The Economics of the Family

Chapter 10: An equilibrium model of marriage, fertility and divorce.

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1 Introduction

This chapter provides a simple model of the marriage market that includes fertility, divorce and remarriage and addresses some of the basic issues associated with the higher turnover in the marriage market. For this purpose, we introduce search frictions, heterogeneity and unexpected shocks to match quality. The model is simple enough to identify the welfare implication of increasing turnover. The main result is that the prospects of remarriage generate multiple equilibria due to a positive feedback whereby a higher *aggregate* divorce rate facilitates remarriage, which, in turn, raises the incentives of each couple to divorce. Moreover, when multiple equilibria exist, an equilibrium with higher divorce and remarriage rates generates higher expected welfare for all participants in the marriage market. This is a direct outcome of the positive search externalities that are embedded in the model. The main lesson is that a high aggregate divorce rate can be beneficial because it facilitates the recovery from negative shocks to match quality, allowing couples to replace bad marriages by better ones. Related papers are Aiyagari et al (2000), Brien et al (2006) and Chiappori and Weiss (2006).

2 A model of the Marriage Market

Consider a society in which there is an equal number of men and women and all individuals are *ex ante* identical and live for two periods. Alone, each person consumes their own income Y. If married, the partners share consumption and each consumes $2Y$. In addition, marriage entails a non monetary return θ that both partners enjoy. This "quality of match" is randomly distributed and different couples draw different values of θ at the time of marriage. However, the future quality of match is uncertain.

Meetings are random. At the beginning of each period, each person randomly meets a person of the opposite sex of his/her age group in a given cohort. We assume that marriage binds for at least one period. At the end of the first period divorce can occur but remarriage is possible only with unattached individuals who never married before or have divorced. In the first period, one meets an eligible partner with certainty. The probability of each individual to meet a single person of the opposite sex in their second period of life equals the proportion in the population of unattached individuals of the opposite sex, divorced or never married. This assumption is crucial for our analysis and implies an "increasing returns meeting technology" whereby, the more singles are around, the easier it is for each single person to find a match. The logic behind this assumption is that meetings often occur at work or school and are "wasted" if the person you meet is already married.

Marriage also provides the partners with the option to produce (exactly) two children (there is no out of wedlock birth). The production of children entails a cost to the parents in the first period, c, and a benefit which both parents enjoy in the subsequent period. The utility of a child is independent of household income but depends on the proximity to their natural parents. It equals q^* if the children live with both natural parents and to q^0 if they live with only one of the parents or in a step family; we assume $q^* > c > q^0$. Both parents treat the utility of the child as a public good and it enters additively into their preferences. Partners with children find divorce more costly, because the welfare of the children is higher if children are raised with their natural parents.

Upon meeting, the quality of match θ is revealed and the matched partners decide whether to marry or not. If they choose to marry, they can further decide whether they wish to have children. Because of the delayed benefits, the production of children is a relevant option only for partners in the first period of their life. During each period, there is a shock ε to the quality of match, which is revealed at the end of the period. Having observed the shock at the end of the first period, the partners decide whether to divorce or not. The random variables θ and ε are assumed to be independent across couples. In particular, for each remarried person the values of θ in the first and second marriage are independent. We denote the distributions of θ and ε by $G(\theta)$ and $F(\varepsilon)$ with densities $g(\theta)$ and $f(\varepsilon)$ respectively. We assume that these distributions have zero mean and are symmetric around their mean.

We assume that *all* goods in the household, consumption, match quality and children are public and both partners enjoy them equally. Hence, by assumption, men and women benefit equally from marriage or divorce. The assumptions of public goods and equal numbers of men and women generate perfect symmetry between genders that allows us to set aside, in this introductory chapter, conflict and bargaining between the partners.

2.1 Individual Choices

2.1.1 The last stage: the remarriage decision

We first analyze the marriage, fertility and divorce decisions of individuals who take the conditions in the marriage market as given. We proceed from the last available choice, marriage at the second period and work backwards. Two unattached individuals who meet at the beginning of the second period will marry if and only if their drawn θ satisfies

$$\theta \geq -Y. \tag{1}$$

That is, conditioned on meeting, marriage occurs whenever the sum of monetary and non monetary gains from marriage is positive. This simple marriage rule holds because each partner gains $Y + \theta$ from the marriage and, if one of the partners has a child then, by assumption, the benefits from that child are the same whether the child lives with a single parent or in a step family. There are thus no costs associated with remarriage.

We denote the probability of remarriage conditioned on a meeting in the second period by

$$\gamma = 1 - G(-Y), \tag{2}$$

and the expected quality of match conditioned on marriage in the second period by

$$\beta = E(\theta/\theta \geq -Y). \tag{3}$$

Note that although the expected value of θ is zero, the expectation conditioned on remarriage, β, is positive, reflecting the option not to marry if the drawn θ is low.

The probability of meeting an unattached person of the opposite sex at the beginning of the second period is denoted by u. The probability that an unattached person will meet an eligible single person whom he or she will choose to marry is $p = u\gamma$. Note that men and women face the same remarriage probability p, because we assume perfect symmetry between men and women. The expected utility of an unattached person, conditioned on having children is, therefore,

$$V_{2,n} = p(2Y + \beta) + (1 - p)Y + nq^0, \tag{4}$$

where $n = 1$ if children are present and $n = 0$, otherwise.

2.1.2 The intermediate stage: the divorce decision

A married person will choose to divorce if and only if the θ drawn at the beginning of the first period and the ε drawn at the end of the first period are such that

$$2Y + \theta + \varepsilon + nq^* < V_{2,n} \ . \tag{5}$$

This can be rewritten as $\varepsilon + \theta < h_n$, where

$$h_n \equiv -Y + p(Y + \beta) - n(q^* - q^0) \tag{6}$$

is the expected *net* gain from divorce.

The probability of divorce for a married couple with initial quality of match θ is given by $F(h_n - \theta)$. This probability depends on both individual circumstances, represented by θ and n, and on market conditions, represented by $p = u\gamma$. Specifically, the probability of divorce rises with the number of singles who are eligible for remarriage, u, and is lower among couples who have children or are well matched. That is, surprises such as shocks to the quality of the match, represented here by ε, are less disruptive if the current marriage is good, the cost of separation is high or remarriage is unlikely. The influence of remarriage prospects on the decision to divorce creates a link between the aggregate divorce rate and the individual decision to divorce. If many choose to divorce then the number of singles, u, is high, which would raise the probability of remarriage, p, and the net gain from divorce, h_n, and thus the probability of divorce.

2.1.3 The first stage: the marriage and fertility decisions

Two unmarried individuals who meet at the beginning of the first period and observe their drawn quality of match, θ, must decide whether to marry and whether to have children upon marriage. Their expected life time utility upon marriage, conditioned on n, is given by

$$W_{1,n}(\theta) = 2Y + \theta - nc + \int_{h_n-\theta}^{\infty} (2Y + nq^* + \theta + \varepsilon) f(\varepsilon) d\varepsilon + F(h_n - \theta) V_{2,n}. \tag{7}$$

17Differentiating $W_{1,n}(\theta)$ with respects to θ yields (details are given in the appendix):

$$\frac{\partial W_{1,n}}{\partial \theta} = 2 - F(h_n - \theta). \tag{8}$$

That expected utility is increasing in the quality of match is intuitively clear, because a couple with high θ can always replicate the divorce and remarriage decisions of a couple with low θ. The value of marrying without children, $W_{1,0}(\theta)$, and the value of marrying with children, $W_{1,1}(\theta)$, are continuous, increasing and convex functions of θ. A person who chooses not to marry at the beginning of the first period has expected lifetime utility given by:

$$V_1 = Y + V_{2,0}. \tag{9}$$

Thus, a first marriage will occur if and only if:

$$\max\left(W_{1,0}(\theta), W_{1,1}(\theta)\right) \geq V_1. \tag{10}$$

This maximum function inherits the properties of the individual $W_{1,n}$ functions; that is, it is continuous, increasing and convex in θ. Because the values of marriage with and without children both rise with θ, the decision whether to marry has the form of a stopping rule. That is, couples will marry if and only if $\theta \geq \theta_m$, where θ_m is determined by the condition that 10 holds as an equality.[1] Because the maximum is an increasing function of θ, θ_m is unique. See Figure 1.

The decision whether to have children can also be represented as a stopping rule, because 8 implies that $\frac{\partial W_{1,1}}{\partial \theta} > \frac{\partial W_{1,0}}{\partial \theta} > 0$ for all θ. That is, the quality of the first match is more important if the couple has children and are thus less likely to divorce (recall that children impede divorce, $h_1 < h_0$). Therefore, there is a unique value of θ, θ_c, that solves $W_{1,1}(\theta) = W_{1,0}(\theta)$; see Figure 1. Thus a very simple rule arises: those couples for whom $\theta < \theta_m$ will not marry. Those couples for whom $\theta \geq \theta_m$ will marry but they may or may not have children, depending on the costs and benefits from having children. If the cost of having children is relatively high then $\theta_c > \theta_m$ and only those married couples for whom $\theta > \theta_c$ will have children while couples for whom $\theta_c > \theta \geq \theta_m$ will choose to marry but have no children. This is the case illustrated in figure 1. If the cost of having children is relatively low then $\theta_c < \theta_m$ and all people that marry will have children. In terms of Figure 1 this is equivalent to moving $W_{1,1}(\theta)$ up until the two curves intersect at a value of θ below θ_m.

An interesting testable implication of this model is that individuals are less selective in their first marriage decision than in their remarriage decision. That is, $\theta_m \leq -Y$ (see

[1] We are here implicitly assuming that the support of θ is wide enough so that some people do not marry.

the appendix). Conditional on θ, marriage in the first period is always more attractive because of the *option* to sample ε. There is no downside risk because one can divorce if ε is low. Such an option is not available in the second period. The option to have children makes this preference for early marriage even stronger.

Another testable result is that individuals become more selective in their first marriage decisions if more eligible singles are available for remarriage in the second period. That is, θ_m is increasing in the remarriage probability, p. This follows directly from the observation that the probability of remarriage has a stronger effect on someone who chose not to marry and is thus sure to be single in the second period than on someone who married and will be single next period with probability less than one. That is,

$$\frac{\partial W_{1,n}}{\partial p} = (Y + \beta)F(h_n - \theta) < Y + \beta = \frac{\partial V_1}{\partial p}. \tag{11}$$

It is also the case that the critical value for having children, θ_c, rises with the probability of remarriage, p, implying that a couple will be less inclined to have children when p is higher. This follows because childless couples are more likely to divorce and therefore the positive impact of p on couples without children is stronger, $\frac{\partial W_{1,0}}{\partial p} > \frac{\partial W_{1,1}}{\partial p}$; see the appendix.

2.1.4 Summary

We have identified two basic forces that guide marriage, divorce, and fertility choices; individual circumstances, represented here by θ and ε and market forces represented here by p. Couples who drew a good match quality upon meeting are more willing to marry and to invest in children because they expect the marriage to be more stable. High turnover in the marriage market has the opposite effect; it discourages marriage and investment in children, because of the higher risk of divorce. These two forces interact and reinforce each other. If individuals expect high turnover, they invest less in children and are therefore more likely to divorce, which raises turnover. High turnover can raise the probability of divorce even in the absence of children because partners are more willing to break a marriage when the prospects for remarriage are good.

2.2 Aggregation

We can now aggregate over couples with different realizations of θ and define the aggregate rate of divorce (per number of individuals in the cohort) assuming that the cost of children is large enough so that $\theta_c > \theta_m$.

$$d = \int_{\theta_m}^{\theta_c} F(h_0 - \theta) g(\theta) d\theta + \int_{\theta_c}^{\infty} F(h_1 - \theta) g(\theta) d\theta. \tag{12}$$

Given the value of p that individuals expect, the implied proportion of singles at the beginning of period 2 is:

$$u = U(\theta_m(p), \theta_c(p)) \equiv G(\theta_m) + d \tag{13}$$

and the aggregate number of remarriages (per number of individuals in the cohort) is $p = \gamma u$.

Our results on individual behavior imply that $U(.,.)$ is increasing in its two arguments. Specifically, from equations 12 and 13 and the fact that children raise the cost of divorce, $h_0 > h_1$, we obtain:

$$\begin{aligned} \frac{\partial U}{\partial \theta_m} &= (1 - F(h_0 - \theta_m)) g(\theta_m) > 0, \\ \frac{\partial U}{\partial \theta_c} &= [F(h_0 - \theta_c) - F(h_1 - \theta_c)] g(\theta_c) > 0. \end{aligned} \tag{14}$$

Having shown that both $\theta_m(p)$ and $\theta_c(p)$ are increasing in the remarriage probability, p, we conclude that $U(\theta_m(p), \theta_c(p))$ is also increasing in p.

2.3 Equilibrium

Equilibrium is defined by the condition that the value of p that individuals expect is the same as the aggregate number of singles implied by the expectation. That is,

$$p = U(\theta_m(p), \theta_c(p)). \tag{15}$$

The function $U(.,.)$ is a non decreasing function from $[0,1]$ to $[0,1]$. Therefore, by the Tarski fixed point theorem (see Mas-Colell *et al* (1995), section MI), there is at least one equilibrium point in the interval $[0,1]$ at which expectations are realized.

We may narrow down the range of possible equilibria, based on some *a priori* information. Because of the advantages of joint consumption and the zero mean and symmetry assumptions on $G(\theta)$ and $F(\varepsilon)$, more than half of the population will choose to marry, and those who subsequently received a sufficiently favorable shock to the quality of match will remain married even if the probability of finding a new mate is 1, implying that $p < 1$ in equilibrium. If there is not much heterogeneity in θ and the support of the shock ε is small, everyone will marry and no one will divorce so that $p = 0$ in equilibrium. However, with sufficiently large variability in θ and ε, an equilibrium p will be positive, because even in the absence of remarriage prospects, couples who draw a sufficiently low quality of match will not marry, and married couples who suffered a large negative shock will divorce, so that $U(\theta_m(0), \theta_c(0)) > 0$.

Because of the positive feedback, whereby an increase in the expected number of singles induces more people to become single, there may be multiple equilibria. Having assumed that all individuals are *ex ante* identical, we can rank the different equilibria based on their common expected value of life time utility:

$$W_1 = E\max(W_{1,1}(\theta), W_{1,0}(\theta), V_1). \tag{16}$$

The expectation is taken at the beginning of the first period prior to any meeting, when the quality of prospective matches is yet unknown. An equilibrium with a higher number of unattached individuals at the beginning of the second period will generally have less marriages, more divorces and fewer couples with children. Despite these apparently negative features, equilibria with higher p are in fact Pareto *superior*, because of the better option for couples who suffered a bad shock to their first marriage to recover by forming a new marriage. To see this, note that by 11, $\frac{\partial W_{1,n}}{\partial p}$ and $\frac{\partial V_1}{\partial p}$ are positive, implying that an increase in p causes an increase in the expected welfare of all members of society, irrespective of the value of θ that they draw. In other words, the search frictions, represented here by random meetings with members of the opposite sex, irrespective of whether or not they are already attached, imply that those who choose to divorce or remain single exert a positive externality on other members of society who find it easier to find a mate for remarriage. This externality dominates the welfare comparisons because all other factors, such as the damage to children, are internalized by the partners. The presence of children implies that married couples are more reluctant to divorce, which yields a lower equilibrium value for p. However, it is still true that all couples, including couples with children, will be better off in an equilibrium with a higher p if multiple equilibria exist.

3 An Example

We now introduce a simple example with multiple equilibria and discuss their properties. Assume that ε takes only two values, $-a$ and $+a$ with equal probability, while θ is distributed uniformly on $[-b, b]$. For this example, we assume that $2a > (q^* - q^0)$; that is, the variance of the match quality shock is large relative to the loss for children from divorce, so that even couples with children may divorce if the revised quality of their match is low enough.

The expected utility profile if marriage takes place, conditional on having children or not $(n = 0, 1)$, is:

$$W_{1,n}(\theta) = \begin{cases} 3Y + \theta + p(Y + \beta) + n(q^0 - c) & if \quad -b \leq \theta < h_n - a \\ \frac{7}{2}Y + \frac{3}{2}\theta + \frac{1}{2}p(Y + \beta) + \frac{1}{2}a + n(\frac{1}{2}q^0 + \frac{1}{2}q^* - c) & if \quad h_n - a \leq \theta \leq h_n + a \\ 4Y + 2\theta + n(q^* - c) & if \quad h_n + a < \theta \leq b \end{cases} . \tag{17}$$

For a given n and conditional on marriage, couples who draw θ such that $\theta + a < h_n$ will divorce for sure at the end of the first period.[2] Couples who draw θ such that $\theta - a > h_n$ will stay married for sure (if they marry). Couples who draw θ in the intermediate range $h_n - a \leq \theta \leq h_n + a$ will divorce if the shock is negative and remain married otherwise.

[2] Marriage followed by certain divorce can occur if the gains from joint consumption are sufficiently large to offset the low quality of the current match $(Y + \theta > 0)$.

Using equations 4 and 9, the value of not marrying in the first period is given by:

$$V_1 = 2Y + p(Y + \beta) \tag{18}$$

which is independent of θ.

We now wish to identify the points θ_m and θ_c that trigger marriage and having children, respectively. For this purpose, it is useful to inspect Figure 2, in which we plot $W_{1,0}(\theta)$, $W_{1,1}(\theta)$ and V_1.[3] Note that the kinks in $W_{1,0}(\theta)$ always appear at higher values of θ than the kinks in $W_{1,1}(\theta)$. This happens because the expected gains from divorce are higher for couples without children, $h_0 - h_1 = q^* - q^0 > 0$. It can be seen that an intersection of the two curves can occur only in the intervals $[h_1 - a, h_0 - a]$ or $[h_1 + a, h_0 + a]$. Moreover, it can be verified that if the costs from having children are relatively high, that is, $q^* > c > \frac{q^*+q^0}{2}$, then the only possible intersection is in the region $[h_1 + a, h_0 + a]$; see the appendix for a proof.[4] We obtain θ_c by equating $W_{1,0}(\theta)$ for the intermediate region $(h_0 - a < \theta < h_0 + a)$ with $W_{1,1}(\theta)$ for $\theta > h_1 + a$. This gives:

$$\theta_c = p(Y + \beta) - Y + a - 2(q^* - c), \tag{19}$$

Using this expression that determines θ_c, we can now determine θ_m. Referring again to Figure 2, we see that $\max(W_{1,0}(\theta), W_{1,1}(\theta))$ is represented by the upper *envelope* of the $W_{1,0}(\theta)$ and $W_{1,1}(\theta)$ profiles. We thus have to consider three segments of this envelope. In the first case (with low V_1), V_1 intersects the envelope at a value of θ below $h_0 - a$, where couples would be indifferent between singlehood and a marriage without children followed by a certain divorce. In the second segment the intersection occurs at $\theta \in [h_0 - a, \theta_c]$,where couples would be indifferent between singlehood and a marriage without children followed by divorce if a negative shock occurs (this is the case illustrated in Figure 2). In the third case, (high V_1) the intersection is above θ_c,where couples would be indifferent between singlehood and a marriage with children that remains intact with

[3] In this figure we have $h_1 + a > h_0 - a$. This follows from the assumption that $2a > (q^* - q^0)$.

[4] The interested readers may try the case with low costs of children, see appendix.

certainty. In the appendix we show that:

$$\theta_m = \begin{cases} -Y & \text{if} \quad p(Y+\beta) > a \\ \frac{1}{3}(p(Y+\beta)-a)-Y & \text{if} \quad a \geq p(Y+\beta) \geq 3(q^*-c)-2a \\ \frac{1}{2}p(Y+\beta)-\frac{1}{2}(q^*-c)-Y & \text{if} \quad p(Y+\beta) < 3(q^*-c)-2a \end{cases}. \tag{20}$$

Note that the assumptions $2a > (q^* - q^0)$ and $c > \frac{q^*+q^0}{2}$ ensure that interval $[3(q^* - c) - 2a, a]$ is non-empty.

From equations (19) and (20) we see that both θ_m and θ_c rise with the expected remarriage rate, p. That is, the likelihood of marrying and having children decline with p. This happens because matched partners anticipate that they are more likely to divorce if the prospect of remarriage rises. Both θ_m and θ_c decline with income, implying that the likelihood of marrying and having children rise with income. This happens in our model because of the complementarity between the incomes of the spouses that is induced by joint consumption of public goods. A dollar increase in Y raises the consumption of *each* married person by 2 dollars, while their consumption as a single will rise by only one dollar.

The proportion of singles at the beginning of the second period that is associated with a given p consists of those who did not marry in the beginning of the first period, $G(\theta_m(p))$ and the divorcees at the end of the first period among the married. These divorcees constitute of all the married for whom $\theta_m < h_0 - a$, half of the married for whom $h_0 - a \leq \theta_m \leq \theta_c$ and none of the married for whom $\theta_m > \theta_c$. Therefore, equation (13) for the proportion of singles at the beginning of period 2 can be written as

$$U(\theta_m(p), \theta_c(p)) = \begin{cases} \frac{1}{2}\left(G\left(h_0(p)-a\right)+G\left(\theta_c(p)\right)\right) & \text{if} \quad p(Y+\beta) > a \\ \frac{1}{2}\left(G\left(\theta_m(p)\right)+G\left(\theta_c(p)\right)\right) & \text{if} \quad a \geq p(Y+\beta) \geq 3(q^*-c)-2a \\ G\left(\theta_m(p)\right) & \text{if} \quad p(Y+\beta) < 3(q^*-c)-2a \end{cases}. \tag{21}$$

Because in this particular example, the reservation rules for marriage and for having children are linear functions of p we obtain under the assumption that $G(.)$ is uniform that $U(\theta_m(p), \theta_c(p))$ is also a piecewise linear function of p. Consequently multiple equilibria can arise. Within the confines of our example, multiple equilibria occur only if there is not too much heterogeneity in the quality of match. We therefore choose a relatively small b and obtain Figure 3. As seen in this figure, there are three equilibria at $p = 0$, at $p = 0.25$ and at $p = 0.5$. Details of these three equilibria are presented in Table 1. In all three equilibria, everyone marries whomever they meet (this holds in both periods[5]), but the higher is the equilibrium level of p, the lower is the proportion of families that choose to have children and the higher is the proportion that divorces. At the low equilibrium, where everyone expects a remarriage rate of $p = 0$, all couples have children and no one divorces. This implies that there will be no singles in the second period, which justifies the expectations. At the equilibrium in which everyone expects a remarriage rate of $p = 0.25$, half of the couples have children and, of those who do not have children, half divorce upon the occurrence of a bad shock. This implies that at the beginning of the second period, a quarter of the population will be single, which justifies the expected remarriage rate. At the equilibrium with $p = 0.5$, no couple has children and half of them divorce upon the realization of a bad shock so, in this case too, expectations are realized. Thus, all three equilibria share the basic property that expectations are fulfilled. However, the intermediate equilibrium at $p = 0.25$ is not stable with respect to an arbitrary change in expectations. That is, if the expected remarriage rate, p, rises (declines) slightly then the aggregate number of singles $U(\theta_m(p), \theta_c(p))$ rises (declines) too.[6]

For these examples, one can easily calculate the equilibrium value of *ex ante* welfare, W_1 (see equation (16)). If $p = 0.5$, $W_{1,0}(\theta)$ is the highest for all θ, implying that all couples marry, have no children and divorce with probability 0.5, so that

$$W_1 = EW_{1,0}(\theta) = \frac{7}{2}Y + \frac{1}{4}Y + \frac{1}{2}a = 4\frac{5}{24}. \tag{22}$$

If $p = 0$, $W_{1,1}(\theta)$ is the highest for all θ, implying that all couples marry, have children and do not divorce, so that

$$W_1 = EW_{1,1}(\theta) = 4Y + (q^* - c) = 4\frac{1}{12}. \tag{23}$$

[5] In the second period, this implies that $\gamma = 1$ and $\beta = 0$.

[6] If b goes to zero and all matches are *ex ante* identical, the middle section disappears and the equilibrium function becomes a step function yielding only two stable equilibria.

The calculation of welfare is a bit more complex if $p = 0.25$. In this case, the maximum is given by $W_0(\theta)$ if $\theta \leq 0$ and by $W_1(\theta)$ if $\theta \geq 0$. Thus,

$$W_1 = \frac{1}{2}(\frac{7}{2}Y + \frac{1}{4}Y + \frac{1}{2}a + \frac{3}{2}E(\theta/\theta \leq 0)) + \frac{1}{2}(4Y + (q^* - c) + 2E(\theta/\theta \geq 0)) = 4\frac{1}{6}. \quad (24)$$

These calculations illustrate that ex-ante welfare rises as we move to equilibrium points with higher p, reflecting the positive externality associated with an increase in the aggregate number of singles.

4 Income uncertainty and ex-post heterogeneity

The simple model assumed perfect symmetry among spouses and that all individuals have the same incomes which remain fixed over time. We now allow income to change over time, which creates income heterogeneity ex-post. As before, all men and women have the same income, Y, in the first period of their life. However, with probability λ income in the second period rises to Y^h and with probability $1-\lambda$ it declines to Y^l. To maintain ex-ante symmetry, we assume that the incomes of men and women follow this same process. To simplify, we shall assume now that the quality of the match, θ, is revealed only at the end of each period. The realized value of θ at the end of the first period can trigger divorce, while the realized value of θ at the end of the second period has no behavioral consequences in our two period model. Since there are gains from marriage, and the commitment is only for one period, everyone marries in the first period. However, in this case, changes in incomes as well as changes in the quality of match can trigger divorce. We continue to assume risk neutrality and joint consumption.

The main difference from the previous model is that at the beginning of the second period there will be *two* types of potential mates, rich and poor. Let α be the expected remarriage rate and π the proportion of high income individuals among the divorcees, and let $y = \pi Y^h + (1-\pi)Y^l$ be the average income of the divorcees. Then the expected values of being unattached in the beginning of the second period for each type are

$$V^j(\alpha, \pi) = Y^j + \alpha y, \quad j = l, h. \quad (25)$$

This expression is obtained because type j consumes Y^j alone and expects to consume $Y^j + y$ when married and the expected value of the quality of a new match θ in the second (and last) period is zero. Clearly, a richer person has a higher expected value from being unattached.

At the end of the first period, the quality of the current match and the new income values (Y^h or Y^l) for each spouse are revealed, and each partner can choose whether to stay in the current match or divorce and seek an alternative mate. An hh couple divorces if:

$$2Y^h + \theta < Y^h + \alpha y \Rightarrow \theta < \alpha y - Y^h \tag{26}$$

An ll couple divorces if:

$$2Y^l + \theta < Y^l + \alpha y \Rightarrow \theta < \alpha y - Y^l \tag{27}$$

Note that, despite the lower value of being unattached for the two spouses, a poor couple is *more* likely to divorce, because the current marriage is less attractive.

In a mixed couple, type h will wish to divorce if

$$Y^h + Y^l + \theta < Y^h + \alpha y,$$

which is the same as condition 27, while type l will wish to divorce if

$$Y^h + Y^l + \theta < Y^l + \alpha y,$$

which is the same as condition 26. But inequality 26 implies inequality 27, which holds for a wider range of θ. Thus, the condition for marital dissolution for mixed couples is 27. For mixed couples there will be disagreement on the divorce decision if

$$\alpha y - Y^h \leq \theta < \alpha y - Y^l.$$

In this case, divorce is always triggered by the *high* income spouse who can do better outside the marriage.

In equilibrium, the expected remarriage rate, α, equals the divorce rate, that is,

$$\alpha = \lambda^2 G(\alpha y - Y^h) + (1 - \lambda^2) G(\alpha y - Y^l). \tag{28}$$

Equation 28 involves two endogenous variables, the expected remarriage rate α and the expected income of a divorcee, y. However, these two variables are interrelated and the equilibrium condition 28 can be reduced to one equation in one unknown, αy, which is the variable part of the expected gains from divorce. Then, we can deduce the separate equilibrium values of both α and y.

As a first step, note that the proportion in the population of high income divorcees of each gender is

$$\alpha\pi = \lambda[\lambda G(\alpha y - Y^h) + (1 - \lambda) G(\alpha y - Y^l)] \tag{29}$$

Taking the difference between 28 and 29, we have

$$\alpha(1 - \pi) = (1 - \lambda) G(\alpha y - Y^l). \tag{30}$$

Using the definition of y, we have

$$1 - \pi = \frac{Y^h - y}{Y^h - Y^l}. \tag{31}$$

Then, substituting from 31 into 30 we get

$$\alpha = \frac{\alpha y}{Y^h} + (1 - \lambda) G(\alpha y - Y^l) \frac{(Y^h - Y^l)}{Y^h}. \tag{32}$$

Finally, eliminating α in 28, we can then rewrite the equilibrium condition as an equation in αy

$$\alpha y = \lambda^2 G(\alpha y - Y^h) Y^h + (1 - \lambda^2) G(\alpha y - Y^l)[\frac{\lambda}{1+\lambda} Y^h + \frac{1}{1+\lambda} Y^l]. \tag{33}$$

To analyze this equation, we note that the expected income of a divorcee, y, is bounded between Y^l (which occurs if only low income individuals divorce, $\pi = 0$) and Y^h (which occurs if only high income individuals divorce, $\pi = 1$) and that the divorce rate α is bounded between 0 and 1. Therefore, αy is bounded between 0 and Y^h. Assuming that

$G(-Y^h) > 0$, equation 33 has a positive solution for αy because the RHS of 33 is positive at $\alpha y = 0$ and smaller than Y^h at $\alpha y = Y^h$ and $G(.)$ is continuous. However, because both sides of 33 are increasing in αy, this equation may have *multiple* solutions. Given an equilibrium value for αy, we can find the equilibrium divorce rate, α, from equation 28 and the equilibrium share of the rich among the divorcees, π, from the ratio of 29 to 28.

The comparative statics of this system are somewhat complicated, but the basic principles are quite clear. An increase in the proportion of the rich in the second period, λ, has two opposing effects on the equilibrium divorce rate. First, it raises the monetary gain from maintaining the current marriage. Second, it raises the average quality of divorcees and thus the prospects of finding a good match, which encourages divorce. The relative importance of these considerations depends on the initial proportions of the two types, the values of low and high income and the distribution of match quality. We cannot provide general results but simulations suggest that the divorce rate tends to increase with the proportion of the rich when the proportion of the rich is low in the second period. An increase in the income of the poor or the rich tends to reduce divorce. The positive income effects reflect the increasing gains from remaining married when consumption is a public good. There is no simple mapping from income risk or income inequality to the rate of divorce, but starting from equality an increase in the difference $Y^h - Y^l$ raises the divorce rate. An increase in the variability of the quality of match generally leads to a rise in the divorce rate.

The simple model outlined above generates *positive* assortative mating in the second period. This happens here because the good matches hh are less likely to break, and all types have the same remarriage probability α. So that there is a larger proportion of h among those who stay married than in the population. This can be immediately seen by noting that the term in square brackets in 29 is smaller than 1, so that $\alpha\pi < \lambda$. Conversely, there is a larger proportion of l among the singles than in the population, because they are more likely to divorce and are equally likely to remain single. This process of selective remarriage, via differential incentives to divorce, is quite different from the usual models (see Burdett and Coles 1999) that are built on the idea that the high type is more selective in the first marriage. In the search model, rejection of unsatisfactory mates is done when one is single, reflecting the assumption that a match is "for ever". In our model, rejection happens when married, after θ is revealed. This reflects our assumption that marriage is an "experience good". It seems that the two approaches lead to the same outcome.

5 Conclusion

The simple models discussed in this chapter make several important points that carry a general message for the empirical and theoretical analysis of the family. First, the marriage, fertility and divorce decisions are closely interrelated. Couples decide to marry and to have children based on the risk of divorce and the prospect of remarriage. Conversely, the fact that couples chose to marry, or have children, has implications for their subsequent divorce decisions. Second, in a marriage market, as in other search markets, individual decisions can be quite sensitive to the choices of others. In particular, if many choose to remain single, not to have children, or to divorce, this will strengthen the incentive of each couple separately to behave in a similar manner. Such markets are susceptible to sudden and large structural changes as may have happened following the introduction of the contraception pill in the 1970's. As we have seen, search externalities may have important policy and welfare implications. In particular, societies with high marital turnover may in fact yield better outcomes for the typical adult, because such an equilibrium allows easier recovery from bad shocks. In this chapter, we assumed that children are always worse off as a consequence of divorce. In the subsequent chapter, we shall discuss child support transfers and show that children are not necessarily harmed by divorce and, conditional on the divorce of their parents, may in fact be better off in a high divorce environment.

6 Appendix

6.1 Properties of the expected utility, with and without children

Using 7, 4 and 6,

$$W_{1,n}(\theta) = 2Y + \theta - nc + \int_{h_n-\theta}^{\infty} (2Y + nq^* + \theta + \varepsilon) f(\varepsilon) d\varepsilon + F(h_n - \theta)V_{2,n},$$

$$V_{2,n} = p(2Y + \beta) + (1-p)Y + nq^0,$$

$$h_n = -Y + p(Y + \beta) - n(q^* - q^0).$$

Hence,

$$V_{2,n} = h_n + 2Y + nq^* \tag{34}$$

Differentiating $W_{1,n}(\theta)$ with respects to θ yields

$$1 + \int_{h_n-\theta}^{\infty} f(\varepsilon)d\varepsilon + (2Y + nq^* + \theta + h_n - \theta) f(h_n - \theta) - f(h_n - \theta)V_{2,n}, \tag{35}$$

where we use the fact that the derivative of an integral with respect to the lower bound equals the value of the integrand at that point. Cancelling and collecting terms, we obtain

$$\frac{\partial W_{1,n}}{\partial \theta} = 2 - F(h_n - \theta), \tag{36}$$

as stated in 8. Note that $1 \leq \frac{\partial W_{1,n}}{\partial \theta} \leq 2$ and that $\frac{\partial W_{1,n}}{\partial \theta}$ is increasing in θ. Hence, the expected values with and without children, $W_{1,1}(\theta)$ and $W_{1,0}(\theta)$ respectively, are increasing and convex functions of of θ, with slopes bounded between 1 and 2. Also, because $h_1 < h_0$, $\frac{\partial W_{1,1}}{\partial \theta} > \frac{\partial W_{1,0}}{\partial \theta}$. Finally, examining the partial impact of p, holding θ fixed we see that

$$\frac{\partial W_{1,n}(\theta)}{\partial p} = \frac{\partial}{\partial p}[\int_{h_n-\theta}^{\infty} (2Y+nq^*+\theta+\varepsilon) f(\varepsilon) d\varepsilon + F(h_n-\theta)(h_n+2Y+nq^*)] = F(h_n-\theta)(Y+\beta), \tag{37}$$

implying that $\frac{\partial W_{1,0}}{\partial p} > \frac{\partial W_{1,1}}{\partial p}$.

6.2 Properties of the trigger for having children, θ_c

The trigger for θ_c is determined by the condition $W_{1,1}(\theta_c) = W_{1,0}(\theta_c)$. If there is a solution for θ_c, it must be unique because $\frac{\partial W_{1,1}}{\partial \theta} > \frac{\partial W_{1,0}}{\partial \theta}$. Using 7 and 34, the requirement that $W_{1,1}(\theta_c) = W_{1,0}(\theta_c)$ implies

$$\begin{aligned} & -c + \int_{h_1-\theta_c}^{\infty} (2Y + q^* + \theta_c + \varepsilon) f(\varepsilon) d\varepsilon + F(h_1 - \theta_c)(h_1 + 2Y + q^*) \\ = & \int_{h_0-\theta_c}^{\infty} (2Y + \theta_c + \varepsilon) f(\varepsilon) d\varepsilon + F(h_0 - \theta_c)(h_0 + 2Y). \end{aligned} \tag{38}$$

or

$$\begin{aligned} & -c + q^* + \int_{h_1-\theta_c}^{\infty} (\theta_c + \varepsilon) f(\varepsilon) d\varepsilon + F(h_1 - \theta_c) h_1 \\ = & \int_{h_0-\theta_c}^{\infty} (\theta_c + \varepsilon) f(\varepsilon) d\varepsilon + F(h_0 - \theta_c) h_0. \end{aligned} \tag{39}$$

By 6,

$$\frac{dh_0}{dp} = \frac{dh_1}{dp} = Y + \beta. \tag{40}$$

Differentiating both sides of 39 with respect to p and θ_c, we obtain

$$(1-F(h_1-\theta_c))d\theta_c + F(h_1-\theta_c)(Y+\beta)dp = (1-F(h_0-\theta_c))d\theta_c + F(h_0-\theta_c)(Y+\beta)dp, \tag{41}$$

implying that

$$\frac{d\theta_c}{dp} = Y + \beta.$$

6.3 Properties of the trigger for marriage, θ_m

By definition,

$$\max(W_{1,1}(\theta_m), W_{1,0}(\theta_m) = V_1 \tag{42}$$

Because $W_{1,1}(\theta_m)$ and $W_{1,0}(\theta_m)$ both increase in θ, while V_1 is independent of θ, the solution for θ_m must be unique if it exists. The solution must also satisfy $\theta_m \leq -Y$, because

$$W_{1,0}(-Y) = V_1 + \int_{p(Y+\beta)}^{\infty} (-p(Y+\beta)+\varepsilon) f(\varepsilon) d\varepsilon \geq V_1. \tag{43}$$

There are two cases to consider.

Case 1, $W_{1,0}(\theta_m) = V_1 > W_{1,1}(\theta_m)$, which implies $\theta_c > \theta_m$. In this case

$$2Y + \theta_m + \int_{h_0-\theta_m}^{\infty} (2Y + \theta_m + \varepsilon) f(\varepsilon) d\varepsilon + F(h_0 - \theta_m) V_{2,0} = Y + V_{2,0}, \tag{44}$$

or

$$Y + \theta_m + \int_{h_0-\theta_m}^{\infty} (2Y + \theta_m + \varepsilon) f(\varepsilon) d\varepsilon = (1 - F(h_0 - \theta_m))(h_0 + 2Y). \tag{45}$$

Differentiating totally both sides of 45 yields

$$\begin{aligned} & [1 + (1 - F(h_0 - \theta_m) + f(h_0 - \theta_m)(2Y + h_0)] d\theta_m - f(h_0 - \theta_m)(h_0 + 2Y)(Y + \beta) dp \\ = \; & f(h_0 - \theta_m)((h_0 + 2Y) d\theta_m + [(1 - F(h_0 - \theta_m)) - (h_0 + 2Y) f(h_0 - \theta_m)](Y + \beta) dp. \end{aligned} \tag{46}$$

Cancelling equal terms and rearranging, we obtain

$$\frac{\partial \theta_m}{\partial p} = (Y + \beta) \frac{1 - F(h_0 - \theta)}{2 - F(h_0 - \theta)} > 0 \text{ if } \theta_c > \theta_m. \tag{47}$$

Case 2, $W_{1,1}(\theta_m) = V_1 > W_{1,0}(\theta_m)$, which implies $\theta_c < \theta_m$. In this case

$$2Y + \theta_m - c + \int_{h_1-\theta_m}^{\infty} (2Y + q^* + \theta_m + \varepsilon) f(\varepsilon) d\varepsilon + F(h_1 - \theta_m) V_{2,1} = Y + V_{2,1}, \tag{48}$$

or

$$Y + \theta_m - c + \int_{h_1-\theta_m}^{\infty} (2Y + q^* + \theta_m + \varepsilon) f(\varepsilon) d\varepsilon = (1 - F(h_1 - \theta_m))(h_1 + 2Y + q^*). \quad (49)$$

Using the same calculations as in the previous case, we obtain

$$\frac{\partial \theta_m}{\partial p} = (Y + \beta) \frac{1 - F(h_1 - \theta)}{2 - F(h_1 - \theta)} > 0 \text{ if } \theta_c < \theta_m. \quad (50)$$

We conclude that

$$\frac{\partial \theta_c}{\partial p} > \frac{\partial \theta_m}{\partial p}. \quad (51)$$

6.4 Calculations for the example

6.4.1 Properties of θ_c in the example

We first prove that if the costs of having children are relatively high, that is if $q^* > c > \frac{q^*+q^0}{2}$, then an intersection of $W_{1,0}(\theta)$ with $W_{1,1}(\theta)$ cannot occur in the region $[h_1 - a, h_0 - a]$. The proof is by contradiction. Assume for some $\theta \in [h_1 - a, h_0 - a]$, $W_{1,0}(\theta) = W_{1,1}(\theta)$. Then this θ must satisfy

$$\begin{aligned} & \frac{7}{2}Y + \frac{3}{2}\theta + \frac{1}{2}p(Y+\beta) + \frac{1}{2}a + (\frac{1}{2}q^0 + \frac{1}{2}q^* - c) \quad (52) \\ = \; & 3Y + \theta + p(Y + \beta). \end{aligned}$$

Solving for θ and denoting the solution by θ_c , we have

$$\theta_c = p(Y + \beta) - Y - a + 2c - (q^* + q^0). \quad (53)$$

Recalling equation (6) for $n = 0$:

$$h_0 = -Y + p(Y + \beta),$$

we obtain, using $c > \frac{q^*+q^0}{2}$,

$$\theta_c = h_0 - a + 2c - (q^* + q^0) > h_0 - a. \tag{54}$$

6.4.2 Properties of θ_m in the example

Proof of (20). Consulting Figure 2 and allowing V_1 to move up or down, we see that we have to consider three cases for equation 42

First, low values of V_1 give an intersection with $W_{1,0}(\theta)$ below $\theta = h_0 - a$. Equating V_1 with $W_{1,0}(\theta)$ this gives:

$$\theta_m = -Y \tag{55}$$

This requires that:

$$\begin{aligned} -Y &= \theta_m \leq h_0 - a = -Y + p(Y+\beta) - a \\ &\Rightarrow p(Y+\beta) \geq a \end{aligned} \tag{56}$$

For intermediate values of $\theta \in [h_0 - a, \theta_c]$ we equate V_1 with $W_{1,0}(\theta)$ evaluated in the intermediate region of equation (17). This gives:

$$\theta_m = \frac{1}{3}(p(Y+\beta) - a) - Y. \tag{57}$$

Since we have $\theta_m \leq \theta_c$ this value and (19) requires that:

$$p(Y+\beta) \geq 3(q^* - c) - 2a \tag{58}$$

Finally we can consider high values of θ, such that $\theta \geq \theta_c$. Equality for equation (10) requires equating V_1 with $W_{1,1}(\theta)$ evaluated for $\theta \geq \theta_c$(that is, the third region of equation (17)). This gives

$$\theta_m = \frac{1}{2}p(Y+\beta) - \frac{1}{2}(q^* - c) - Y \tag{59}$$

This case requires $\theta_m > \theta_c$ which gives:

$$p(Y+\beta) < 3(q^* - c) - 2a \tag{60}$$

6.4.3 Properties of the proportion of singles, $U(\theta_m(p), \theta_c(p))$, in the example (proof of 21)

The proportion of singles at the beginning of the second period consists of those who did not marry in the beginning of the first period, $G(\theta_m(p))$, and of the divorcees at the end of the first period among the married. The proportion of divorcees depends on the location of θ_m. If V_1 is low and intersects $W_{1,0}(\theta)$ below $h_0 - a$, then all of the married for whom $\theta_m < \theta < h_0 - a$ divorce for sure, and all of the married for whom $h_0 - a < \theta < \theta_c$ divorce upon a bad shock, i.e. with a probability of $\frac{1}{2}$, while those married with children for whom $\theta > \theta_c$ do not divorce. Therefore,

$$\begin{aligned} U(\theta_m(p), \theta_c(p)) &= G(\theta_m(p)) + (G(h_0(p) - a) - G(\theta_m(p))) + \frac{1}{2}(G(\theta_c(p)) - G(h_0(p) - a)) \\ &= \frac{1}{2}[G(\theta_c(p)) + G(h_0(p) - a)]. \end{aligned} \tag{61}$$

For intermediate values of V_1, the intersection with $W_{1,0}(\theta)$ is in the range $[h_0 - a, \theta_c]$, where the married with children for whom $\theta_m < \theta < \theta_c$ divorce upon the occurrence of a bad shock. In this case,

$$\begin{aligned} U(\theta_m(p), \theta_c(p)) &= G(\theta_m(p)) + \frac{1}{2}(G(\theta_c(p)) - G(\theta_m(p))) \qquad (62) \\ &= \frac{1}{2}(G(\theta_c(p)) + G(\theta_m(p))). \end{aligned}$$

Finally, for high values of V_1, the intersection is with $W_{1,1}(\theta)$ above θ_c, where all married people have children, and no one divorces. In this case,

$$U(\theta_m(p), \theta_c(p)) = G(\theta_m(p)). \tag{63}$$

6.5 Low costs of raising children, $\frac{q^* + q^0}{2} > c > q^0$

For completeness, we discuss briefly the case with low costs of raising children. In this case, the intersection is at $\theta \in [h_1 - a, h_0 - a]$. Therefore, we equate $W_{1,0}(\theta)$ evaluated in the first region of equation (17) with $W_{1,1}(\theta)$ evaluated in the intermediate region of equation (17), implying

$$\theta_c = p(Y+\beta) - Y - a + 2c - q^* - q^0, \tag{64}$$

and

$$\theta_m = \begin{cases} -Y & \text{if} \quad p(Y+\beta) > a + q^0 + q^* - 2c \\ \frac{1}{3}(p(Y+\beta) - a + 2c - q^* - q^0) - Y & \text{if} \quad a + q^0 + q^* - 2c \geq p(Y+\beta) \geq c + q^* - 2q^0 - 2a \\ \frac{1}{2}p(Y+\beta) - \frac{1}{2}(q^* - c) - Y & \text{if} \quad p(Y+\beta) < c + q^* - 2q^0 - 2a \end{cases} . \tag{65}$$

Note that the assumptions $2a > (q^* - q^0)$ and $c < \frac{q^*+q^0}{2}$ ensure that interval $[c + q^* - 2q^0 - 2a, a + q^0 + q^* - 2c]$ is non-empty.

The aggregate number of singles associated with a given p is

$$U(\theta_m(p), \theta_c(p)) = \begin{cases} \frac{1}{2}(G(h_1(p) + a) + G(\theta_c(p))) & \text{if} \quad p(Y+\beta) > a + q^0 + q^* - 2c \\ \frac{1}{2}(G(h_1(p) + a) + G(\theta_m(p))) & \text{if} \quad a + q^0 + q^* - 2c \geq p(Y+\beta) \geq c + q^* - 2q^0 - \\ G(\theta_m(p)) & \text{if} \quad p(Y+\beta) < c + q^* - 2q^0 - 2a \end{cases} \tag{66}$$

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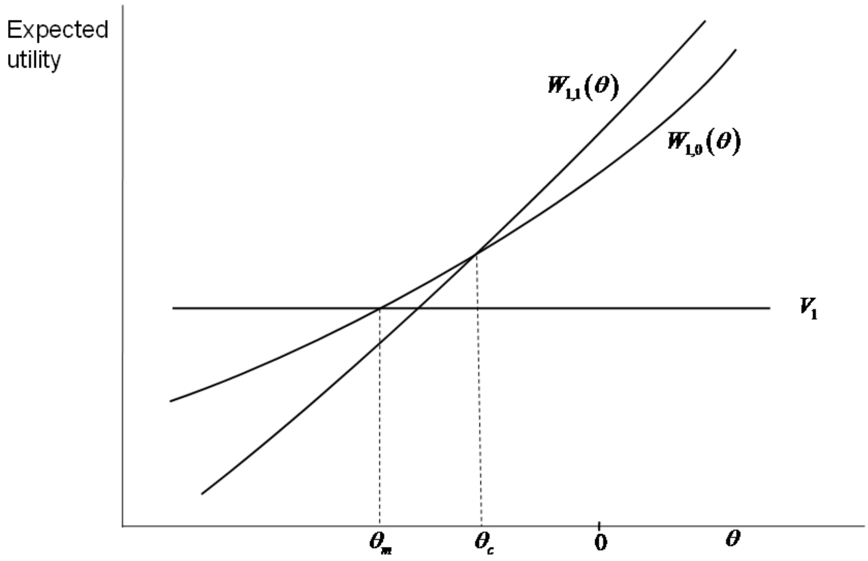

Figure 1: Expected utility profiles.

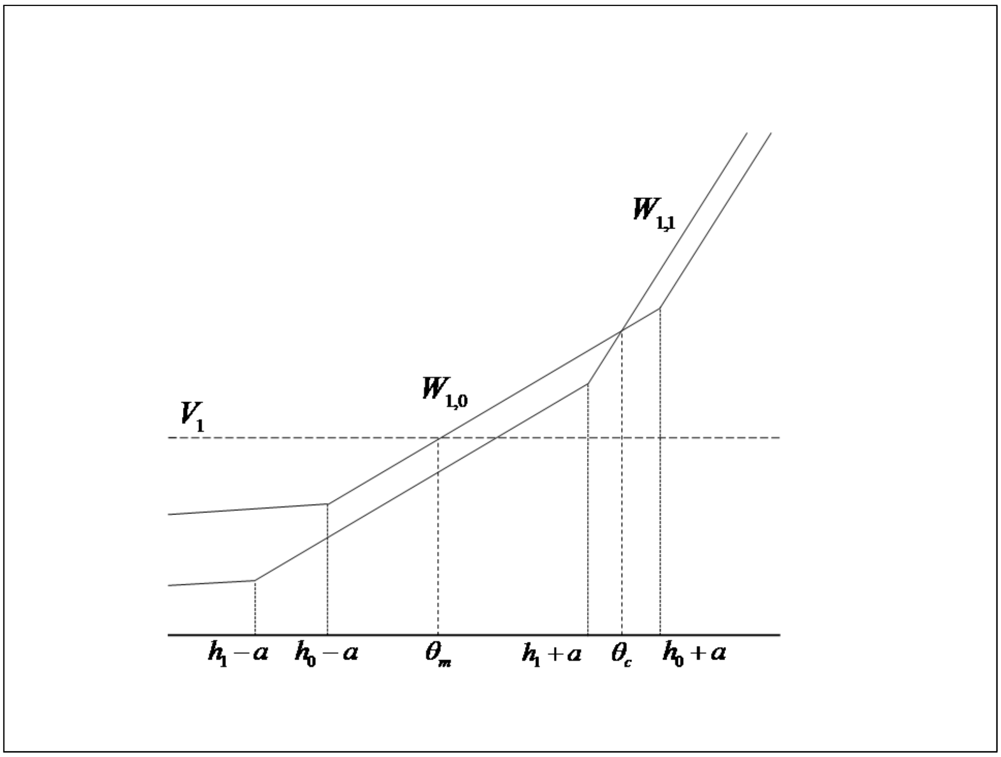

Figure 2: Expected utility profiles for example

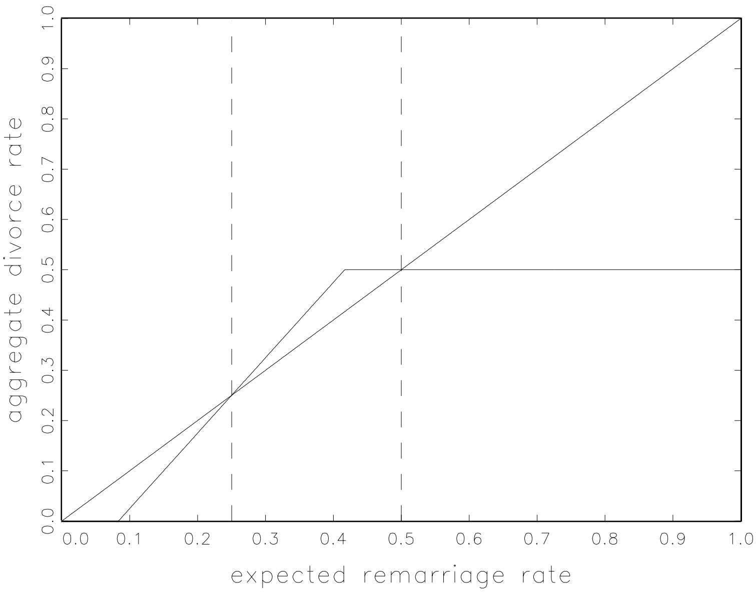

Figure 3: Equilibrium Divorce Rates

Table 1: Example with Multiple Equilibria

	$p = 0.0$	$p = 0.25$	$p = 0.5$
Critical value for marriage, θ_m	-1.31	-1.22	-1.14
Critical value for children, θ_c	$-.25$	.00	.25
Percent married	100	100	100
Percent with children	100	50	0
Percent divorced with children	0	0	0
Percent divorced without children	–	50	50
Percent single	0	25	50
Lifetime utility	4.083	4.094	4.208

Parameter values

Income, $Y = 1$

Range for the match quality, $\theta \in [-\frac{1}{6}, \frac{1}{6}]$

Size of shock to match quality, $a = \pm\frac{11}{12}$

Utility of children in intact family, $q^* = 1$

Utility of children following divorce, $q^0 = 0$

Cost of raising children, $c = \frac{11}{12}$

Probability of remarriage, $\gamma = 1$

Expected quality of match conditioned on remarriage,

$\beta = E(\theta/\theta \geq -Y) = 0$