Corporate Finance: Debt renegotiation

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A Theory of Workouts and the Effects of Reorganization Law
The model

- The timing:
  - **Period 0**
    - The firm has a cash flow $Y < B + D$
    - The firm can restructure its debt
  - **Period 1**
    - The firm needs to pay $B$ to a bank and $qD$ to bondholders
    - The firm can invest $I$ in a project
  - **Period 2**
    - The firm needs to pay $(1-q)D$ to bondholders
    - The project yields $X \sim [0, \infty)$

- The dist. of $X$ is $f(x)$ and the CDF is $F(X)$

- The mean of $X$ is $\hat{X} = \int_{0}^{\infty} XdF(X)$
Bank debt restructuring

- Y < I+B+qD ⇒ without restructuring the firm cannot invest
- The firm must raise I+B+qD-Y in period 1 in order to cover I
- The Trust Indenture Act of 1939 requires debtholders unanimity to change the interest, principal, or maturity of pubic debt ⇒ renegotiation of public debt is very hard
- Bank debt restructuring:
  - the bank gives the firm cash worth I+B+qD-Y so the firm can meet all its obligations in period 1
  - the face value of the new bank’s debt is higher to ensure that the bank makes a profit
- Simplifying assumption: bankruptcy occurs in period 2 iff
  \[ X < Z \equiv I + B + qD - Y + (1 - q)D \]
  Bank debt public debt
- This is a simplifying assumption since the face value of the new bank debt is actually higher
Bank debt restructuring – period 2

**Solvency: X > Z**

\[ Z \equiv I + B + qD - Y + (1-q)D = I + B + D - Y \]

⇒ The firm and the bank split \( X - (1-q)D \)

**Bankruptcy: X < Z**

\[ \frac{I + B + qD - Y}{Z} X \quad \text{New bank debt} \]

\[ \frac{(1-q)D}{Z} X \quad \text{Public debt} \]
Condition for bank restructuring

\[ \int_{0}^{Z} \frac{I + B + qD - Y}{Z} XdF(X) \]

Period 2 payoff in bankruptcy

\[ + \int_{Z}^{\infty} \left( X - (1 - q)D \right)dF(X) - \left( I + qD - Y \right) \geq \frac{B}{B + D} Y \equiv L_B \]

Period 1 extra cash outflow

Payoff in period 1 under bankruptcy (absent restructuring)

\[ \int_{0}^{Z} X + \frac{I + B + qD - Y - Z}{Z} XdF(X) \]

\[ - (1 - q)D / Z \]

\[ + \int_{Z}^{\infty} \left( X - (1 - q)D \right)dF(X) - (I + qD) \geq \frac{B}{B + D} Y - Y = - \frac{D}{B + D} Y \equiv -L_D \]
The bank will agree to restructure B:

\[
\hat{X} - I \geq qD + \int_{0}^{Z} \frac{(1-q)D}{Z} XdF(X) + \int_{Z}^{\infty} (1-q)DdF(X) - L_D
\]

\[V_D = \text{the value of public debt under restructuring}\]

- The condition for restructuring:

\[
\hat{X} - I \geq V_D - L_D
\]

- \(V_D - L_D\) is positive or negative
Investment with bank debt

- Invest iff: \( \hat{X} - I \geq V_D - L_D \)

- No investment is efficient
- Investment is efficient

- \( V_D - L_D > 0 \):
  - Don’t invest
  - Invest

- \( V_D - L_D < 0 \):
  - Don’t invest
  - Invest

- We can have underinvestment (debt overhang) or overinvestment (asset substitution)
The effect of public debt maturity

- The value of debt under restructuring:
  \[ V_D = qD + \int_0^Z \frac{(1-q)D}{Z} XdF(X) + \int Z(1-q)DdF(X) \]

- \[ V_D = D \text{ if } q = 1 \]

- How does \( q \) affect \( V_D \)?
  \[
  \frac{\partial V_D}{\partial q} = D - \int_0^Z \frac{D}{Z} XdF(X) - \int Z(1-q)DdF(X) \\
  > D - \int_0^Z \frac{D}{Z} ZdF(X) - \int Z(1-q)DdF(X) = 0
  \]
The effect of public debt maturity

- \( q \uparrow \Rightarrow V_D \uparrow \Rightarrow V_D - L_D \uparrow \)

- Investment takes place iff: \( \hat{X} - I \geq V_D - L_D \)

- Since \( q \uparrow \Rightarrow V_D \uparrow \), underinvestment is likely when \( q \to 1 \) (short maturity) and overinvestment is likely when \( q \to 0 \) (long maturity)

- \( q \uparrow \) exacerbates underinvestment (debt overhang) but alleviates overinvestment (asset substitution)

\[ \hat{X} - I \]

Bad effect:
- Don’t invest
- Invest

Good effect:
- Don’t invest
- Invest
New capital infusions from a new bank or by issuing equity

- Invest iff: \( \hat{X} - I \geq V_D - L_D + B - L_B \) (+)

- The new cash infusion exacerbates underinvestment (debt overhang) but alleviate overinvestment (asset substitution).

- If \( V_D - L_D > 0 \): Don’t invest
- If \( V_D - L_D < 0 \): Invest

- The new cash infusion is via equity (debt has priority over equity during bankruptcy in period 2)
  \( \Rightarrow \) The firm will issue debt, not equity (equity subsidizes public debt and is therefore wasteful)
New senior debt

- Suppose the firm can issue in period 1 senior debt with face value $D'$ which is due in period 2 (existing debt is not protected by seniority covenants).

- If $D' \rightarrow \infty$, the firm always defaults in period 2 so the new senior debtholders will get the entire period 2 cash flow.

- Existing debt gets 0 in period 2.

⇒ The value of the existing debt is only equal to the period 1 payment $qD < V_D$. 
New senior debt

- Invest iff: $X - I \geq qD - L_D$

The new senior debt alleviates underinvestment (debt overhang) but exacerbates overinvestment (asset substitution).

Seniority covenants (which prevent the firm from issuing senior debt) are worthwhile if overinvestment is likely ($q \to 0$) but are a bad idea if underinvestment is likely ($q \to 1$).
Existing public debt is junior to bank debt

- If the firm goes bankrupt in period 1, junior debtholders get \([Y-B]^+ < L_D\).

- Invest iff: \(\hat{X} - I \geq V_D - [Y - B]^+ > V_D - L_D\).

- When existing debt is junior, underinvestment (debt overhang) is exacerbated (likely when \(q \to 1\)) but overinvestment (asset substitution) is alleviated (likely when \(q \to 0\)).
Public Debt Exchange
Public debt exchange

- Suppose the firm can restructure its public debt (despite the difficulties) through an exchange (tender your old debt and get a new debt or cash)

- The firm faces a cash shortage:

  \[
  I + B < Y < I + B + qD
  \]

  - Money to pay bank and invest
  - Money needed to stay solvent in period 1 and invest

- Timing:
  - Stage 1: the firm makes TIOLI offer to the bank
  - Stage 2: the firm offers an exchange of existing public debt with new public debt due in period 2 whose face value is pD (investors who refuse keep their old securities)
Public debt exchange – stage 2

- Suppose the bank rejects the TIOLI
- The firm offers an exchange of existing public debt with new senior public debt due in period 2 and with face value pD
- The firm can set p s.t. old debtholders get nothing in period 2
- Suppose that

\[
\hat{X} + Y - I - B \geq qD \iff \hat{X} - I \geq qD - Y + B
\]

Max. period 2 payoff of new public debtholds (they get everything since p is set high)

Period 1 payoff of debtholders who reject the exchange (their period 2 payoff is 0 since p is high)

- If the condition holds, then there exists an equil. in which all public debtholders accept

\[ \Rightarrow \] If the condition holds, the bank gets B if it rejects the TIOLI \( \Rightarrow \) to induce the bank to accept the TIOLI, the firm must offer the bank at least B
Bank debt restructuring in stage 1

- The restructured bank debt, \(B'\), is senior to public debt.

- The firm makes a TIOLI to the bank s.t.:

\[
\int_{0}^{B'} XdF(X) + \int_{B'}^{\infty} B'dF(X) + Y - I - qD = B
\]

- Rewriting:

\[
\int_{0}^{B'} (X + (Y - I - qD) - B)dF(X) = \int_{B'}^{\infty} (B - B' - (Y - I - qD))dF(X)
\]

- The bank accepts the TIOLI only if \(B'\) satisfies the above equation.
Back to public debt exchange (the bank’s debt remains B)

- Suppose that \( \hat{X} - I \geq qD - Y + B \) so the firm can exchange its public debt.

- Suppose the firm offers new debt with face value \( pD \) (\( p \) need not be so high that bankruptcy occurs for sure) - solvency in period 2:

\[
\hat{X} - I \geq qD - Y + B
\]

\[
X + Y - I - B = \text{extra cash from period 1}
\]

\[
pD = \text{face value of new debt}
\]

\[
pD \text{ is paid in full}
\]

\[
Y - I - B
\]
Back to public debt exchange (the bank’s debt remains B)

- Solvency in period 2 (the face value of new debt is pD):
  \[ X + Y - I - B \geq pD \quad \Rightarrow \quad X \geq I + B + pD - Y \equiv X_b \]

- The value of old public debt:
  \[ OD = qD + \int_0^\infty (1-q)DdF(X) \]
  \[ \text{Period 1 payoff} \]

- The value of new public debt (paid only in period 2):
  \[ ND = \int_0^{X_b} \left( X + Y - I - B \right) dF(X) + \int_{X_b}^\infty pDdF(X) \]
  \[ \text{Public debt is not paid in full} \quad \text{Public debt is paid in full} \]
Public debt exchange

- The minimal $p$ needed to ensure acceptance is defined implicitly by $ND = OD$:

$$\int_{0}^{X_b} (X + Y - I - B)dF(X) + \int_{X_b}^{\infty} pDdF(X) = qD + \int_{X_b}^{\infty} (1 - q)DdF(X)$$

- Reorganizing, $p$ which ensures that public debt can be exchanged, is defined by:

$$\int_{0}^{X_b} (X + Y - I - qD - B)dF(X) = \int_{X_b}^{\infty} (1 - p)DdF(X)$$
Summary

- The bank accepts the TIOLI only if $B'$ satisfies:

$$\int_{0}^{B'} \left( (X + (Y - I - qD) - B) \right) dF(X) = \int_{B'}^{\infty} \left( (B - B' - (Y - I - qD)) \right) dF(X)$$

- Public debtholders accept the public debt exchange only if $p$ satisfies:

$$\int_{0}^{X_b} \left( (X + Y - I - qD - B) \right) dF(X) = \int_{X_b}^{\infty} \left( (1 - p)D \right) dF(X)$$
Exchange or restructure B?

- **Equityholders’ payoff:**
  - In a public debt exchange: \([X-X_b]^+\), where \(X_b \equiv I+B+pD-Y\)
  - In bank debt restructuring: \([X-B'-(1-q)D]^+\)

- Exchange is more profitable iff

\[
X_b < B' + (1-q)D \quad \Rightarrow \quad B' > \underbrace{X_b - (1-q)D}_{I+B+pD-Y}
\]

- We’ll show that this inequality holds by writing \(B' = X_b - (1-q)D + \varepsilon\) and showing that \(\varepsilon > 0\)
Exchange or restructuring of B?

The condition for bank debt restructuring:

\[
\int_{B'} \left( X + (Y - I - qD) - B \right) dF(X) = \int_{B'} \left( B - B' - (Y - I - qD) \right) dF(X)
\]

- Recall that \( B' \equiv X_b - (1-q)D + \epsilon \) and \( X_b \equiv I + B + pD - Y \). Then the RHS becomes:

\[
\int_{B'} \left( B - B' - (Y - I - qD) \right) dF(X)
\]

\[
= \int_{B'} \left( B - \left[ \frac{X_b}{B' = x_b - (1-q)D + \epsilon} \right] - (1-q)D + \epsilon \right)(Y - I - qD) dF(X)
\]

\[
= \int_{B'} \left( \frac{X_b - pD}{I + B - Y - X_b + D - \epsilon} \right) dF(X) = \int_{B'} ((1 - p)D - \epsilon) dF(X)
\]
Exchange is more profitable

- The TIOLI to the bank:
  \[ \int_0^{B'} \left( X + (Y - I - qD) - B \right) dF(X) = \int_{B'}^{\infty} ((1 - p)D - \varepsilon) dF(X) \]

- The condition for public debt exchange:
  \[ \int_0^{X_b} \left( X + (Y - I - qD) - B \right) dF(X) = \int_{X_b}^{\infty} (1 - p)D dF(X) \]

- Suppose that \( \varepsilon < 0 \). Then, \( B' \equiv X_b - (1-q)D + \varepsilon < X_b \)

  \( \Rightarrow \) The LHS of the 2\textsuperscript{nd} equation is larger but the RHS of the 1\textsuperscript{st} equation is larger \( \Rightarrow \) Contradiction!

  \( \Rightarrow \) \( \varepsilon > 0 \) \( \Rightarrow \) \( B' > X_b - (1-q)D \) \( \Rightarrow \) exchange is more profitable