MEANING AND MEASURABILITY OF NONLOCAL QUANTUM STATES

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ABSTRACT

A method for the instantaneous measurements of nonlocal quantum states of composite physical system is presented. The general form of those states whose measurement (at a well defined time) is consistent with relativistic causality is derived. Some new varieties of quantum measurement are discussed.

In this work we shall try to clarify the question: Can we make sense out of the quantum state in relativistic quantum mechanics? To make sense in our approach, is to be able to measure: To be physically meaningful, for us here, shall be to be measurable.

As early as 1931, Landau and Peierls(1) found that the theory of relativity produces new restrictions on the measurement process. After their work it was thought that in relativistic quantum theory only local variables are measurable; but lately it has emerged(2,3) that this is not so: Certain nonlocal states *can* be verified by experiment. On the other hand, *not* all of them are measurable, and indeed there are states whose measurement would contradict the principle of casuality.

In this work we start with the *assumption* that we can measure any *local* operator, and we *investigate* which *nonlocal* operators and states are measurable. We are interested in the following question: Does it have physical meaning to speak about nonlocal variables at a particular time? The measuring procedures we seek, in response to this question, will consequently be instantaneous. First we need to explain what we *signify* by "measurement." It is different from the usual definition of measurement in quantum mechanics. We define a measurement here as the nondemolishing verification that a certain variable A has a given value a. If before the measurement the observed variable has the value a, then the experiment will produce the result "yes," and the state of the system will *not* change. If our initial state is a linear combination of eigenstates of the observed operator with different eigenvalues of A, then the experiment will produce the result "yes" or "no" with appropriate probability. In case the answer is "yes," the final state

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will be the projection of the initial state on the degenerate space of eigenstates of A with eigenvalue a. If the answer is "no," then the final state will be orthogonal to that space. The difference between this and the usual definition of measurement in quantum mechanics is that we *not* require that the other eigenstates of the observed variable (with *other* eigenvalues than a) be unaltered during the measuring process. The only requirement is that, if we start with a state wherein the $A \neq a$, then $A \neq a$ at the *end* of the measurement as well.

Our first nonlocal variable will be the sum of the local variables A_1 and A_2 that are related to spatially separate parts of the system. We are interested in nonlocal measurement; that is, after the measurement we should like to know the value of the sum $A_1 + A_2$ *without* knowing the values of A_1 and A_2 separately.

We have to verify that $A_1 + A_2 = a$. By redefining $A_2 \rightarrow A_2 - a$, we see that our problem is the verification that

$$A_1 + A_2 = 0. \tag{1}$$

Our measuring device consists of two separate parts which have canonical coordinates q_1 and q_2. We prepare this composite device in the nonlocal state

$$\begin{aligned} q_1 - q_2 &= 0, \\ \pi_1 + \pi_2 &= 0, \end{aligned} \tag{2}$$

where π_i is the momentum conjugate to q_i. We can do this by local interaction when the two parts of the measuring device are initially brought together. Then we separate those parts and position them at the appropriate parts of the observed system.

The next stage of our measurement procedure is the local interaction between appropriate components of the measuring device and those of the observed system. The interactions are short and simultaneous. The time of the measurement is defined by the time of this interaction. The Hamiltonian of the interaction is

$$H_{int} = g(t)\ (q_1A_1 + q_2A_2), \tag{3}$$

where g(t) is nonzero only during a short interval of time $[t_o, t_o + \epsilon]$ and it fulfills the normalization condition

$$\int_{t_o}^{t_o+\epsilon} g(t)\ dt = 1. \tag{4}$$

Then, in the Heisenberg picture,

$$\dot{\pi}_1 = -g(t)A_1,$$
$$\dot{\pi}_2 = -g(t)A_2, \qquad (5)$$

A_1 and A_2 are not changed by the interaction (3), and we can take ϵ small enough so that A_1 and A_2 will not be changed (as a result of their *own* dynamics) during the time of the interaction. Then, using the initial condition (2) and the normalization (4) of g(t), we find from (5) that

$$(\pi_1+\pi_2)_{t>t_o+\epsilon} = -(A_1+A_2)_{t=t_o}. \qquad (6)$$

The last step of our measuring procedure consists of local measurements of π_1 and π_2. We will perform those measurements immediately after the local interactions at time $t = t_o + \epsilon$. This completes the measurement of $A_1 + A_2$.

Indeed, we see from (6) that knowing π_1 and π_2 after the interaction gives us the value of $A_1 + A_2$. At time $t = t_o + \epsilon$ there is no local observer who knows the value of $A_1 + A_2$. Such knowledge would require bringing the results of the local measurements of π_1 and π_2 together, and that would require some additional finite period of time. But, since the values π_1 and π_2 have been indelibly recorded at time $t = t_o + \epsilon$ (by means of those final local measurements), the measurement of $A_1 + A_2$ (given that the "measurement" of $A_1 + A_2$ is taken to mean the indelible recording, in some macroscopic form, of the value $A_1 + A_2$ at time $t_o + \epsilon$) is ambiguously completed at that time.

We can generalize this method to the measurement of the sum of N local operators $\Sigma_{i=1}^{n} A_i$ related to N separate parts of the composite system.

A more general class of nonlocal variables that we can measure are the modular sum of local variables:

$$(\Sigma_{i=1}^{N} A_i) \bmod a.$$

Now we are going to describe the method for measuring nonlocal states using sets of measurements of the type considered above. What we mean by measurement of the state $|\phi\rangle$ is the nondemolishing verification that the state of the system is $|\phi\rangle$. If we start with the state

$$|\psi\rangle = \alpha|\phi\rangle + \beta|\phi_\perp\rangle,$$

where $|\phi_\perp\rangle$ is orthogonal to $|\phi\rangle$, then the measurement will produce the result "yes" with probability $\|\alpha\|^2$, and the final state will in those cases be $|\phi\rangle$; it will produce the result "no" with probability $\|\beta\|^2$, and the final state in those cases will be orthogonal to $|\phi\rangle$ (but will *not* necessarily be $|\phi_\perp\rangle$). To begin with, we study a system that consists of two separate parts with K orthogonal states in each. We shall designate local bases in each part as $|i\rangle_1$ and $|j\rangle_2$, i, j = 1,2,...,K. The general state of the system can be written as

$$|\psi\rangle = \Sigma_{i,j=1}^{K}\beta_{ij}\ |i\rangle_1\ |j\rangle_2. \tag{7}$$

We can always find new bases in the separate local parts such that the state $|\psi\rangle$ will have the following form (we will call it *canonical*):

$$|\psi\rangle = \Sigma_{i=1}^{K}\ \alpha_i|i\rangle_1\ |i\rangle_2. \tag{8}$$

Therefore, any state $|\psi\rangle$ can be brought into the canonical form (8). Now we give the measuring procedure that verifies the state $|\phi\rangle$ (the canonical form of which is)

$$|\phi\rangle = \frac{1}{K}\ \Sigma_{i=1}^{K}\ |i\rangle_1\ |i\rangle_2. \tag{9}$$

The measurement of the state will include measurements of two nonlocal operators. The first is verification that $A_1 + A_2 = 0$, where

$$\begin{aligned} A_1\ |i\rangle_1 &= -i|i\rangle_1, \\ A_2\ |i\rangle_2 &= i|i\rangle_2. \end{aligned} \tag{10}$$

This measurement is a nondemolishing verification that our state has a canonical form in given local bases without defining the coefficients α_i. The next measurement has to specify α_i; in our case, it has to verify that all α_i are equal.

We define unitary local operators that will act in every local part of the system:

$$\begin{aligned} U_1|i\rangle_1 &= |i+1\rangle_1, \quad U_1|K\rangle_1 = |1\rangle_1, \\ U_2|i\rangle_2 &= |i+1\rangle_2, \quad U_2|K\rangle_2 = |1\rangle_2. \end{aligned} \tag{11}$$

It is easy to see that among the states that have canonical form in our basis, only the state $|\phi\rangle$ (9) will not change under the transformation U_1U_2:

$$U_1U_2\ |\phi\rangle = |\phi\rangle. \tag{12}$$

Now we define B_1 and B_2:

$$e^{iB_1} = U_1, \quad e^{iB_2} = U_2. \tag{13}$$

Then, taking into account that B_1 and B_2 commute, we have $U_1U_2 = e^{i(B_1+B_2)}$ and therefore Eq.(12) is equivalent to

$$(B_1 + B_2) \bmod 2\pi = 0. \tag{14}$$

Thus the second measurement that will complete the verification of our state is the measurement of the modular sum of B_1 and B_2. B_1 and B_2 are Hermitian local operators; therefore one can relate them to physical variables. We can generalize this method to the composite system that has M>2 separated parts. One can measure the state

$$|\phi\rangle = \frac{1}{\sqrt{K}}\ \Sigma_{i=1}^{K}\ |i\rangle_1\ |i\rangle_2\ .\ .\ .\ |i\rangle_M. \tag{15}$$

The last example of what one can measure will be a nonlocal operator with nondegenerate eigenstates: We define the operator by its eigenstates:

$$\begin{aligned}
|\phi_1\rangle &= \tfrac{1}{\sqrt{2}}(|1\rangle_1\ |1\rangle_2 + |2\rangle_1\ |2\rangle_2),\\
|\phi_2\rangle &= \tfrac{1}{\sqrt{2}}(|1\rangle_1\ |1\rangle_2 - |2\rangle_1\ |2\rangle_2),\\
|\phi_3\rangle &= \tfrac{1}{\sqrt{2}}(|1\rangle_1\ |2\rangle_2 + |2\rangle_1\ |1\rangle_2),\\
|\phi_4\rangle &= \tfrac{1}{\sqrt{2}}(|1\rangle_1\ |2\rangle_2 - |2\rangle_1\ |1\rangle_2),
\end{aligned} \tag{16}$$

the eigenvalues may be arbitrary but no two of them are equal. The measurement procedure will be the following:

We take the local operators A_1 of Eq.(10); then $|\phi\rangle_1$ and $|\phi\rangle_2$, as well as $|\phi\rangle_3$ and $|\phi\rangle_4$, will be degenerate eigenstates of the operator $(A_1 + A_2)$ mod 2 which we know how to measure. Next we perform the appropriate local unitary transformations

$$T = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

and measure the operator $(A_1 + A_2)$ mod 2 as it is defined in the new bases. Now, the degenerate eigenstates will be $|\phi\rangle_1$ and $|\phi\rangle_3$ as well as $|\phi\rangle_2$ and $|\phi\rangle_4$. For the measurement that consists of these two measurements, the states $|\phi\rangle_i$ are eigenstates and they are nondegenerate.

Let us give an example of a nonlocal operator the measurability of which would violate causality. The operator will have the following set of nondegenerate eigenstates:

$$\begin{aligned} |\psi_1\rangle &= \frac{1}{\sqrt{2}}(|1\rangle_1 \, |1\rangle_2 + |2\rangle_1 \, |2\rangle_2), \\ |\psi_2\rangle &= \frac{1}{\sqrt{2}}(|1\rangle_1 \, |1\rangle_2 - |2\rangle_1 \, |2\rangle_2), \\ |\psi_3\rangle &= |1\rangle_1 \, |2\rangle_2, \\ |\psi_4\rangle &= |2\rangle_1 \, |1\rangle_2. \end{aligned} \tag{17}$$

We may contradict the principle of causality in the following way:

(i) preparing state $|2\rangle_2$ in part 2 at time $t \ll t_o$,

(ii) preparing state $|1\rangle_1$ or $|2\rangle_1$ at time $t = t_o - \epsilon$,

(iii) measurement of the operator at time $t = t_o$,

(iv) local verification of the state $|2\rangle_2$ at time $t = t_o + \epsilon$.

The probability of the result of the local measurement (iv) in part 2 at time $t_o + \epsilon$ *will depend on our choice at time $t_o - \epsilon$ in part 1*, albeit part 1 is separated from part 2 by an arbitrary distance.

This contradiction of the causality principle concludes the proof. Based on the same idea, we can derive relativistic restrictions on measurable variables and thereby prove that every measurable nonlocal state must take the canonical form (8) with all $\|\alpha_i\|$ equal.

We can generalize the statement to composite systems with many parts. One can divide any system into two subsystems, and then the measurable states have to be of the form

$$|\phi\rangle = \frac{1}{\sqrt{K}} \Sigma_{i=1}^{K} \, |\psi^i\rangle_1 \, |\psi^i\rangle_2, \tag{18}$$

where $|\psi^i\rangle_\alpha$ $i = 1,2,\ldots,k$ are orthonormal states in part $\alpha = 1,2$. Also, for any measurable state of a composite system, the density matrices in each separate part of the system must be similar to the diagonal matrix in which all nonvanishing values must be equal. Therefore, the state

$$|\phi\rangle = \alpha_1\ |1\rangle_1\ |1\rangle_2 + \alpha_2\ |2\rangle_1\ |2\rangle_2, \quad \|\alpha_1\| \neq \|\alpha_2\| \neq 0, \tag{19}$$

is unmeasurable. We can prove that the measurability of $|\phi\rangle$ contradicts the principle of causality. But there are other kinds of measurement for which $|\phi\rangle$ is measurable (and it is this fact which gives us the possibility of speaking about the state $|\phi\rangle$).

First, one can *prepare* state $|\phi\rangle$. We prepare locally the states

$$\alpha_1|1\rangle_1 + \alpha_2|2\rangle_1 \quad \text{and} \quad \frac{1}{\sqrt{2}}\ (|1\rangle_2 + |2\rangle_2). \tag{20}$$

Then the initial state of the system will be

$$\frac{1}{\sqrt{2}}(\alpha_1|1\rangle_1 + \alpha_2|2\rangle_1)\ (|1\rangle_2 + |2\rangle_2)$$
$$= \frac{1}{\sqrt{2}}(\alpha_1|1\rangle_1|1\rangle_2 + \alpha_2|2\rangle_1 2\rangle_2) + \frac{1}{\sqrt{2}}\ (\alpha_1|1\rangle_1|2\rangle_2 + \alpha_2|2\rangle_1|1\rangle_2). \tag{21}$$

Now we can verify by our measurement procedure that the state has a canonical form. This will give the answer "yes" with probability 1/2, and in these cases the final state will be $|\phi\rangle$. The experiment, of course, may or may not be successful; indeed we do not know of any measurement that will prepare the state with probability 1.

Another variety of measurement is a particular kind of nondemolishing verification that the state is $|\phi\rangle$. We know how to accomplish this for *every* nonlocal state $|\phi\rangle$. This verification measurement does not satisfy all the requirements of our definition of state measurement. It is non-demolishing for the state $|\phi\rangle$, but this time the final state will be $|\phi\rangle$ in *any* case, *without* dependence on the initial state.

In this measurement we use a measuring device that has a Hilbert space isomorphic to our system and we prepare it in a state $|\underline{\phi}\rangle$ that corresponds under the isomorphism to $|\phi\rangle$. Then we switch on some local simultaneous interactions that will produce an "exchange" between the state of the system and the state of the measuring device. The interactions that will do that are interactions between every separate part t of the system and the corresponding part of the measuring device. These are described by the transformation

$$|i\rangle_t\ |\underline{j}\rangle_t \rightarrow |j\rangle_t\ |\underline{i}\rangle_t, \tag{22}$$

where $|\underline{i}\rangle_t$ is a set of orthogonal states in one separate component t of the measuring device.

We see, indeed, that this transformation leads to exchanging of the states

$$|\psi\rangle|\underline{\phi}\rangle \rightarrow |\phi\rangle|\underline{\psi}\rangle. \tag{23}$$

In some sense it is difficult to claim that the measuring process is complete after the exchange. It is not only that there is no local observer who can immediately know the result of the measurement (that we have encountered before), but also that we cannot perform local measurements on the measurement device and thereby obtain a set of results (in separated places) that, after being brought together, will give us the answer. We need to bring the parts of the measuring device itself together in one place. Then the state of the measuring device will be local, and we have assumed that we can measure any local state.

This "exchange" measurement has another limitation as well. It may be used only as a state measurement. We cannot produce an "exchange" measurement of an operator. This is true not only for the usual definition of an operator measurement but also for verification of the given value of the operator in the case that there are degenerate eigenstates with this value. We can perform this verification using the methods described in the beginning for a quantum system that is correlated with another system. We can do this without destroying the correlation. However, this is something that clearly cannot be accomplished by an "exchange" measurement without touching the "other" system.

In this work we have presented a method for the measurement of nonlocal states in composite systems that have N separate parts with K orthogonal states in every part. The general form of the measurable states is

$$|\phi\rangle = \frac{1}{\sqrt{K}} \Sigma_{i=1}^{K} \Pi_{j=1}^{N} |i\rangle_j . \tag{24}$$

Some of those states are familiar ones. If $K = N = 2$, Eq.(24) is the EPR-Bohm state that was used later by Bell in his original paper about the Bell inequality. If $N = 2$ and $K \to \infty$, then the state is similar to the original EPR state. We proved that, at least for $N = 2$, these are the only measurable nonlocal states, all of which have the following local property: Any local measurement in any separate part has the same probability to produce *any* given result. In other words, the density matrix in all separate parts is proportional to the unit matrix. This explains why these measurements do not contradict causality. Finally we saw new kinds of measurements that are suitable for any nonlocal state.

REFERENCES

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