Marriage with Labor Supply

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March 2011

Abstract

We propose a search-matching model of the marriage market and of the collective intrahousehold allocation of leisure time and consumption. The model extends the seminal work of Sattinger (1995), Lu and McAfee (1996), Shimer and Smith (2000) to allow for labour supply decisions. Contrary to Choo, Seitz, and Siow (2008b,a), who develop a similar project, instead of assuming a frictionless environment and model matching as an efficient assignment mechanism, we assume that singles meet randomly and sequentially and describe the marriage market as the steady-state equilibrium of an economy with forward looking agents where the only source of risk is exogenous divorce. The estimated matching probability resulting from the steady-state flow condition is strongly increasing in both male and female wages. We estimate that the marriage externality (household production) is shared between husbands and wifes in a way that is more favourable to the partner with the highest wage, and much more so for men than for women. The model is also able to explain why married women work less and married men more by making leisure an inferior good for men and a normal good for women.

Keywords: Marriage search model, Collective labor supply, Structural estimation.

JEL classification: C78, D83, J12, J22.

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1 Introduction

One of the key issues in understanding how tax policies affect labour supply is intra-household allocation of time and consumption. This is in particular the case of welfare benefits aimed at providing a safety net against poverty and work incentives at the same time, such as the Working Family Tax Credit programme in the UK and the Earned Income Tax Credit in the US. The models used to address these issues typically take the household as a unit with aggregate preferences. The collective models of the family (Chiappori, 1988, 1992),[1] go one step further by describing intrahousehold resource allocation as a Pareto equilibrium for the exchange economy comprising each family member endowed with his/her own preferences. The limitation of the Collective framework for policy evaluation lies in the multiplicity of equilibria and the lack of a selection device that could tell not only how a welfare policy affects resource allocation for a given sharing rule but also how it affects the sharing rule itself.

To this end, it is important that the model also describe the mecanisms of the determination of the threats that each household member can summon in the negotiation process.[2] This requires a model of the matching process that can link the distributions of the characteristics relevant for marriage decisions with the relative capacity of each household member to extract rent for one's own account. The role of distribution factors such as the sex ratio or rules about divorce in the definition of the bargaining power of household members is now well established (see Grossbard-Shechtman, 1984, Brien, 1997, Chiappori, Fortin, and Lacroix, 2002, **?**, Amuedo-Dorantes and Grossbard, 2007, Seitz, 2009). However, these papers do not contruct a matching model that would provide a mechanism explaining why it is so.

Several matching models of the marriage market have recently appeared in the literature to analyse various applied questions such as birth control policies and female bargaining power (Chiappori and Oreffice, 2008), divorce and remarriage (Brien, Lillard, and Stern, 2006, Chiappori and Weiss, 2006, 2007, Chiappori, Iyigun, and Weiss, 2008), sorting in the marriage market and inequality (Fernandez, Guner, and Knowles, 2005, Chiappori, Iyigun, and Weiss, 2009), labour market participation and fertility (Caucutt, Guner, and Knowles, 2002), etc. All these papers have a long-run macroeconomic interest and feature ad hoc multiperiod, heterogeneous agent models of equilibrium choice with a strong emphasis on one particular type of control and one particular dimension of het-

[1]Chiappori's seminal contributions generated a long list of papers building on the model of the family as a Pareto equilibrium. We can only cite a few of them: Browning, Bourguignon, Chiappori, and Lechene (1994), Fortin and Lacroix (1997), Browning and Chiappori (1998), Chiappori, Fortin, and Lacroix (2002), Mazzocco (2004, 2007), Blundell, Chiappori, Magnac, and Meghir (2007), etc. Note also that the assumption of efficient allocations within the family has been disputed, in particular, consistently, by DelBoca and Flinn (2005, 2006, 2009).

[2]See Lundberg and Pollak (1996) for a review of bargaining games and their empirical implications for the distribution within the family.

erogeneity. In contrast, Choo, Seitz, and Siow (2008b,a) integrate Chiappori's collective model in an assignment framework in a way that is not contingent to a specific empirical question. They build their synthesis on the recent works of Choo and Siow (2006), Siow (2009),[3] who develop a general empirical framework of matching and sorting in the marriage market, that is an assignment model with no frictions, subsuming individuals characteristics in a single index with an additive stochastic component. The econometric model then takes the form of a multinomial logit model. A potential drawback of the frictionless cooperative framework is that matching is necessarily perfect, so there is thus no potential reason for divorce and remarriage besides shocks to the stochastic component of utility. More generally, it is not clear that this framework can be extended to a dynamic envronment in a tractable way. Nevertheless, Choo, Seitz and Siow have already derived some important results. In the first paper (2008a) they provide a rationale for the link between distribution factors and the sharing rule. In the second paper (2008b) they show that marriage decisions bring new (over)identifying restrictions on the sharing rule. Moreover, these additional restrictions can identify the sharing rule when the spouse does not work.

In this paper, we aim at a similar target as Choo, Seitz and Siow: we also want to integrate the collective model within a matching framework. However, instead of assuming a frictionless environment, we follow the lead of the more macroeconomically-oriented afore-mentioned papers and assume that single individuals randomly search for a partner and they can test only one partner at a time. We design a search-matching model of the marriage market with labour supply by building on the seminal works of Sattinger (1995), Lu and McAfee (1996), Shimer and Smith (2000). The analytical advantage of random search on frictionless, complete-information assignment is considerable. We will see that it is easy to solve for the equilibrium even with forward-looking agents and divorce shocks.

Tractability yet follows from the crucial assumption of a steady-state environment. Although an important application of the matching framework is to explain long-term demographic changes such as the increasing divorce and remarriage rates, we assume that these changes are slow enough for a steady state to hold at least approximately. We then show that the steady-state flow conditions deliver important identifying restrictions on matching probabilities, and indirectly on the relationship between transfers – or the sharing rule defined as the ratio of male to female transfers – and partners' wages. In that respect, we draw similar conclusions as Choo et al.

The steady-state flow conditions connect the distributions of individual characteristics that determine marriage decisions amongst singles and amongst married couple. The probability that a given pair of female and male characteristics yields a marriage or not is thus essentially identifiable as the ratio of the joint density of this pair amongst couples

[3] See also Chiappori, Salanie, and Weiss (2010).

divided by the product of the marginal densities of each component of the pair amongst single females and amongst single males. This rationalises the link between distributional factors and marriage decisions in a very simple way.

This is the main, in some way trivially simple, contribution of this paper. The other identification recipes are less original and rest on the comparison of hours worked by married individuals and hours worked by singles at same wages. However, we hope to contribute slightly more incrementally by proving that numerical techniques such as Chebishev polynomials, Clenshaw-Curtis quadrature and Discrete Cosine Transform can be very useful to straightforwardly transform the nonparametric identification formulas into nonparametric estimators. We apply these estimators to data on wages and hours drawn from the US SIPP.

The main results are as follows. We find that though husbands' and wifes' are not strongly correlated (25%), the estimated matching probability resulting from the steady-state flow condition is strongly increasing in both male and female wages. We estimate that the marriage externality (household production) is shared between husbands and wifes in a way that is more favourable to the partner with the highest wage, and much more so for men than for women. The model is also able to explain why married women work less and married men more by making leisure an inferior good for men and a normal good for women. The only noticeable failure of the model is a relative incapacity to predict the higher labour market participation of high-wage men married to low-wage women and the corresponding lower participation of women.

The layout of the paper is as follows. First we construct the model. Second we study identification. Third we estimate the model. The last section concludes.

2 The Model

2.1 Setup

We consider a marriage market with L_m males and L_f females. The number of married couples is denoted as N and the respective numbers of single males and single females are $U_m = L_m - N$ and $U_f = L_f - N$.

Individuals differ in labor productivity, denoted as $x \in [\underline{x}, \overline{x}]$ for males and $y \in [\underline{y}, \overline{y}]$ for females. Let $\ell_m(x)$ and $\ell_f(y)$ denote the densities of the measures of males of type x and females of type y, with $L_m = \int \ell_m(x)\, \mathrm{d}x$ and $L_f = \int \ell_f(y)\, \mathrm{d}y$. The corresponding densities of wages in the sub-populations of singles are denoted as $u_m(x)$ and $u_f(y)$, with $U_m = \int u_m(x)\, \mathrm{d}x$ and $U_f = \int u_f(y)\, \mathrm{d}y$. The density of couples of type (x, y) is denoted as $n(x,y)$, with $N = \iint n(x,y)\,\mathrm{d}x\,\mathrm{d}y$ and

$$\ell_m(x) = \int n(x,y)\,\mathrm{d}y + u_m(x)\,, \tag{1}$$

$$\ell_f(y) = \int n(x,y)\,\mathrm{d}x + u_f(y). \tag{2}$$

We assume that only singles search for a partner, ruling out "on-the-mariage" search. The number of meetings per period is measured by a meeting function $M(U_m, U_f)$ and $\lambda_i = \frac{M(U_m,U_f)}{U_i}$ is the instantaneous probability that a searching individual of gender i meet with a single of the other sex. We also denote $\lambda = \dfrac{M(U_m, U_f)}{U_m U_f}$.

All meetings do not result in a match. We assume that there exists a function $\alpha(x,y) \in [0,1]$ indicating the probability that a match (x,y) be consumated. The matching probability is an equilibrium outcome that will be later determined. The matching set is the support of distribution α.

Matches are exogenously dissolved with instantaneous probability δ .

2.2 Flow equations

In steady state flows in and out of the stocks of married couples must exactly balance each other out. This means that, for all (x,y),

$$\delta n(x,y) = u_m(x)\,\lambda_m \frac{u_f(y)}{U_f}\alpha(x,y) = \lambda u_m(x)\,u_f(y)\,\alpha(x,y). \tag{3}$$

The left-hand side is the flow of divorces. The right-hand side measures the flow of new marriages as the product of the measure of male singles of type x with the probability of a contact and the probability of drawing a female single of type y.

Integrating over y:

$$\delta \int n(x,y)\,\mathrm{d}y = \lambda u_m(x) \int u_f(y)\alpha(x,y)\,\mathrm{d}y$$

and using equation (1) to substitute $\int n(x,y)dy$ out of this equation, yields the following equilibrium conditions for u_m:

$$\delta\left[\ell_m(x) - u_m(x)\right] = \lambda u_m(x) \int u_f(y)\alpha(x,y)\,\mathrm{d}y,$$

or, equivalently,

$$u_m(x) = \frac{\delta \ell_m(x)}{\delta + \lambda \int u_f(y)\alpha(x,y)\,\mathrm{d}y}. \tag{4}$$

By symmetry, the equation defining the equilibrium distribution of wages in the population of single females is:

$$u_f(y) = \frac{\delta \ell_f(y)}{\delta + \lambda \int u_m(x)\alpha(x,y)\,\mathrm{d}x}. \tag{5}$$

2.3 Utility flows

Individuals draw utility from consumption and leisure. Let

$$v_m(x, xT + t) = \frac{xT + t - A_m(x)}{B_m(x)}, \tag{6}$$

denote the indirect utility of male wage x and non-labour income t. T denotes total time endowment, $B_m(x)$ is an aggregate price index, with $B_m(\underline{x}) = 1$ (arbitrarily), and $A_m(x)$ is a minimum expenditure to attain a positive utility, with $A_m(\underline{x}) = 0$.[4]

To keep the model easily tractable we rule out labour market nonparticipation. So, hours worked follow by Roy's identity as

$$h_m(x, xT + t) = T - A'_m(x) - b'_m(x)[xT + t - A_m(x)], \tag{7}$$

where $b_m(x) = \log B_m(x)$ and a prime (such as in b'_m and A'_m) denotes a derivative. A standard specification is $A_m(x) = xa_m$ and $B_m(x) = x^{b_m}$. In which case,

$$h_m = T - a_m - \frac{b_m}{x}[xT - xa_m + t].$$

We use symmetric definitions for females. In particular,

$$h_f(y, yT + t) = T - A'_f(y) - b'_f(y)[yT + t - A_f(y)], \tag{8}$$

2.4 Optimal rent sharing between spouses

Let $W_m(v, x)$ denote the present value of marriage for a married male of type x receiving a flow utility v and let $W_m(x)$ denote the value of singlehood (derived in the next section). Equating annuities to expected income flows links values as

$$rW_m(v, x) = v + \delta\left[W_m(x) - W_m(v, x)\right],$$

where r is the discount factor. Then, define male surplus as

$$S_m(v, x) = W_m(v, x) - W_m(x) = \frac{v - rW_m(x)}{r + \delta}.$$

[4]The indirect utility function maximises the utility of consumption c and labour supply h subject to the budget constraint $c = xh + t$ and the participation constraints $c > 0$ and $h < T$.

We assume that spouses share ressources cooperatively using Nash bargaining, whereby transfers t_m and t_f solve

$$\max_{t_m, t_f} S_m\left(v_m(x, xT + t_m), x\right)^{\beta} S_f\left(v_f(y, yT + t_f), y\right)^{1-\beta}$$

subject to the condition

$$t_m + t_f \leq C(x, y) + z$$

where $C(x, y) + z$ is the value of the public goods that are produced in the household. It is supposed to be a function of wages x and y and of a match-specific component z that is drawn from a zero-mean distribution G independently of x and y. Note that transfers can be positive or negative. However, in equilibrium, both transfers should be positive. Otherwise, one is better off remaining single.

Such a public good may be the home production of food products that is sold in local markets, or less tangible services that make certain purchases redundant. For example, the possibility of talking to each other is a source of untertainment that makes the purchase of books or theater tickets less imperative to married couples. In the same way as one imputes housing services to home owners using hedonic prices, one can think of $C(x, y) + z$ as the imputed provision for all the public goods and services produced in the marriage. Children are usually thought of as an example of these "public goods". This may not be the right analogy. Single individuals can adopt children. It is yet more difficult for them to find children to adopt and to rear them. Marriage (or cohabitation) can be seen as a more efficient technology to produce children, the value of which could be imputed by looking at the cost of adoption.

Of course, our framework is overly simplistic. One should not characterise individuals only by labour market productivity. For example, the child rearing technology is of lesser value for divorced individuals with children from first marriage. Indeed, the main reason why we have introduced the match-specific component of the public good, z, which allows some couples with wages (x, y) to match or others not to, is that there are multiple personal characteristics that enter the definition of attractiveness, which we do not take into account.[5] Designing empirically tractable multidimensional matching models with random search is definitely a promising area for further research.

With quasi-linear utility functions, the solution is trivially found to be such that:

$$t_m(x, y, z) - s_m(x) = \beta[C(x, y) + z - s_m(x) - s_f(y)], \tag{9}$$

and

$$t_f(x, y, z) - s_f(y) = (1 - \beta)[C(x, y) + z - s_m(x) - s_f(y)], \tag{10}$$

[5] It also allows to smooth out the discontinuity at the boundary of the matching set.

where we denote

$$s_m(x) = B_m(x) r W_m(x) - xT + A_m(x),$$
$$s_f(y) = B_f(y) r W_f(y) - yT + A_f(y).$$

These functions are different from zero if singles can expect a return from marriage. In other words, they are the Average Treatment Effect for the decision to search (given the individual's wage).

Two dating singles with x and y decide to match if the overall surplus is positive, i.e.

$$s(x, y) + z > 0, \tag{11}$$

with

$$s(x, y) = C(x, y) - s_m(x) - s_f(y). \tag{12}$$

The matching probability can then be calculated as

$$\begin{aligned} \alpha(x, y) &= \Pr\{s(x, y) + z > 0 | x, y\} \\ &= 1 - G\left(-s(x, y)\right). \end{aligned} \tag{13}$$

2.5 The value of singlehood

The value of being single, for males, solves the option value equation:

$$\begin{aligned} rW_m(x) &= v_m(x, xT) + \lambda \iint \max\{S_m\left(v_m(x, xT + t_m(x, y, z)), x\right), 0\} \, \mathrm{d}G(z) \, u_f(y) \, \mathrm{d}y \\ &= v_m(x, xT) + \frac{\lambda}{r + \delta} \iint \max\{v_m(x, xT + t_m(x, y, z)) - rW_m(x), 0\} \, \mathrm{d}G(z) \, u_f(y) \, \mathrm{d}y. \end{aligned}$$

Equivalently,

$$s_m(x) = \frac{\lambda\beta}{r + \delta} \iint \max\{z + C(x, y) - s_m(x) - s_f(y), 0\} \, \mathrm{d}G(z) \, u_f(y) \, \mathrm{d}y. \tag{14}$$

A similar expression can be derived for females:

$$s_f(y) = \frac{\lambda(1 - \beta)}{r + \delta} \iint \max\{z + C(x, y) - s_m(x) - s_f(y), 0\} \, \mathrm{d}G(z) \, u_m(x) \, \mathrm{d}x. \tag{15}$$

2.6 Equilibrium

An equilibrium is a fixed point (u_m, u_f, w_m, w_f) of the following system of equations where the first two equations determine equilibrium distributions of wages amongst singles; the last two equations determine equilibrium values:

$$u_m(x) = \frac{\ell_m(x)}{1 + \frac{\lambda}{\delta}\int u_f(y)\alpha(x,y)\,\mathrm{d}y} \tag{16}$$

$$u_f(y) = \frac{\ell_f(y)}{1 + \frac{\lambda}{\delta}\int u_m(x)\alpha(x,y)\,\mathrm{d}x} \tag{17}$$

$$s_m(x) = \frac{\frac{\lambda\beta}{r+\delta}\iint \max\{z + C(x,y) - s_f(y), s_m(x)\}\,\mathrm{d}G(z)\,u_f(y)\,\mathrm{d}y}{1 + \frac{\lambda\beta}{r+\delta}U_f} \tag{18}$$

$$s_f(y) = \frac{\frac{\lambda(1-\beta)}{r+\delta}\iint \max\{z + C(x,y) - s_m(x), s_f(y)\}\,\mathrm{d}G(z)\,u_m(x)\,\mathrm{d}x}{1 + \frac{\lambda(1-\beta)}{r+\delta}U_m} \tag{19}$$

where

$$U_m = \int u_m(x)\,\mathrm{d}x, \quad U_f = \int u_f(y)\,\mathrm{d}y$$
$$\lambda = \frac{M(U_m, U_f)}{U_m U_f}$$
$$\alpha(x,y) = 1 - G\left(s_m(x) + s_f(y) - C(x,y)\right).$$

Note that equations (18), (19) rewrite equations (14), (15) so that s_m and s_f are now fixed points of contracting operators (given u_m and u_f; see Shimer and Smith (2000)).

Shimer and Smith (2000) prove the existence of an equilibrium for a simpler version of the search-matching equilibrium. They consider a symmetric equilibrium with a quadratic matching function (λ constant). The common distribution of singles ($u = u_m = u_f$) is the solution to an equation similar to equations (16) or (17):

$$u(x) = \frac{\ell(x)}{1 + \frac{\lambda}{\delta}\int u(y)\alpha(x,y)\,\mathrm{d}y}, \tag{20}$$

that can be shown to be contracting once u is reparametrised as $v = \log(u)$. Nevertheless, the general equilibrium that involves α as well as u is shown to exist but is may not be unique as the existence proof relies on Schauder theorem. Tröger and Nöldeke (2009) prove the existence of an equilibrium in u for all α (the first step of Shimer and Smith's proof) for the linear matching case ($\lambda = 1/\sqrt{U}$). They do not derive any contraction property and rely on Schauder theorem to prove existence but not unicity.

When solving for this equilibrium, the standard fixed-point iteration algorithm, $x_{n+1} = Tx_n$, worked well in practice for estimated parameter values (see below), even starting far from the equilibrium (like with $s_m(x) = 0$ and $u_m(x) = \ell_m(x)$). [6]

[6] Sometimes it is useful to "shrink" steps by using iterations of the form

$$x_{n+1} = Hx_n \equiv x_n + \alpha(Tx_n - x_n)$$

with $\alpha \in (0,1]$. A stepsize $\alpha < 1$ may help if T is not everywhere strictly contracting.

3 Data

In this section we present the data we use for estimating the model. We also present a few salient facts on wage and hour distribution that the model is challenged to replicate.

3.1 Demographic data on marriages and divorces

In the US, the 2001 Census shows (Kreider, 2005) that 30.1% of men (24.6% women), 15 years and plus, are not married and 21 % (23.1% women) are divorcees. The median age at first marriage is 24 for men and 21.8 for women. Table 1 displays the percent ever married by age for men and women and for various cohorts. As time passes one tends to marry less early and women marry more early than men.

The median duration of first marriages is 8.2 years (men) and 7.9 (women). The median age at first divorce is 31.5 for men and 29.4 for women. The median duration between first divorce and remarriage, for those married two times, is 3.3 years (men) and 3.5 (women) and second marriages last 9.2 and 8.1 years. Now, about 75-80% of first marriages, depending on cohorts, reach 10 years, 60-65% 20 years, 50-60% 30 years. This is consistent with a separation rate of around 2.5% per year. For second marriages 70-80% reach 10 years, 55% 15 years and 50% 20. The separation rate is slightly higher, around 3% annual.

According to survival data the median marriage length should hence be of 23-28 years instead of 8-9 years. The Poisson assumption that we use in the model is somewhat at odds with the data because a large proportion of marriages never end, and those who end in divorce do it relatively fast, in the first two years. One way of making divorce rates non stationary is to permit z to change rapidly. Thus marriages resulting from a very large z would end fast if new, likely lower values are soon drawn.

3.2 Wage and labour supply data

We use the US Survey of Income and Program Participation (SIPP) from 1996-1999. For every quater that an individual is in the panel we collect information on the labour market state at the time of the survey, wages if employed, the number of hours worked, gender, and the corresponding information for the respondent's spouse if married. Our sample is restricted to individuals who are not self-employed or in the military, between the ages of 21 and 65. We assume the environment stationary and calculate individuals' mean wages over employment spells, and mean hours worked over all quarters including non-employment spells. Thus, we somewhat reduce the transitory noise in wages and hours, and we reduce the number of labour-market non-participation spells (zeros). Then we drop all observations with zero hours worked (individuals and individuals' spouses never employed in the 4-year period). This is definitely not a satisfactory procedure but the

	cohort			
age	1945 to 1949	1950 to 1954	1955 to 1959	1960 to 1964
	Men			
20 years	20.4	23.0	17.6	15.8
25	66.6	59.2	49.9	45.0
30	79.7	74.0	68.8	65.6
35	86.2	81.7	78.5	76.6
40	89.6	85.9	83.6	
45	91.5	88.2		
50	93.1			
	Women			
20 years	44.8	40.5	36.6	30.2
25	78.7	70.1	66.0	59.5
30	85.4	80.7	78.1	74.4
35	88.3	86.2	84.5	83.0
40	90.9	89.1	87.7	
45	92.1	90.6		
50	93.0			

Table 1: Percent ever married by age (Source: US Census Bureau, SIPP, 2001 Panel, Wave 2 Topical Module)

model cannot deal (at the moment) for both the extensive and the intensive margins of labour market participation. We also trim the 1% top and bottom wages.

In our sample, we have $2N/(2N+U_m+U_f) = 50.3\%$ of the population that is married, and there is a slight deficit of single males vis-a-vis single females: $U_m/U_f = 90.0\%$ ($N = 6827, U_m = 6386, U_f = 7098$).

Let (x, y, h_m^1, h_f^1) denote an observation for a married couple and let (x, h_m^0) and (y, h_f^0) denote observations for single males and females. By definition, $h_m^1 \equiv h_m(x, xT + t_m(x, y, z))$, $h_m^0 \equiv h_m(x, xT)$, with symmetric expressions for h_f and h_f^0. We set the maximal number of hours T equal to the upper bound of hours in the sample, i.e. $T = 667$ hours per month (28 full 24-hour days!).

Wage distributions.

Figure 1, panel (a), shows the Gaussian-kernel density estimates for the wage distributions in four subpopulations differing by gender and marriage status. A clear stochastic ordering appears: married males have higher wages on average and more dispersed wages than single males. Single males, and single and married females exhibit strikingly similar wage distributions. Panel (b) displays the corresponding CDFs. The wage scale is in logs so as to emphasize the non-normality of the distributions: both tails are fatter than for a normal distribution.

Then we consider the joint distribution of wages among married couples, also estimated using a Gaussian kernel density (Figure 1). The most salient aspect of this distribution is its very large support. Virtually no wage configuration, like a low male wage and a high female wage or vice versa, seems impossible. Spouses' wages are only weakly correlated (25%), but the wage density is clearly oriented along the dominant diagonal (see the flat projection in the panel (b)). These features (wide support, low correlation) make it is important to allow for a match-specific externality component (z).

At this stage, the data seem like an impossible challenge for the theory. Such a low correlation between x and y tends to indicate a very little amount of sorting based on wages. However, the estimation of the model has some interesting surprises in store.

Hours.

Figure 3 displays nonparametric kernel estimates of mean hours given one's own wage for single and married individuals. Married males work more than single males, who work more than single females, who work much more than married females. Apparently, cohabitation allows men to specialise in wage-work and women to specialise in household production. The labour supply profiles of singles tend to tilt upward, being more like married females at low wages and more like married males at high wages.

Figure 4 shows mean hours given both spouses' wages for married males and females. There is some evidence of complementarity: Male hours are highest and female hours lowest for high wage men married to low wage women, and male hours are lowest and female hours highest for high wage women married to low wage men.

4 Identification

Let the distribution of z have cdf $G(z) = \Phi(z/\sigma)$, where Φ is a distribution with mean 0 and variance 1. Let us also suppose that the support of Φ, $[v_{min}, v_{max}]$, is large enough, maybe equal to the whole real line, for the matching probability to be strictly between 0 and 1 for all (x, y):

$$0 < \alpha(x, y) = 1 - \Phi(-s(x, y)/\sigma) < 1.$$

We now study the identification of the model using the following data:

- the distribution of the age at first marriage for men and women,
- the joint cross-sectional distributions of wages x, y and hours h_m^0, h_f^0 for single men and women,
- the joint cross-sectional distribution of spouses' wages x, y and hours h_m^1, h_f^1 for married couples.

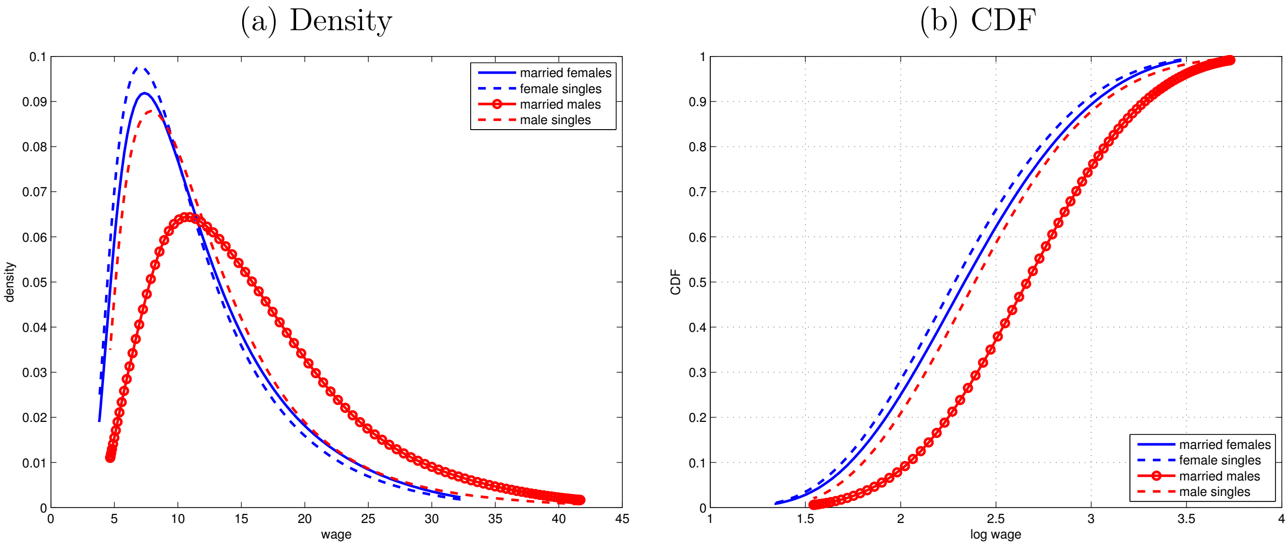

Figure 1: Marginal wage distributions

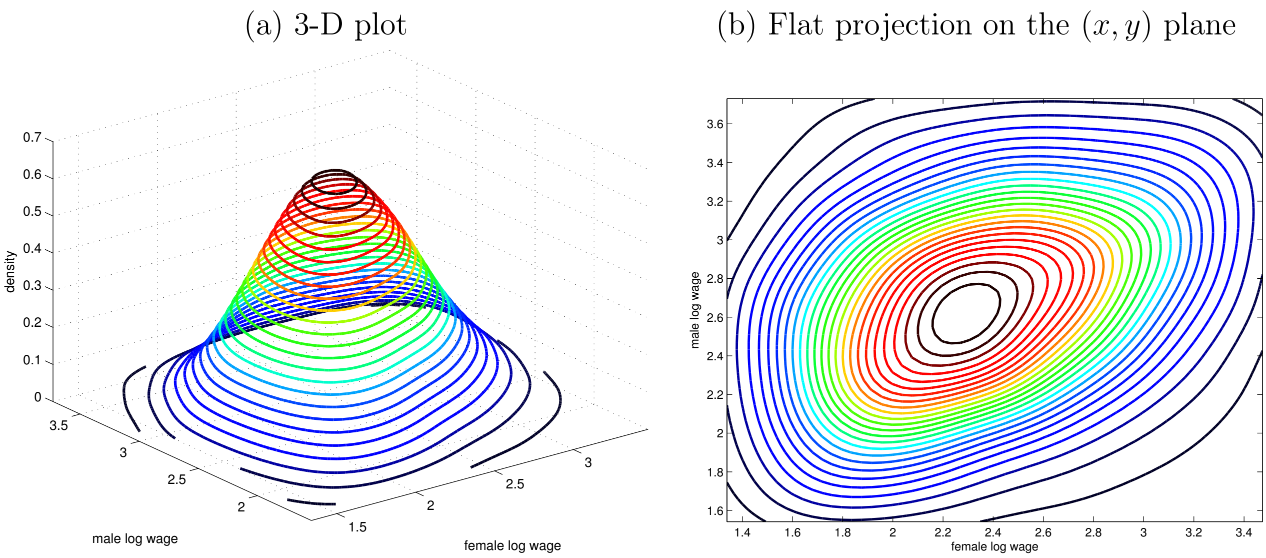

Figure 2: Joint log-wage density for married couples

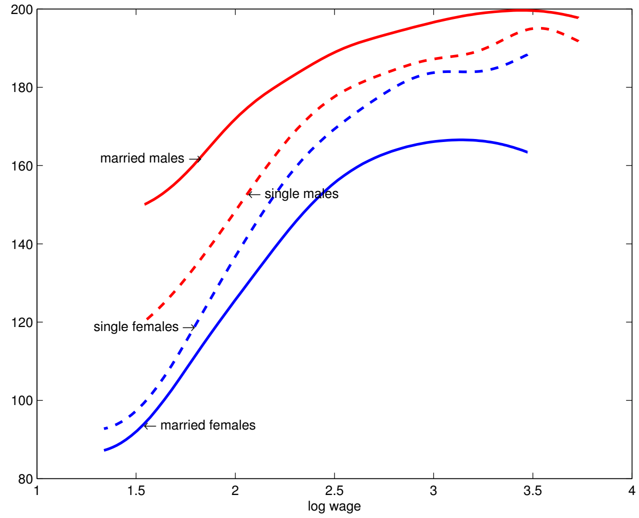

Figure 3: Mean hours supplied given own wage

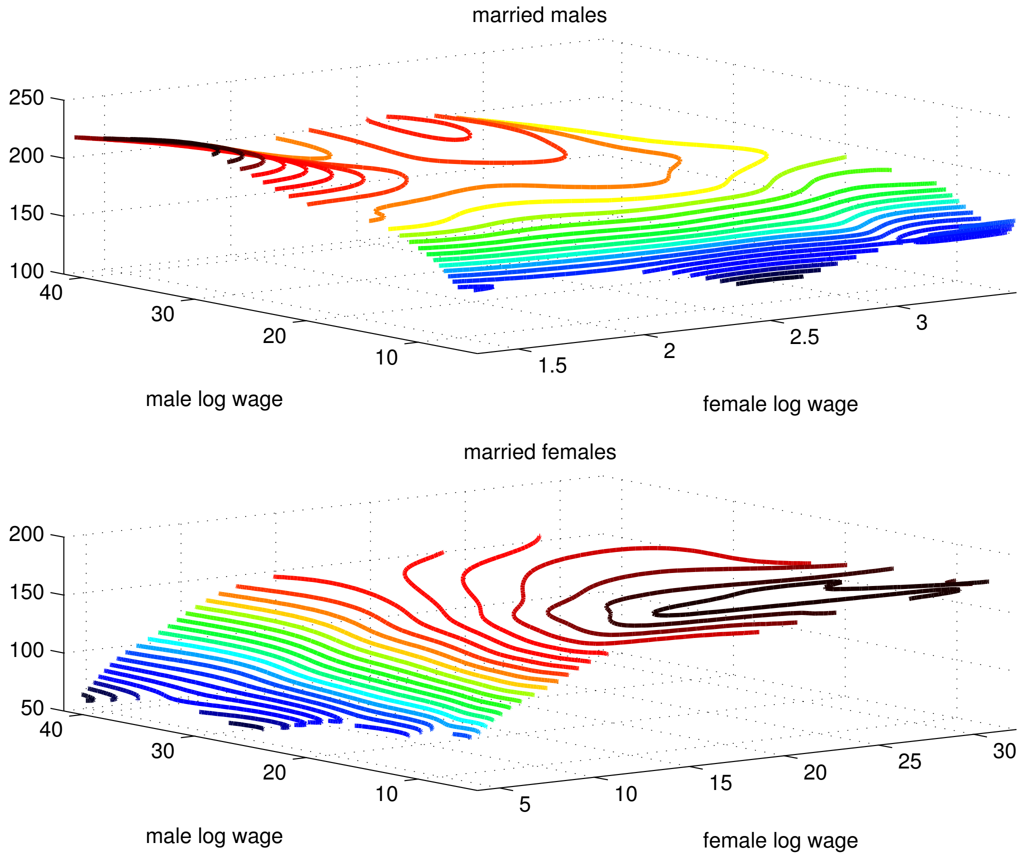

Figure 4: Hours supplied for married couples

4.1 The divorce rate

The average matching probability for a single man of type x of randomly meeting a single woman is equal to

$$\begin{aligned}\mu_m(x) &\equiv \lambda_m \int \frac{u_f(y)}{U_f}\alpha(x,y)\,\mathrm{d}y \\ &= \delta \int \frac{n(x,y)}{u_m(x)}\,\mathrm{d}y = \delta\frac{\ell_m(x) - u_m(x)}{u_m(x)}\end{aligned}$$

with $\lambda_m = \lambda U_f$ and with a similar formula for single women. Now, the average marriage rate among single men is

$$\mu_m \equiv \int \mu_m(x)\frac{u_m(x)}{U_m}\,\mathrm{d}x = \delta\frac{L_m - U_m}{U_m}.$$

Using the survival data on the duration of singlehood before first marriage for the 1955-1959 cohort displayed in Table 1, one can estimate both an age at which individuals start searching for a partner (age_{0m} and age_{0f}) and δ by running jointly the regressions of log survival probabilities:

$$\begin{aligned}\log Survival_m(age_m) &= -\delta(age_m - age_{0m}),\\ \log Survival_f(age_f) &= -\delta(age_f - age_{0f}).\end{aligned}$$

We estimate starting ages $age_{0m} = 17.3$ years and $age_{0f} = 11.8$ years. The estimated divorce rate is $\delta = 8.0\%$ annual. This yields a median waiting time before marriage for men of 8.1 years (mean = 11.7) and of 9.0 years for women (mean = 13.0). The implied median marriage duration is 8.7 years (mean = 12.6 years), which is remarkably similar to value that can be directly estimated from data on marriage duration. The fit is good (see Figure 5).

Figure 6 plots the implied average durations before (re)marriages by gender and wage. Low wage-individual have to wait for a very long time, and women more than men. The waiting time decreases with the wage.

4.2 Inference from wages

Proposition 1. *$\lambda\alpha(x,y)$ is identified from wage distributions.*

The equilibrium flow condition implies that

$$\lambda\alpha(x,y) = \delta\frac{n(x,y)}{u_m(x)u_f(y)}. \tag{21}$$

So the matching probability is identified up to the multiplicative factor λ.

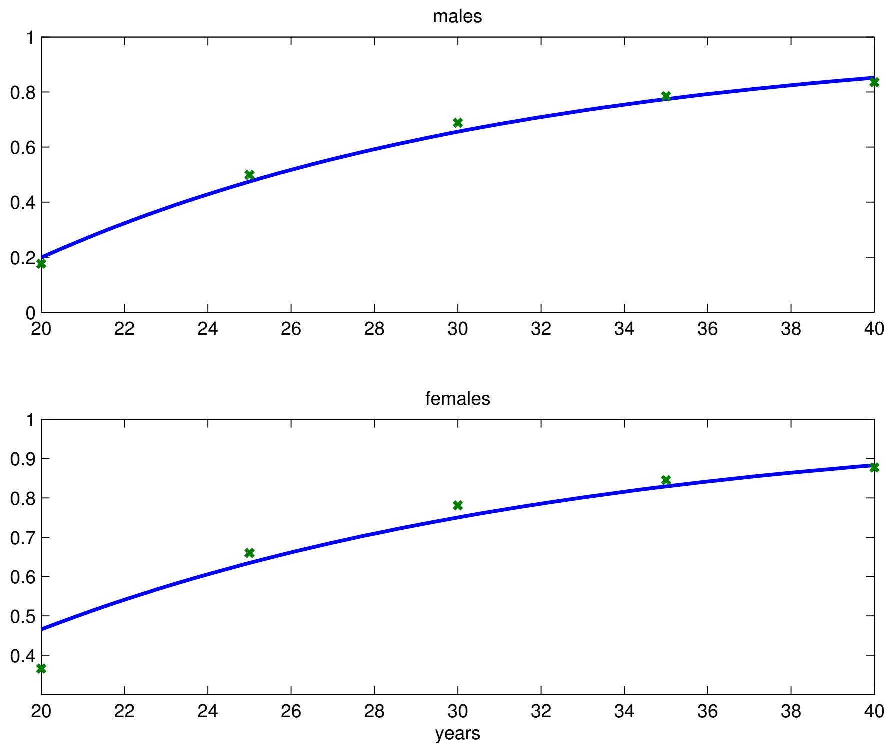

Figure 5: Fit of percent ever married by age (x-marks show 1955-1959 cohort data)

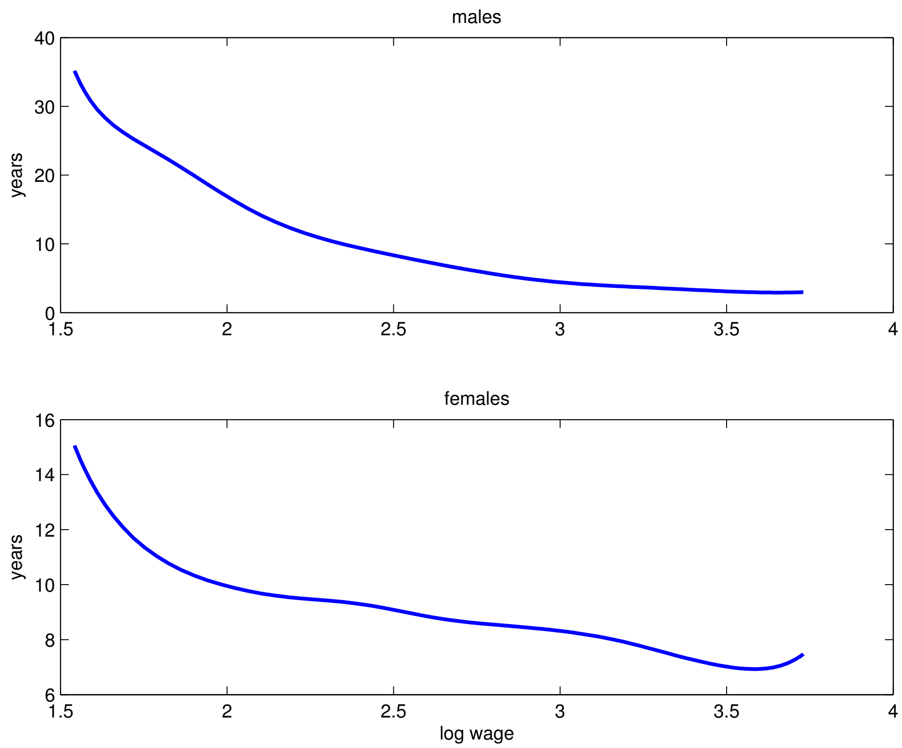

Figure 6: Median duration before marriage, by gender and wage

Define conditional mean transfers for couples as

$$\bar{t}_m(x,y) \equiv \mathbb{E}(t_m(x,y,z)|x,y,s(x,y)+z>0),$$
$$\bar{t}_f(x,y) \equiv \mathbb{E}(t_f(x,y,z)|x,y,s(x,y)+z>0).$$

Proposition 2. *$\frac{s(x,y)}{\sigma}$, $\frac{s_m(x)}{\beta\sigma}$ and $\frac{s_f(y)}{(1-\beta)\sigma}$, hence $\frac{\bar{t}_m(x,y)}{\beta\sigma}$ and $\frac{\bar{t}_f(x,y)}{(1-\beta)\sigma}$, are identified given Φ and λ from wage distributions, and $\frac{C(x,y)}{\sigma}$ is identified from given Φ, λ and β.*

It follows from equation (13) that

$$s(x,y) = -\sigma\Phi^{-1}(1-\alpha(x,y)), \tag{22}$$

and from equation (14) that

$$s_m(x) = \beta\frac{\lambda}{r+\delta}\int\left(\int \max\{z+s(x,y'),0\}\,\mathrm{d}G(z)\right)u_f(y')\,\mathrm{d}y'\Bigg],$$

where

$$\begin{aligned}\int \max\{s(x,y)+z,0\}\,\mathrm{d}G(z) &= s(x,y)\alpha(x,y) + \int_{-s(x,y)}^{\sigma v_{max}} z\,\mathrm{d}G(z)\\ &= \sigma\left[-\alpha\Phi^{-1}(1-\alpha) + \int_{\Phi^{-1}(1-\alpha)}^{v_{max}} v\,\mathrm{d}\Phi(v)\right]\\ &= \sigma\mu_\Phi(\alpha(x,y)) \quad \text{(say)}.\end{aligned}$$

Hence

$$s_m(x) = \beta\sigma\frac{\lambda}{r+\delta}\int \mu_\Phi(\alpha(x,y'))u_f(y')\,\mathrm{d}y', \tag{23}$$

and by symmetry,

$$s_f(y) = (1-\beta)\sigma\frac{\lambda}{r+\delta}\int \mu_\Phi(\alpha(x',y))u_m(x')\,\mathrm{d}x'. \tag{24}$$

It follows that $\frac{s(x,y)}{\sigma}$, $\frac{s_m(x)}{\beta\sigma}$ and $\frac{s_f(y)}{(1-\beta)\sigma}$ are identified given Φ and λ

Then,

$$\begin{aligned}\bar{t}_m(x,y) &= s_m(x) + \beta\mathbb{E}[s(x,y)+z|x,y,s(x,y)+z>0]\\ &= \beta\sigma\left[\frac{\lambda}{r+\delta}\int \mu_\Phi(\alpha(x,y'))u_f(y')\,\mathrm{d}y' + \frac{\mu_\Phi(\alpha(x,y))}{\alpha(x,y)}\right]\end{aligned} \tag{25}$$

and, symmetrically,

$$\begin{aligned}\bar{t}_f(x,y) &= s_f(y) + (1-\beta)\mathbb{E}[s(x,y)+z|x,y,s(x,y)+z>0] \\ &= (1-\beta)\sigma\left[\frac{\lambda}{r+\delta}\int \mu_\Phi(\alpha(x',y))u_m(x')\,\mathrm{d}x' + \frac{\mu_\Phi(\alpha(x,y))}{\alpha(x,y)}\right]. \end{aligned} \tag{26}$$

Hence, $\frac{\bar{t}_m(x,y)}{\beta\sigma}$ and $\frac{\bar{t}_f(x,y)}{(1-\beta)\sigma}$ are also identified given Φ and λ. Note that the bargaining power parameter β determines the level of transfers, not their shape.

Lastly, household production is such that

$$\begin{aligned}C(x,y) &= s(x,y) + s_m(x) + s_f(y) \\ &= \sigma\Bigg[-\Phi^{-1}(1-\alpha(x,y)) + \beta\frac{\lambda}{r+\delta}\int \mu_\Phi(\alpha(x,y'))u_f(y')\,\mathrm{d}y' \\ &\quad + (1-\beta)\frac{\lambda}{r+\delta}\int \mu_\Phi(\alpha(x',y))u_m(x')\,\mathrm{d}x'\Bigg]. \end{aligned} \tag{27}$$

Hence, contrary to transfers, the shape of $C(x,y)$ depends on β.

4.3 Inference from hours

Proposition 3. *$b'_m(x)\beta\sigma$ and $b'_f(y)(1-\beta)\sigma$ are identified given Φ and λ from hour and wage distributions.*

Matching hours worked by married males with hours worked by single males on same wages, equation (8) implies that

$$h^1_m(x,y) - h^0_m(x) = -b'_m(x)t_m(x,y,z),$$

and integrating over z and married couples given (x,y)

$$\Delta_m(x,y) \equiv \mathbb{E}(h^1_m|x,y) - \mathbb{E}(h^0_m|x) = -b'_m(x)\beta\sigma\frac{\bar{t}_m(x,y)}{\beta\sigma}.$$

By symmetry, we also have:

$$h^1_f(x,y) - h^0_f(x) = -b'_f(y)t_f(x,y,z),$$

and

$$\Delta_f(x,y) \equiv \mathbb{E}(h^1_f|x,y) - \mathbb{E}(h^0_f|y) = -b'_f(y)(1-\beta)\sigma\frac{\bar{t}_f(x,y)}{(1-\beta)\sigma}.$$

Hence $b'_m(x)\beta\sigma$ and $b'_f(y)(1-\beta)\sigma$ are identified given Φ.[7]

[7]Note that the argument still applies if hours are measured with error and measurement errors have

Because only one private good's expenditure is recorder, it is not possible to to separate b'_m, b'_f from β and σ.

Proposition 4. *$A_m(x), A_f(y), B_m(x), B_f(y)$ are identified from from wages and hours given Φ, λ, β and σ for initial conditions $A_m(0) = A_f(0) = 0$ and $B_m(0) = B_f(0) = 1$.*

Given an arbitrary choice of β and σ, $A_m(x)$ and $A_f(y)$ can be estimated by solving the linear ordinary differential equations:

$$\frac{\mathrm{d}[xT - A_m(x)]}{\mathrm{d}x} - b'_m(x)[xT - A_m(x)] = \mathbb{E}(h^0_m|x),$$
$$\frac{\mathrm{d}[yT - A_f(y)]}{\mathrm{d}y} - b'_f(y)[yT - A_f(y)] = \mathbb{E}(h^0_f|y).$$

That is,

$$xT - A_m(x) = B_m(x) \int_0^x \frac{\mathbb{E}(h^0_m|x')}{B_m(x')} \,\mathrm{d}x', \tag{28}$$
$$yT - A_f(y) = B_f(y) \int_0^y \frac{\mathbb{E}(h^0_f|y')}{B_f(y')} \,\mathrm{d}y', \tag{29}$$

where

$$B_m(x) = \exp \int_0^x b'_m(x') \,\mathrm{d}x',$$
$$B_f(y) = \exp \int_0^y b'_f(y') \,\mathrm{d}y',$$

using initial conditions $A_m(0) = A_f(0) = 0$ and $B_m(0) = B_f(0) = 1$.

4.4 Identification of Φ and λ

Making use of

$$h^1_m(x, y) - h^0_m(x) = -b'_m(x) t_m(x, y, z),$$

we have that

$$\frac{h^1_m(x, y) - h^0_m(x)}{-b'_m(x)\beta\sigma} = \frac{s_m(x)}{\beta\sigma} + \frac{s(x, y)}{\beta} + \frac{z}{\sigma}$$
$$= \frac{\bar{t}_m(x, y)}{\beta\sigma} + \frac{z}{\sigma} - \mathbb{E}\left(\frac{z}{\sigma}\middle| x, y, \frac{z}{\sigma} > -\frac{s(x, y)}{\sigma}\right),$$

zero mean conditional on x, y, z.

and, symmetrically,

$$\frac{h_f^1(x,y)-h_f^0(y)}{-b_f'(y)(1-\beta)\sigma} = \frac{s_f(y)}{(1-\beta)\sigma} + \frac{s(x,y)}{1-\beta} + \frac{z}{\sigma}$$
$$= \frac{\bar{t}_f(x,y)}{(1-\beta)\sigma} + \frac{z}{\sigma} - \mathbb{E}\left(\frac{z}{\sigma}\middle| x,y,\frac{z}{\sigma} > -\frac{s(x,y)}{\sigma}\right).$$

These two equations relate $h_m^1(x,y) - h_m^0(x)$ and $h_f^1(x,y) - h_f^0(y)$ to x, y and $\frac{z}{\sigma}$, which distribution among married couples is distribution Φ truncated below by $\frac{z}{\sigma} > -\frac{s(x,y)}{\sigma}$. It follows that Φ and λ are two unknowns of two complicated nonlinear equations.

Suppose that neither $b_m'(x)\beta\sigma$ nor $\frac{s_m(x)}{\beta\sigma} + \frac{s(x,y)}{\beta}$ depend on Φ and λ, and likewise for the other equation, then the distribution of z/σ, alias Φ, is identified if $\alpha(x,y)$ tends to one for some limiting value of (x,y). Now, for all other x, y, the truncation $\frac{z}{\sigma} > -\frac{s(x,y)}{\sigma}$ makes z/σ depend on $\frac{s_m(x)}{\beta\sigma} + \frac{s(x,y)}{\beta}$. So, these equations look like standard deconvolution equations but are in reality much more complex. The convergence rate of any nonparametric estimator of Φ (complicated to construct) is thus bound to be low.

For any given Φ, however, it is straightforward to identify λ by matching standard moments such as the variance of hours for married couples:

$$\mathrm{Var}\left(\frac{h_m^1 - h_m^0}{-b_m'(x)\beta\sigma} - \frac{\bar{t}_m(x,y)}{\beta\sigma}\right) = \mathbb{E}\left[\mathrm{Var}\left(\frac{z}{\sigma}|x,y,\frac{z}{\sigma} > -\frac{s(x,y)}{\sigma}\right)\right]$$
$$= \mathrm{Var}\left(\frac{h_f^1 - h_f^0}{-b_f'(y)(1-\beta)\sigma} - \frac{\bar{t}_f(x,y)}{(1-\beta)\sigma}\right).$$

In addition, the equality between the first and third moments provides a specification test for the model.

5 Estimation

In this section we implement the results of the preceding section to provide information on the shape of the preference parameters, the marriage externality and transfers.

5.1 Numerical details

We discretise the set of wages using a grid of $n+1$ (non equally distant) Chebyshev points defined as:

$$x_j = \frac{x_{min} + x_{max}}{2} + \frac{x_{max} - x_{min}}{2}\cos\frac{j\pi}{n}, \quad 0 \le j \le n.$$

Choosing Chebyshev polynomials to approximate smooth functions on a compact set is convenient, as we can then use the Clenshaw-Curtis quadrature to approximate the integrals in the fixed-point and identifying conditions as

$$\int f(x)\,\mathrm{d}x \simeq \frac{x_{max}-x_{min}}{2}\sum_{j=0}^{n} w_j f(x_j),$$

where weights w_j are easily and efficiently calculated using Fast Fourier Transform (FFT).[8]

The fact that CC quadrature relies on Chebyshev polynomials of the first kind allows to interpolate functions very easily between points $y_0 = f(x_0), ..., y_n = f(x_n)$ using Discrete Cosine Transform (DCT):

$$f(x) = \sum_{k=0}^{n} Y_k \cdot T_k(x), \tag{30}$$

where Y_k are the OLS estimates of the regression of $y = (y_0, ..., y_n)$ on Chebishev polynomials

$$T_k(x) = \cos\left(k \arccos\left(\frac{x - \frac{x_{min}+x_{max}}{2}}{\frac{x_{max}-x_{min}}{2}}\right)\right),$$

but are more effectively calculated using FFT. The fact that the grid $(x_0, ..., x_n)$ is not uniform and is denser towards the edges of the support interval allows to minimise the interpolation error and thus avoids Runge's phenomenon.[9]

Another advantage of DCT is that, having calculated $Y_0, ..., Y_n$, then polynomial projections of $y = (y_0, ..., y_n)$ of any order $p \leq n$ are obtained by stopping the summation in (30) at $k = p$. Another advantage is that it is easy to approximate the derivative f' or the primitive $\int f$ simply by differentiating or integrating Chebyshev polynomials.[10]

[8] Trefethen 2008 explains why, though CC quadrature looks theoretically dominated by Gauss quadrature, which is exact for poynomials of degree $2n + 1$ when CC quadrature is exact only up to degree n, it is not in practice. Moreover, CC quadrature is much more easily implemented as finding the weights for Gauss quadrature requires finding the eigenvalues of a matrix. Instead, the weights for CC quadrature are straightforwardly obtained using FFT. The following MATLAB code can be used (Waldvogel, 2006):

```
function [nodes,wcc] = cc(n)
nodes = cos(pi*(0:n)/n);
N=[1:2:n-1]'; l=length(N); m=n-l;
v0=[2./N./(N-2); 1/N(end); zeros(m,1)];
v2=-v0(1:end-1)-v0(end:-1:2);
g0=-ones(n,1); g0(1+l)=g0(1+l)+n; g0(1+m)=g0(1+m)+n;
g=g0/(n^2-1+mod(n,2)); wcc=real(ifft(v2+g));
wcc=[wcc;wcc(1)];
```

[9] A MATLAB code for DCT is, with $y = (y_0, ..., y_n)$:

```
Y = y([1:n+1 n:-1:2],:);
Y = real(fft(Y/2/n));
Y = [Y(1,:); Y(2:n,:)+Y(2*n:-1:n+2,:); Y(n+1,:)];
f = @(x) cos(acos((2*x-(xmin+xmax))/(xmax-xmin))*(0:n))*Y(1:n+1);
```

[10] Note that

$$\cos(k \arccos x)' = \frac{k \sin(k \arccos x)}{\sin(\arccos x)}$$

In practice we tried values such as $n = 50, 100, 500$ on a laptop without running into any memory or computing time difficulty.

5.2 Calibration of non-identified parameters

Bargaining power is assumed to be evenly distributed between men and women, i.e. $\beta = 0.5$. The standard deviation of xT is 8440 and that of yT is 6171. We arbitrarily fix $\sigma = 1000$, the order of magnitude. This calibration affect the levels but not the shape of most estimated parameters.

Moreover, although Φ is in principle identified, constructing an estimator is complicated and its rate of convergence is likely to be very slow. We therefore do not attempt at estimating Φ and postulate a standard normal distribution instead; in which case,

$$\mu_\Phi(\alpha) = -\alpha\Phi^{-1}(1-\alpha) + \phi \circ \Phi^{-1}(1-\alpha),$$

where ϕ is the PDF of the standard normal distribution.

We postpone detailing the estimation of λ to the end of this section. For the moment, let us think of λ as being calibrated so that the meeting rate is 11.7 times per year for men and 13 for women, or $\lambda/\delta = .022$.

5.3 The matching probability

The equilibrium flow condition implies that

$$\alpha(x, y) = \frac{\delta}{\lambda} \frac{n(x, y)}{u_m(x)u_f(y)}.$$

The PDFs $n(x, y)/N$, $u_m(x)/U_m$ and $u_f(y)/U_f$ are estimated by Gaussian kernel density. We use twice the usual bandwidth to smooth the density functions in the tails. This is important as we divide n by $u_m u_f$ to calculate α. Additional smoothing is thus required. Figure 7 displays the shape of the matching probability function thus estimated. It is

and

$$\int \cos(k \arccos x)\,\mathrm{d}x = \begin{cases} x & \text{if } k = 0 \\ \frac{x^2}{2} & \text{if } k = 1 \\ \frac{\cos(k+1)x}{2(k+1)} - \frac{\cos(k-1)x}{2(k-1)} & \text{if } k \geq 2 \end{cases}$$

In calculating an approximation of the derivative, it is useful to smoothen the function by summing over only a few polynomials. Derivatives are otherwise badly calculated near the boundary. Moreover, our experience is that the approximation:

$$\int_{\underline{x}}^{x} \mathbf{1}\{t \leq x\} f(x)\,\mathrm{d}x \simeq \sum_{k=0}^{n} w_k \mathbf{1}\{t \leq x_k\} f(x_k)$$

gave similar results as integrating the interpolated function.

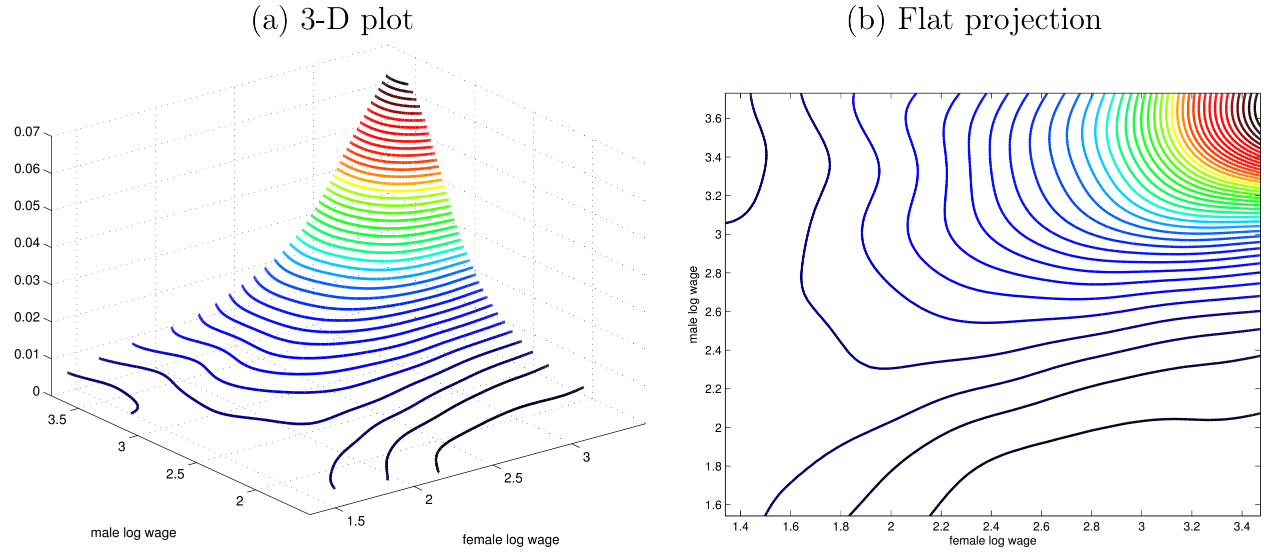

Figure 7: Matching probabilities $\alpha(x, y)$

unambiguously increasing in both wages. The region of high-female-low-male wages displays particularly low matching probability. Moreover, the matching probability increases exponentialy with both wages and is relatively flat for wages below the median.

5.4 Marriage externalities and transfers

We estimate conditional mean transfers using equations (25) and (26).

Figure 8, panel (a), shows the average household production function,

$$C(x, y) + \mathbb{E}[z|x, y, s(x, y) + z > 0] = \bar{t}_m(x, y) + \bar{t}_f(x, y),$$

while panel (b) displays the average household earnings from wage-work for comparison. It has essentially the same shape as the matching function $\alpha(x, y)$. Note that household production is less sensitive to female wage increases as it is to male wage increases. The externality that is shared between household members is estimated about 8-15% of total household earnings. Low income households receive more from marriage than rich households, as a proportion of labour earnings.

Figure 9 displays the estimated sharing rule, i.e. the transfer ratio $\bar{t}_m/\bar{t}_f$. The shape of the sharing rule is identified but not its absolute level, proportional to $\beta/(1-\beta)$. Using $\beta = 1/2$, the range of transfer ratios is estimated 0.5 - 2. The ratio of transfers increases faster in the male wage dimension than in the female wage dimension (see 2-D projections in panels (c) and (d)), in a way that is consistent with the marriage externality increasing faster with x than with y. Ressource allocation thus seems to be more profitable to men's wages than women's.

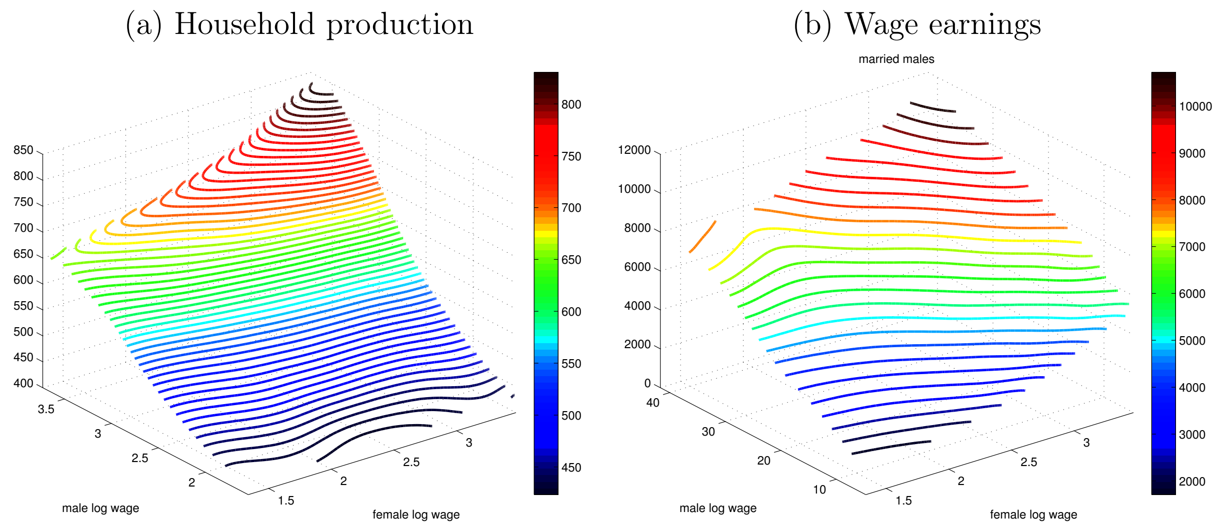

Figure 8: Total mean marriage externality $(\bar{t}_m(x, y) + \bar{t}_f(x, y))$

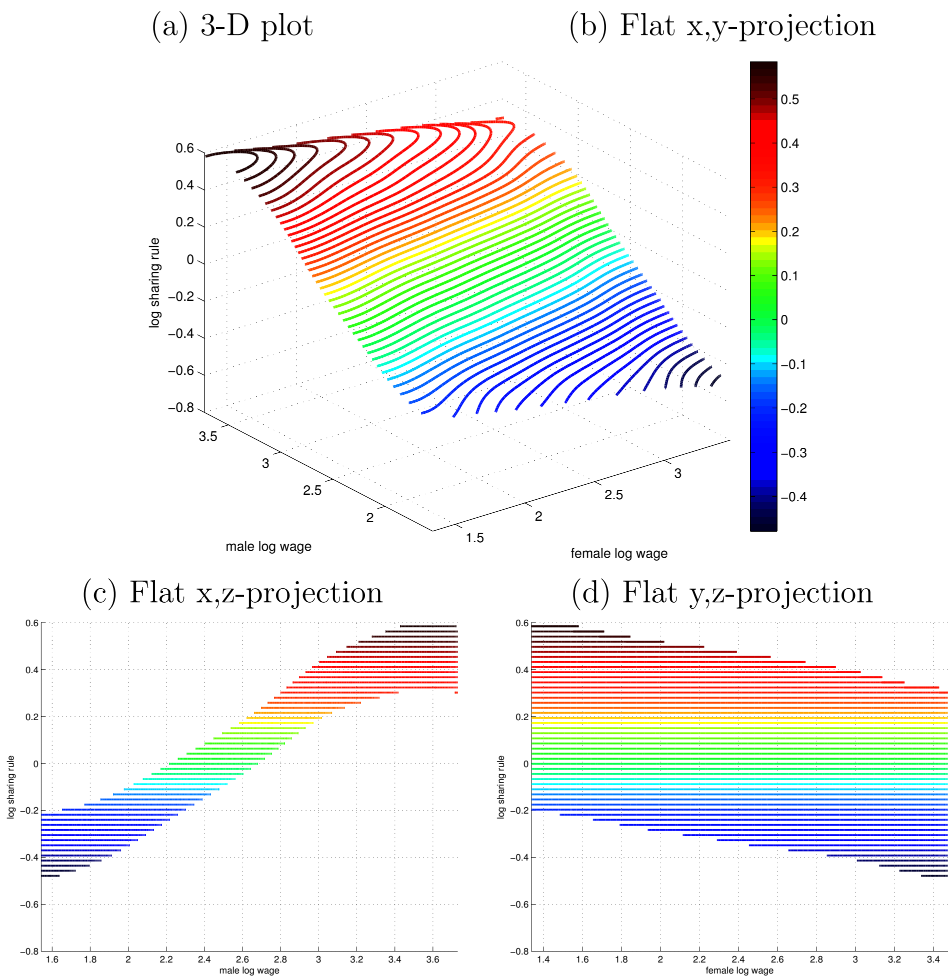

Figure 9: Sharing rule $(\bar{t}_m/\bar{t}_f)$

5.5 Preference parameters

Figure 11 shows the estimated values of income effects: $-b'_m(x)$ and $-b'_f(y)$, together with $B_m(x)$ and $B_f(y)$. Figure 12 shows estimates of price effects $T - A'_m(x)$ and $T - A'_f(y)$, as well as $A_m(x)$ and $A_f(y)$. A low-order polynomial approximation is shown (dashed curves) for comparison.

We estimate $b'_m(x)$ by regressing $\Delta_m(x, y)$ on $\bar{t}_m(x, y)$:

$$b'_m(x) = \frac{\int \Delta_m(x,y)\bar{t}_m(x,y)n(x,y)\,\mathrm{d}y}{\int \bar{t}_m(x,y)^2 n(x,y)\,\mathrm{d}y},$$

and similarly for women:

$$b'_f(x) = \frac{\int \Delta_f(x,y)\bar{t}_f(x,y)n(x,y)\,\mathrm{d}x}{\int \bar{t}_f(x,y)^2 n(x,y)\,\mathrm{d}x},$$

where the integrals are approximated using Clenshaw-Curtis quadrature. Then A and A' are calculated using the formulas of the identification section.

The fit of conditional mean hours is of course perfect when one conditions only on one's wage given the semi-nonparametric nature of the estimation. However the ability of the model to fit the conditional mean hours given both spouses' wages is limited by the form of the transfer functions. We find that the model fails to some extent fitting hours for high wage men and low wage women (see Figure 5.5).

For men, leisure (household production) is an inferior good – higher transfers increase hours worked – whereas for women, it is a normal good. This explains why married men work more than singles and married women less. The propensity of higher transfers to induce men's work is lower for higher wages, whereas it is the reverse for women.

5.6 The meeting rate λ

Parameter λ is estimated by minimising the Euclidian norm of

$$\begin{pmatrix} \mathrm{Var}\left(\frac{h_m^1 - h_m^0}{-b'_m(x)\beta\sigma} - \frac{\bar{t}_m(x,y)}{\beta\sigma}\right) - \mathbb{E}\left[\mathrm{Var}\left(\frac{z}{\sigma}|x,y,\frac{z}{\sigma} > -\frac{s(x,y)}{\sigma}\right)\right] \\ \mathrm{Var}\left(\frac{h_f^1 - h_f^0}{-b'_f(y)(1-\beta)\sigma} - \frac{\bar{t}_f(x,y)}{(1-\beta)\sigma}\right) - \mathbb{E}\left[\mathrm{Var}\left(\frac{z}{\sigma}|x,y,\frac{z}{\sigma} > -\frac{s(x,y)}{\sigma}\right)\right] \end{pmatrix}$$

after functions $x, y \mapsto b'_m(x), b'_f(y), \bar{t}_m(x,y), \bar{t}_f(x,y), s(x,y)$ have been estimated given λ. The two hour variances are estimated by empirical variances calculated from the subsample of 6827 married couples' observations. Otherwise, we compute

$$\mathrm{Var}\left(\frac{z}{\sigma}|x,y,\frac{z}{\sigma} > -\frac{s(x,y)}{\sigma}\right) = \frac{\phi\left(\frac{s(x,y)}{\sigma}\right)}{\alpha(x,y)}\left[\left(\frac{s(x,y)}{\sigma}\right)^2 + 1 - \frac{\phi\left(\frac{s(x,y)}{\sigma}\right)}{\alpha(x,y)}\right]$$

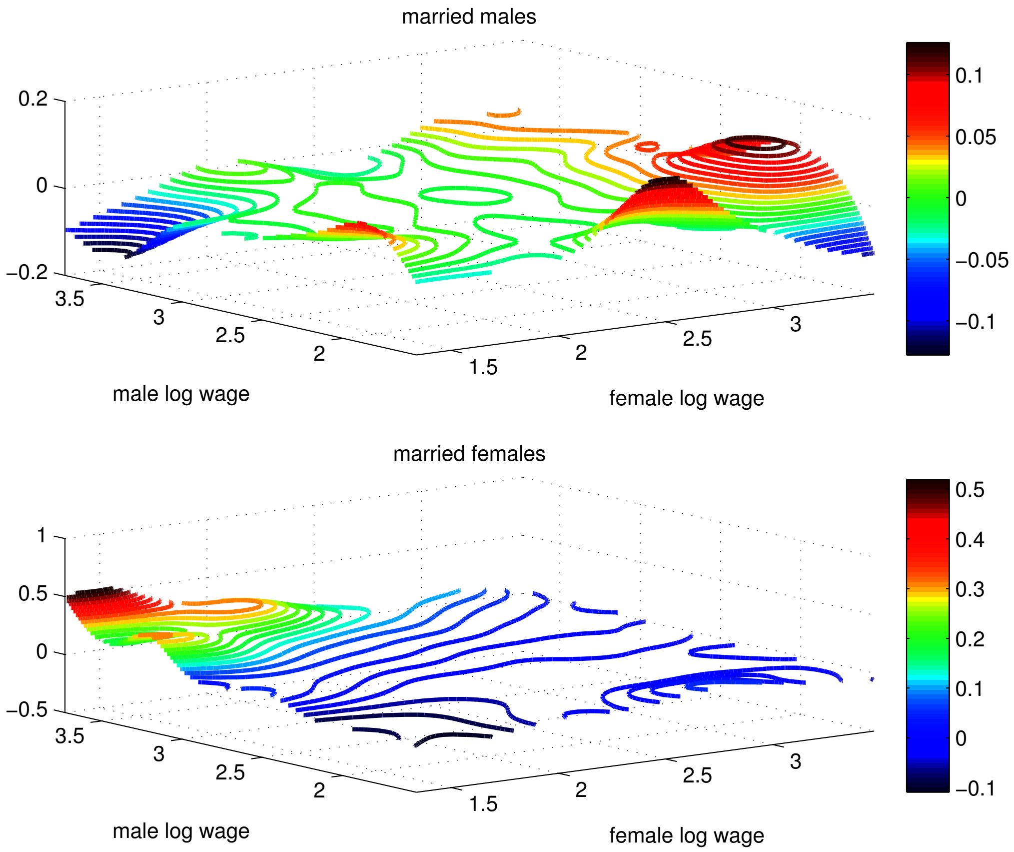

Figure 10: Fit of conditional mean hours $(\widehat{h}(x,y)/h(x,y)-1)$.

with $\alpha(x,y) = \Pr\left\{\frac{z}{\sigma} > -\frac{s(x,y)}{\sigma}\right\}$, and

$$\mathbb{E}\left[\mathrm{Var}\left(\frac{z}{\sigma}|x,y,\frac{z}{\sigma} > -\frac{s(x,y)}{\sigma}\right)\right] = \iint \mathrm{Var}\left(\frac{z}{\sigma}|x,y,\frac{z}{\sigma} > -\frac{s(x,y)}{\sigma}\right)\frac{n(x,y)}{N}\,\mathrm{d}x\mathrm{d}y$$
$$= \frac{\lambda}{\delta N}\iint \phi\left(\frac{s(x,y)}{\sigma}\right)\left[\left(\frac{s(x,y)}{\sigma}\right)^2 + 1 - \frac{\phi\left(\frac{s(x,y)}{\sigma}\right)}{\alpha(x,y)}\right] u_m(x)u_f(y)\,\mathrm{d}x\mathrm{d}y.$$

We find that the criterion is minimised for an aggregate matching probability of

$$\iint \alpha(x,y)\frac{u_m(x)}{U_m}\frac{u_f(y)}{U_f}\frac{n(x,y)}{N}\,\mathrm{d}x\mathrm{d}y = .7\%.$$

Given the estimated divorce rate, this implies about 12 meetings per year on average for singles.

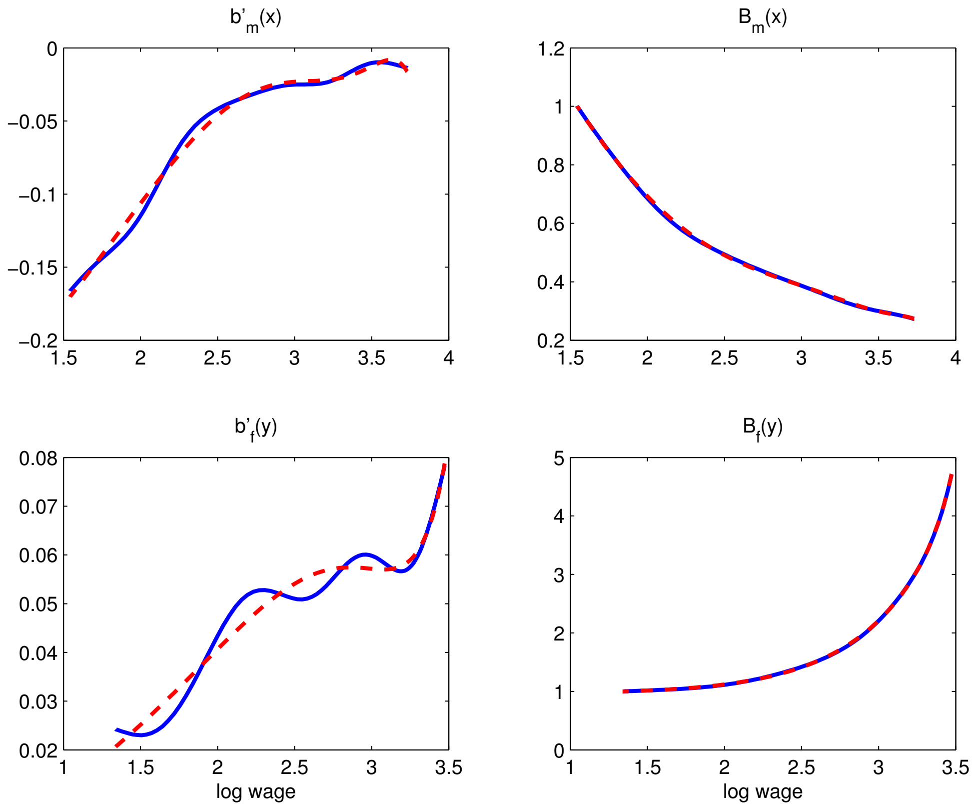

Figure 11: Income effects (dotted line: 4th order approximation)

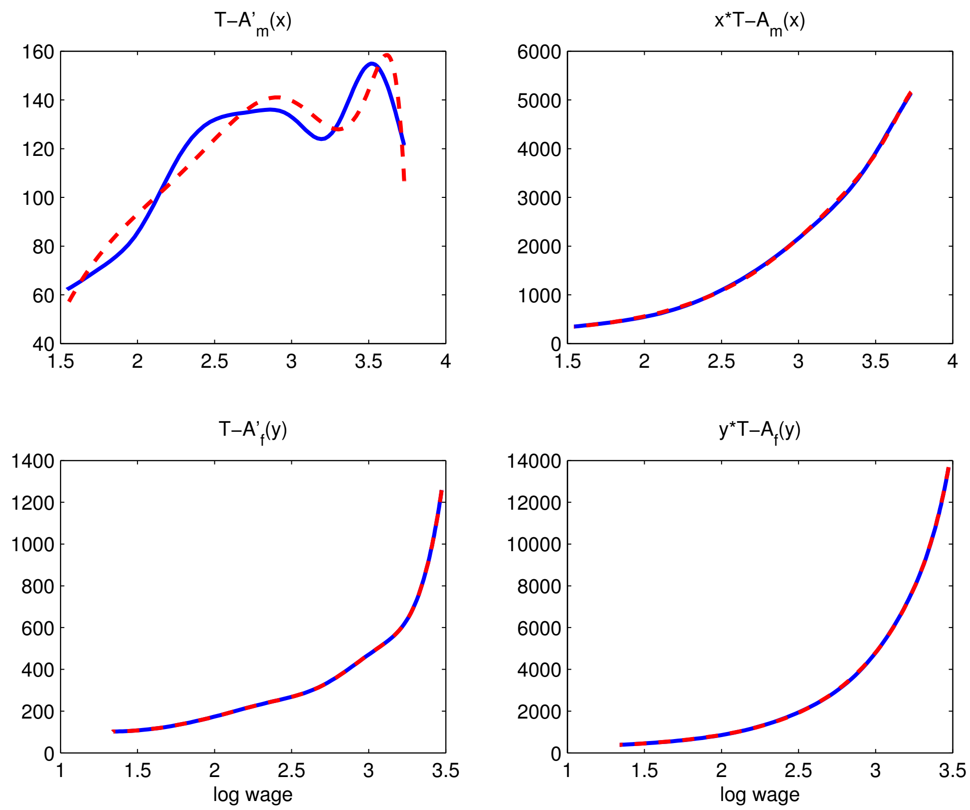

Figure 12: Price effects (dotted line: 4th order approximation)

The corresponding estimated variance are:

$$\mathrm{Var}\left(\frac{h_m^1 - h_m^0}{-b_m'(x)\beta\sigma} - \frac{\bar{t}_m(x,y)}{\beta\sigma}\right) = 13.4$$
$$\mathrm{Var}\left(\frac{h_f^1 - h_f^0}{-b_f'(y)(1-\beta)\sigma} - \frac{\bar{t}_f(x,y)}{(1-\beta)\sigma}\right) = 7.4$$
$$\mathbb{E}\left[\mathrm{Var}\left(\frac{z}{\sigma}|x,y,\frac{z}{\sigma} > -\frac{s(x,y)}{\sigma}\right)\right] = 10.8$$

Calculating a formal test of the equality of these numbers is difficult given the complexity of the estimation, but do not seem sufficiently far appart to cast doubts on the quality of the adequation of the model to the data.

6 Simulation

Now, we take the estimated externality function $C(x,y)$ and kernel density estimates of the unconditional wage distribution $\ell_m(x)$ and $\ell_f(y)$ and we simulate the equilibrium. Because λ should be a equilibrium parameter, we postulate a Cobb-Douglas meeting function $M(U_m, U_f) = M_0 U_m^{1/2} U_f^{1/2}$ and we estimate M_0 as $M_0 = \lambda U_m^{1/2} U_f^{1/2}$ for the value of λ that we used in estimation. The fixed point equations (16)-(19) are solved using the standard iteration algorithm.

Simulating the model using the full nonparametric estimates of the structural parameters yields, as expected a perfect fit. What is more challenging is to use a small-order polynomial approximation of the functional parameters. We thus start by selecting the first four coefficient of the DCT of $b_m'(x_i)$ and $b_f'(y_i)$ for $n+1$ Chebyshev nodes. The thus truncated DCT produces fourth-order polynomial approximations of $b_m'(x)$ and $b_f'(y)$. Then we integrate the fourth-order polynomial approximations to obtain $b_m(x) = \log B_m(x)$ and $b_f(y) = \log B_f(y)$.

We also select the first four coefficient of the DCT of $h_m^0(x_i)/B_m(x_i)$ and $h_f^0(y_i)/B_f(y_i)$, $i = 0, ..., n$. The truncated DCT then produces a fourth-order polynomial approximation of functions $h_m^0(x)/B_m(x)$ and $h_f^0(y)/B_f(y)$. Then we calculate $A_m(x)$ and $A_f(y)$ from equations (28) and (29) by analytical integration of these polynomial aproximations of $h_m^0(x)/B_m(x)$ and $h_f^0(y)/B_f(y)$. Finally $A_m'(x)$ and $A_f'(y)$ follow by analytical differentiation. Figures 11 and 12 show how far off the nonparametric estimates the 4th order polynomial approximations are.

Then, we solve for the equilibrium. We estimate $U_m = 6222, U_f = 6971, N = 6753$ (instead of $U_m = 6386, U_f = 7098, N = 6827$). The estimation error is esentially due to the fact that kernel density estimates on a discrete grid do not exactly sum to one. Figures 13 and 14 show the corresponding fit for univariate densities and conditional hours. The fit is quite good. Note that the suspicious ondulations easily disappear by increasing the order of the polynomial approximation.

7 Conclusion

We have developed a prototypical version of a search-matching model of the marriage market with labour supply. We also work out new (in applied economics) and useful numerical approximations techniques in order to estimate nonparametrically the main functional parameters of the model. We study identification and estimation from cross-section data. We collapse the SIPP panel in one single cross-section first to reduce the prevalence of labour-market non-participation, that this first version of the model cannot allow for. Second, although individuals base their marriage decisions on expectations of future transfers, the only source of risk in the model is an exogenous divorce probability. This thus makes the model as close to a static model a dynamic model can be. We leave to further work the task of developing a proper dynamic environment allowing, in particular, for a correct description of individual dynamics of divorces and remarriages.

Despite these limitations, the model is rich of interesting lessons. We first show that wage distributions of single men and women and of married couples provide useful information on structural matching parameters in a way that had not been previously recognised in the literature on collective models of the family. We find that though the joint distribution of wages across married couples is not strongly elleptical, the estimated matching probability resulting from the steady-state flow condition is strongly increasing in both male and female wages. By matching hours worked by married individuals to hours worked by single individuals with same wages, we can estimate the relationship between transfers between mates and wages. We find that marriage generates an externality that is (tentatively) valued between 8 and 15% of household earnings. This externality is then shared between husband and wife in a way that is more favourable to the richer partner, and much more so for men than for women. The model is also able to explain why married women work less and married men more by making leisure an inferior good for men and a normal good for women. The only noticeable failure of the model is a relative incapacity to predict the higher labour market participation of high-wage men married to low-wage women and the corresponding lower participation of women.

Many possible extensions of the model easily come to mind. One could seek to endogenize divorce, either thru shocks to z, the match-specific component of the marriage externality, or via on-the-marriage search. Another important extension will be to allow for other dimensions of heterogeneity but wages. A third obvious extension is to introduce children.

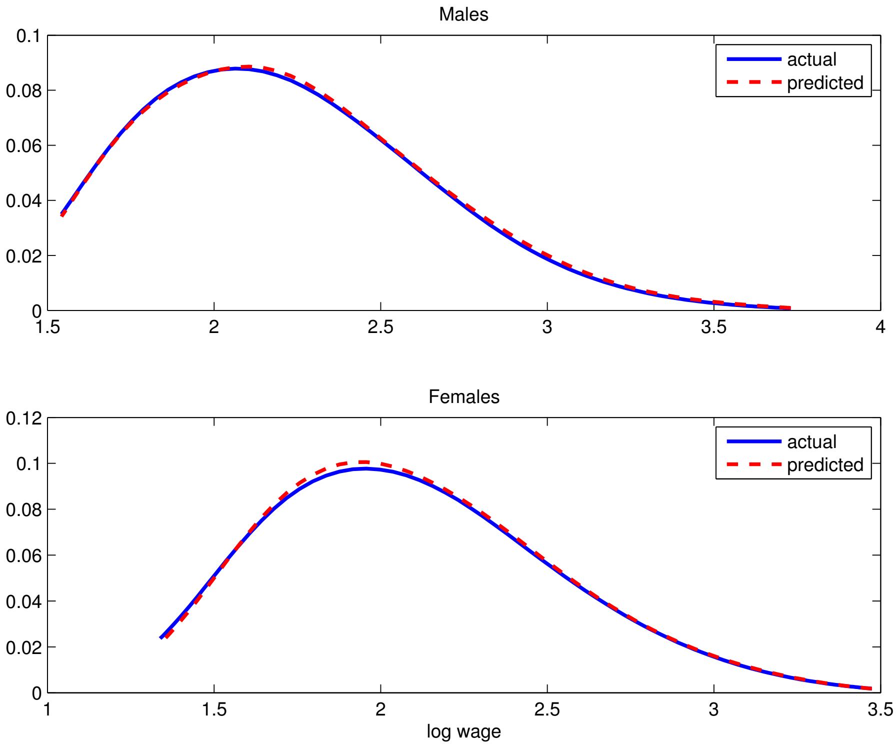

Figure 13: Fit of wage densities for singles - 4th order approximation

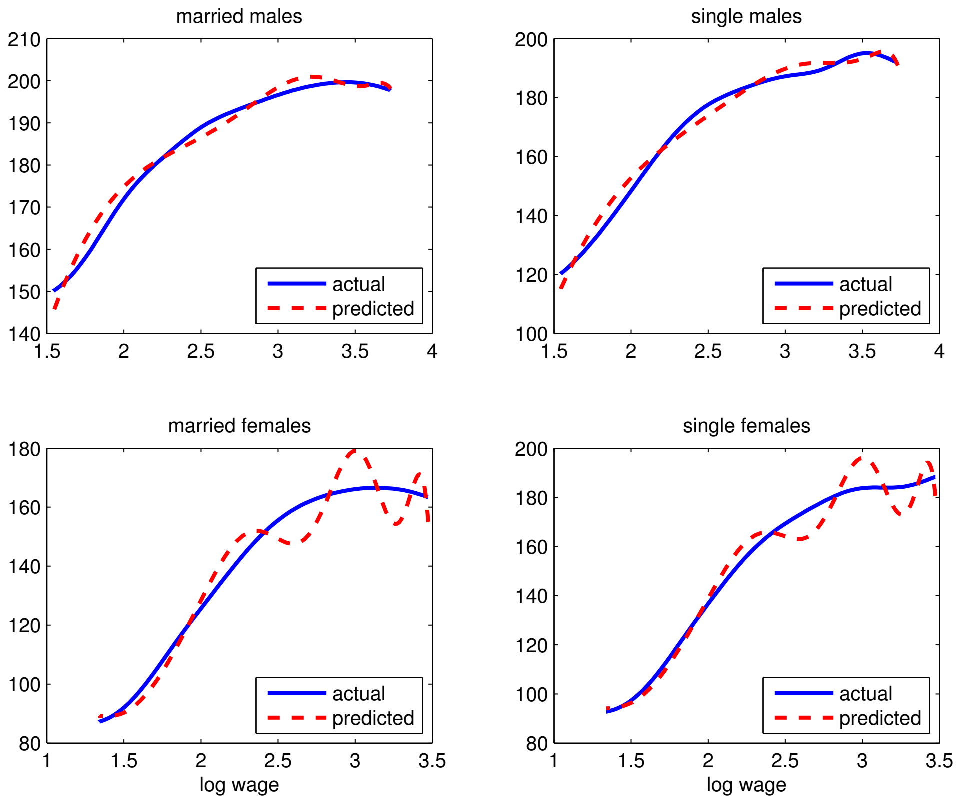

Figure 14: Fit of conditional mean hours - 4th order approximation

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