An Algorithm to Find Overlapping Subgraphs of a Graph and Applications to Preconditioning

David Fritzsche* Andreas Frommer† Daniel B. Szyld*

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Abstract

Additive and multiplicative Schwarz preconditioners are known to be an efficient class of preconditioners for problems involving a geometric domain. They are based on decomposing the domain into a set of overlapping subdomains, in such a way that the restrictions of the original problem to the parallel subdomains are easily solvable. Algebraic Schwarz preconditioners work without an underlying geometric domain by restricting the linear operator to overlapping subsets of the coordinates. A new graph theoretical algorithm, called OPABLO (Overlapping PArametrized Block Ordering), to find such overlapping subsets is presented. The graph formulation of the problem is to find overlapping subgraphs, also called blocks, of the graph of the matrix. The OPABLO algorithm is based on the PABLO family of algorithms; see [3], [1], and [2]. It retains the basic structure of the PABLO algorithm and two of the main properties: The time complexity is linear in the number of nonzeros and one block is built at a time. Furthermore, certain useful constraints, like limits on the amount of overlap for a single block and for single nodes, can be enforced in a rather natural way. Numerical experiments show how these OPABLO-based preconditioners perform and how their performance compare to and improve on the performance of XPABLO-based block diagonal and block triangular preconditioners. Comparisons with ILUT preconditioners are also shown.

*Department of Mathematics, Temple University, Philadelphia, PA 19122-6094, {fritzsche,szyld}@temple.edu.

†Faculty of Mathematics and Science, Bergische Universität Wuppertal, D-42097 Wuppertal, Germany, frommer@math.uni-wuppertal.de

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