Using Perturbed QR Factorizations to Solve Linear Least-Squares Problems

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Abstract

We propose and analyze a new tool to help solve sparse linear least-squares problems $\min_x \|Ax-b\|_2$. Our method is based on a sparse QR factorization of a low-rank perturbation $\tilde{A}$ of $A$. More precisely, we show that the $R$ factor of $\tilde{A}$ is an effective preconditioner for the least-squares problem $\min_x \|Ax-b\|_2$, when solved using LSQR. We purpose applications for the new technique. When $A$ is rank deficient we can add rows to ensure that the preconditioner is well-conditioned without column pivoting. When $A$ is sparse except for a few dense rows we can drop the dense rows in $\tilde{A}$. Another application is solving an updated or downdated problem. If $R$ is a good preconditioner for the original problem $A$, it is a good preconditioner for the updated/downdated problem $\tilde{A}$. We can also solve what-if scenarios, where we want to find the solution if a column of the original matrix is changed/removed. We present a spectral theory that analyzes the generalized spectrum of the pencil $(A^*A, R^*R)$ and analyze the applications.

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