Corporate Finance: Capital structure as a disciplining device

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Grossman and Hart, 1982

"Corporate Financial Structure
and Managerial Incentives"

The model

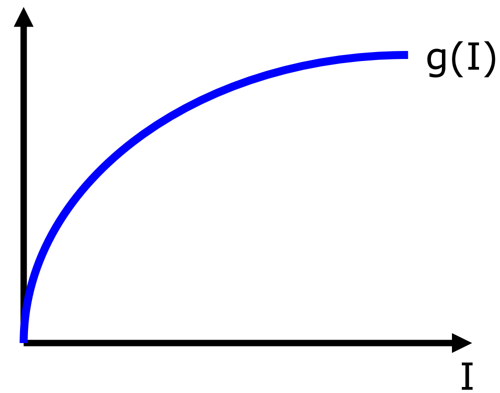
- The timing:

Period 1

A firm is established:
- raises R dollars
- issues debt D
- invests I

Period 2

The investment yields $zg(I)$, $R-I$ is consumed as perks provided that the firm is solvent

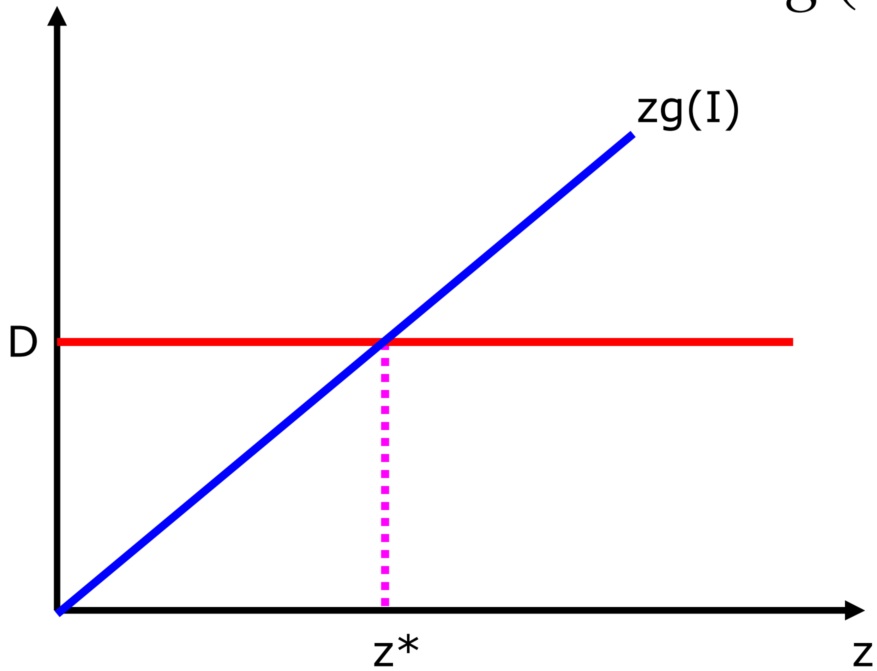


- $g'(I)>0>g''(I)$ and $g(0) = 0$
- $g'(0) = \infty$ and $g'(\infty) = 0$ (interior sol'n)
- $z \sim [0, \infty)$ according to $f(z)$ with CDF $F(z)$

Period 2

- The firm is solvent iff:

$$zg(I) \geq D \quad \Rightarrow \quad z \geq z^* \equiv \frac{D}{g(I)}.$$



Comparative statics:

- D↑ ⇒ z*↑
- I↑ ⇒ z*↓

The manager's problem

- The manager's payoff at the optimum:

$$U(I,D)=\int_{z^*}^{\infty}(R-I)dF(z)=(R-I)(1-F(z^*))$$

- The F.O.C for the manager's problem

$$\frac{\partial U(I,D)}{\partial I}=\underbrace{-(R-I)f(z^*)\overbrace{\frac{\partial z^*}{\partial I}}^{(-)}}_{\substack{\text{Marginal benefit: } I\uparrow\Rightarrow z^*\downarrow \\ \text{so the manager enjoys R-I} \\ \text{with higher prob.}}}-\underbrace{(1-F(z^*))}_{\substack{\text{Marginal cost: } I\uparrow \\ \text{leaves smaller perks;} \\ \text{the prob. that perks are} \\ \text{enjoyed is 1-F(Z*)}}}=0$$

The F.O.C for the manager's problem

- The effect of I on z*:

$$z^* = \frac{D}{g(I)} \quad \Rightarrow \quad \frac{\partial z^*}{\partial I} = -\frac{Dg'(I)}{\left(g(I)\right)^2} = -\frac{z^* g'(I)}{g(I)}$$

- Rewriting the F.O.C for I:

$$-\left(1 - F\left(z^*\right)\right) = \left(R - I\right) f\left(z^*\right) \frac{\partial z^*}{\partial I} = -\left(R - I\right) f\left(z^*\right) \frac{z^* g'(I)}{g(I)}$$

$$\Rightarrow \frac{g(I)}{\left(R - I\right) g'(I)} = \frac{z^* f\left(z^*\right)}{1 - F\left(z^*\right)} \equiv z^* H(z^*)$$

- z*H(z*) is strictly increasing

How does D affect I?

- Fully differentiating the F.O.C for I:

$$\frac{\partial}{\partial I}\left(\frac{g(I)}{(R-I)g'(I)}\right)\partial I = \frac{\partial}{\partial z^*}\left(z^* H(z^*)\right)\frac{\partial z^*}{\partial I}\partial I + \frac{\partial}{\partial z^*}\left(z^* H(z^*)\right)\frac{\partial z^*}{\partial D}\partial D$$

- Hence,

$$\frac{\partial I}{\partial D} = \frac{\overbrace{\frac{\partial}{\partial z^*}\left(z^* H(z^*)\right)}^{(+)}\overbrace{\frac{\partial z^*}{\partial D}}^{(+)}}{\frac{\partial}{\partial I}\left(\frac{g(I)}{(R-I)g'(I)}\right)\underbrace{-\underbrace{\frac{\partial}{\partial z^*}\left(z^* H(z^*)\right)}_{(+)}\underbrace{\frac{\partial z^*}{\partial I}}_{(-)}}_{(+)}}$$

How does D affect I?

- The effect of I on the LHS of the F.O.C for I:

$$\frac{\partial}{\partial I}\left(\frac{g(I)}{(R-I)g'(I)}\right)=\frac{\overbrace{(R-I)(g'(I))^2}^{(+)}-\overbrace{\left(\overbrace{(R-I)g''(I)}^{(-)}-\overbrace{g'(I)}^{(-)}\right)g(I)}^{(+)}}{((R-I)g'(I))^2}$$

- Hence,

$$\frac{\partial I}{\partial D}=\frac{\overbrace{\frac{\partial}{\partial z^*}(z^*H(z^*))\frac{\partial z^*}{\partial D}}^{(+)}}{\underbrace{\frac{\partial}{\partial I}\left(\frac{g(I)}{(R-I)g'(I)}\right)}_{(+)}-\underbrace{\frac{\partial}{\partial z^*}(z^*H(z^*))\frac{\partial z^*}{\partial I}}_{(+)}}>0$$

The owner's problem

- The value of debt (debt is "long-term"):

$$B(D) = \int_{z^*}^{\infty} D dF(z)$$

- The value of equity:

$$E(D) = \int_{z^*}^{\infty} \left(zg(I^*) - D\right) dF(z)$$

- The total value of the firm:

$$V(D) = \int_{z^*}^{\infty} zg(I^*) dF(z)$$

- D affects matters through z* and I

The optimal choice of D

- The F.O.C for D:

$$V'(D) = -z^* g(I^*) f(z^*) \left(\underbrace{\frac{\partial z^*}{\partial D}}_{(+)} + \underbrace{\frac{\partial z^*}{\partial I}}_{(-)} \right) + \underbrace{\int_{z^*}^{\infty} z g'(I^*) dF(z)}_{(+)} \underbrace{\frac{\partial I^*}{\partial D}}_{(+)} = 0$$

- At D=0, z*=0, so

$$V'(0) = \underbrace{\int_{0}^{\infty} z g'(I^*) dF(z)}_{(+)} \underbrace{\frac{\partial I^*}{\partial D}}_{(+)} > 0$$

- D*>0 $\Rightarrow$ The firm will always issue some debt