EXCERCISE: RUNAWAY EIGENVALUES

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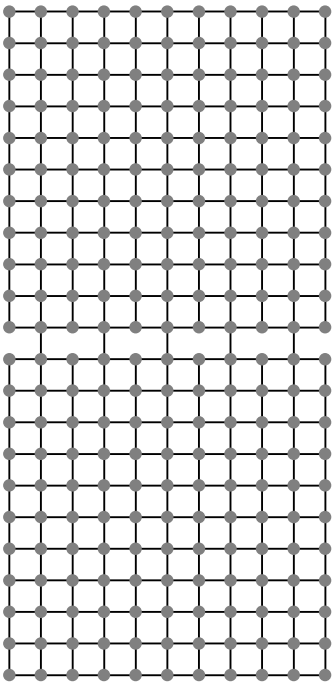
This exercise explores the effect of dropping a small number of edges from a preconditioner. We will use the following mesh in the experiments. Generate the incidence factor of its Laplacian using the script `ex_runaway.m`. The mesh contains two 11-by-11 unit-weight square meshes connected by five inter-mesh edges. Three of these inter-mesh edges have weight $\epsilon = 10^{-5}$, one have weight 10, and one weight 100. (The weights are the magnitude of the elements of the corresponding column in the incidence factor.) The last three columns in the incidence factor correspond to inter-mesh edges with weights 10, 100, and ϵ.

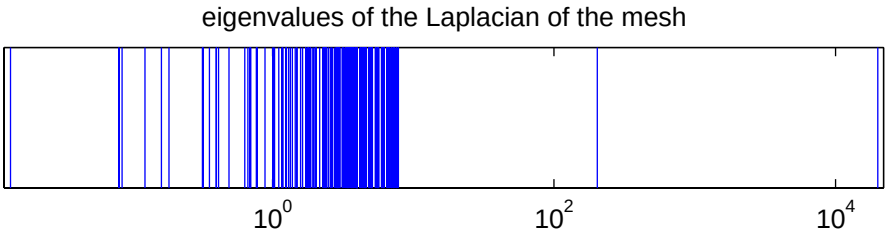


In this exercise, use the function `eigendistribution_plot` that we provide to visualize the spectrum of a matrix or generalized spectum of a preconditioned system. To compute the eigenvalues reliably and to visualize them, use the following method in MATLAB,

```
>> n = size(U,1); % number of unknowns/mesh vertices
>> ev = (svd(full(U))).^2;
>> eigendistribution_plot(ev(1:(n-1)));
>> title('eigenvalues of the Laplacian');
```

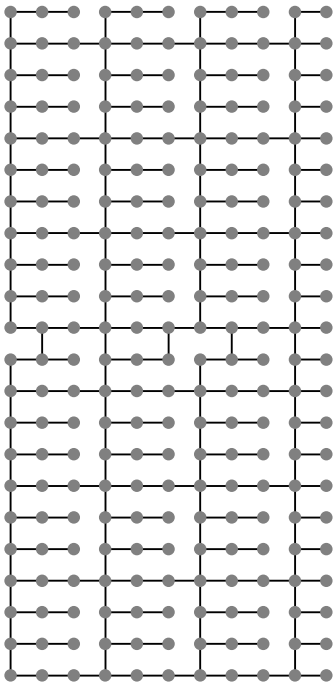
```
... % pause or print the graph, or open a new figure
>> gev = (svd(pinv(full(V)) * full(U))).^2;
>> eigendistribution_plot(gev(1:(n-1)));
>> title('generalized preconditioned eigenvalues');
```

Here U is the incidence factor of the Laplacian of the mesh and V is the incidence factor of the preconditioner. We always drop the first eigenvalue, which is 0, from the plot. Plotting the eigenvalues of the mesh itself should give the following output:



In all the questions below, explore the generalized spectra, describe your findings, and try to explain your findings.

(1) Use the incidence factor of the mesh (which is computed for you by `ex_runaway.m`) to create the incidence factor V_ϵ whose graph is the original mesh except for one weight-ϵ inter-mesh edge.
(2) Now drop the edge with weight 100 and check what happens to the generalized spectrum.
(3) The script `ex_runaway.m` also computes the incidence factor of a Joshi-type preconditioner for the mesh. All the inter-mesh edges are included in the preconditioner. Inspect the generalized eigenvalues and use them to estimate how good the preconditioner is.



(4) Drop one heavy inter-mesh edge from the Joshi-type preconditioner and analyze the generalized spectrum. How good is the preconditioner? Repeat with the other heavy edge.
(5) Now drop both of the heavy edges and check what happens to the spectrum. How much does the dropping degrade the preconditioner?