Comment on "Which-way information in a nested Mach-Zehnder interferometer"

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Potoček and Ferenczi [Phys. Rev. A **92**, 023829 (2015)] provided an analysis of the experimental evidence obtained by Danan *et al.* [Phys. Rev. Lett. **111**, 240402 (2013)] for the surprising behavior of photons passing through an interferometer, in particular, motion on disconnected paths. Potoček and Ferenczi reproduced the results of the experiment, but when analyzing its modification, they claimed that the reasoning of Danan *et al.*, which led to disconnected paths, is erroneous. It is argued here that the criticism of Potoček and Ferenczi is unfounded.

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Potoček and Ferenczi (PF) [1] presented a very detailed and correct analysis of the experiment by Danan *et al.* [2] in terms of classical optics, significantly extending the analysis of Saldanha [3]. These analyses agree completely with a brief analysis in the framework of classical optics appearing in Ref. [2]. PF correctly state that the classical optics analysis explains all the results without the need for disconnected photon paths. Classical optics does not have a concept of a photon, let alone a postselected photon, so of course there are no photon paths, connected or not.

The novel results of PF are related to a modification of Danan *et al.*'s experiment. They correctly show that the relative heights of the peaks in Ref. [2] depend on the three path lengths, and, in particular, can be tuned such that, without blocking any of the paths, the trace of only mirror C is present in the output signal. The error of PF is their assertion that the lack of signals from mirrors A and B in their modification of the experiment is of the same kind as the lack of signals from mirrors E and F in the experiment of Ref. [2].

Quantum mechanics has a concept of a photon, and there is a clear meaning for the location of the wave packet of a (preselected) photon. However, standard quantum mechanics, as classical optics, has no definition of the location of a pre- and postselected photon. Danan *et al.*'s experiment tested the paths of photons inside the interferometer according to the *definition*, given in Ref. [4], as *a place where the photon leaves a weak trace*. The nested interferometer in Ref. [2] is surprising, mainly because, for a particular setup, the weak trace definition disagrees with the "common sense" definition of Wheeler [5]: *The photon was present only in the paths through which it could pass*.

Since observing the trace left by the pre- and postselected photons on other systems is an extremely difficult task, in the experiment of Ref. [2], a degree of freedom of the photons themselves served as a pointer variable for measuring the trace. This was the transversal momentum of the photon which was read off through the positions of the photons on the quad-cell detector.

The two-state vector formalism [6] (TSVF) describes a pre-and postselected photon by a two-state vector. The TSVF provides a simple way to characterize the weak trace which the pre- and postselected photon leaves. It vanishes outside the overlap of the forward and the backward evolving states. The strength of the trace in a particular location is characterized by the weak value of the projection on this location. In the experiment of Ref. [2], there were nonzero weak values of the projection on the mirrors inside the inner interferometer, in spite of the fact that, due to the interference, the photons passing through the inner interferometer could not reach the detector. The weak values of the projections on different mirrors were given by Eq. (3) of Ref. [2]:

$$(\mathbf{P}_A)_w = (\mathbf{P}_C)_w = 1, \quad (\mathbf{P}_B)_w = -1, \quad (\mathbf{P}_E)_w = (\mathbf{P}_F)_w = 0. \tag{1}$$

The weak values of the projections show an additional surprising feature. Although there is a trace inside the inner interferometer, there is no trace leading towards and out of it.

Let us consider now the modification of PF in the framework of the TSVF. To eliminate signals at A and B, they suggest to add a phase to path C, equal to the Gouy phase $\zeta(z_D)$, without changing anything else. For simplicity, assume that the the detector is far away, so the Gouy phase is $\zeta(z_D) = \frac{\pi}{2}$. Adding phase $\frac{\pi}{2}$ to path C changes the two-state vector describing the photon at the intermediate time. Instead of Eq. (1) of Ref. [2], it is

$$\langle\Phi||\Psi\rangle = \frac{1}{\sqrt{3}}(\langle A| + i\langle B| + \langle C|)\ \frac{1}{\sqrt{3}}(|A\rangle + i|B\rangle + i|C\rangle). \tag{2}$$

At mirrors E and F, the forward and backward evolving states do not overlap, as in the original case. Then, the weak values of the projections on different mirrors are

$$(\mathbf{P}_B)_w = -(\mathbf{P}_A)_w = i, \quad (\mathbf{P}_C)_w = 1, \quad (\mathbf{P}_E)_w = (\mathbf{P}_F)_w = 0. \tag{3}$$

These values explain the null result in the quad-cell detector at the frequencies of all detectors except C, but for very different reasons. At frequencies f_E and f_F, the null result is because there is no trace in E and F. At frequencies f_A and f_B, the null result is because the quad-cell detector at large distance provides the real part of the weak value of the projections, which happens to be zero. The effect of the imaginary part of the weak value on the measuring device is a shift of the conjugate variable, the lateral shift of the beam [see Eq. (10) of Ref. [6]]. For large z_D, it is negligible in comparison to the lateral shift due to the change in the direction of the beam. However, when the detector is not very far, the effect of the imaginary part is significant. In fact, in the experiment of

Ref. [2], a serious effort was required to keep the phases stable, to avoid imaginary weak values of the projections.

The null signals at frequencies f_A and f_B in the PF modification of the experiment are not because the photons do not leave a trace at mirrors A and B, but because the measurement procedure, with a particular distance to the detector, fails to detect it. Changing the position of the detector reveals the frequencies f_A and f_B. In contrast, keeping destructive interference towards mirror F ensures null signals at f_E and f_F at any position of the detector.

A subtle point worth repeating [7–9] is that when all mirrors vibrate, none of the signals, including f_E and f_F, are exactly zero. And if we postulate that in the experiment there is a zero trace at E and F, it is impossible to have signals at f_A and f_B. What allows us to say that there is a trace in A and B, but to disregard the trace in E and F, is that the ratio between the strengths of the traces becomes arbitrarily large at the weak limit, while the ratio between the trace in C and the trace in A and B remains constant. If all mirrors vibrate with the same amplitude proportional to a small parameter ϵ, then the traces in C, A, and B are proportional to ϵ, while the traces in E and F are proportional to ϵ^2.

The photons in the interferometer of Danan *et al.* leave a trace which includes a disconnected path. The experiment in Ref. [2] faithfully shows this (together with a continuous path C). In the PF modification of Danan *et al.*'s experiment, the photons leave a trace with a disconnected path, too. The null signals at f_A and f_B in PF's modified experiment arise from an inappropriate method of observing this trace. Therefore, the argument of PF against Danan *et al.*'s conclusions does not hold.

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