Teleportation of quantum states

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The recent result of Bennett *et al.* [Phys. Rev. Lett. **70**, 1895 (1993)] of teleportation of an unknown quantum state is obtained in the framework of nonlocal measurements proposed by Aharonov and Albert [Phys. Rev. D **21**, 3316 (1980); **24**, 359 (1981)]. The latter method is generalized to the teleportation of a quantum state of a system with continuous variables.

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Recently, Bennett, Brassard, Crepeau, Jozsa, Peres, and Wootters (BBCJPW) [1] have shown how to transfer ("teleport") an unknown quantum spin state by using prearranged correlated quantum systems and transmission of classical information. This result can be achieved also using a method of Aharonov and Albert [2] for nonlocal measurements. We will review briefly these methods and will conclude by suggesting a method for teleportation of states of systems with continuous variables.

We call a measurement *nonlocal* if it cannot be reduced to a set of local measurements. An example is a measurement of a sum of variables A_1 and A_2 related to two separate locations 1 and 2. The method of Aharonov and Albert for nonlocal measurements uses only local interactions. The measurement is described by interaction Hamiltonian

$$H = g(t)P_1 A_1 + g(t)P_2 A_2 , \tag{1}$$

where $g(t)$ is normalized function with a compact support at the time of the measurement; P_1, P_2 are conjugate momenta of the pointer variables of two parts of measuring device which locally interact at locations 1 and 2. In order to perform a nonlocal measurement (and not two local measurements), the initial state of the measuring device has to be

$$Q_1 + Q_2 = 0 , \quad P_1 - P_2 = 0 . \tag{2}$$

After the interaction is completed,

$$A_1 + A_2 = Q_1 + Q_2 , \tag{3}$$

and therefore, local measurements of Q_1 and Q_2 yield the value of $A_1 + A_2$.

A set of measurements of nonlocal variables can serve as a verification of a nonlocal (entangled) state. The EPR-Bohm state of two spin-$\frac{1}{2}$ particles (completely anticorrelated state) can be verified using two consecutive measurements: first of $\sigma_{1x} + \sigma_{2x}$ and then of $\sigma_{1y} + \sigma_{2y}$, see Fig. 1. Here and below we will use the units of $\hbar/2$ for the spin components, such that each component can have values ± 1. If the outcomes are

$$\sigma_{1x}(t_1) + \sigma_{2x}(t_1) = 0 , \quad \sigma_{1y}(t_2) + \sigma_{2y}(t_2) = 0 , \tag{4}$$

where $t_2 > t_1$, then, after time t_2, the system is in the EPR-Bohm state. Had we started at time $t < t_1$ with the EPR-Bohm state, we would be certain to obtain the outcomes (4).

The method of Aharonov and Albert is applicable also to measurements which are nonlocal not only in space but also in time. The interaction Hamiltonian has to be modified such that local interactions in separate locations will take place at different times. For example, for a measurement of the sum $A_1(t_1) + A_2(t_2)$ the Hamiltonian is

$$H = g(t - t_1)P_1 A_1 + g(t - t_2)P_2 A_2 , \tag{5}$$

where $g(t)$ has compact support around zero. It has been shown [3] that sums and also modular sums of local variables are measurable. For measuring a sum modulo a the measuring device has to be set in the following initial state:

$$\begin{aligned} &(Q_1 + Q_2) \mathrm{mod} a = 0 , \\ &P_1 - P_2 = 0 , \\ &P_1 \mathrm{mod} \frac{2\pi\hbar}{a} = 0 . \end{aligned} \tag{6}$$

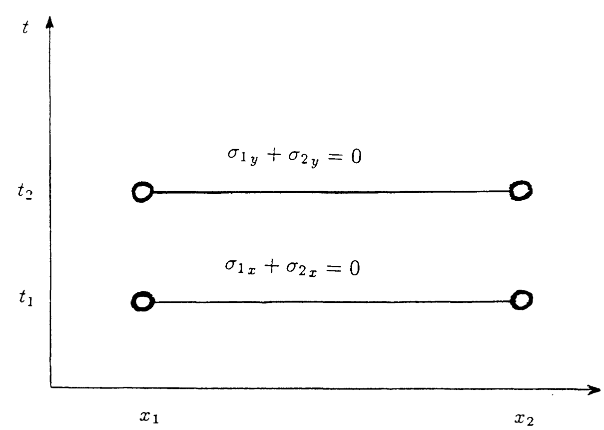

FIG. 1. If the system is in the EPR-Bohm state then the outcomes of the nonlocal measurements have to be as shown in the figure. Conversely, if these are the outcomes of the nonlocal measurements then, after the measurements, the system is in the EPR-Bohm state.

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Let us turn now to the problem of *teleportation* of an unknown spin state of a spin-$\frac{1}{2}$ particle. Let us assume that the state of particle 1 is $\alpha|\uparrow\rangle+\beta|\downarrow\rangle$ and we have to teleport it to particle 2. To this end consider the "crossed" measurements of $\sigma_{1x}(t_1)-\sigma_{2x}(t_2)$ and $\sigma_{1y}(t_2)-\sigma_{2y}(t_1)$, see Fig. 2. If the outcomes are

$$\sigma_{1x}(t_1)-\sigma_{2x}(t_2)=0\ ,\quad \sigma_{1y}(t_2)-\sigma_{2y}(t_1)=0\ , \tag{7}$$

then, taking into account the measurement interaction (5) with $A_1=\sigma_{1x}$ ($A_1=\sigma_{1y}$) and $A_2=-\sigma_{2x}$ ($A_2=-\sigma_{2y}$), the initial state of the measuring device (2), and the outcomes of the local readings of the measuring device [which are also described by Eq. (2)], we obtain, after straightforward calculation, that the final state of particle 2 is $\alpha|\uparrow\rangle+\beta|\downarrow\rangle$, i.e., we have succeeded in teleporting the state of particle 1 to particle 2.

However, this procedure is not good enough, since the nonlocal measurements might not yield the specific outcomes (7). The difference between the spin components might be equal ± 2 and in that case we destroy the state without teleporting it. In order to obtain *reliable* teleportation (such as the one suggested by BBCJPW) we must measure, instead, the following nonlocal observables:

$$\begin{aligned}&[\sigma_{1x}(t_1)-\sigma_{2x}(t_2)]\mathrm{mod}4\ ,\\&[\sigma_{1y}(t_2)-\sigma_{2y}(t_1)]\mathrm{mod}4\ .\end{aligned} \tag{8}$$

A null outcome reduces to the previous case. If, however, the outcome of one of the above is 2, then we can convert it to 0 by appropriate rotation of the coordinate frame of the second particle (for example, $\sigma_{2x}=-\sigma_{2x'}$, for $\hat{x}'=-\hat{x}$). Thus, for any set of outcomes of the nonlocal measurements (8) the spin state is teleported; in some cases the state is rotated, but the resulting rotation can be inferred from the nonlocal measurements. We can complete, then, the teleportation by the following transforma-

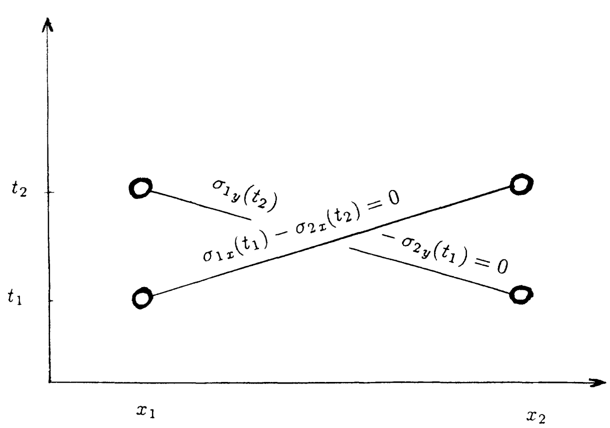

FIG. 2. Teleportation via "crossed" space-time nonlocal measurements. The measurements with the outcomes in the figure cause the state of particle 2 after t_2 to be the state of particle 1 before t_1 (and the state of particle 1 after t_2 to be the state of particle 2 before t_1). For *reliable* teleportation the nonlocal measurements to be performed are the measurements of the differences of the spin components modulo 4 accompanied by appropriate local rotations.

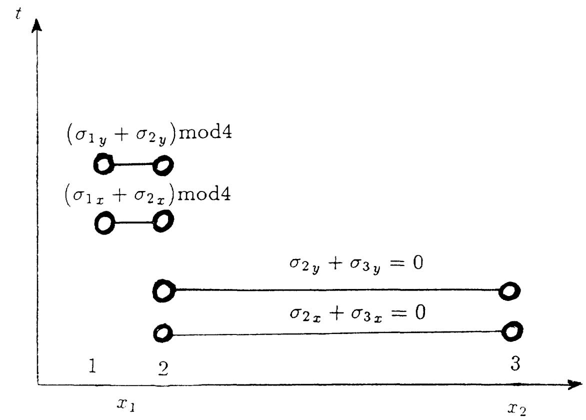

FIG. 3. Nonlocal measurements of the BBCJPW teleportation scheme. The state of particle 1 is teleported to particle 3. To this end the particle 3 is prepared in the EPR-Bohm state with particle 2 located near particle 1 using nonlocal measurements. Then the measurements of the composite system consisting of particles 1 and 2 causes the teleportation (possibly, with rotation).

tions. In the case of two null outcomes no additional transformation is needed; in three other cases a transformation of rotation by the angle π is necessary: the rotation around the y axis for the outcome (2,0), around the x axis for (0,2), and around the z axis for the outcome (2,2).

The Aharonov-Albert method for nonlocal measurement contains the following elements: (i) a preparation of an entangled state of the measuring device, (ii) local interactions with separate parts of the system, (iii) local readings of the separate parts of the measuring device resulting in a set of numbers obtained in the respective space-time locations of the parts of the system. These numbers represent classical information which must be transmitted for completing the teleportation. (In our example, the information tells us which rotation must be performed). The initial entanglement of the measuring device, which is the core of the method, may employ pairs of spin-$\frac{1}{2}$ particles in the EPR-Bohm state (see Sec. IV of Ref. [3]), making this method very similar to the BBCJPW proposal [4]. The latter can be presented in our language as in Fig. 3. The EPR-Bohm pair which is employed by BBCJPW can be created via two (successful) measurements (4). The measurement "in the Bell operator basis" [Eqs. (1) and (2) of Ref. [1]] at the location 1, performed on the composite system consisting of particle 1 and one member of the EPR-Bohm pair, is equivalent to two consecutive measurements of the modular sums:

$$\begin{aligned}&[\sigma_{1x}(t_1)+\sigma_{2x}(t_1)]\mathrm{mod}4\ ,\\&[\sigma_{1y}(t_2)+\sigma_{2y}(t_2)]\mathrm{mod}4\ .\end{aligned} \tag{9}$$

The four different combinations of the outcomes of the nonlocal measurements (9) correspond to the four outcomes of local measurement of BBCJPW. The procedure of teleportation is completed by appropriate rotation according to these results. After the teleportation, particle 1 is in a mixed state and contains no information. This is

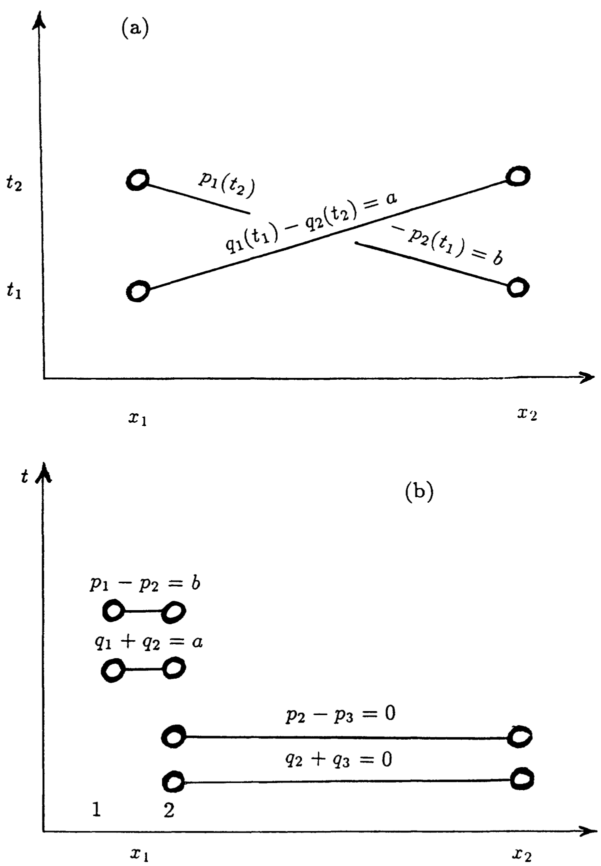

FIG. 4. Teleportation of an unknown quantum state of a system with continuous variables: (a) the method of "crossed" space-time nonlocal measurements, (b) the method analogues to the BBCJPW proposal. In both cases the final state (before the back shifts) is $e^{ibq}\Psi(q+a)$.

in contrast with the crossed measurements method in which (after the appropriate rotation) the final state of particle 1 is the initial state of particle 2.

In the framework of nonlocal measurements there is a natural way of extending the teleportation scheme to the systems with continues variables. Consider two similar systems located far away from each other and described by continuous variables q_1, q_2 with corresponding conjugate momenta p_1 and p_2. In order to teleport a quantum state $\Psi(q_1)$ we perform the following crossed nonlocal measurements [see Fig. 4(a)] obtaining the outcome a and b:

$$q_1(t_1)-q_2(t_2)=a \ , \quad p_1(t_2)-p_2(t_1)=b \ . \tag{10}$$

Straightforward calculation shows that these nonlocal crossed measurements correlate the state of particle 1 before t_1 and the state of particle 2 after t_2, thus teleporting the quantum state to the second particle up to a shift of $-a$ in q and $-b$ in p. These shifts are known (after the results of local measurements have been transmitted), and can easily be corrected by appropriate back shifts even if the state is unknown, thus completing a reliable teleportation of the state $\Psi(q_1)$ to $\Psi(q_2)$.

A generalization of the BBCJPW scheme to the case of continuous variables is also possible, see Fig. 4b. The method contains the following stages: first, the preparation of the EPR state of particles 2 and 3,

$$q_2+q_3=0 \ , \quad p_2-p_3=0 \ . \tag{11}$$

Second, the consecutive measurements performed on particles 1 and 2 yielding the outcomes a and b,

$$q_1+q_2=a \ , \quad p_1-p_2=b \ . \tag{12}$$

Each pair of measurements (11), (12) causes an anticorrelation, thus the anticorrelation between particles 2 and 3 together with the anticorrelation between particles 1 and 2 lead to a correlation between particles 1 and 3. The only difference between the states is due to the shifts both in q and in p

$$q_3=q_1-a \ , \quad p_3=p_1-b \ . \tag{13}$$

If the initial state of particle 1 is $\Psi(q_1)$, then the state of particle 3, after the measurements (11) and (12) have been performed, is $e^{ibq_3}\Psi(q_3+a)$, which is exactly the state obtained after the crossed measurements (10). The final stage of teleportation is appropriate back shifts of the state in p and q. The essential ingredients of this calculation appear (in another context) elsewhere [5]. Note again that while the crossed measurements yield two-way teleportation, we have obtained now only a one-way teleportation: from particle 1 to particle 3.

We have shown how nonlocal measurements can be used for teleportation of unknown quantum states, first for an example of a spin state, and then for a system with a continuous variable. We presented "crossed measurement" method which yields two-way teleportation. The method avoids the necessity of measuring Bell's type operator which currently represents a serious experimental challenge. But our formalism also suggests a way of splitting the Bell's operator measurement into two apparently simpler measurements.

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[1] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).

[2] Y. Aharonov and D. Albert, Phys. Rev. D **21**, 3316 (1980); **24**, 359 (1981).

[3] Y. Aharonov, D. Albert, and L. Vaidman, Phys. Rev. D **34**, 1805 (1986).

[4] In this case we also need to transmit just two bits of classical information, which is the minimal information for teleportation of a spin state, as has been proven in Ref. [1].

The number of "nonlocal channels" in our method is two instead of just one in the BBCJPW method. This is because we have accomplished two-way teleportation. (Obviously, for teleporting also the state of particle 2 to particle 1 we need to send another two bits of classical information from site 2 to site 1.)

[5] Y. Aharonov, D. Albert, and C. K. Au, Phys. Rev. Lett. **47**, 1029 (1981).