Counterfactuality of "counterfactual" communication

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The counterfactuality of the recently proposed protocols for direct quantum communication is analyzed. It is argued that the protocols can be counterfactual only for one value of the transmitted bit. The protocols achieve a reduced probability of detection of the particle in the transmission channel by increasing the number of paths in the channel, but it is not below the probability of detection of a particle actually passing through such a multi-path channel.

I. INTRODUCTION

Penrose [1] coined the term "counterfactuals" for describing quantum interaction-free measurements (IFM) [2].

> Counterfactuals are things that might have happened, although they did not in fact happen.
>
> Penrose 1994

He argued that in the IFM, an object is found because it might have absorbed a photon, although actually it did not. The idea of the IFM has been applied to "counterfactual computation" [3], a setup in which one particular outcome of a computation becomes known in spite of the fact that the computer did not run the algorithm. Noh [4], combining the IFM with the BB84 cryptographic protocol [5], created a counterfactual cryptography: a method for secret key distribution using events in which there was no particle present in the transmission channel.

It was argued [6], that a modification of the counterfactual computation proposal which included quantum Zeno effect achieves counterfactuality for all outcomes of the computation. Recently, this idea has been used for performing "counterfactual communication" [7], which supposedly allowed to send information from Bob to Alice without transferring any physical system between them, i.e. without a particle being present in the transmission channel. The transmission happens in a counterfactual way: a mere possibility of transmitting the particle allows transmission of the value of the bit without actually transmitting the particle.

I find all these results very paradoxical. They contradict physical intuition of causality: information is usually transmitted continuously in space. I argued [8] that to resolve the paradoxical feature of the IFM one has to adopt the many-worlds interpretation (MWI) of quantum mechanics [9, 10] in which the particle touches the object in a parallel world restoring causality on the level of the physical universe which includes all the worlds. However, I believe that a protocol which is capable of transmitting both values of a bit without a particle present in the transmission channel is an impossible task irrespectively of the interpretation of quantum mechanics one adopts. I have expressed this opinion already [11, 12], but more protocols were suggested [13] and the controversy remains open [14–16]. Clarification of these conceptual issues becomes particularly important due to recent increased interest in applications of counterfactual protocols [17–22]. Here I discuss the issue in more details in hope to resolve the controversy.

II. COMMUNICATION WITH QUANTUM PARTICLES

There is a surprisingly low bound on the number of bits which can be sent using 1 qubit: The Holevo bound of 1, when the qubit is not entangled [23], and 2, when entanglement is allowed [24]. This is when we transmit one particle with an internal structure of a qubit. In the question we want to analyze, the particle does not have an internal structure: the information is encoded in the presence or absence of the particle.

Consider Bob, in a closed region, who wants to communicate a classical bit to Alice. For a bit value 1, he places a shutter in a particular position, while for value 0, the shutter is absent. Alice is everywhere outside Bob's closed region. In order to know the value of Bob's bit, she can send a particle through the place where the shutter might be, see Fig. 1. If we define the boundary of Bob's region as the transmission channel, then Alice's particle crosses it twice: when it moves towards and from Bob. Note that for value 1, there is no particle crossing the transmission channel moving from Bob to Alice, but, of course, the particle moving from Alice to Bob remains.

Quantum mechanics, via IFM allows, at least sometimes, to transmit the bit 1 such that the particle does not appear in the transmission channel also on the way to Bob, see Fig. 2. Alice arranges a Mach-Zehnder interferometer (MZI) tuned to destructive interference in one of the ports such that one arm of the interferometer crosses the place where Bob's shutter might be. Detection of the particle in the dark port of the interferometer tells us with certainty that bit is equal to 1 (the shutter is present).

There are several arguments for the claim that in the cases the particle is detected in the dark port, it was not present in the transmission channel. The simplest one is: "If it were in the transmission channel, it could not be detected by Alice". In my view, this argument cannot be

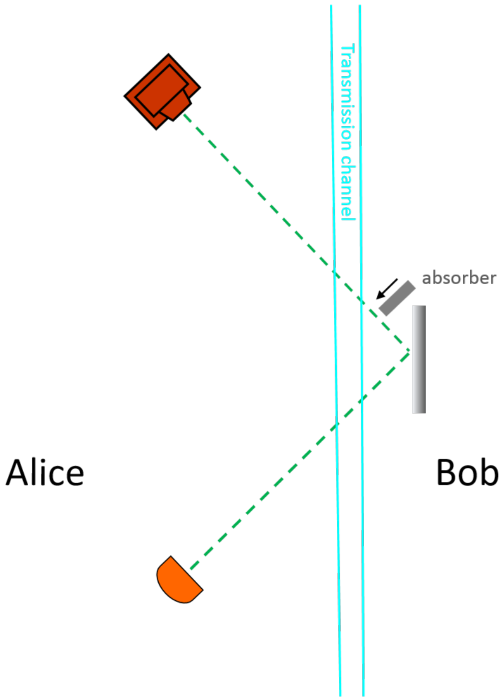

FIG. 1: Simple communication with a quantum particle. Alice sends a particle to Bob and knows the bit chosen by Bob through observation if the particle comes back to her or not.

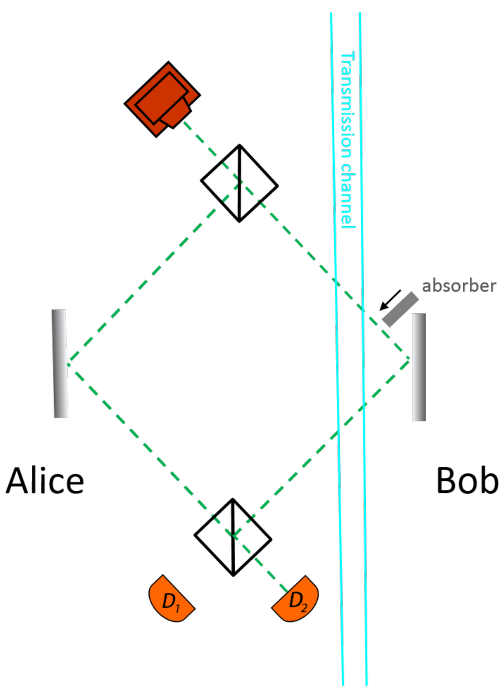

FIG. 2: A single bit value communication with IFM. The interferometer is tuned in such a way that detector D_1 never clicks if the paths are free. Alice knows that Bob chose bit 1 (blocked the path) when she observes the click in D_1.

used for claims about quantum particles [25]. I, instead, suggest to rely on the fact that the particle does not leave any trace in the transmission channel.

For transmitting just one bit value, one can use a classical particle which will not be present in the transmission channel [26]. Alice and Bob agree in advance that at a particular time, for a particular value of a bit, Bob sends the particle to Alice, while for the other bit value, he sends nothing. Note, however, that this classical protocol cannot achieve the task of the quantum IFM. In the IFM, Alice learns about the shutter in Bob's place without prior agreement with Bob and without Bob knowing that she acquired this knowledge.

In the IFM method of Fig. 2, Alice, getting the click in the bright port, does not obtain a definite knowledge of the classical bit 0. Without the shutter the click happens with certainty, but with the shutter it might happened too, with probability 25%. This protocol is also not the most efficient method for communication of bit 1. In half of the cases the particle is lost (then we get the information that the bit is 1 but not in a counterfactual way) and in one quoter of cases, when the particle is detected by a bright port, we get no decisive information.

The quantum method can be modified to be a reliable transmission of both values of the classical bit. To this end, instead of the shutter, Bob inserts a half-wave plate (HWP), Fig. 3. This transforms the dark port to bright port and vice versa. However, half of the wave always passes through the communication channel, so one cannot argue that this is a counterfactual communication.

Combining the quantum Zeno effect with the IFM [27] allows to perform a counterfactual transmission of bit 1 with probability arbitrary close to 1. The device consists of a chain of N unbalanced interferometers with identical beam splitters having small transmittance, see Fig. 4a. I define the transmittance T_1 using parameter α: $T_1 = \sin^2\alpha$ and use the notation such that each one of the beam splitters in the chain performs the following unitary evolution of the state of the particle:

$$\begin{aligned} |L\rangle &\rightarrow \cos\alpha|L\rangle + \sin\alpha|R\rangle, \\ |R\rangle &\rightarrow -\sin\alpha|L\rangle + \cos\alpha|R\rangle. \end{aligned} \quad (1)$$

A straightforward calculation shows that n beam splitters perform the following evolution of the wave packets of the particle entering the chain:

$$\begin{aligned} |L\rangle &\rightarrow \cos n\alpha\; |L\rangle + \sin n\alpha\; |R\rangle, \\ |R\rangle &\rightarrow -\sin n\alpha|L\rangle + \cos n\alpha\; |R\rangle. \end{aligned} \quad (2)$$

We set parameter of transmittance $\alpha = \frac{\pi}{2N}$. Then, the chain of the interferometers moves the wave packet of the particle from one side to the other: $|L\rangle \rightarrow |R\rangle$, $|R\rangle \rightarrow -|L\rangle$.

The Zeno effect takes place when the right arms of the interferometers are blocked, Fig. 4b. The state remains $|L\rangle$ with probability $\cos^{2N}\frac{\pi}{2N}$. When N is large,

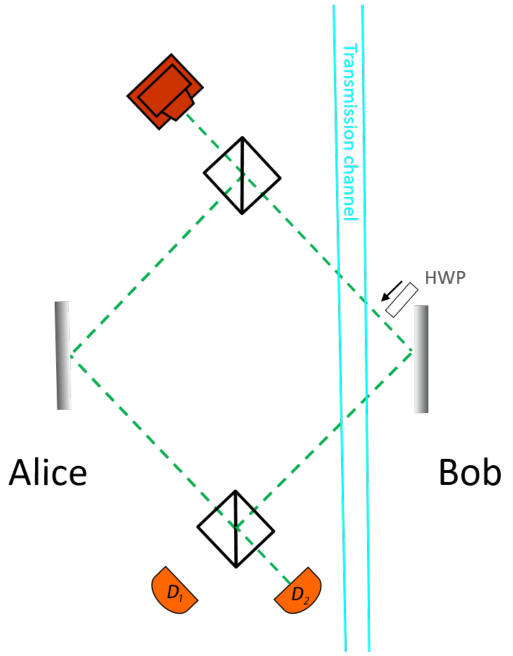

FIG. 3: Communication with MZI and HWP. The interferometer is tuned to destructive interference towards D_1. Bob communicates the bit to Alice by changing the destructive interference to detector D_2 by inserting the HWP on the right path of the particle.

$\cos^{2N}\frac{\pi}{2N} \simeq 1-\frac{\pi^2}{4N}$, so the the probability becomes close to 1.

In summary, when the bit is 0, Bob does nothing and Alice gets the click with certainty at the right port in the detector D_2. When the bit is 1, Bob blocks the interferometers and Alice gets click with a very high probability at the left port in the detector D_1.

III. "DIRECT COUNTERFACTUAL QUANTUM COMMUNICATION"

In this section I will describe a recent protocol by Salih et al. [7]. The prototype of this method is "Counterfactual quantum computation through quantum interrogation" [6, 11]. To explain it, let us first consider MZI nested inside another MZI, see Fig. 5. The inner interferometer is tuned in such a way that when it is undisturbed, there is destructive interference toward the second beam splitter of the external interferometer, see Fig. 5a. The external interferometer is tuned in such a way that there is a destructive interference toward D_2 when the lower path of the inner interferometer is blocked, see Fig. 5b. This configuration provides (sometimes) definite information about value 0 of the classical bit. Indeed, click in D_2 is possible only if the shutter is not present. One can naively argue that Alice obtains this information in a counterfactual way, since for free inner interferometer the particle cannot pass through Bob's site and reach D_2. However, as I already mentioned above and as I will discuss more in Section V, the presence of the weak trace inside inner interferometer makes this claim incorrect.

Salih et al. [7] argued that a modification of the scheme with nested interferometer based on quantum Zeno effect leads to a protocol for transmitting both values of clas-

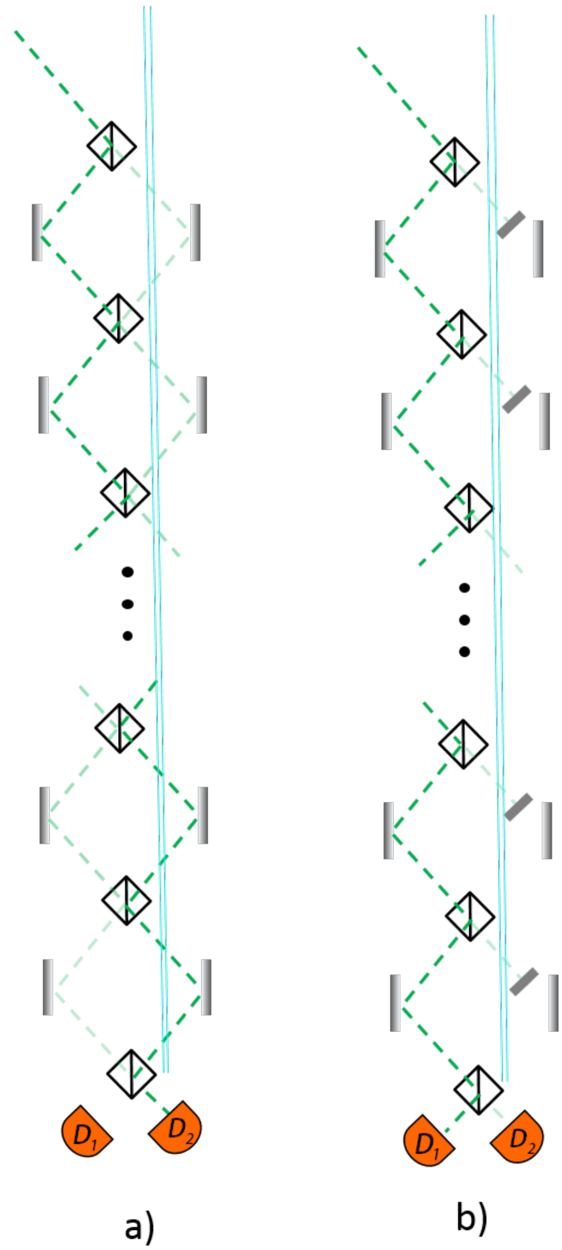

FIG. 4: Efficient communication using IFM and quantum Zeno effect. a). The chain of interferometers wih highly reflective beam splitters is tuned such that the particle has destructive interference towards D_1. b). Bob blocks the interferometers and then, due to Zeno effect, the particle reaches detector D_1 with probability close to 1.

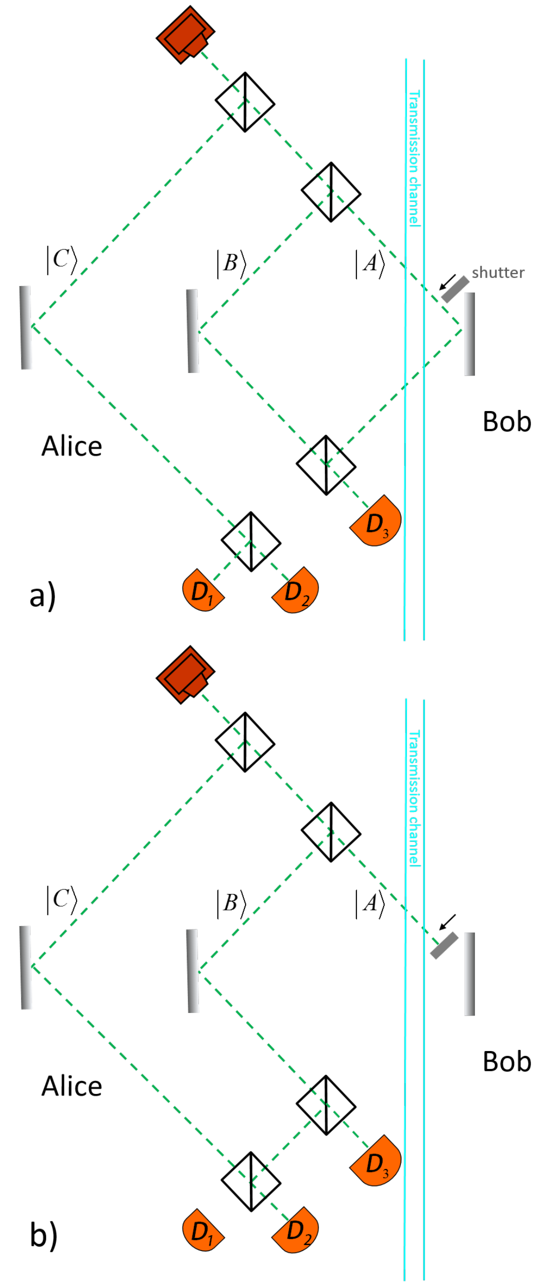

FIG. 5: Communication with nested MZIs. a). The inner interferometer is tuned such that the particle cannot path through the upper arm of the large interferometer. b). There is a destructive interference towards D_1 when the right path of the inner interferometer is blocked.

sical bit in a counterfactual way with efficiency arbitrarily close to 100%. In the protocol, $M-1$ chains of the $N-1$ interferometers described in Fig. 4 are inserted into another chain of interferometers with M beam splitters performing similar unitary operation with beam splitters having small albeit different transmittance. The relation between the number of beam spitters and their transmittance is again, $T_1 = \sin^2 \frac{\pi}{2M}$. To simplify the analysis, I modify Salih et al. protocol making it slightly less efficient, but still good according to their line of arguments. The modification is the replacement of the mirrors of the external chain of interferometers by highly reflective beam splitters. Transmitted waves are lost, but the modification balances the losses in the inner chains when shutters are introduced, such that the state of particles which are not lost is described by (2).

The setup is described in Fig. 6. The external chain of interferometers has M beam splitters with transmittance $T_2 = \sin^2 \frac{\pi}{2M}$ and the transmittance of the side beam splitters serving as mirrors is $T_3 = 1 - \cos^{2N} \frac{\pi}{2N}$.

All right mirrors of the internal chains are in the hands of Bob. He knows that Alice sends a particle at a particular time in the top of the external chain in the state $|L\rangle$. If Bob wants to communicate bit 1, he blocks all inner interferometers, see Fig. 6a. Then, after m beam splitters of the external interferometer the *normalized* quantum state is

$$|\Psi_m^{(1)}\rangle = \cos^{(m-1)N} \frac{\pi}{2N} \left(\cos \frac{m\pi}{2M} |L\rangle + \sin \frac{m\pi}{2M} |R\rangle \right) + ... \tag{3}$$

and after all M beam splitters the state is

$$|\Psi_M^{(1)}\rangle = \cos^{(M-1)N} \frac{\pi}{2N} |R\rangle + ... \tag{4}$$

In both equations "..." signify states orthogonal to the term presented explicitly. If $1 \ll M \ll N$, then the norm of the leading term in (4) is close to 1:

$$\cos^{2(M-1)N} \frac{\pi}{2N} \simeq 1 - \frac{\pi^2 M}{4N}. \tag{5}$$

This means that in the limit of large N, Bob's choice of bit 1, corresponding to blocking of the inner interferometers, leads to click of Alice's detector D_2. Since the state $|\Psi_M^{(1)}\rangle$ is orthogonal to state $|L\rangle$ at the output port of the interferometer, there is zero probability for a click of D_1. There is a probability of not detecting the particle by detectors D_1 and D_2. It can be lost on Alice's or Bob's sides. However, this probability vanishes for large N.

If Bob wants to communicate a bit 0, instead, he does nothing, Fig. 6b. Then, every wave packet entering any of the inner chains of the interferometers follow evolution (2) inside this chain and does not come back to the external interferometer. At the output of the interferometer, the normalized quantum state is

$$|\Psi_M^{(0)}\rangle = \cos^{(M-1)N} \frac{\pi}{2N} \cos^M \frac{\pi}{2M} |L\rangle + ... \tag{6}$$

Under the condition $1 \ll M \ll N$, the norm of the leading term in (6) is also close to 1:

$$\cos^{2(M-1)N} \frac{\pi}{2N} \cos^{2M} \frac{\pi}{2M} \simeq 1 - \frac{\pi^2}{4} \left(\frac{M}{N} + \frac{1}{M} \right). \tag{7}$$

Detection of the particle in detector D_1 tells Alice that Bob sends bit 1. Note, that there is a non zero probability for a failure which might become large if the condition

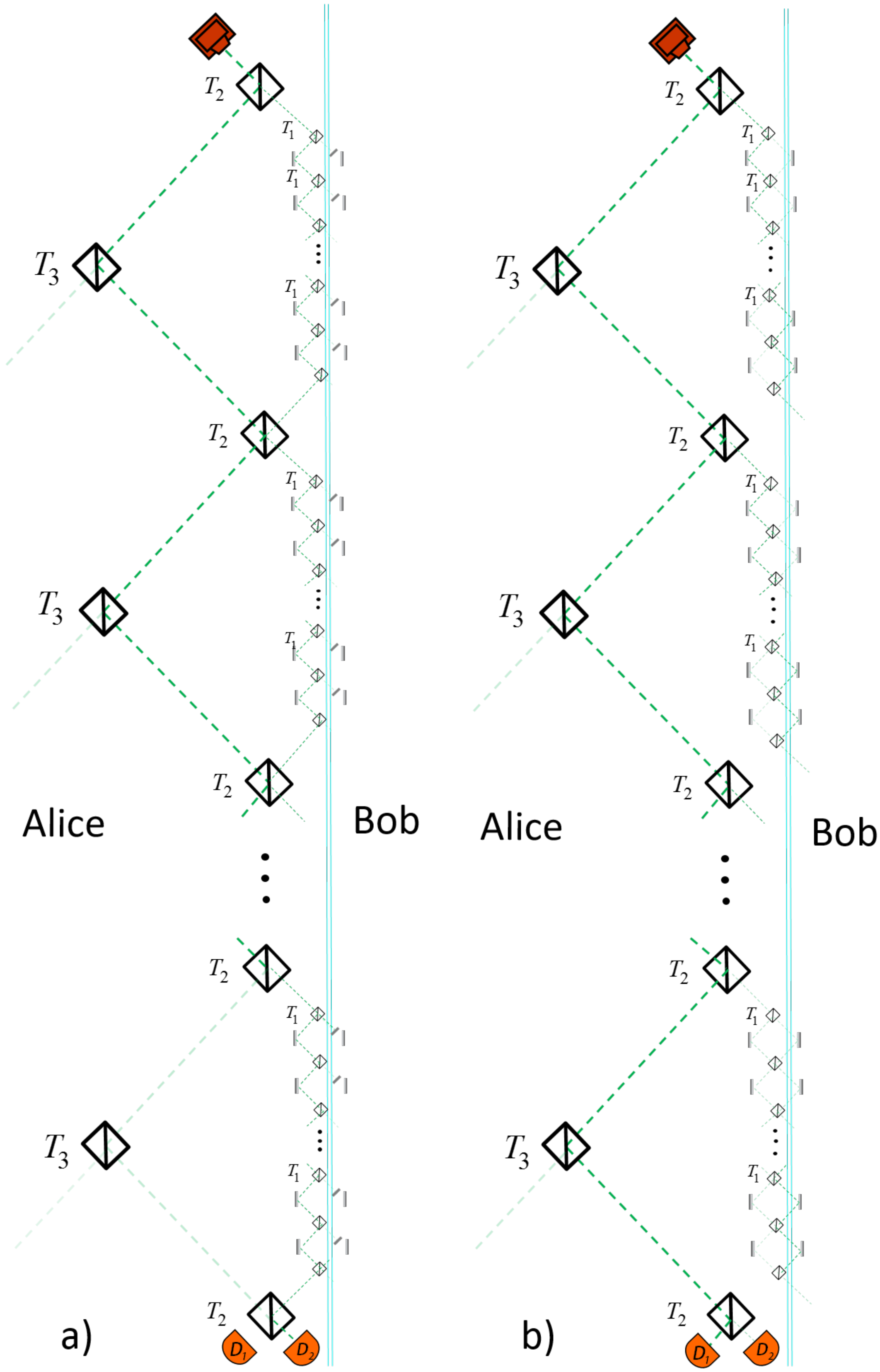

FIG. 6: "Direct counterfactual quantum communication" a). For bit 1 Bob blocks all inner interferometers. In this case detector D_2 clicks with probability close to 1, while D_1 cannot click. b). For bit 0 Bob leaves all inner interferometers undisturbed. For large N, D_1 clicks with probability close to 1, while the probability for a click in D_2 goes to zero.

$1 \ll M \ll N$ is not fulfilled. The particle can be lost and the particle can be detected by D_2. However, if the condition holds, the probability for a failure is vanishingly small. The probability for loosing the particle and getting no result is of order $\frac{\pi^2 M}{4N}$ for bit 1 and somewhat larger, $\frac{\pi^2}{4}\left(\frac{M}{N}+\frac{1}{M}\right)$ for bit 0. The click of D_2 tells Alice with certainty that the bit is 1, and the click of D_1 tells that the bit is 0 with only a very small probability for an error: about $\frac{\pi^2}{4M^2}$. We have shown that this is a good direct communication protocol.

IV. "DIRECT QUANTUM COMMUNICATION WITH ALMOST INVISIBLE PHOTONS"

As in the simple example in Sec. II (Fig.3), using HWPs instead of absorbing shutters leads to a communication protocol which is theoretically free of errors. Li et al. [13] suggest such a protocol and argue that it has "arbitrarily small probability of the existence of the particle in the transmission channel".

The configuration is similar to the experiment of Salih et al. [7]: a chain of $M-1$ interferometers with inner chains of $N-1$ interferometers (but now M, N have to be even numbers). Without absorbers the evolution is unitary and Zeno effect is not used in this protocol. There is no need to modify the protocol as I did in the previous section by replacing ideal mirrors with partially reflecting mirrors, because there are no losses to compensate.

Another difference in the protocol is the transmittance of the beam splitters in the inner chains. The parameter α is bigger by a factor of 2: $\alpha = \frac{\pi}{N}$. As a result, the chain (without the HWPs) works as two consecutive inner chains of the protocol discussed in the previous section. The first half of the chain moves the wave packet to the right side and the second brings it back to the left. From (2) we obtain the transformation of the wave packet in the inner chain of the interferometers $|L\rangle \rightarrow -|L\rangle$, see Fig 7a.

When the HWPs are inserted in every interferometer of the inner chain, see Fig. 7b, they cause the π phase change of every state $|R\rangle$ and, consequently, every second beam splitter reverses the operation of the previous one:

$$|L\rangle \rightarrow \cos\alpha|L\rangle + \sin\alpha|R\rangle \rightarrow \cos\alpha|L\rangle - \sin\alpha|R\rangle \rightarrow |L\rangle. \tag{8}$$

Since every chain has an even number of beam splitters, the transformation of the wave packet in the inner chain is $|L\rangle \rightarrow |L\rangle$.

The setup for sending bit 0 is described on Fig. 8a. Bob leaves the inner interferometers untouched. Then, each inner chain of the interferometers changes the phase of the quantum state of the particle: $|L\rangle \rightarrow -|L\rangle$. State $|L\rangle$ of the inner interferometer is a state $|R\rangle$ of an external interferometer. Thus, the operation of the first external interferometer is

$$|L\rangle \rightarrow \cos\alpha|L\rangle + \sin\alpha|R\rangle \rightarrow \cos\alpha|L\rangle - \sin\alpha|R\rangle \rightarrow -|L\rangle. \tag{9}$$

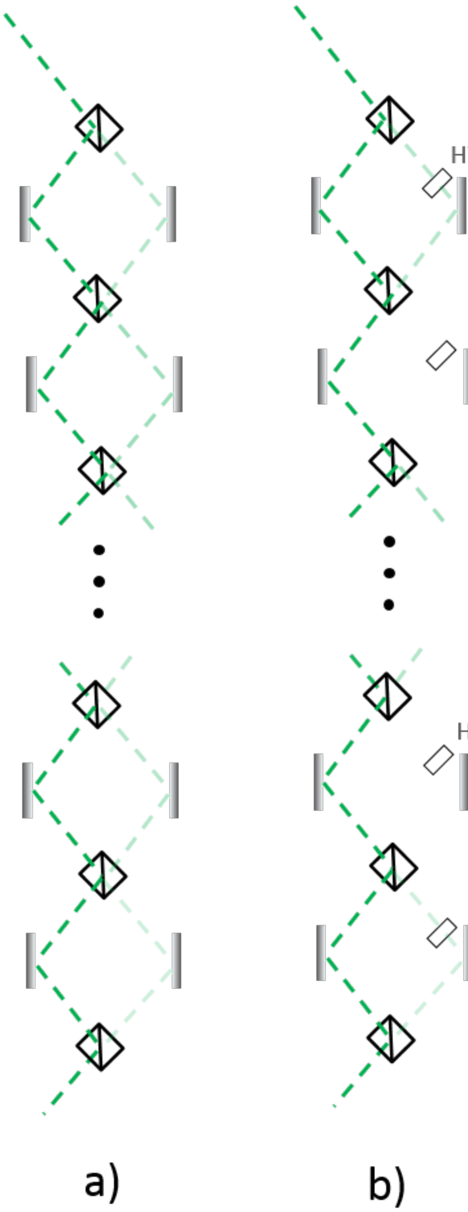

FIG. 7: The chain of interferometers with highly reflective beam splitters manipulated by the half-wave plates a). An empty chain moves the wave packet of the particle from left to right and then to left again but adding the phase of π. b) Half wave plates "undo" the transformation on every second interferometer ending up with the original state on the left without additional phase.

Since the number of beam splitters in the external chain is even, at the end of the process the particle is on the left side and it is detected by detector D_1 with certainty.

If Bob wants to communicate bit 1, instead, he inserts HWPs in all the interferometers of the inner chains Fig. 8b. Now, after every two beam splitters of the inner chain, the wave packet comes back to the left side

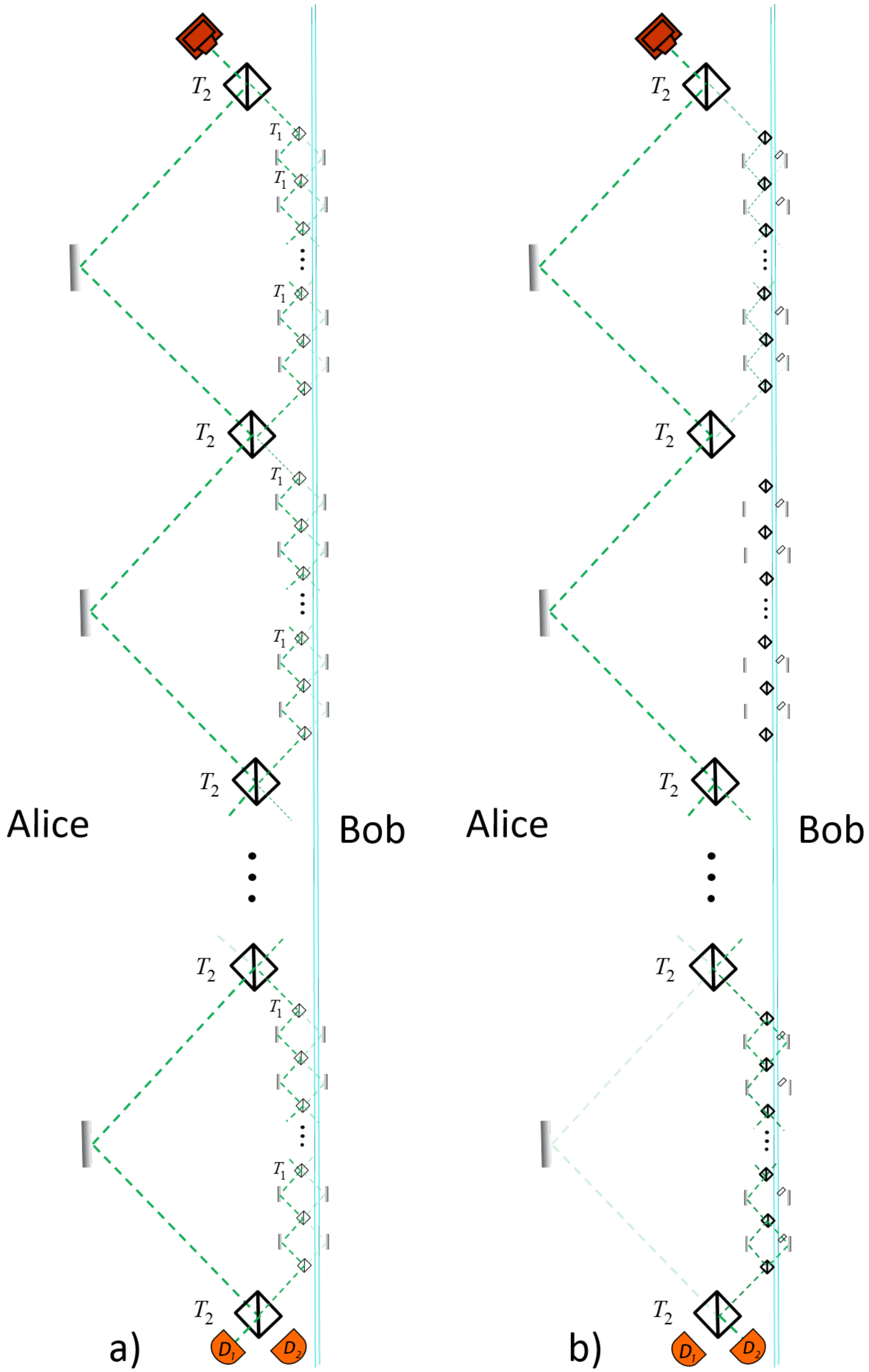

FIG. 8: "Direct quantum communication with almost invisible photons' a). The chains of the interferometers are tuned such that D_1 clicks with probability 1. b). If Bob inserts HWPs in all inner interferometers, then D_2 clicks with probability 1.

without acquiring additional phase. Thus, every inner chain works as a mirror and the external chain of the interferometers moves the particle from left to right to be detected with certainty by detector D_2. Alice knows with certainty the choice of Bob by observing which detector clicks. This is an ideal direct communication protocol: theoretically, when there are no losses, there is zero probability for an error.

V. CRITERIA FOR COUNTERFACTUALITY

The question I want to answer in this paper: Can the protocols of Sections III and IV be considered as *counterfactual* communication protocols? Let us consider the following three statements which try to capture the meaning of counterfactuality.

1) The probability of finding the particle in the transmission channel is zero or can be made arbitrarily small.

2) The particle did not pass through the transmission channel.

3) The particle was not present in the transmission channel (or the probability of the existence of a photon in the transmission channel can be made arbitrarily small).

In the papers on counterfactual communication [7, 13] all these statements were considered to be interchangeable, i.e. all are true and each one of them represents counterfactuality. I argue that the situation is more subtle and clarification is needed.

Statement (1). A non-demolition measurement of the presence of the particle in the transmission channel disturbs completely the operation of the communication protocols which are considered. When such a measurement is present, Bob does not transmit information to Alice by his actions. So, the truth or falsehood of this statement is not a decisive indication of the counterfactuality of the protocols.

However, since there is a separate controversy about the validity of this statement for the two protocols under discussion, it should be clarified too. The papers on counterfactual communication state that this is a correct statement while I [12] claim that in these protocols the probability of finding the particle in the transmission channel is 1.

The source of this controversy is our different assumptions. The communication protocols involve preparation and detection of the particle. I considered the probability of finding the particle in the transmission channel on the condition of the same final detection of the particle as in the protocol without intermediate measurement. Under this condition, the probability of finding the particle is exactly 1, since if it is not found, the result of the final detection cannot be that of the undisturbed protocol. On the other hand, without condition on the final detection, the probability to detect the particle in the transmission channel is vanishingly small. These are two correct statements about probability to find the particle in the intermediate measurement for different conditions: probability 1, conditioned on the proper preparation and the proper final detection of the particle, and vanishingly small probability when conditioned only on the proper preparation of the particle. None of these statements shed much light on the issue of counterfactuality of the protocols *without* intermediate non-demolition measurements.

Statement (2). In contrast to such a claim for a classical particle, the meaning of this statement for a quantum particle is not rigorously defined. For a classical particle, the operational meaning of (2) is statement (1), but as discussed above, (1) is not helpful for the quantum case. In a double slit experiment with a screen, there is no good answer through which slit the particle passed and through which it did not pass. However, if the detector which finds the particle is placed after one of the slits and the wave packet passing through the other slit does not reach the detector, then it is frequently declared, following Wheeler [28], that the particle did not pass through the second slit. (Note that this contradicts the textbook picture, attributed to von Neumann, according to which the wave passes through both slits and then collapses to the location of the detector.) If we adopt Wheeler's definition, then the statement (2) is correct for the protocol of Section III. The wave packets of the particle passing through the transmission channel do not reach the detector which detects the particle in this protocol. I, however, argued that we should not adopt Wheeler's definition for discussing the past of a quantum particle [25].

The concept of quantum particle passing through a channel does not have a meaning in standard quantum mechanics, since particles do not have trajectories. It is rigorously defined in the framework of Bohmian mechanics. But the effect of "surrealistic trajectories" [29, 30] makes this interpretation not useful for analyzing issues of communication. The fact that Bohmian trajectory does not pass through transmission channel does not tell us that Eve, which has an access to this channel, cannot get some information about this communication.

To summarize: statement (2) is not defined in standard quantum mechanics. One has to add something else for assigning meaning to (2). Adopting Bohmian trajectories or Wheeler's postulate makes (2) a correct statement, but I argue that this move is not helpful for the analyses of communication protocols.

Statement (3). Outside the framework of Bohmian mechanics, quantum mechanics does not provide a rigorous meaning also for statement (3). It seems to me that without clear ontological definition the way to go is to introduce an operational meaning. We cannot rely on operational definition based on statement (1) since strong (even if nondemolition) measurements change the setup we want to analyze. So, my proposal is to look at the *weak* trace the particle leaves.

All particles interact locally with the environment. If the particle is present in a particular place, it leaves some trace there, and it does not leave any trace in a place in which it was not present. Therefore, we can run the protocol we want to analyze and then look at the trace left on the environment. If in a particular region there is no trace, we will say that the particle was not there. There is a logical possibility that the particle changed the local environment due to nonvanishing local interactions, but then changed it back to its original state. It does not sound probable, but in any case, my main claim is a type of negation of this statement for which this criticism does not hold: *If a particle left a trace in a particular place, one cannot claim that the particle was not there.*

We analyze interference experiments. The trace left by particles has to be small, otherwise the experiment is disturbed. When the trace is small one may argue that it can be neglected. I argue that the fact that it is small is not enough. It can be neglected only if it small compare to a trace which a single particle being in this place would leave given the same coupling with the environment. This is the main technical question of this paper: comparison between the trace left in the transmission channel in the "counterfactual" communication protocols [7, 13] and the trace left by a single particle passing through the channel. In the next section I will analyze a minimal trace left by a single particle being in a transmission channel. The comparison will be made in the following Sections.

VI. THE TRACE LEFT BY A SINGLE PARTICLE

As I mentioned above, in the framework of standard quantum mechanics there is no rigorous way to decide about the validity or negation of statement (3): the particle present or not present in a transmission channel. In a two-slit experiment it is not clear whether the particle was in a particular slit. However, if a particle starts on one side of a plate with two slits and is found later on the other side, it is not controversial to say that the particle was in the two slits. We do not know if it was in the two slits together, or in one of them, but we firmly believe that it cannot be that it was not present in both.

In the following we will analyze the trace in several simple examples in which we are certain, using this approach, that the particle was present in the transmission channel. We will build a model of a single-path transmission channel and will consider transmitting the particle from Alice to Bob using various setups consisting of several such paths.

a) *A single particle is present in the transmission channel in the form of a single localized wave packet.* The localized wave packet passes from Alice to Bob, bounces on a mirror, and is detected by Alice, see Fig. 9. Since localized wave packet behaves here as a classical particle, we can reasonably say that on the way to Bob the particle was present in the transmission channel once.

Let us model the interaction of the particle with the transmission channel as von Neumann measurement with a Gaussian probe. The pointer variable is x and the

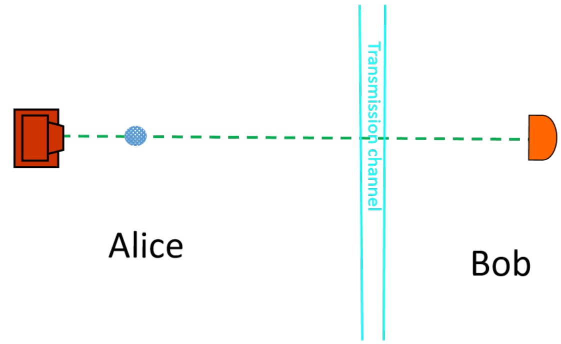

FIG. 9: A single particle in a single localized wave packet passes from Alice to Bob in the transmission channel. Some trace is invariably left in the channel. We model it as a shift of local degree of freedom of the channel described by (11).

initial wave function of the pointer is

$$|\Phi_0\rangle = \frac{1}{\sqrt{\Delta\sqrt{\pi}}} e^{-\frac{x^2}{2\Delta^2}}. \qquad (10)$$

When one particle is present in the transmission channel, the interaction causes a shift by δ. Thus, in case (a), the state of the measuring device after the interaction will be:

$$|\Phi\rangle = \frac{1}{\sqrt{\Delta\sqrt{\pi}}} e^{-\frac{(x-\delta)^2}{2\Delta^2}} = \sqrt{1-\epsilon^2}|\Phi_0\rangle + \epsilon|\Phi_\perp\rangle, \qquad (11)$$

where $|\Phi_\perp\rangle$ is orthogonal to the initial pointer state and $\epsilon = \sqrt{1 - e^{-\frac{\delta^2}{\Delta^2}}}$.

How to quantify the trace? One option is to consider theoretical probability of detecting the particle in the channel in this procedure, ϵ^2. Another option is just to consider the shift of the pointer via the parameter $\left|\frac{\delta}{\Delta}\right|$.

For strong trace, the probability criterion does not represent the trace well: it remains almost 1 for $\left|\frac{\delta}{\Delta}\right| = 10$ and also for $\left|\frac{\delta}{\Delta}\right| = 1000$. In practice, however, it is plausible that in a realistic experiment, when in addition to the quantum uncertainty of the pointer there is an uncertainty of the grid on which we read the pointer, only very large $\left|\frac{\delta}{\Delta}\right|$ can be observed.

For small value of $\left|\frac{\delta}{\Delta}\right|$, the probability of detection is proportional to the square of this parameter. However, again, if in the experiment we observe the pointer variable x, then due to uncertainty of the the grid, the shift seems to be a more relevant parameter, than its square.

b) *Single particle is present in the channel several times.* If the particle bounces on Bob's side, returns to Alice and passes again repeating the procedure N times, see Fig. 10, we can reasonably say that the particle was in the transmission channel on the way to Bob N times.

If our "transmission channel" is a collection of N identical local paths similar to the path of case (a), each having its own degree of freedom making separate von

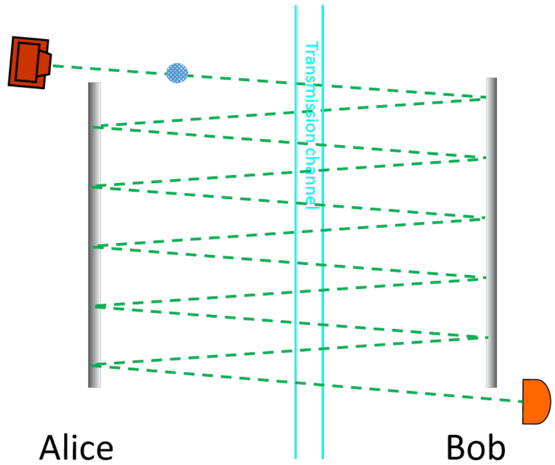

FIG. 10: A single particle in a single localized wave packet passes from Alice to Bob several times. We model the trace as local shifts of separate degrees of corresponding to each time the particle is present in the transmission channel. (For simplicity only cases of motion toward Bob are considered.)

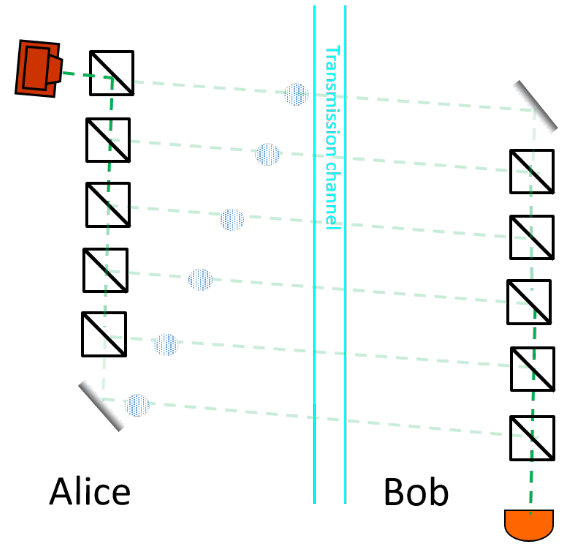

FIG. 11: A single particle in a superposition of several localized wave packets passes from Alice to Bob. We assume that the beam splitters are arranged in such a way that all packets have equal amplitudes and beam splitters on Bob's side are tuned to interfere constructively toward the detector.

Neumann measurements of the presence of the particle, then the process is described by

$$\prod_{i=1}^{N}|\Phi_0\rangle_i \rightarrow \prod_{i=1}^{N}(\sqrt{1-\epsilon^2}|\Phi_0\rangle_i + \epsilon|\Phi_\perp\rangle_i). \qquad (12)$$

The probability for detection of the particle is $1-(1-\epsilon^2)^N$. For a small coupling, it is, as expected, N times ϵ^2, the probability of case (a), but it is not true when the probability is large. If we observe the shifts of pointer variables x_i, the expectation value of each one of them is exactly as in case (a). The reasonable trace parameter for this case is $\sum_{i=1}^{N}\left|\frac{\langle x_i\rangle}{\Delta}\right| = N\left|\frac{\delta}{\Delta}\right|$.

c) *A single particle moves in the transmission channel in the form of a superposition of several localized wave packets.* If we split the wave packet of a particle into N equal parts and pass them to Bob through the N channel, Fig. 11, we can again say that the particle was in transmission channel once.

The quantum state of the particle entering the transmission channel is $|\Psi_{in}\rangle = \frac{1}{\sqrt{N}}\sum_{i=1}^{N}|i\rangle$ where $|i\rangle$ signifies the wave packet of the particle inside path i of the transmission channel. After the particle passes the transmission channel, the state of the particle and (our model of) the transmission channel is

$$\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\prod_{j\neq i}|\Phi_0\rangle_j(\sqrt{1-\epsilon^2}|\Phi_0\rangle_i + \epsilon|\Phi_\perp\rangle_i)|i\rangle. \qquad (13)$$

The probability to detect the particle in the transmission channel is the probability to find one of the states $|\Phi_\perp\rangle_i$. It is ϵ^2 as in case (a). The sum of the expectation values of shifts of x_is is also the same as in case (a), $\sum\langle x_i\rangle = \delta$.

It is important to consider post-selection measurement of the state of the particle reaching Bob's hands. Let us assume that Bob post-selects the undisturbed state $|\Psi_{fin}\rangle = \frac{1}{\sqrt{N}}\sum_{i=1}^{N}|i\rangle$. This corresponds to detection by Bob's detector in Fig. 11. For a good channel, the probability to find this state is very close to 1. Then, the state of the transmission channel becomes

$$\frac{N\sqrt{1-\epsilon^2}\prod_{j=1}^{N}|\Phi_0\rangle_j + \epsilon\sum_{i=1}^{N}|\Phi_\perp\rangle_i\prod_{j\neq i}|\Phi_0\rangle_j}{\sqrt{N^2(1-\epsilon^2)+N\epsilon^2}}. \qquad (14)$$

At this stage, the probability to find one of the states $|\Phi_\perp\rangle_i$ is reduced dramatically. This is because the failure of post-selection leads to the probability 1. For small ϵ the probability to detect the particle in the transmission channel after this post-selection is approximately $\frac{\epsilon^2}{N}$.

VII. THE WEAK TRACE.

We are interested in the analysis of protocols based on interference. Therefore, in the regime they work properly, there might be only a tiny trace left on the environment. So, we will analyze the protocols presented at the beginning of the paper assuming that the trace is very small. We use the same model: in every path of the transmission channel, the presence of the particle causes a shift of the Gaussian wave function of the channel degree of freedom (11). And we assume that the coupling is weak: $\delta \ll \Delta$, and consequently, $\epsilon \ll 1$. For simplicity we will consider the trace created by particles moving from Alice to Bob and will disregard the trace created on the way from Bob to Alice.

The first example, described in Fig. 1, is essentially the case (a) described in Fig. 9. The trace in the communication channel is the shift δ of the channel variable and probability to discover the presence of the particle observing the trace is ϵ^2.

In the case (c) with post-selection, described in Fig. 11, the probability to discover the presence of the particle observing the trace is $\frac{\epsilon^2}{N}$. It is interesting that in this case, for weak coupling, the post-selection of the particle state does not change the expectation value of the sum of the pointer variables $\langle \sum x_i \rangle = \delta$. One way to see this is to note that $\langle \sum x_i \rangle = \delta$ is proportional to the weak value [31] of the sum of the projections on all parts of the channel. It equals 1 because the initial state is the eigen state of the sum of the projections with the eigenvalue 1 [32]. This means that whatever post-selection of the state of the particle takes place, there will be always a trace manifested in shifts of expectation values $\langle x_i \rangle$. The direction of the shift is not relevant if we are interested in how much trace is left. So the relevant parameter is $\sum_i |\langle x_i \rangle|$. We have found a lower bound, $\sum_i |\langle x_i \rangle| \geq \delta$.

In the communication using IFM, Fig. 2, we have to consider a few cases. Assume that the bit is 0, Bob does not block the path. Then, after the particle passes the transmission channel, but before passing through the second beam splitter, the state of the particle and the channel is

$$\frac{1}{\sqrt{2}} \left[\Phi_0\rangle |L\rangle + (\sqrt{1-\epsilon^2} |\Phi_0\rangle + \epsilon |\Phi_\perp\rangle) |R\rangle \right]. \qquad (15)$$

The particle is detected by D_2 with probability close to 1. In this case, the quantum state of the channel is

$$\sqrt{\frac{2}{1+\sqrt{1-\epsilon^2}}} \left[\frac{1+\sqrt{1-\epsilon^2}}{2} |\Phi_0\rangle + \frac{\epsilon}{2} |\Phi_\perp\rangle \right]. \qquad (16)$$

This exhibits the shift of the probability distribution of the channel variable $\frac{\delta}{2}$ and the probability of detection of $|\Phi_\perp\rangle$, which is a decisive prove of the presence of the particle in the channel, $\frac{\epsilon^2}{4}$. There is also probability of approximately $\frac{\epsilon^2}{4}$ of an error: D_1 clicks. In this case the trace $|\Phi_\perp\rangle$ will be found almost with certainty.

We see that for bit 0 the trace is comparable to the case of a single particle present in the channel. However, in the IFM communication protocol of Fig. 2, bit 0 is not of interest, since Alice cannot know this bit value with high probability. If Bob wants to communicate bit 1 and he blocks the path, then the state before passing through the second beam splitter is

$$\frac{1}{\sqrt{2}} \left[\Phi_0\rangle |L\rangle + (\sqrt{1-\epsilon^2} |\Phi_0\rangle + \epsilon |\Phi_\perp\rangle) |absorbred\rangle \right]. \qquad (17)$$

If the particle is absorbed by Bob, the trace in the channel is exactly as in case (a) when a single particle is present in the channel: shift by δ and probability to find the trace is ϵ^2. In contrast, if the particle is detected by Alice, in a good case by D_1 which tells her the bit, or in the bad case by D_2, there will be no any trace in the transmission channel. The wave packet "tagged" by an orthogonal state of the channel, $|\Phi_\perp\rangle$ cannot reach Alice.

In the experiment with HWP instead of the absorber, Fig. 3, the case of not putting the HWP in, is identical to the IFM case for bit 0. But the trace in the case of bit 1, when the HWP is present in the path of the interferometer, is the same as for bit 0. When the correct detector clicks (with probability close to 1), the trace is comparable to the case of the presence of a single particle: probability of $\frac{\epsilon^2}{4}$ to detect the trace (instead of ϵ^2) and half the shift of the distribution of the channel degree of freedom, $\frac{\delta}{2}$. When there is an error, i.e. the wrong detector clicks, which happens with tiny probability of $\frac{\epsilon^2}{4}$, then the detection of the trace is almost certain and the distribution of the channel degree of freedom is changed completely, it is not a Gaussian anymore.

Let us turn to the analysis of the IFM experiment with the chain of the interferometers, Fig. 4. We will consider first the communication of bit 0, when Bob leaves the interferometers undisturbed. The exact expressions are complicated, but for weak coupling, we can consider only first order in ϵ, which is simple. From (2) we see that neglecting the coupling to the channel, the state of the particle in the nth interferometer is

$$\cos \frac{n\pi}{2N} |L\rangle + \sin \frac{n\pi}{2N} |R\rangle. \qquad (18)$$

The wave packet $|R\rangle$ "tagged" at the nth interferometer by state $|\Phi_\perp\rangle_n$ in the transmission channel interferes only with itself and reaches detectors in the state

$$\cos \frac{(N-n)\pi}{2N} |R\rangle - \sin \frac{(N-n)\pi}{2N} |L\rangle. \qquad (19)$$

In the experiment, the particle ends up in detector D_2 (state $|R\rangle$) with probability close to 1. Then, the state of the transmission channel is

$$\mathcal{N} \left(\prod_{n=1}^{N-1} |\Phi_0\rangle_n + \epsilon \sum_{n=1}^{N-1} \sin^2 \frac{n\pi}{2N} \prod_{j\neq n} |\Phi_\perp\rangle_n \right), \qquad (20)$$

with normalization parameter $|\mathcal{N}|$ close to 1. Therefore, the probability to detect the particle in the transmission channel, i.e., to find at least one of the states $|\Phi_\perp\rangle_n$ is, approximately

$$\epsilon^2 \sum_{n=1}^{N-1} \sin^4 \frac{n\pi}{2N} \sim \epsilon^2 N \frac{2}{\pi} \int_0^{\frac{\pi}{2}} \sin^4 x dx = \frac{3\epsilon^2 N}{8}. \qquad (21)$$

It is much larger than the minimal probability to find a single particle present in this multiple-path channel, which can be as low as $\frac{\epsilon^2}{N}$. For bit 0, it is definitely not a "counterfactual" communication. It can be seen also by calculating the shifts of the degrees of freedom of the transmission channel. The shifts are proportional to the expectation value of projection on the paths of the transmission channel. The shift in path n is $\delta \sin^2 \frac{n\pi}{2N}$ and the sum of all shifts is approximately $\delta \frac{N}{2}$, much larger than δ, the etalon of the presence of a single particle in the channel.

The situation is different for communication of bit 1. Bob blocks the paths of the interferometers, Fig. 4b, but due to Zeno effect, the probability of absorbtion by Bob is negligible. Detector D_1 clicks with probability close to 1 telling Alice that the bit value is 1. In this case there is no trace in the communication channel. It is a counterfactual communication for bit value 1.

It might seem that the post-selection of detecting the particle by Alice is important here. Without the post-selection, the probability to detect the particle in the transmission channel is not zero but of order $\frac{\epsilon^2}{N}$ (always correlated with Alice failing to see the particle). As we showed above, for a transmission channel which has N paths, such probability was also possible in the case of a particle present in the transmission channel, but this was the case with post-selection. In my view, since the probability of detection of the particle by Alice is close to 1 and then the trace is absent, it is legitimate to name such a protocol counterfactual. The criterion of the sum of shifts of degrees of freedom of the transmission channel tells us the same. When the particle is present, the sum is at least δ even if the channel has multiple paths. In our case, even without post-selection, it is of the order of $\frac{\delta}{N}$. Still, with post-selection the situation is conceptually better: the shift is exactly zero, the transmission channel is undisturbed.

Let us turn now to the case of nested interferometers, Fig. 5. The case which is particularly interesting is described in Fig. 5a. Bob does not put the absorber in, and detector D_1 clicks. Alice knows that the bit is 0, but contrary to the cases above, naively, it is a counterfactual communication since the particle "could not pass through the transmission channel". Indeed, the wave packet entering nested interferometer does not reach detector D_1.

The interferometer was defined only by its general properties regarding destructive interferences in particular situations. In order to make quantitative predictions, we have to specify the beam splitters of the interferometer. A possible implementation of the interferometer [33] is such that the first two beam splitters transform the localized wave packet entering the interferometer into a state:

$$|\Psi_{in}\rangle \to \frac{1}{\sqrt{3}}(|A\rangle + |B\rangle + |C\rangle). \tag{22}$$

Then, the other two beam splitters provide transformations:

$$\begin{aligned} |A\rangle &\to \frac{1}{\sqrt{3}}|1\rangle - \frac{1}{\sqrt{6}}|2\rangle + \frac{1}{\sqrt{2}}|3\rangle, \\ |B\rangle &\to -\frac{1}{\sqrt{3}}|1\rangle + \frac{1}{\sqrt{6}}|2\rangle + \frac{1}{\sqrt{2}}|3\rangle, \\ |C\rangle &\to \frac{1}{\sqrt{3}}|1\rangle + \sqrt{\frac{2}{3}}|2\rangle, \end{aligned} \tag{23}$$

where state $|i\rangle$ signify a wave packet entering detector D_i. It is is easy to see that these rules ensure destructive interferences as stated.

After the interaction, the state of the particle and the channel is

$$\frac{1}{\sqrt{3}}[|A\rangle(\sqrt{1-\epsilon^2}|\Phi_0\rangle + \epsilon|\Phi_\perp\rangle) + (|B\rangle + |C\rangle)|\Phi_0\rangle]. \tag{24}$$

Detection of the particle in detector D_1 is the post-selection of the state $\frac{1}{\sqrt{3}}(|A\rangle - |B\rangle + |C\rangle)$. Therefore, the state of the channel after the post-selection is

$$\sqrt{1-\epsilon^2}|\Phi_0\rangle + \epsilon|\Phi_\perp\rangle. \tag{25}$$

This is exactly the same state of the channel as in the case that a single particle passing through it, see (11). Thus, the scheme with nested interferometers does not provide counterfactual communication [11].

VIII. THE WEAK TRACE IN "DIRECT COUNTERFACTUAL QUANTUM COMMUNICATION"

Now we are ready to analyze the trace left in the "Direct counterfactual quantum communication". The case of bit 1, Fig. 6a, is simple. The trace in the communication channel is correlated to the final location of the particle. If it is absorbed by Alice, which happens with the probability close to 1 and corresponds to the proper operation of the protocol, the trace is zero. The wave packets "tagged" by orthogonal states of the channel, $|\Phi_\perp\rangle_{m,n}$, cannot reach Alice. The trace is present only if the particle is absorbed by one of the Bob's shutters which happens with vanishing probability. This is a counterfactual communication protocol for bit 1.

The interesting case is that of bit 0, when Bob leaves the interferometer undisturbed, Fig. 6b. We assume that the interaction with the channel is small, $\epsilon \ll 1$. Moreover, there are other small parameters in the problem, $\frac{1}{M}$, $\frac{1}{N}$, and we assume that ϵ is much smaller than these parameters too, so only the first order in ϵ should be considered.

The amplitude of the wave packet of the particle in the nth path of mth chain of the inner interferometers (n, m), is

$$\cos^{(m-1)N}\frac{\pi}{2N}\sin\frac{\pi}{2M}\cos^{(m-1)}\frac{\pi}{2M}\sin\frac{n\pi}{2N}. \tag{26}$$

A particle present in the path (m, n) creates changes in the state of the corresponding degree of freedom of the transmission channel according to (11):

$$|\Phi_0\rangle_{m,n}|m,n\rangle \to \left(\sqrt{1-\epsilon^2}|\Phi_0\rangle_{m,n} + \epsilon|\Phi_\perp\rangle_{m,n}\right)|m,n\rangle. \tag{27}$$

The wave packet $|m,n\rangle$ "tagged" by the orthogonal state $|\Phi_\perp\rangle_{m,n}$ interferes only with itself and leaves the inner chain in the state, see (19):

$$|m,n\rangle \rightarrow \sin\frac{n\pi}{2N}|R\rangle - \cos\frac{n\pi}{2N}|L\rangle. \quad (28)$$

State $|R\rangle$ is lost and the state $|L\rangle$ of the last inner interferometer of the chain m enters from the right the remaining $M-m$ large interferometers. The transformation of this state is

$$|m,N\rangle \rightarrow \sin\frac{\pi}{2M}\cos^{(M-m)N}\frac{\pi}{2N}\cos^{(M-m-1)}\frac{\pi}{2M}\times \times \left(\cos\frac{\pi}{2M}|L\rangle + \sin\frac{\pi}{2M}|R\rangle\right) + ... \ , \quad (29)$$

where "..." signify wave packets which do not reach the detectors. (Note that $|L\rangle$ in (28) is $|R\rangle$ in (29).)

In the protocol, the particle is found with probability close to 1 by detector D_1. After the detection of the particle, the amplitude of the term $|\Phi_\perp\rangle_{m,n}$ corresponding to detection of the particle in the path (m,n) can be found by collecting factors from (26-29). It is

$$\frac{\epsilon}{2}\cos^{(M-1)N}\frac{\pi}{2N}\sin^2\frac{\pi}{2M}\sin\frac{n\pi}{N}\cos^{(M-2)}\frac{\pi}{2M}. \quad (30)$$

As it explained in Section III, the protocol works properly if $1 << M << N$. With this condition

$$\cos^{(M-1)N}\frac{\pi}{2N} \sim 1, \quad \cos^{(M-2)}\frac{\pi}{2M} \sim 1, \quad (31)$$

so we can approximate the probability that one of the orthogonal states $|\Phi_\perp\rangle_{m,n}$ will be found in the transmission channel by

$$\sum_{m,n}\frac{\epsilon^2}{4}\sin^4\frac{\pi}{2M}\sin^2\frac{n\pi}{N} \sim \frac{\epsilon^2\pi^4N}{2^7M^3}. \quad (32)$$

The number of paths in the channel is approximately MN. We have seen that a single particle present in such a channel can be found with probability as low as $\frac{\epsilon^2}{MN}$ which is smaller than the probability of detection of the particle in the protocol by a factor of approximately $\frac{N^2}{M^2}$. Since the protocol works well only when $N \gg M$, the trace in the protocol is larger than the trace of a single particle present in the transmission channel.

Another criterion is the sum of displacements of the degrees of freedom in all paths of the channel, the etalon for which is δ. It can be found by calculating the absolute values of weak values of all projections: $\delta\sum_{m,n}|(\mathbf{P}_{m,n})w|$ with pre- and post-selection specified by the protocol. The scalar product in the denominator of the weak value is close to 1 since the probability of post-selection is close to 1. The amplitude of the forward evolving state at path (m,n) is given by (26) and, similarly the amplitude of the backward evolving state there is

$$\cos^{(M-m-1)N}\frac{\pi}{2N}\sin\frac{\pi}{2M}\cos^{(M-m)}\frac{\pi}{2M}\sin\frac{(N-n)\pi}{2N}. \quad (33)$$

Thus, the weak value of the projection on the path (m,n) can be approximated as

$$(\mathbf{P}_{m,n})_w \sim \frac{\pi^2}{8M^2}\sin\frac{n\pi}{N}, \quad (34)$$

and the sum of al the shifts of the degrees of freedom of the transmission channel is

$$\delta\sum_{m,n}|(\mathbf{P}_{m,n})_w| \sim \delta\frac{\pi^2N}{16M}. \quad (35)$$

Since for the proper operation of the protocol we need $N \gg M$, the trace in the protocol is much larger than the "etalon" of the trace of a single particle present in such multiple-path channel. Therefore, the "Direct counterfactual quantum communication" protocol of Salih et al. [7], when executed for bit value 0, cannot be considered counterfactual.

IX. THE WEAK TRACE IN "DIRECT QUANTUM COMMUNICATION WITH ALMOST INVISIBLE PHOTONS"

Let us turn to the "Direct quantum communication with almost invisible photons" protocol [13]. When Bob transmits 0, i.e. does nothing, Fig. 8a, the amplitude in the path (m,n) is

$$\begin{array}{lll} \sin\frac{\pi}{2M}\sin\frac{n\pi}{N} & \text{for} & m \ \text{odd}, \\ 0 & \text{for} & m \ \text{even}. \end{array} \quad (36)$$

The wave packet $|m,n\rangle$, "tagged" by the orthogonal state $|\Phi_\perp\rangle_{m,n}$, interferes only with itself and it leaves the inner chain in the state

$$|m,n\rangle \rightarrow \sin\frac{n\pi}{N}|R\rangle - \cos\frac{n\pi}{N}|L\rangle. \quad (37)$$

The state $|R\rangle$ is lost and the state $|L\rangle$ of the last inner interferometer of the chain m enters from the right the remaining $M-m-1$ large interferometers. Based on a) "tagging" takes place only for odd m, b) the number of the large interferometers is odd (M, the number of beam splitters is even), and c) after every second beam splitter the wave function repeats itself, we can conclude that the wave packet leaves the last beam splitter of the last inner chain with the same amplitude. The wave packet entering the last beam splitter of the large interferometers transforms into

$$\cos\frac{\pi}{2M}|R\rangle - \sin\frac{\pi}{2M}|L\rangle. \quad (38)$$

In the protocol, the particle is detected with probability 1 by detector D_1 where goes the state $|L\rangle$. Thus, after detection of the particle, the amplitude of the term $|\Phi_\perp\rangle_{m,n}$ corresponding to the detection of the particle in the path (m, n) can be found by collecting factors from (36-38) and factor ϵ due to the interaction:

$$\frac{\epsilon}{2}\sin\frac{2n\pi}{N}\sin^2\frac{\pi}{2M}. \tag{39}$$

This expression holds only for odd m, the amplitude in the paths with even m vanishes. Summing the probabilities of finding the record the particle leaves in all the paths, i.e. summing on odd ms up to $M-1$ and on integers n up to $N-1$, we obtain approximate probability of detection of the particle:

$$\sum_{m,n}\frac{\epsilon^2}{4}\sin^4\frac{\pi}{2M}\sin^2\frac{2n\pi}{N}\sim\frac{\epsilon^2\pi^4 N}{2^8 M^3}. \tag{40}$$

A single particle in this system present in the transmission channel can be found with probability of order $\frac{\epsilon^2}{MN}$. Depending on the ratio $\frac{N}{M}$, it can be smaller or larger than (40).

Compare now the sum of displacements of the degrees of freedom in all paths of the channel. It can be found by calculating the absolute values of weak values of all projections: $\delta\sum_{m,n}|(\mathbf{P}_{m,n})_w|$ with pre- and post-selection specified by the protocol. In this case, the pre- and post-selected states are identical, so the weak values are just the expectation values:

$$(\mathbf{P}_{m,n})_w=\begin{cases}\sin^2\frac{\pi}{2M}\sin^2\frac{n\pi}{N} & \text{for } m \text{ odd,}\\ 0 & \text{for } m \text{ even.}\end{cases} \tag{41}$$

Therefore, the sum of displacements can be approximated as

$$\delta\sum_{m,n}|(\mathbf{P}_{m,n})_w|\sim\delta\frac{\pi^2 N}{16M}. \tag{42}$$

The sum of the displacements when one particle in the transmission channel is δ, so the ratio $\frac{N}{M}$ tells us when the sum of displacements in the protocol (42) is smaller or larger than that of a single particle present in the channel.

Let us now repeat the analysis for the case of bit 1, when Bob puts HWPs in every inner interferometer. Now the amplitude in the path (m, n) is

$$\begin{matrix}\sin\frac{m\pi}{2M}\sin\frac{\pi}{N} & \text{for} & n \text{ odd,}\\ 0 & \text{for} & n \text{ even.}\end{matrix} \tag{43}$$

The wave packet $|m, n\rangle$, can be "tagged" by the orthogonal state $|\Phi_\perp\rangle_{m,n}$ only for odd n. Thus, due to the presence of the HWPs, the wave packet comes back unchanged every second beam splitter. It leaves the chain of the inner interferometers in the state

$$|m, n\rangle\rightarrow\cos\frac{\pi}{N}|R\rangle-\sin\frac{\pi}{N}|L\rangle. \tag{44}$$

State $|R\rangle$ is lost and the state $|L\rangle$ of the last inner interferometer of the chain m enters from the right the remaining $M-m-1$ large interferometers. In the chain of the large interferometers it performs usual evolution (2) and ends up in the state

$$\sin\frac{m\pi}{2M}|R\rangle-\cos\frac{m\pi}{2m}|L\rangle. \tag{45}$$

In the protocol, the particle is detected with probability 1 by detector D_2 where goes the state $|R\rangle$. Thus, after detection of the particle, the amplitude of the term $|\Phi_\perp\rangle_{m,n}$ corresponding to the detection of the particle in the path (m, n) can be found by collecting factors from (43 -45) and the factor ϵ due to the interaction:

$$\frac{\epsilon}{2}\sin^2\frac{\pi}{N}\sin^2\frac{m\pi}{2M}. \tag{46}$$

This expression holds only for n odd, the amplitude in the paths with n even, vanishes. Summing probabilities of finding the record the particle leaves in all the paths, i.e. summing on odd ns up to $N-1$ and on integers m up to $M-1$, we obtain approximate probability of detection of the particle:

$$\sum_{m,n}\frac{\epsilon^2}{4}\sin^4\frac{m\pi}{2M}\sin^4\frac{\pi}{N}\sim\frac{\epsilon^2 3\pi^4 M}{2^6 N^3}. \tag{47}$$

Again, as in the case of bit 0, the ratio $\frac{N}{M}$ tells us if it is larger or smaller than the minimal probability of detection in case of a single particle present in the transmission channel. However, the dependence is opposite: if for bit 0 the probability of detection in the protocol was smaller than single-particle etalon, for bit 1 it will be larger and vice versa.

Let us consider the displacements of the degrees of freedom criterion. The sum of displacements of the degrees of freedom in all paths of the channel can be found by calculating the absolute values of weak values of all projections: $\delta\sum_{m,n}|(\mathbf{P}_{m,n})_w|$ with pre- and post-selection specified by the protocol. Also in this case, the pre- and post-selected states are identical and the weak values are just expectation values:

$$(\mathbf{P}_{m,n})_w=\begin{cases}\sin^2\frac{m\pi}{2M}\sin^2\frac{\pi}{N} & \text{for } n \text{ odd,}\\ 0 & \text{for } n \text{ even.}\end{cases} \tag{48}$$

Therefore, the sum of displacements can be approximated as

$$\delta\sum_{m,n}|(\mathbf{P}_{m,n})_w|\sim\delta\frac{\pi^2 M}{4N}. \tag{49}$$

We see that the displacement criterion leads to the same conclusion as the probability of detection criterion. The ratio $\frac{N}{M}$ tells us when the sum of displacements in the protocol (49) is smaller or larger than that of the one particle. If it is smaller for bit 1, it is larger for bit 0 and vice versa.

X. CONCLUSIONS

Standard quantum formalism does not specify the position of a quantum particle. Thus, it does not provide an unambiguous answer to the question: Is a particular communication protocol counterfactual? i.e.: Was the particle present in the transmission channel? In this paper I analyzed the approach to answer this question based on the weak trace the particle leaves in the channel. I compared the trace left in the channel in recently proposed protocols claimed to be counterfactual with the trace in the protocol constructed to transmit a single particle in the same channel.

In the analysis, I considered two criteria for comparing the traces. First, the probability of finding a conclusive evidence for the presence of the particle, and second, the expectation value of the sum of total shifts of some variables of the channel.

The question of counterfactuality of the protocols is considered in cases the protocols work properly, i.e. when the particle is detected by the right detector. It means that the particle is pre- and post-selected. The protocols were compared to transmission of a single pre- and post-selected particle. In all these cases the probability of post-selection was closed to 1.

The analyses using the two criteria of the trace led to the same conclusion. *It is possible to communicate only one value of a bit in a counterfactual way.*

The protocol "Direct counterfactual quantum communication" [7] is fully counterfactual for bit value 1. The trace is identically 0. Nothing changes in the transmission channel and therefore there is zero probability to detect the particle in the transmission channel.

However, the protocol is not counterfactual for the bit value 0. It is true that by increasing the number of paths in the channel, the probability of finding a conclusive evidence of the presence of the particle reduces, but increasing the number of paths also reduces the probability to find a single quantum particle when it passes the channel. The probability to find the presence of the particle in the transmission channel in the event of successful operation of the protocol is larger than the probability to detect a particle successfully passing through this channel.

The criterion of the shifts of variables of the channel tells us the same. The expectation value of the sum of the shifts is zero for bit 1, but for bit value 0 it is larger than the sum of the shifts when a single particle passes the channel.

The protocol "Direct quantum communication with almost invisible photons" is not fully counterfactual for any bit value. Some trace is always left in the transmission channel. However, if we are ready to consider a protocol as counterfactual when it leaves a trace which is much smaller than the trace of a single particle passing through this channel, then we can arrange that it will be counterfactual for one of the bit values. By playing with the numbers N and M of inner and external interferometers respectively, the protocol can be made counterfactual for value 0 or value 1 of the bit. It *cannot* be made counterfactual for both.

The two criteria agree about counterfactuality of the protocols. Note that the criterion of the sum of the shifts of the transmission channel variables in all its paths is easier to apply since it does not depend on the number of paths in the channel, while the probability of detecting the particle reduces with the number of paths in the channel.

The main conclusion of the paper is that recent protocols claiming to be counterfactual for both values of transmitted classical bit are not counterfactual according to the trace they leave in the transmission channel. We do not have counterfactual protocols for direct communication.

This result limits the arguments of cryptographic security of such protocols. However, combining the method with the BB84 protocol does allow secure counterfactual quantum key distribution [4]. The analysis of transmission of a particle in a multi-path channel of Section IX which showed that the probability of detection in the channel of the successfully transmitted particle can be reduced by increasing the number of paths, might also be useful for quantum cryptography.

I find another benefit of this study in deeper understanding the reality of quantum particles. It helps to understand my answer the question: "Where are particles passing through interferometers?" [25, 34].

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