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The three-box paradox revisited

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Abstract
The classical three-box paradox of Kirkpatrick (2003 *J. Phys. A: Math. Gen.* **36** 4891) is compared to the original quantum three-box paradox of Aharonov and Vaidman (1991 *J. Phys. A: Math. Gen.* **24** 2315). It is argued that the quantum three-box experiment is a 'quantum paradox' in the sense that it is an example of a classical task which cannot be accomplished using classical means, but can be accomplished using quantum devices. It is shown that Kirkpatrick's card game is analogous to a different game with a particle in three boxes which does not contain paradoxical features.

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1. Introduction

The three-box paradox is an example presented by Aharonov and Vaidman [1] of a pre- and post-selected quantum system that exhibits highly counterintuitive behaviour. It has become the focus of a long-lived debate, including attempts to demystify and refute the paradox (see, for example, [2–6]). One such attempt in particular, presented by Kirkpatrick [7], involves a 'classical analogue' of the three-box paradox in the form of a card game. In this paper, we analyse the paradoxical features of the three-box experiment and argue that it does not have a classical analogue. We analyse Kirkpatrick's card game and show that it does not reproduce the paradoxical features of the original three-box experiment.

The paradox in the three-box experiment is that at a particular time we can claim that a particle is in some sense both with certainty in one box, A, and with certainty in another box, B. Now, if a particle is certainly in A, then it is certainly not in B, and vice versa. Therefore, if a single particle is both certainly in A and certainly in B we have a paradox. The difficulty is not unlike the one presented by a two-slit interference experiment, where we have to admit that a single particle passes simultaneously through two separated slits, but it is more acute. In the three-box experiment, the particle is *certain* to be found in A if searched for in A, and *certain* to be found in B if searched for in B instead.

Today we do not know of any real 'paradoxes' in physics. A true physical paradox would be a prediction of a current physical theory that contradicts experimental results. Such a paradox would necessitate a modification of the current theory, i.e. progress in physics. What we mean by 'quantum paradox' is a phenomenon that *classical* physics cannot explain. The three-box paradox describes a particular task that cannot be accomplished using classical means.

An example of this type of quantum paradox is the Greenberger–Horne–Zeilinger [8] (GHZ) set-up modified by Mermin [9], in which a team equipped with quantum technology can win with certainty a game with separated players that can be won by a team employing classical (relativistic) physics only with probability 0.75 [10]. The GHZ game is one of numerous games [11, 12] based on quantum entanglement, a phenomenon that enables correlations between remote observers that are stronger than those possible classically [13]. The three-box experiment is one of the only quantum paradoxes which do not employ entanglement between remote locations, so it is a genuinely new paradox[1]. The existence of a classical system which can perform the three-box task would remove the paradox, and thus it is important to analyse the validity of Kirkpatrick's argument.

In the next section we consider some unusual properties of a pre- and post-selected spin-$\frac{1}{2}$ particle, in order to clarify the main paradoxical feature of the quantum three-box experiment described later in section 3. Section 4 is devoted to a presentation of Kirkpatrick's game as given in his paper. In section 5, we strip Kirkpatrick's game of those elements which are not relevant to the analogy with the three-box paradox and describe other simple classical games with similar properties. This allows us to show that Kirkpatrick's game does not present a classical analogue to the quantum three-box paradox. Section 6 concludes the paper with a broader view on related issues.

2. Paradoxical features of the pre- and post-selected spin-$\frac{1}{2}$ particle

In quantum theory, unlike classical theory, an initial (or final) state alone does not provide all the information about a system during the time interval between two measurements. For pre- and post-selected quantum systems, a complete description is given by a two-state vector [15] that takes into account both initial and final states. This is the basis for all the examples in which we can make apparently contradictory statements about a quantum system at the time between its pre- and post-selection.

For example, consider a spin-$\frac{1}{2}$ particle pre-selected in the state $|{\uparrow_x}\rangle$ and post-selected in the state $\langle{\uparrow_z}|$. An intermediate measurement of σ_x or of σ_z is in either case certain to yield +1. The spin of the particle is *certain* to be 'up' in the x direction if a projection measurement onto $|{\uparrow_x}\rangle$ is performed, and *certain* to be 'up' in the z direction if a projection measurement onto $|{\uparrow_z}\rangle$ is performed.

A naive classical analogy of a spin 'up' in the x direction is a classical pointer, an arrow, pointing in the x direction. An arrow pointing in the x direction certainly does not point in the z direction, so apparently we have obtained a paradox of the kind associated with pre- and post-selection. However, it is not so, since there is no classical task involved. Spin measurement is genuinely quantum, and within the framework of quantum theory there is no paradox. For, when the spin is in the state $|{\uparrow_x}\rangle$, we cannot claim that it is not pointing in the z direction; even without post-selection there is a 50% chance of finding it in the state $|{\uparrow_z}\rangle$.

[1] One can also consider interaction-free measurement [14] as a quantum paradox which is not based on classically impossible correlations. There, a gedanken device is considered which explodes whenever a particle 'touches' it. Quantum strategy allows us to detect the device without it exploding.

An example of a situation that is paradoxical within the framework of quantum theory is a pre- and post-selected spin-$\frac{1}{2}$ particle that is certain to be 'up' in the x direction, certain to be 'up' in the y direction *and* certain to be 'up' in the z direction [16]. Here, we discuss another paradoxical experiment; it is based on the mathematical structure of the three-box paradox. Would you be surprised by the following? Suppose that we give you a box, and arrange a *single* pre- and post-selected particle so that:

> If the particle is searched for in a box with its spin 'up' in the z direction, it is to be found with certainty, while if the same particle is searched for in the same box with its spin 'down' in the z direction, it is also to be found with certainty.

These two properties are contradictory not only in the naive classical analogue (of an arrow pointing in two opposite directions) but also in standard quantum theory. The usual 'eigenvalue–eigenstate' link suggests that a particle which is to be found with certainty, if searched for, in a box with its spin 'up' in the z direction has the quantum state $|{\uparrow_z}\rangle$, and thus cannot be found in the box with its spin 'down' in the z direction.

It is possible for a pre- and post-selected system to yield definite outcomes for intermediate measurements that are not ensured by either the pre-selection or the post-selection alone. Still, a situation in which contradictory outcomes are obtained with certainty, if measured, is surprising. Let us show how this is achieved.

The system consists of a spin-$\frac{1}{2}$ particle and two boxes, A and B. The particle is pre-selected in the state

$$|\psi\rangle = \frac{1}{\sqrt{3}}(|A, \uparrow_z\rangle + |A, \downarrow_z\rangle + |B, \uparrow_z\rangle), \tag{1}$$

and post-selected in the state

$$\langle\phi| = \frac{1}{\sqrt{3}}(\langle A, \uparrow_z| + \langle A, \downarrow_z| - \langle B, \uparrow_z|), \tag{2}$$

where $|A, \uparrow_z\rangle$ represents the particle in box A with spin 'up' in the z direction. We give you box A and offer you to either look for a particle with spin 'up' or to look for a particle with spin 'down' inside. To do this, you have to perform an ideal projection measurement. Therefore, if you look for the particle with spin 'up' and find it, the particle's quantum state after your measurement becomes $|A, \uparrow_z\rangle$, but if you do not find it, its state becomes $\frac{1}{\sqrt{2}}(|A, \downarrow_z\rangle + |B, \uparrow_z\rangle)$. The latter is orthogonal to the post-selected state, however, so the post-selection cannot succeed if the particle is not found. And because the pre- and post-selections are symmetric with respect to the states $|A, \uparrow_z\rangle$ and $|A, \downarrow_z\rangle$, the particle is also to be found, if searched for, with its spin 'down' in the same box.

Although this spin example is more counterintuitive than the previous one, it too does not fall into the category of 'quantum paradoxes' as defined above, since there is no classical task involved. In both examples we discuss the results of spin measurements which are inherently quantum. The measurements in the second example are particularly difficult. In the first example, a suitably oriented Stern–Gerlach apparatus can be used, but in the second example this is no longer appropriate—a Stern–Gerlach measurement in box A would distinguish between all three states $|A, \uparrow_z\rangle$, $|A, \downarrow_z\rangle$ and $|B, \uparrow_z\rangle$, and therefore would not be a projection measurement.

3. The three-box paradox

The original three-box experiment [1] involves no spin, only a particle and three separate boxes. The particle is pre-selected in the state

$$|\psi\rangle = \frac{1}{\sqrt{3}}(|A\rangle + |B\rangle + |C\rangle), \tag{3}$$

and post-selected in the state

$$\langle\phi| = \frac{1}{\sqrt{3}}(\langle A| + \langle B| - \langle C|), \tag{4}$$

where the mutually orthogonal states $|A\rangle$, $|B\rangle$ and $|C\rangle$ denote the particle being in boxes A, B and C, respectively.

In-between pre- and post-selection, an observation takes place either of box A or of box B. A successful observation of box A corresponds to the particle being found in the box and is represented by the projection $P_A = |A\rangle\langle A|$. An unsuccessful observation of box A corresponds to the particle *not* being found and is represented by $\mathbf{1} - P_A = |B\rangle\langle B| + |C\rangle\langle C|$. Similar definitions hold for an observation of box B. As before, each of the two intermediate observations is certain to succeed. Indeed, not finding the particle in A leads to the quantum state $\frac{1}{\sqrt{2}}(|B\rangle + |C\rangle)$, and not finding it in B leads to the state $\frac{1}{\sqrt{2}}(|A\rangle + |C\rangle)$, both of which are orthogonal to the post-selected state $\langle\phi|$.

The three-box example can be presented as a classical task which cannot be achieved using classical means but which can be achieved using quantum preparation and verification measurements. Consider the following game. Alice, equipped with quantum devices, prepares the particle in the system of three boxes and passes the first two boxes on to Bob. Bob, who is unaware of Alice's quantum machinery, is told to look in one of the boxes before returning them both to Alice, and that he wins if he does *not* find anything. However, Alice gets to decide according to her post-selection measurement whether the game 'counts' or not. Now, Bob has no *a priori* reason not to agree to play; his chances of winning appear to be $1/2$. It is obviously so before Alice performs the post-selection measurement, and since Bob is careful not to leave any mark disclosing the results of his measurement, Alice apparently cannot gain anything from the post-selection. Bob will find, however, that Alice somehow manages to discard all runs of the game in which he does not find the particle, and so Bob will always lose.

4. Kirkpatrick's game

Kirkpatrick [7] claims that '... the three-box example is neither quantal nor a paradox ...', and supposedly demonstrates this by means of its reproduction in an elaborate playing card game.

Kirkpatrick's classical system is constructed using ordinary playing cards. Each playing card has two marks, a 'face' (e.g., Jack, Queen, King) and a 'suit' (e.g., Spades, Diamonds, Hearts), and these are treated as system variables: *Face* (with values J, Q, K) and *Suit* (with values S, D, H). The deck of cards is divided into two parts called *These* and *Others*, and the system consists also of a memory $\mathcal{M}$. Kirkpatrick defines the following procedures, in which a system variable is generically referred to as P and its values as $\{p_j\}$.

Preparation. The system is 'prepared' in a particular 'state' $P = p_j$ (e.g., $Face = Q$) by (1) placing all the cards with $P = p_j$ (i.e., all the Queens) in *These* and the remainder of the deck in *Others* and (2) setting the memory to the variable name $\mathcal{M} \equiv P$ (e.g., $\mathcal{M} \equiv Face$).

Observation. The variable P is 'observed' in the following manner. If $\mathcal{M} = P$, (1) select a card at random from *These* and (2) report the value p_j of the variable P of this card (i.e., its 'face' or 'suit'). If $\mathcal{M} \neq P$, then (1) select a card at random from *Others*, (2) report the value p_j of the variable P of this card and (3) prepare the system in the state $P = p_j$ according to the procedure defined above.

Partial observation. In order to reproduce the three-box experiment, it should be possible to perform a partial (as opposed to complete) observation, in which one observes only whether

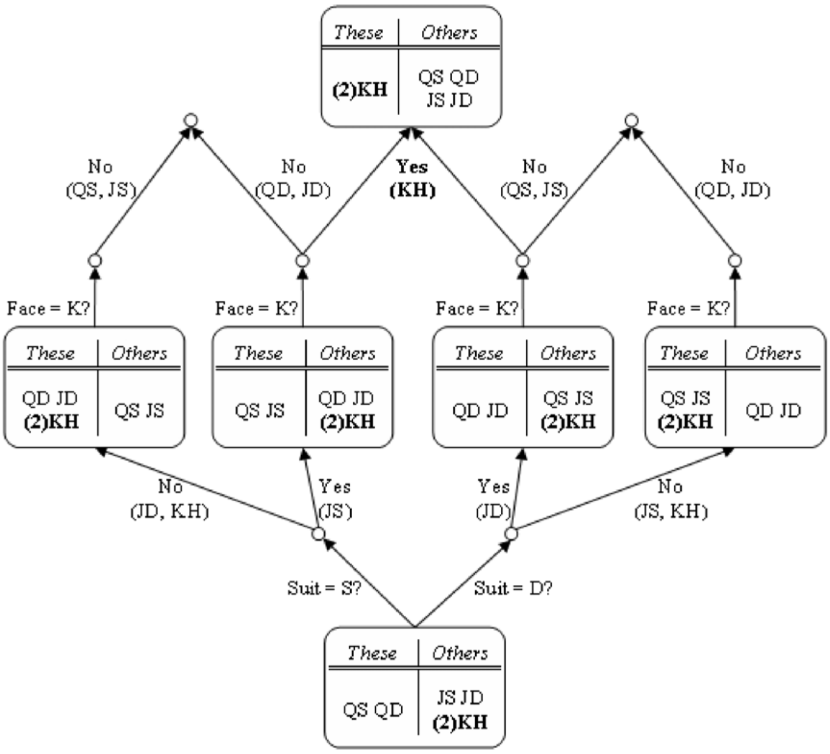

Figure 1. Kirkpatrick's card game. The initial $Face = Q$ state ensures that all outcomes are possible in the subsequent measurement of *Suit*. The final $Face = K$ state requires that a *KH* be selected from *Others* in the final measurement of *Face*. This, in turn, means that a *JS* must be selected in a measurement of whether $Suit = S$, and a *JD* must be selected in a measurement of whether $Suit = D$.

or not the variable P has a particular value p_j. In a partial measurement of P one selects a card from the appropriate part of the deck (as determined by the content of $\mathcal{M}$) and reports the value p_j if the card's value of P is indeed p_j, and $\widetilde{p_j}$ if the card's value of P is *not* p_j. If $\mathcal{M} \neq P$, one prepares the system according to the outcome. To prepare the system in the state $P = \widetilde{p_j}$, place all the cards with $P = \widetilde{p_j}$ in *These* and all other cards (i.e., those with $P = p_j$) in *Others*. The variable $\mathcal{M}$ is, as before, set to $\mathcal{M} \equiv P$.

The role of the variable $\mathcal{M}$ is apparently to distinguish between 'repeated' and 'new' observations. A repeated measurement is certain to yield the same outcome as obtained previously, whereas a 'new' measurement (which by definition must be a measurement of the other variable) causes the system's state to be reset.

The need to define partial observations has to do with the three-box analogue. In the three-box experiment, projection measurements are involved that measure only whether or not the particle is in a particular box. Such measurements are considered to be 'partial', whereas a 'complete' measurement would consist of, e.g., looking in all three boxes.

The three-box analogue is supposedly obtained (see figure 1) by choosing the following six cards for the deck: a Jack of Spades (JS), a Jack of Diamonds (JD), a Queen of Spades (QS), a Queen of Diamonds (QD) and two Kings of Hearts ($(2)KH$). (The reason for two Kings of Hearts is to have each value of each variable appear the same number of times in the deck). The system is prepared in the state $Face = Q$. One of two partial observations of the variable *Suit* then takes place: either of whether or not $Suit = S$ (corresponding to an observation of box A) or of whether or not $Suit = D$ (corresponding to an observation of box B). Finally, a post-selection measurement of *Face* is performed resulting in the final state $Face = K$.

The initial state $Face = Q$ ensures that all outcomes are possible in the following observation of *Suit* (since the JS, the JD and the $(2)KH$ are in *Others*). It is the post-selected, final state that ensures that if the intervening observation was of whether $Suit = S$, then the JS must have been selected, and if it was of whether $Suit = D$, then the JD must have been selected. This is because in the final measurement of *Face* a KH must be selected from *Others*, and whereas in the states $Suit = S$ and $Suit = D$ the $(2)KH$ remain in *Others*, in the states $Suit = \widetilde{S}$ and $Suit = \widetilde{D}$ they are moved to *These*. Therefore, given the post-selection, the JS and the JD are each certain to be selected, and the system's intermediate state is certain to be $Suit = S$ and certain to be $Suit = D$, depending on what is measured. This, claims Kirkpatrick, is analogous to the particle in the three-box experiment being found with certainty either in box A or in box B.

5. Simplification of Kirkpatrick's game and similar proposals

The large part of Kirkpatrick's game is extraneous to the three-box analogy. It is intended to mimic quantum mechanical phenomena in a more general sense. In another work [17], Kirkpatrick uses a modified version of the game presented here, claiming that it illustrates the ordinary nature of much of quantum probability, including incompatibility of observables, interference, etc. We are not persuaded by his arguments, but the discussion of these issues goes beyond the scope of this paper. We therefore consider Kirkpatrick's game without the complications of defining (and using the terminology of) system states and variables.

Consider a smaller deck of cards consisting of a Jack of Spades (JS), a Jack of Diamonds (JD) and a King of Hearts (KH), still distributed between two groups *These* and *Others* (see figure 2). Suppose that initially all three cards are in *Others*. A card is then selected at random from *Others*, and as before a 'partial observation' is performed either of whether or not it is a spade (i.e., the JS), or of whether or not it is a diamond (the JD). In either case, if the outcome is positive, the selected card is placed in *These* while the others remain in *Others*. Otherwise, the selected card is returned to *Others* while the other cards are moved to *These*. Finally, a card is again selected at random from *Others*, and our post-selection requirement is that this final card is the KH. This requirement ensures that the previously selected card must have been the JS if spade was searched for, and must have been the JD if diamond was searched for instead.

Leifer and Spekkens [18] have suggested yet a simpler game to demonstrate this classical phenomenon. Their system (see figure 3) consists of a ball and a box. The box can be split into two half-boxes, either lengthwise or widthwise, by placing a double partition inside and separating the resulting halves. The two half-boxes can be reassembled into a single box by joining them together and removing the partition. The halves of one division are denoted 'front' and 'back', and of the other, 'right' and 'left'. One measures whether the ball is in the front half of the box by partitioning the box into front and back halves and shaking the front half to hear whether the ball is inside. If the ball is found in this case, its left/right position is randomized, but if it is not found then its left/right position is undisturbed. Similar definitions apply for measurements of whether the ball is in the back, on the right or on the left. Now suppose that this system is pre-selected so that the ball is in the front half, and post-selected so that the ball is in the back half, where the intervening measurement consists of looking for

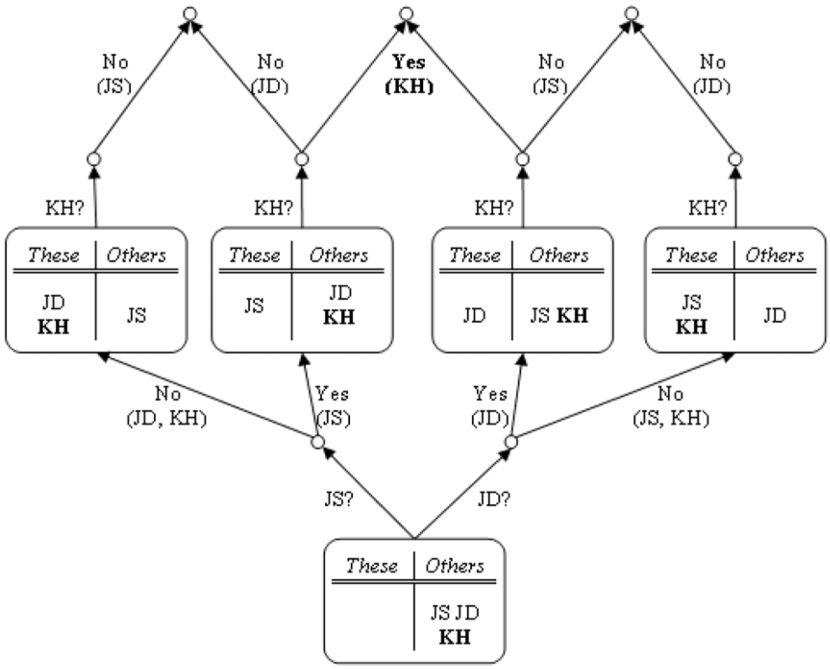

Figure 2. Simplification of Kirkpatrick's game. Initially three cards *JS*, *JD* and *KH* are in *Others*, allowing for any of the relevant values of *Suit* to be selected. The final, post-selection requirement is that a *KH* be selected from *Others*. Therefore, a *JS* must be selected if it is searched for, and that a *JD* must be selected if it is searched for instead.

the ball either on the right or on the left. Then, the ball is necessarily found in the intervening measurement, since otherwise there is no disturbance of the initial front state and no way for the ball to be transferred to the back half. Thus, the ball is certain to be found on the right if that is where it is looked for, and certain to be found on the left if that is where it is looked for instead.

What these games have in common is that they use classical measurement disturbance to effectively encode the outcome of the measurement into the system. In both cases, parts of the system are moved around in such a way that the post-selection becomes impossible whenever the measurement is unsuccessful. Obviously, there is nothing paradoxical about such mechanisms, but neither are they equivalent to the three-box experiment. In the three-box experiment, the intermediate observation consists of just that—observation. A classical system, on the other hand, is not disturbed by observation, therefore the measurement must involve additional actions.

We can suggest another three-box experiment, or game, which utilizes this kind of classical measurement disturbance and is entirely non-paradoxical. Suppose we put a classical ball in one of three boxes, and have another observer look inside either the first box or the second (see figure 4). So long as the box is only observed, there is no post-selection measurement we can perform that will ensure the ball has been found. Therefore, let us have the observer place the ball in the third box whenever he does not find it in the observed box. This makes it easy to ensure the ball is found by post-selecting that finally the ball is not in the third box. It is this three-box game, rather than the original one, to which Kirkpatrick's game is analogous.

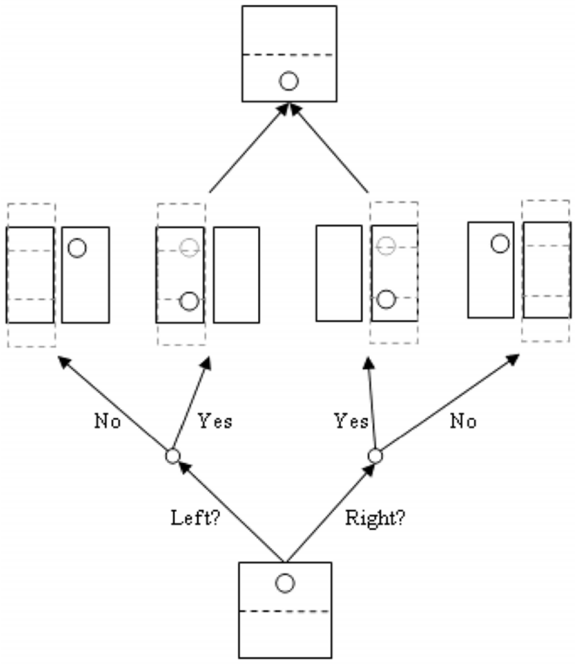

Figure 3. Leifer and Spekkens' game. The ball is pre-selected in the front half and post-selected in the back half. Its movement from the front to the back ensures that the ball is found in the intervening measurement either of whether it is on the left or of whether it is on the right.

In all these games, the 'observation' includes leaving a mark that is later 'read' in the post-selection process. The observation in the original three-box experiment, which consists of opening a box, does not leave such a mark in the framework of classical physics. Note that, contrary to the supposedly analogous games, a successful post-selection in the three-box experiment is possible even with no intermediate measurement (i.e., if none of the boxes are opened).

6. Quantum paradox and beyond

The quantum three-box experiment, beyond its paradoxality, is very effective in demonstrating concepts that are related specifically to pre- and post-selected quantum systems. While we agree (see [19]) with Kastner [2] that in the three-box experiment there is no ontological property, or any kind of hidden variable, corresponding to the particle 'being in box A', we believe that consistent and useful operational definitions can be obtained for the 'elements of reality' [20] and 'weak-measurement elements of reality' [21] associated with the particle 'being in box A'. The former corresponds to 'if observed in A, is to be found there with certainty', and the latter to the *weak value* associated with a *weak measurement* [15] of the particle in A.

In the case that we observe the particle in A weakly (i.e., with negligible disturbance), these concepts become particularly useful, for it is then possible to weakly observe the particle in B as well, at the same time. As it turns out, the 'elements of reality' and 'weak-measurement elements of reality' are the same as if there were indeed two particles, one in A and another in B [22].

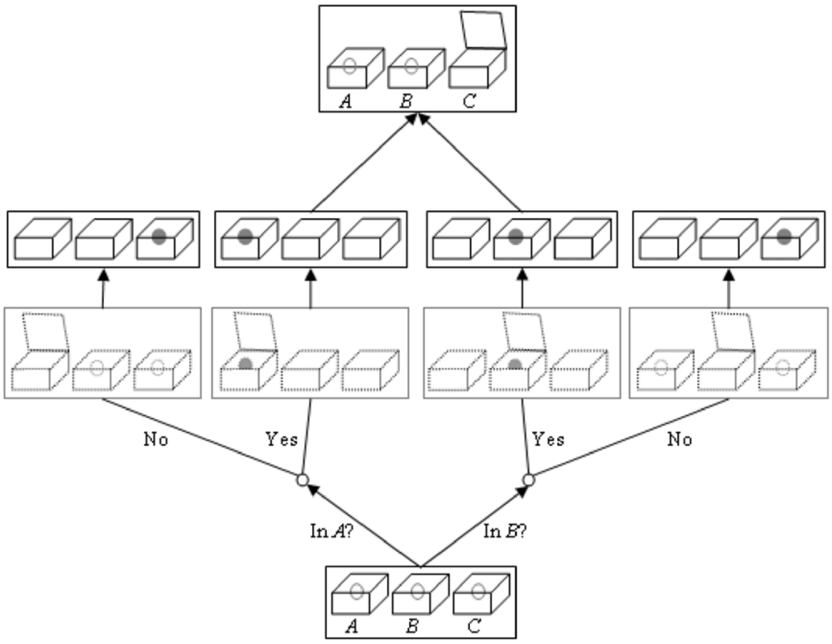

Figure 4. The three-box game to which Kirkpatrick's game is analogous. Initially the ball is in one of the three boxes. If the observer does not find the ball in one of the first two, he moves the ball to the third box. The post-selection is that the ball is not in the third box.

As we have shown, Kirkpatrick's system does not reproduce the three-box paradox because it contains no contradiction. In [2], Kastner claims that the original three-box experiment does not contain a contradiction either. Her argument is that because only one of the boxes is actually observed, the properties of 'being in box A' and of 'being in box B' cannot be attributed simultaneously to the same single particle. Kastner argues that once the particle is found, one cannot consider what would have happened had the other box been observed instead. This implies that when the particle is found it actually becomes 'in the box' as a result of the observation, i.e., the measurement disturbance. Thus, although Kastner does not dispute the quantum mechanical nature of the three-box experiment, Kirkpatrick's game seems to provide a good demonstration of her argument.

Kastner and Kirkpatrick apparently have the following idea in common: that if disturbance is understood to be inherent to measurement, then the difficulty with regard to the three-box experiment is removed. This view is shared by Leifer and Spekkens, who show [18] that any attempt to create a classical analogue of the three-box paradox requires measurement disturbance. But it is precisely because the classical observation in the three-box experiment is *non-disturbing* that the experiment cannot be explained by classical physics or accomplished using classical means. This is what causes the three-box experiment to be a 'quantum paradox'.

Acknowledgments

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