Exploiting Algebraic Dependences Between Local Partial Derivatives in Jacobian Accumulation

Andrew Lyons
lyonsam@gmail.com

We introduce a novel undirected graph model for the optimal Jacobian accumulation problem. This new graph addresses the need illustrated in [1] for accumulation techniques capable of exploiting algebraic dependences between the local partial derivatives.

Let \( G \) be a linearized computational graph (LCG). The corresponding path-clique graph (CPG) \( G^p \) has vertex set \( V^p = E \in G \); two vertices \( i, j \in V^p \) are adjacent iff their corresponding edges in \( G \) lie on a common path. We allow two vertices to be merged if their respective labels are identified as equivalent.

![Diagram of LCG and CPG with edge contractions](image)

Figure 1: An example LCG \( G \) (a), the corresponding CPG \( G^p \) (b), \( G^p \) after vertices \( d_1 \) and \( d_2 \) have been merged (c), \( G^p \) after the contraction of edges \( \{a, d\} \) (d), \( \{ad, b\} \) (e), and \( \{ad, c\} \) (f).

A sequence of edge contraction operations reduces a CPG to an edgeless graph whose vertices yield the values of Jacobian entries. Each edge contraction implies a single fused multiply-add operation in the generated derivative code.

References


*Abstract for poster presentation, CSC07