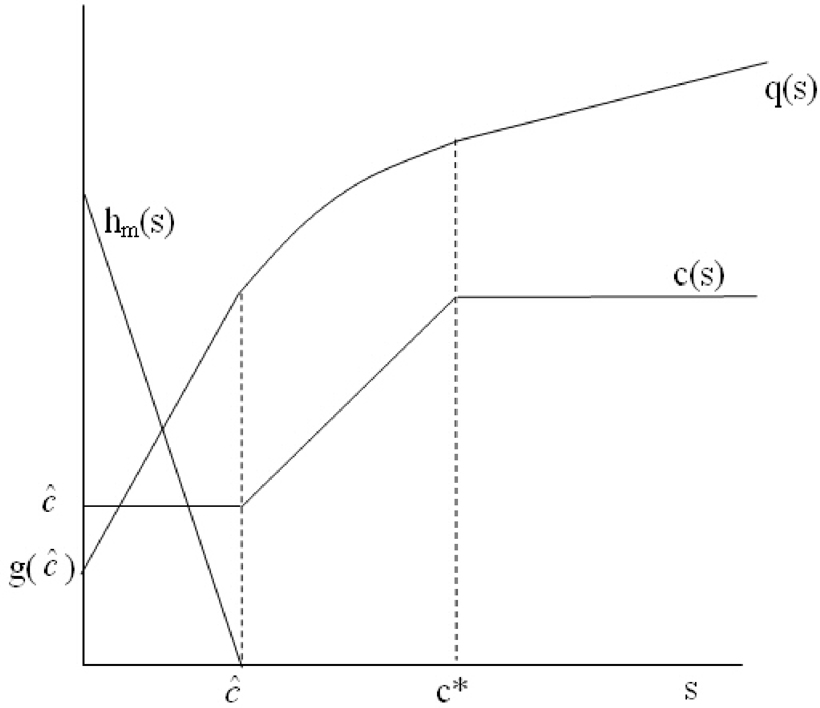

Figure 1: Effect of Transfer on Mother's Work, Child's Consumption and Child's Utility

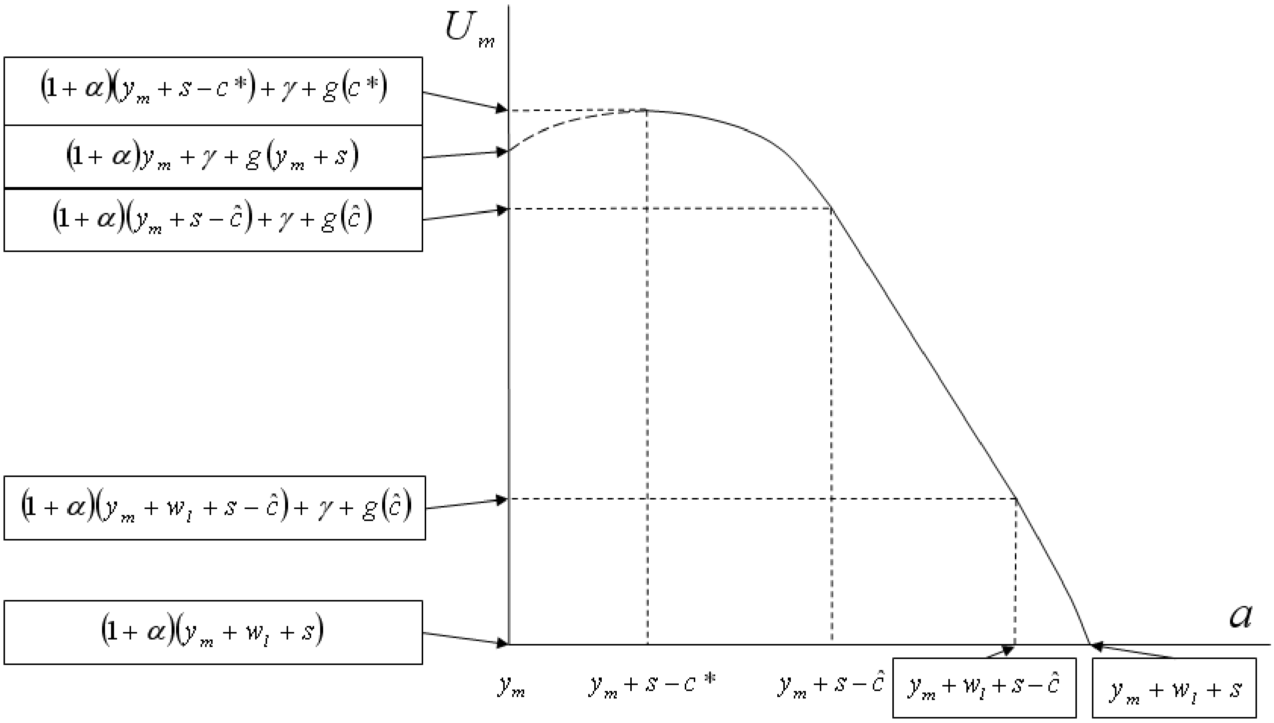

Figure 2: The Pareto Utility Frontier

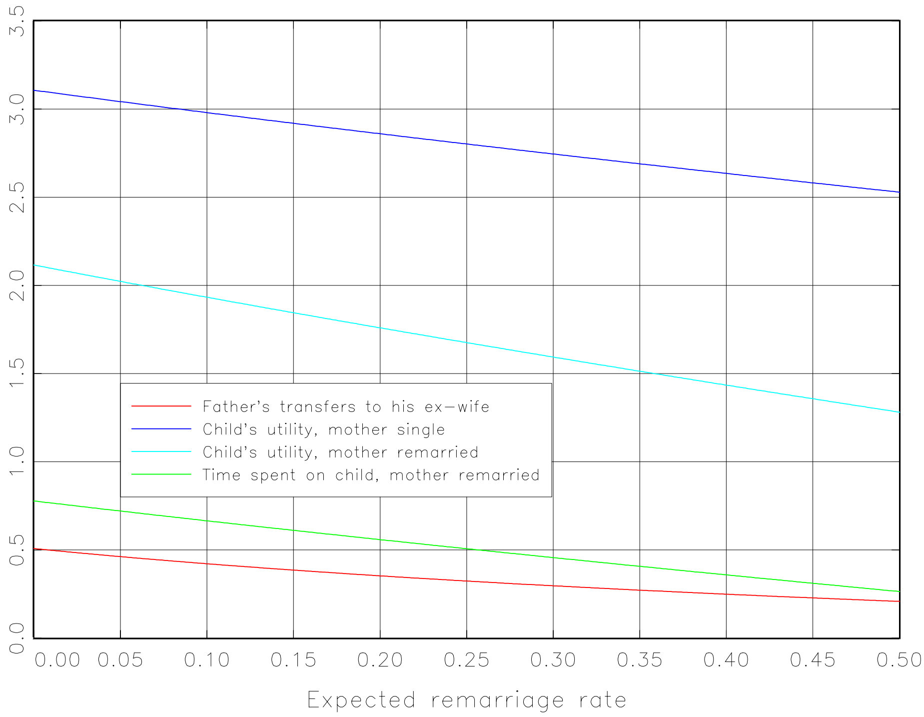

Figure 3: Optimal Transfers and Child's Utility if the Mother Remmaries or Remains Single

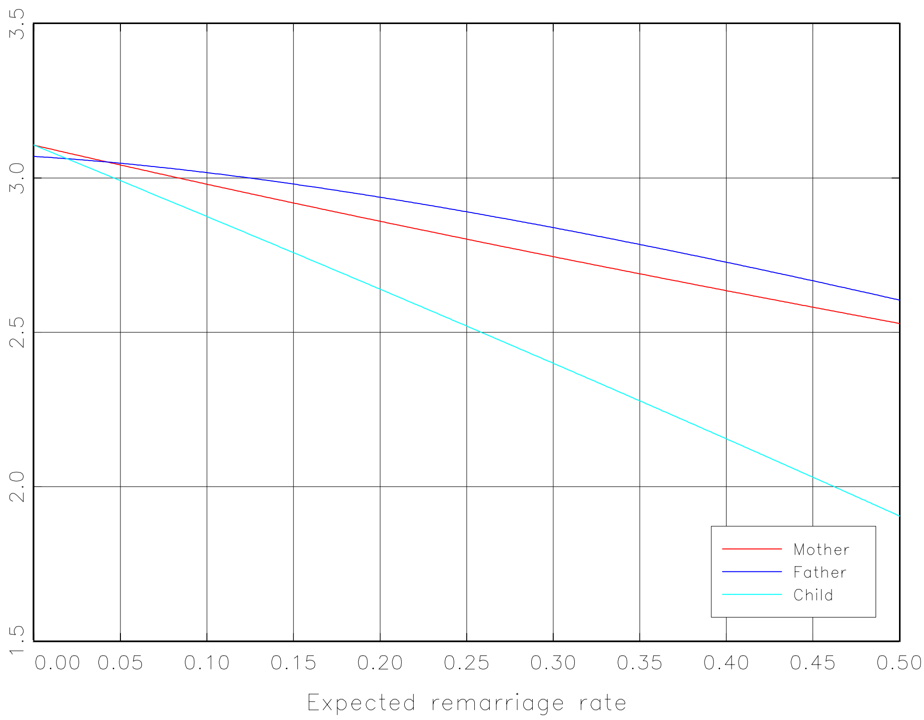

Figure 4: Expected Utilities of Father, Mother and Child at the Time of Divorce

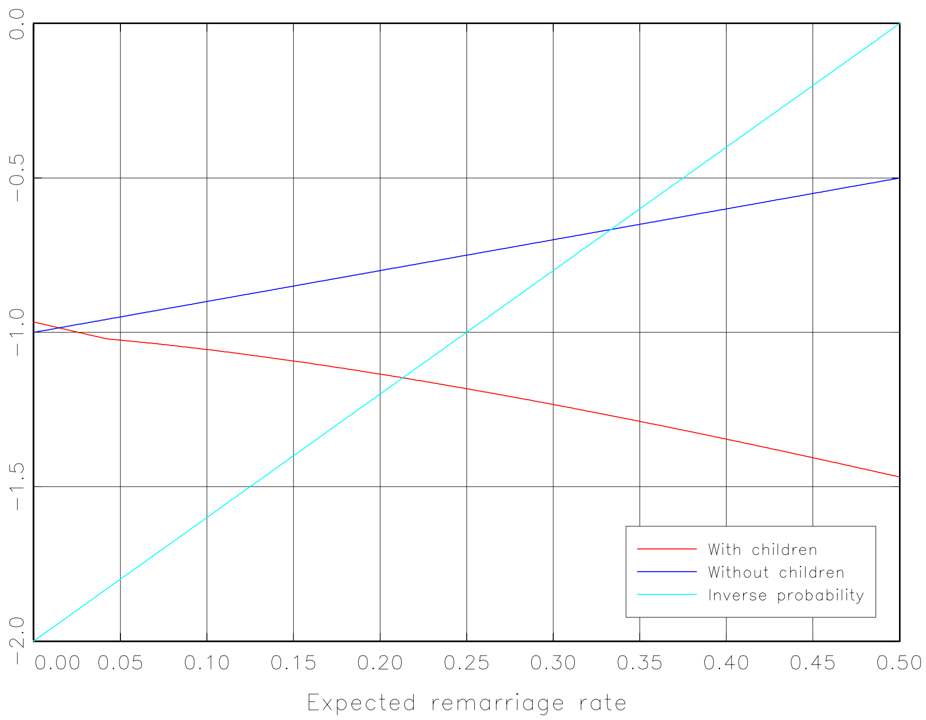

Figure 5: Potential Marriage Market Equilibria, with and without Children

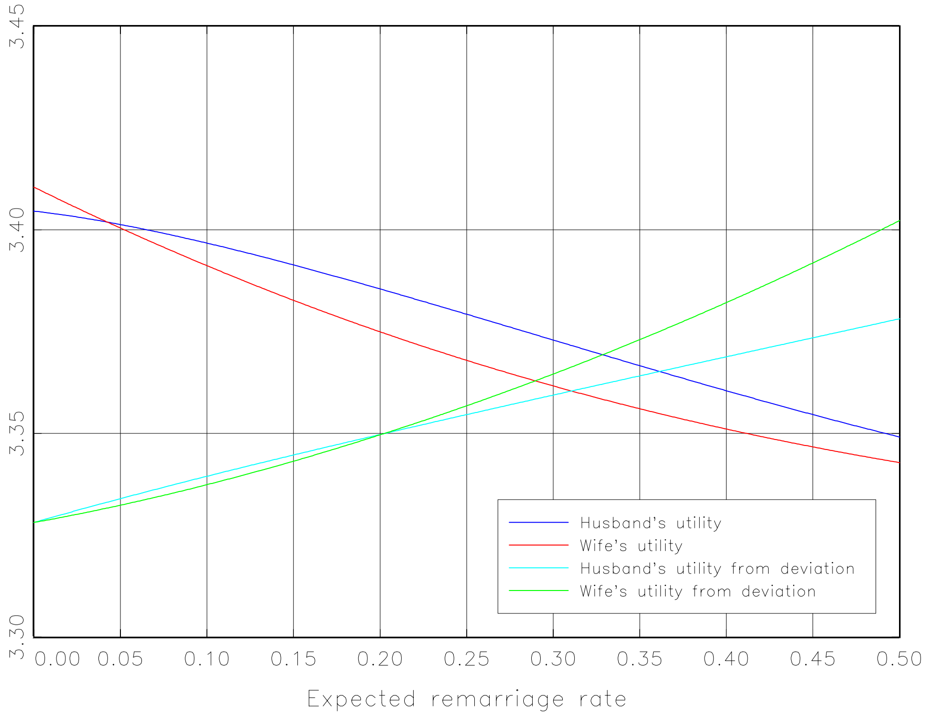

Figure 6: The Impact of Deviation to not having a Child on the Expected Life Time Utility when all other Couples have a Child

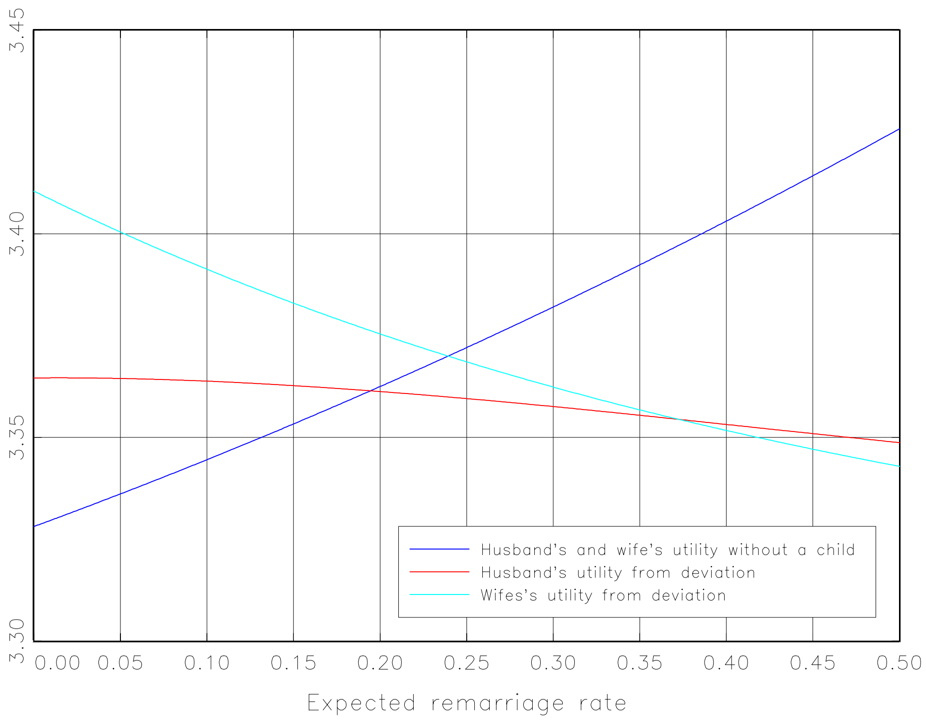

Figure 7: The Impact of Deviation to Having a Child on the Expected Life Time Utility when all other Couples Have no Child

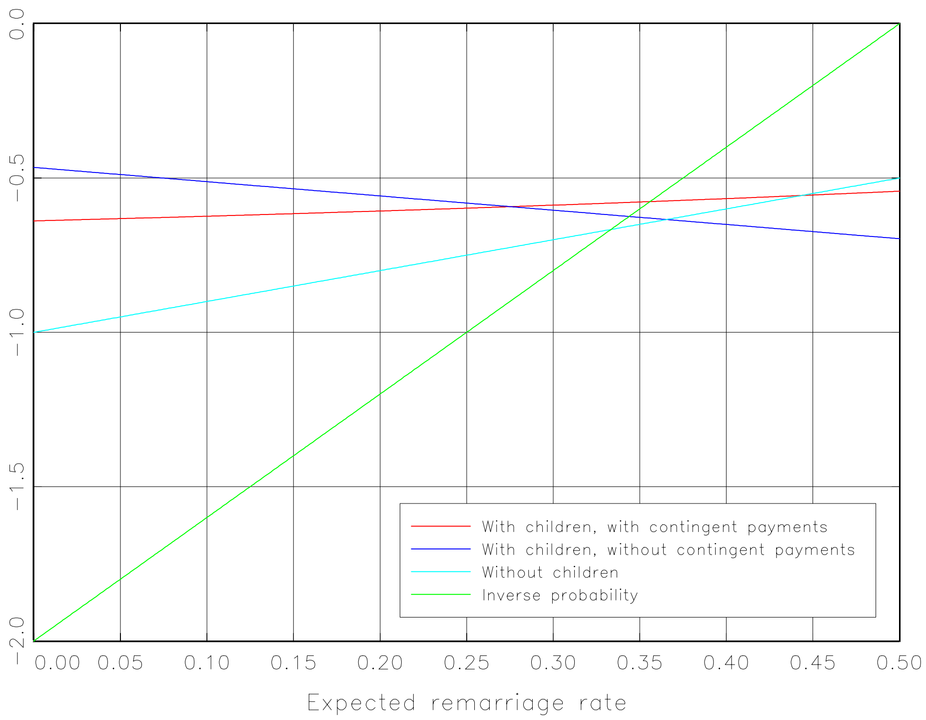

Figure 8: with Children under Different Payment Schemes and without Children

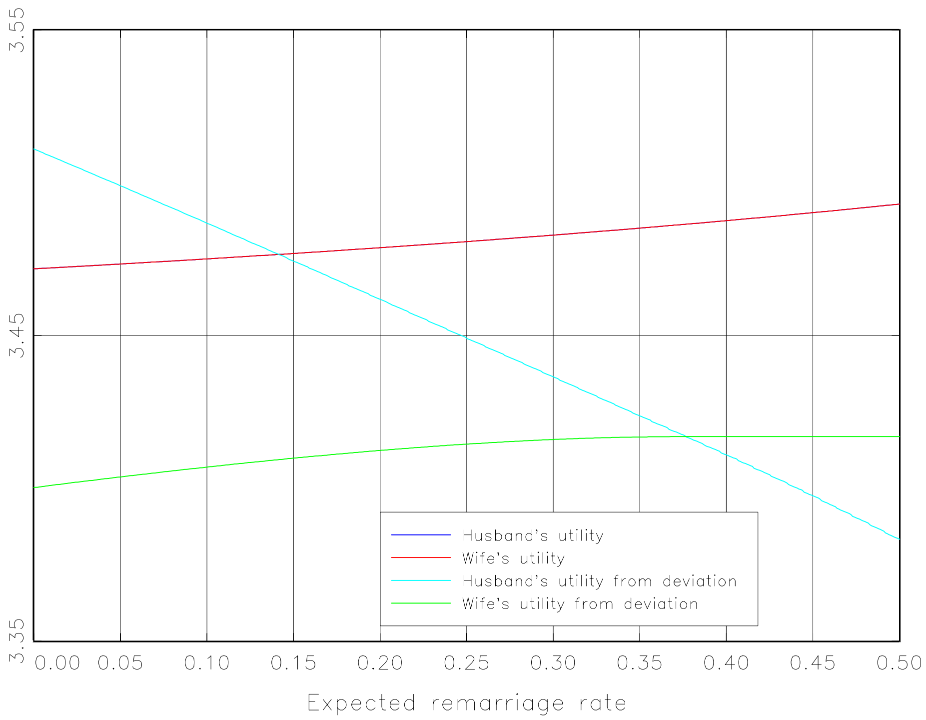

Figure 9: The Impact of Deviation to a Fixed Payment when all other Fathers give a Contingent Payment

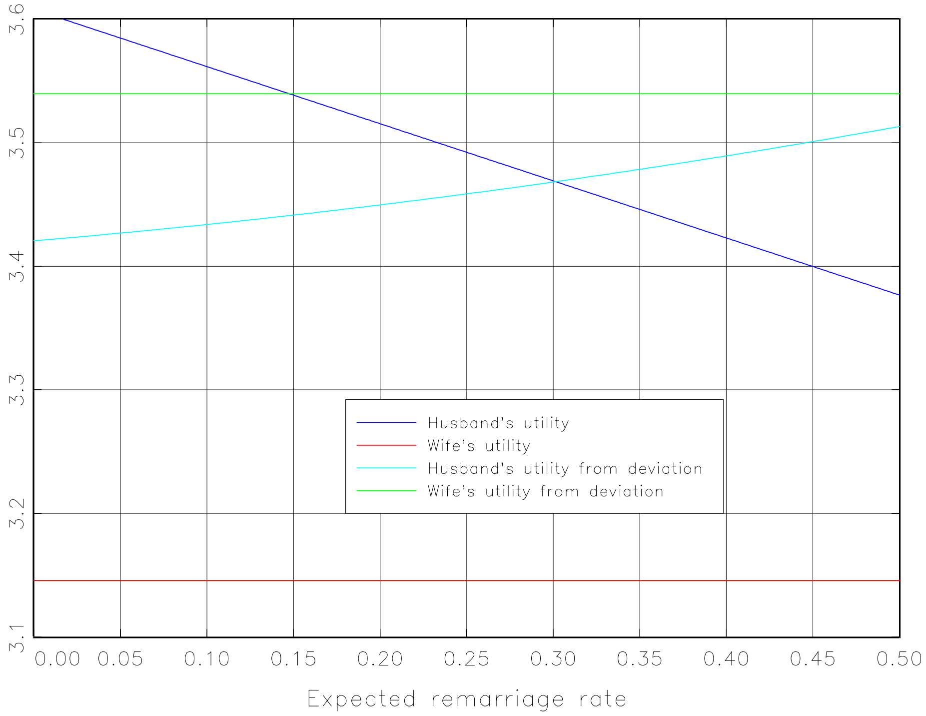

Figure 10: The Impact of Deviation to a Contingent Payment when all other Fathers give no Contingent Payment