Interplay of Aharonov-Bohm and Berry Phases for a Quantum Cloud of Charge

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The Aharonov-Bohm (AB) effect is a simple and topological: an electron encircling a solenoid containing a magnetic flux \( \Phi \) acquires a geometric phase equal to \( \Phi n/\hbar \), where \( n \) is equal to the winding number of the electron around the solenoid. However, when a solenoid enters a quantum cloud of charge and there is no way to associate a well-defined path to the electron, the consequences of the AB effect might be complicated. For example, consider an electron found in a potential well \( V \), in an energy eigenstate. A solenoid crossing the well will move the electron to a slightly lower or higher energy eigenstate. In principle, the precise path of the solenoid and its velocity, and the value of the enclosed magnetic flux \( \Phi \), will determine the final state of the electron (once the solenoid has left the well). This suggests that the AB effect might be observable in a quantum cloud of charge in a non-eigenstate of the cloud, where the AB effect might be observed in a non-eigenstate of the cloud. However, for a solenoid containing exactly half a flux quantum \( \Phi = (\pi) = (\pi)2\hbar/e \) when it adiabatically crosses the quantum "cloud" of an electron in a non-eigenstate of the cloud, the AB effect might be observed in a non-eigenstate of the cloud.
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Consider an electron in a nondegenerate energy eigenstate of an arbitrary potential well $\Gamma$ and a solenoid moving adiabatically on a fixed path $C$ that crosses the electron cloud. We take the solenoid to be liminal, that is, infinitely narrow and long. According to the adiabatic approximation, the solenoid does not induce transitions and the final state of the electron is identical to the initial one, up to a phase $e^{i\phi}$. Now, $e^{i\phi}$ contains a dynamical phase $\phi_N$ and a geometrical phase $\phi_G$ with

$$\phi = \phi_N + \phi_G.$$  \hspace{1cm} (1)

We are interested in the geometrical part of the phase. For simplicity, we first consider the two-dimensional situation illustrated in Figure 1. Two limiting cases,

![Figure 1](image)

**Figure 1.** For simplicity, we consider an infinitely deep potential well such that the electron cloud vanishes outside it.

where we know how much change the solenoid encircles, are easily computed. When the solenoid moves along the path $C_1$, it does not encircle any charge at all. The $AB$ phase is $0$. On the other hand, when the solenoid moves along $C_2$, the electron is encircled with certainty and the $AB$ phase is $\pi/2h$, which in the case of a solenoid (half a quant of flux) yields $\pi$. However, how are we to interpolate the phase for more moderate paths that cross the electron cloud? Apparently, as the solenoid moves

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on paths extending larger and larger portions of the cloud, the effective change it incurs gradually increases from 0 to ε. Thus, the AB phase should gradually change from 0 to ε. Closer inspection, though, lead to a different conclusion. The reason is the physics manifests time-reversal symmetry. The initial wave function of the electron is non-degenerate and therefore unchanged under time reversal. (Assume that, initially, the quantisation is infinitely far from the electron and no other vector fields act on it.) Under time reversal, the magnetic field inside the solenoid changes sign and thus also the magnetic flux (\( \Phi \rightarrow -\Phi \)); however, in the particular case of half a flux quantum, this change is not observable as long as the electron cannot penetrate into the solenoid because the difference between \((\frac{\pi}{2})h\) and \(-\frac{\pi}{2}h\) is exactly a flux quantum. Consequently, for any path \(C\), the geometric phase \(e^{i\gamma}\) must be the same in whatever direction the solenoid moves. On the other hand, \(\Phi\) must change sign when the solenoid changes direction (because it can be written as a line integral along the path \(C\)). Thus, we obtain

\[ e^{i\gamma} = e^{-i\theta} \tag{2} \]

implying \(e^{i\gamma} = \pm 1\). This result, corresponding to \(\Phi\) equal to an integer multiple of \(\Phi_0\), contradicts our naive expectation that the AB phase gradually changes from 0 to ε.

What happened? Let us try to interpolate between the paths \(C1\) and \(C2\). We can gradually distort the path \(C1\) into \(C2\) by many steps that change the loop by an infinitesimal region. In a certain region, the phase factor jumps from \(i\) to \(-i\). We probe this particular infinitesimal region until we come to a point \(P\) where the geometry of the phase jump by what \(P\)'s combination. However, what is the phase when the solenoid moves on a path crossing \(P\)? Our best guess is that the phase is not well defined. Our assumption that the solenoid moves adiabatically breaks down on this path. In other words, although initially the wave function of the electron was a non-degenerate energy eigenstate, it is no longer non-degenerate when the solenoid goes through the point \(P\). Thus, the solenoid induces a degeneracy.

Thus, we find a clue to the puzzling abrupt phase changes. Indeed, we suspect that our argument for a gradual change in the Aharonov-Bohm phase was correct. However, we neglected a second contribution in the geometric phase. As the solenoid crosses, it distorts the wave function of the electron and generates a Berry phase that adds to the AB phase:

\[ \Psi = \psi_0 \times \text{Berry phase} \tag{3} \]

The Berry phase is responsible for maintaining the total geometric phase factor \((1 + i)\) despite gradual changes in the AB phase. The Berry phase is also responsible for the jump in the total geometric phase around the point \(P\) as Berry showed, isolated energy degeneracies add to the geometric phase.

Returning to our original problem, we can add a few details. First, the existence of points of degeneracy, for any arbitrary potential well \(V\), can be proved by using time-reversal symmetry in the context of a Born-Oppenheimer approximation. Nevertheless, finding both points appears to be a difficult problem; the only explicit examples we know are for rotationally symmetric potential wells and for wells with even discrete rotational symmetries \(\{\Phi = n\} \rightarrow \Phi = n + 2\pi/2\Phi\), where the center of the well is such a point. Second, it is clear that there might be more than one such
point. Any odd number of points is consistent with the physics of the external paths \( C_1 \) and \( C_2 \). Third, the adiabatic approximations might break down not only at some isolated point \( P \), but in a whole region if the initial nondimensional model becomes degenerate with states in the continuum. Last, but not least, a similar effect of phase jumps and energy level crossings arises even if the solenoid is not straight and also when several solenoids, each carrying half a flux quantum, enter the electron cloud.

We find a simple rule for the geometric phase of an atom with a heavy nucleus, initially in a spherically symmetric eigenstate, moving around a solenoid. (In this case, geometrical phases arise for both the electron and the nucleus, but the wave function of the nucleus is much more concentrated and semiholons barely penetrate it. Thus, its geometrical phase is simply the usual Aharonov phase, which we neglect in the following.) The role that we can allow the electron cloud to change at the center of the atom and the semiholons with “shadow” fluxes. A shadow flux is a point at which two electronic energy levels cross, if the center of the atom sits there. The winding number of the path of the point charge around the shadow fluxes gives the geometric phase accumulated by the atom.

To derive this rule, consider two straight and parallel solenoids situated at a distance \( L \) apart. Two extreme cases are easily solved. When the distance between the solenoids is much larger than the size of the atom, we can move the atom in the vicinity of one of the solenoids without the electron cloud touching the other solenoid. In this case, the atom reflects a phase of \( \pi \) each time its center crosses the solenoid, exactly as if the other solenoid were not present. There are thus two shadow fluxes, coinciding with the original solenoids. On the other hand, for \( L = 0 \), the two solenoids are at the same point, with their magnetic fluxes adding to an integer flux quantum. However, an integer flux quantum has no effect on an electron. There are therefore no energy level crossings and thus no shadow fluxes. When the solenoids are slightly separated, they do affect energy levels, but, by continuity, this effect is small and does not induce energy level crossing; rather, a minimal distance \( L^* \) is required. Thus, we conclude that, in adiabatic motion, the geometric phase accumulated by the atom due to two parallel solenoids is zero once their separation is less than some critical distance \( L^* \). We can now interpolate between these two extreme cases [large and small \( L \)] (Figure 2). When the distance between the solenoids is very large, the shadows coincide with the original solenoids. When the distance is still large, but comparable to the size of the atom, the shadow fluxes no longer coincide with the original solenoids. Instead, the shadow fluxes associated to each solenoid is shifted towards the other solenoid. When the two solenoids are at a critical distance \( L^* \), their shadow fluxes and the classical fluxes need not coincide on the atom. For separations smaller than \( L^* \), the shadow fluxes disappear.

It is amusing to consider more general patterns of semiholons carrying half a quantum of flux and the resulting shadows. Even in the case of a single solenoid, the shadow need not coincide with the original, if the solenoid is not straight. For example, a solenoid in the bran of a ring should have a circular shadow, but of smaller radius, just in the case of two parallel solenoids, there is a critical radius for the ring (depending on the electron cloud) below which there will be no shadow at all. As a consequence, there will be no topological scattering of the atoms from small toroidal solenoids. For two interesting straight solenoids, we expect hyperbolic shadows situated in the plane of the solenoids, in the acute angles. When the
Figure 2. S1 and S2 represent the original electrodes and S10 and S20 represent their shadows (very close to b - b' for varying length of L).
solenoids are perpendicular to each other, the shadows will coincide with the solenoids.

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