On the role of potentials in quantum theory

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There is a consensus today that the the main lesson of the Aharonov-Bohm effect is that a picture of electromagnetism based on the local action of the field strengths is not possible in quantum mechanics. It is argued here that this statement is correct only when a quantum particle is considered together with classical system(s). If all systems are considered in the framework of quantum theory, everything can be explained without the notion of potentials. The core of the Aharonov-Bohm effect is the same as the core of quantum entanglement: the quantum wave function describes all systems together.

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Before the Aharonov-Bohm effect [1] (AB) had been discovered, the general physical picture was that particles change their motion due to fields in their locations, the fields created by other particles. The main revolutionary aspect of the AB effect was that this is not generally true, i.e. that there can be situations in which two particles, prepared in identical states, move in a region with the same fields but end up in different final states. In particular, there can be zero electromagnetic field in every place where the electron has been, but its motion is affected by the electromagnetic interaction. The AB effect states that the motion of an electron is completely defined by the potentials in the region of its motion and not just by the fields. The potentials depend on the choice of gauge, which cannot affect the motion of particles, but there are gauge invariant properties of the potentials (apart from the fields) that specify the motion of particles. The validity and the meaning of the AB effect has been extensively discussed [2–14]. I believe that the effect is correct, but that the essential mechanism which leads to the effect is different from what is commonly accepted and that we should change our understanding of the nature of physical interactions back to that of the time before the AB effect was discovered. The quantum wave function changes due to local actions of fields.

The discussion will be on the level of gedanken experiments, without questioning the feasibility of such experiments in today's laboratory. Consider a Mach-Zehnder interferometer with an electron tuned in such a way that it always ends up in detector B, see Fig. 1. We can change the electric potential in one arm of the interferometer in such a way that there will be no electromagnetic field in the location of the wave packets of the electron but, nevertheless, the electron will change its behavior and sometimes (or it can be arranged that always) will end up in detector A. This is the electric AB effect. Alternatively, in the magnetic AB effect, the interference picture can be changed due to a solenoid inside the interferometer which produces no electromagnetic field outside.

Let us start our analysis with the electric AB effect. In the original proposal, the potential was created using conductors, capacitors etc. While those are closer to a practical realization of the experiment, a precise theoretical description of such devices is difficult. I consider, instead, two charged particles, the fields of which cancel each other at the location of the electron.

For simplicity of presentation, instead of the Mach Zehnder interferometer, I shall consider a one dimensional interferometer, see Fig. 2. (In fact, for an observer moving with constant velocity in a perpendicular direction, this interferometer looks very much like the Mach Zehnder interferometer described above.) The electron wave packet starts moving to the right toward a barrier which transmits and reflects equal weight wave packets toward mirrors A and B. After reflection from the mirrors, the wave packets split again on the barrier. The interferometer is tuned in such a way that the electron reaches mirror B with certainty.

Another modification is designing special mirrors for the electron which make it spend a long period of time τ "touching" the mirror. A simple model for the mechan-

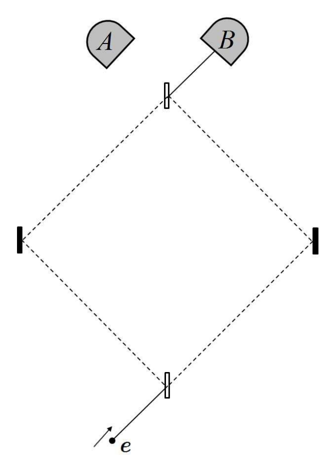

FIG. 1: **Mach-Zehnder interferometer with electron as a test bed of the AB effect.** Introducing relative electric potential between the arms of the interferometer or introducing a solenoid inside the interferometer spoils the destructive interference in detector A.

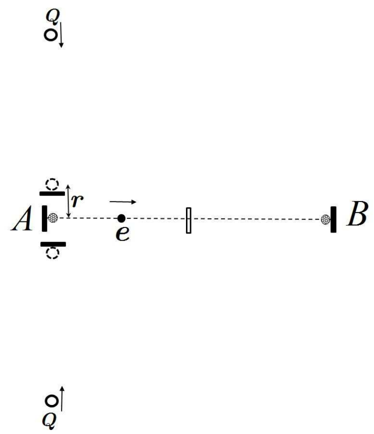

FIG. 2: **A realization of the electric AB effect.** Identical charges brought symmetrically to the electron wave packet in the left arm of the interferometer create potential for the electron without creating electric field in its location.

ical potential energy of the electron as a function of the distance from the mirror is shown on Fig. 3. It goes to infinity at the surface of the mirror, smoothly becomes constant value V at $x \in (0, d)$, and smoothly goes to zero for $x > d$. The energy of the electron is only slightly higher than V. The dimensions of the interferometer are much larger than d and I say that the electron "touches" the mirror when $x \in (0, d)$.

The source of the AB potential will be two particles of mass M and charge Q placed symmetrically on the perpendicular axis at equal large distances from mirror A. They have equal initial velocities toward the location of mirror A. At equal distance r from the mirror, the charged particles bounce back due to other similarly designed mirrors, which make the charges spend a period of time T "touching" these mirrors. This happens during the time the electron's wave packets "touch" their mirrors, $T < \tau$.

We can approximate the potential that the electron in the left arm feels as $\frac{-2eQ}{r}$ for the period of time T.

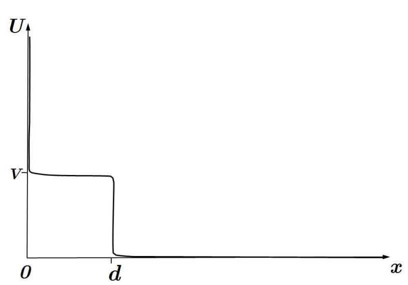

FIG. 3: **The potential of the mirror forces.** A particle with an energy slightly higher than V spends long time "touching" the mirror, i.e. being at a distance less than d from it.

Indeed, when the charges are far away their potential can be neglected, and the time the charges travel toward and from the mirror is much smaller than T. Thus, the phase difference between the two wave packets of the electron is:

$$\phi_{AB} = \frac{-2eQT}{r\hbar}. \tag{1}$$

The electron does not feel an electric field in any place where its wave packet passed, but it shows an interference pattern which is different from the pattern obtained in such an experiment by an "electron" without electric charge.

How can this result be understood if we consider all particles? There are two branches. In the first one the electron is on the left and in the other it is on the right. The energy in the left and in the right branches are equal, so energetic considerations cannot explain the phase difference. The electron does not experience any electric force so it cannot provide the source of the effect. The charges Q, however, do feel different forces in different branches. Thus, their wave packets in the left branch are slightly shifted relative to their wave packets in the right branch.

Let us calculate the shift of position of the wave packet of one charge Q due to its electromagnetic interaction with the electron. The shift is developed during the time T when the charge Q "touches" its mirror. The interaction with the electron leads to a small perturbation in the motion of the charge and, since $d \ll r$, the velocity of the charge during this time, v, can be considered to be constant. The change in the kinetic energy of the charge due to its interaction with the electron allows us to find the change in its velocity and thus the shift δx we are looking for:

$$\frac{-eQ}{r} = \delta(\frac{Mv^2}{2}) \simeq Mv\delta v \qquad \Rightarrow \qquad \delta x = \frac{-eQT}{Mvr}. \tag{2}$$

To observe the interference in the AB experiment, this shift should be made much smaller than the position uncertainty of the charges. It is comparable to the de Broglie wavelength of the charge $\lambda = \frac{h}{Mv}$. Both charges Q are shifted in the same way creating the AB phase:

$$2\frac{\delta x}{\lambda}2\pi = \phi_{AB}. \tag{3}$$

Note that entanglement between the electron and the charges, which could have been created if the uncertainty in the velocity of the charges when they "touch" their mirrors is smaller than δv, disappears when the charges travel far away.

It is interesting to consider a particular value of the charge of the external particles, $Q = 4e$. What is special about this choice is that, in the configuration of the two charges Q and the electron in between, the total electric field at the location of each particle created by other particles is zero. So, apparently, we get the AB effect without fields at the locations of the particles. This, however, is not so. The charges Q do feel fields: not in the left branch in which the electron is near the charges, but in the right branch, in which the electron is far away. The charges Q feel the fields of each other which are not canceled by the field of the electron.

Let us make an additional modification. Now, the charges Q do not automatically perform their motion toward mirror A and back, but only when the electron on the path A triggers this motion, i.e., only in the left branch. In this case, neither the electron, nor the charges Q feel an electromagnetic field in any of the branches. There will be no AB effect in this setup in spite of the fact that the electron of the left branch has an electric potential, while the electron of the right branch has not. The original treatment of the AB effect is invalid since we do not have here a motion of an electron in a classical electromagnetic field. It is an example of a recently introduced "private potential" [15].

Let us turn now to the magnetic AB effect. I will show that the AB effect arises from different shifts of the wave packets of the source which feels different local electric fields created by the left and the right wave packets of the electron.

Consider the following model. The solenoid consists of two cylinders of radius r, mass M, large length L, and charges Q and $-Q$ homogenously spread on their surfaces. The cylinders rotate in opposite directions with surface velocity v. The electron encircles the solenoid with velocity u in superposition of being in the left and in the right sides of the circular trajectory of radius R, see Fig. 4.

The flux in the solenoid due to the two cylinders is:

$$\Phi = 2\ \pi r^2\ \frac{4\pi}{c}\frac{Qv}{2\pi r L} = \frac{4\pi Q v r}{cL}. \tag{4}$$

Thus, the AB phase, i.e., the change in the relative phase between the left and the right wave packets due to the electromagnetic interaction is:

$$\phi_{AB} = \frac{e\Phi}{c\hbar} = \frac{4\pi e Q v r}{c^2 L\hbar}. \tag{5}$$

To simplify the alternative calculation based on direct action of the electromagnetic field, we assume $r \ll R \ll L$. Before entering the circular trajectory, the electron moves toward the axis of the solenoid and thus it provides zero total flux through any cross section of the solenoid. During its motion on the circle, the electron provides magnetic flux through a cross section of the solenoid seen at angle θ, see Fig. 5:

$$\Phi(\theta) = \frac{\pi r^2 e u \cos^3\theta}{cR^2}. \tag{6}$$

By entering one arm of the circle, the electron changes magnetic flux and causes an electromotive force on the charged solenoids which changes their rotational velocity. In order to calculate this change in the velocity we have to integrate the impulse exerted on all thin slices of the charged cylinder. The integration on the angle at which the slice is seen yields:

$$\delta v = \frac{1}{M}\int \frac{\pi r^2 e u \cos^3\theta}{c^2R^2}\frac{1}{2\pi r}\frac{R}{\cos^2\theta}2\pi r\frac{Q}{2\pi r L}d\theta = \frac{uQer}{c^2MRL}. \tag{7}$$

Then, the shift of the wave packet of a cylinder during the motion of the electron is:

$$\delta x = \delta v\frac{\pi R}{u} = \frac{\pi Q e r}{c^2 M L}. \tag{8}$$

I consider here the rotational cylinder motion as a linear

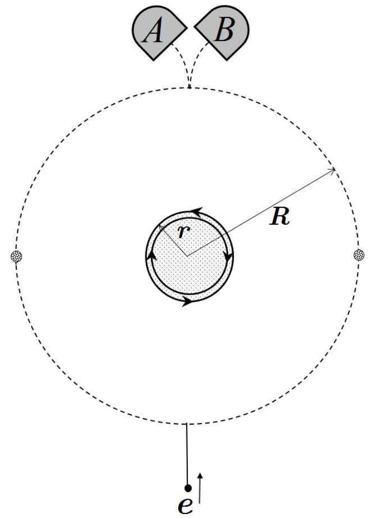

FIG. 4: **The magnetic AB effect.** The electron wave packet coming directly toward the solenoid splits into a superposition of two wave packets and, after encircling the solenoid in the center, interfere on the beamsplitter toward detectors A and B.

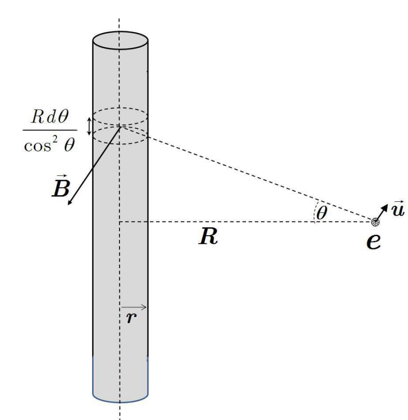

FIG. 5: **The magnetic field of the electron at the cross section of the solenoid.** The infinitesimal slice of the solenoid is seen at angle θ.

motion. The relevant wavelength of de Broglie wave of each cylinder is $\lambda = \frac{h}{Mv}$. For calculating the AB phase we should take into account that both cylinders are shifted and that they are shifted (in opposite directions) in both branches. This leads to a factor 4 and provides the correct expression for the AB phase:

$$4\frac{\delta x}{\lambda}2\pi = \phi_{AB}. \tag{9}$$

If the uncertainty in the velocity of the cylinders is smaller than δv, then, during the electron circular motion, the electron and the cylinders become entangled. But when the electron leaves the circular trajectory, it exerts an opposite impulse on the cylinders and this entanglement disappears.

In all our examples, when all systems are considered in the framework of quantum mechanics, the AB effect is explained through actions of local fields on the quantum wave function. The explanation of the AB effect is as follows. The electron in a superposition of two states $|L\rangle_e$ and $|R\rangle_e$ causes, via action of its electromagnetic field, different evolutions for the quantum state of the source: $|\Psi_L\rangle_S$ and $|\Psi_R\rangle_S$. At the end of the process, the difference between the states of the source is just the AB phase. In the total wave function of the electron and the source,

$$\frac{1}{\sqrt{2}}\left(|L\rangle_e|\Psi_L\rangle_S + |R\rangle_e|\Psi_R\rangle_S\right), \tag{10}$$

the AB phase belongs to all systems and thus it can be observed in the interference experiment of the electron.

The most common manifestation of a quantum wave function for a combined system is the nonlocal correlations which are generated by entangled states. The AB effect is conceptually different, since the effect can appear even if in the state (10) there is almost no entanglement at all times.

I believe that we can find an explanation of the kind presented above for any model of the AB experiment. However, the pictorial explanation of the creation of relative phase due to spatial shifts of wave packets disappears when we go beyond the physics of moving charges. We can replace charged cylinders by a line of polarized neutrons producing magnetic flux due to quantum spins. In this case there will be no spatial shift of wave packets and the magnetic field of the electron changes the phase of the neutrons directly. This is also an explanation of the Aharonov-Casher (AC) effect [16]: the local field acting on the neutron is responsible for appearance of the AC phase. But it does not lead to a classical lag of the center of mass of the neutron [17, 18].

The AB effect shows that the description of the evolution of a quantum particle in a classical electromagnetic field can be done by describing potentials at all locations where the wave function of the particle does not vanish (and cannot be done by describing fields at these locations). But, the classical description of the electromagnetic interaction is just an approximation (which sometimes provides exact results) of an underlying quantum reality. The present work states that the evolution of a composite system of charged particles *can* be described completely by fields at locations of all particles. The potentials are just a useful auxiliary mathematical tool after all.

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