COMBINATORIAL PRECONDITIONERS FOR SCALAR ELLIPTIC FINITE-ELEMENTS PROBLEMS

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ABSTRACT. We present a new preconditioner for linear systems arising from finite-elements discretizations of scalar elliptic partial differential equations (PDE's). The solver splits the collection $\{K_e\}$ of element matrices into a subset $E(t)$ of matrices that are approximable by diagonally-dominant matrices and a subset of matrices that are not approximable. The approximable K_e's are approximated by diagonally-dominant matrices L_e's that are scaled and assembled to form a global diagonally-dominant matrix $L = \sum_{e \in E(t)} \alpha_e L_e$. A combinatorial graph algorithm approximates L by another diagonally-dominant matrix M that is easier to factor. The sparsification M is scaled and added to the inapproximable elements; the sum $\gamma M + \sum_{e \notin E(t)} K_e$ is factored and used as a preconditioner. When all the element matrices are approximable, which is often the case, the preconditioner is provably efficient. Experimental results show that on problems in which some of the K_e's are ill conditioned, our new preconditioner is more effective than an algebraic multigrid solver, than an incomplete-factorization preconditioner, and than a direct solver.

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