Corporate Finance: Credit rationing

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The Theory of Corporate Finance

The model

- The timing:

Period 0	Period 1	Period 2
An entrepreneur has A dollars and needs to invest in a project that costs I > A	The entrepreneur exerts effort to boost the prob. of success. If he does not exert effort he gets private benefits B	If the project succeeds it yields R; if it fails, it yields 0

- Effort raises the prob. of success from p_L to p_H
- $\Delta p \equiv p_H - p_L$
- The project is viable only if there's effort:

$$\underbrace{p_H R - I}_{\text{NPV}} > 0 > \underbrace{p_L R - I + B}_{\text{NPV + Benefits}} \quad \Rightarrow \quad \Delta p R > B$$

The loan agreement

- The loan can be debt or equity (the model cannot distinguish between them)
- Incentive compatibility (to ensure effort):

$$\underbrace{p_H R_b}_{\substack{\text{Entrepreneur's} \\ \text{expected payoff} \\ \text{with effort}}} > \underbrace{p_L R_b + B}_{\substack{\text{Entrepreneur's} \\ \text{expected payoff} \\ \text{without effort}}} \Rightarrow R_b > \frac{B}{\underbrace{p_H - p_L}_{\text{Cost of MH}}} \equiv \frac{B}{\Delta p}$$

- Creditor's individual rationality:

$$p_H \underbrace{(R - R_b)}_{\substack{\text{Maximal pledgeable} \\ \text{income}}} \equiv p_H \underbrace{\left(R - \frac{B}{\Delta p}\right)}_{(+)} \geq \underbrace{I - A}_{\text{Required funds}}$$

Credit rationing

- Creditor's individual rationality:

$$p_H\left(R-\frac{B}{\Delta p}\right)\geq I-A \quad \Rightarrow \quad A\geq \overline{A}\equiv \underbrace{p_H\frac{B}{\Delta p}-\overbrace{\left(p_H R-I\right)}^{\text{NPV with effort}}}_{(+)\text{ by assumption}}$$

- An entrepreneur must have Ā to get funds
- When A < Ā, we get credit rationing: the creditor gets too little ex post to agree to give the entrepreneur I-A
- Credit rationing is "more severe" when B is large: there's more agency problem or MH

Entrepreneur's payoff

- When A < Ā, the project is not funded so U = 0
- When A ≥ Ā, the project is funded; if the entrepreneur has all the bargaining power, the creditor simply breaks even:

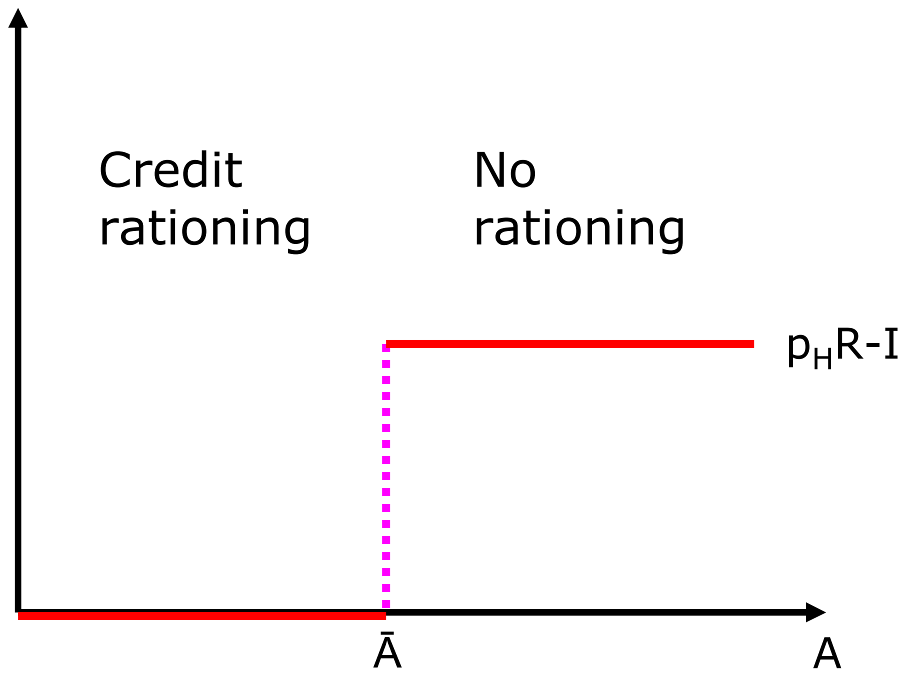
$$\underbrace{p_H R_l}_{\text{Creditor's expected payoff}} = I - A \quad \Rightarrow \quad R_l = \underbrace{\frac{I-A}{p_H}}_{\text{Min payment to creditor given effort}}$$

- The entrepreneur's net payoff (above and beyond A which he can consume anyway by not investing):

$$U = p_H\left(R - R_l\right) - A = p_H\left(R - \frac{I-A}{p_H}\right) - A = \underbrace{p_H R - I}_{\text{NPV with effort}}$$

- Since the creditor breaks even, the entrepreneur captures the entire NPV

The entrepreneur's net payoff (above and beyond A) - illustration



- The entrepreneur either gets all the NPV or nothing $\Rightarrow$ the entrepreneur is indifferent to A above $\bar{A}$

Overborrowing

- Suppose the firm can ↑ the prob. of success by τ by investing J which it borrows from a new creditor

- Assumption: the investment is <u>inefficient</u>: $J > \tau R$

⇒ No point in investing if effort stays the same (investment ↓ NPV and hence ↓ the entrepreneur's payoff); the investment's role is to transfer value from the original creditor

- The entrepreneur invests J only if it induces him to exert no effort (the alternative is to forgo J and exert effort):

$$\underbrace{(p_L + \tau)R_b - J}_{\substack{\text{The entrepreneur's payoff} \\ \text{w/o effort when the new} \\ \text{creditor breaks even}}} + B > \underbrace{p_H R_b}_{\substack{\text{No overinvestment} \\ \text{and effort}}}$$

Overborrowing

- The condition for overborrowing:

$$(P_L+\tau)\underbrace{(R-R_l)}_{R_B}-J+B>p_H\underbrace{(R-R_l)}_{R_B} \Rightarrow \overbrace{\underbrace{(p_H-(p_L+\tau))R_l}_{\downarrow \text{ in the expected payoff of the initial creditor}}+B}^{\text{Benefit}} > \overbrace{\underbrace{\Delta pR}_{\text{Loss due to lack of effort}}+\underbrace{J-\tau R}_{\text{Net cost of new investment}}}^{\text{Cost}}$$

- Overborrowing is worthwhile only if it transfers enough value from the initial creditor to compensate for the resulting inefficiencies

- If the condition holds, the initial creditor must impose a no-extra investment/loan covenant

- $R_l \uparrow \Rightarrow$ overborrowing is more tempting

- But $R_l = (I-A)/p_H$; hence, $A\downarrow \Rightarrow R_l\uparrow \Rightarrow$ overborrowing is more likely when A is low and hence covenants are needed more

Debt overhang

- Suppose the firm has initial secured debt with face value D ≤ A

- The creditor's IR constraint:

$$\underbrace{p_H\left(R-\frac{B}{\Delta p}\right)-D}_{\text{Net pledgeable income}} \geq \underbrace{I-A}_{\text{Size of loan}}$$

- D makes investment less likely

Debt restructuring

- Suppose that R is large enough so the entrepreneur can get a loan without debt but not with the debt:

$$p_H\left(R-\frac{B}{\Delta p}\right)-D<I-A\leq p_H\left(R-\frac{B}{\Delta p}\right)$$

- Absent restructuring, the investment is not made and the creditor gets A
- To induce investment D must be lowered to d such that

$$p_H\left(R-\frac{B}{\Delta p}\right)-d=I-A$$

Multiple projects

- 2 identical projects
- Suppose that the entrepreneur gets R_2 if both projects succeed and gets 0 otherwise (can also pay R_1 is one project succeeds and R_0 if none succeeds but R_2 is sufficient since the entrepreneur is risk neutral)
- Incentive compatibility:

$$\underbrace{p_H^{\,2} R_2}_{\substack{\text{Entrepreneur's}\\ \text{payoff with effort}\\ \text{on both projects}}} > \underbrace{p_L^{\,2} R_2 + 2B}_{\substack{\text{Entrepreneur's payoff}\\ \text{without effort}}} \Rightarrow \left(\frac{p_H + p_L}{2}\right)\underbrace{(p_H - p_L)}_{\Delta p} R_2 > B$$

$$\underbrace{p_H^{\,2} R_2}_{\substack{\text{Entrepreneur's}\\ \text{payoff with effort}\\ \text{on both projects}}} > \underbrace{p_H\, p_L R_2 + B}_{\substack{\text{Entrepreneur's payoff}\\ \text{with effort on a single}\\ \text{projects}}} \Rightarrow p_H \Delta p R_2 > B$$

- The first IC constraint implies the second

The creditor's IR

- Creditor's individual rationality (IR):

$$\underbrace{p_H{}^2 2R + 2p_H(1-p_H)R}_{\text{Expected return}} - \underbrace{p_H{}^2 R_2}_{\text{Entrepreneur's payoff}} = 2p_H R - p_H{}^2 R_2 \geq 2(I-A)$$

- From entrepreneur's IC:

$$R_2 \geq \frac{1}{p_H + p_L}\frac{2B}{\Delta p}$$

- Substituting from IC into creditor's IR:

$$2p_H R - p_H{}^2 \frac{2B}{(p_H + p_L)\Delta p} \geq 2(I-A) \quad \Rightarrow \quad p_H\left[R - \left(\frac{p_H}{p_H + p_L}\right)\frac{B}{\Delta p}\right] \geq I - A$$

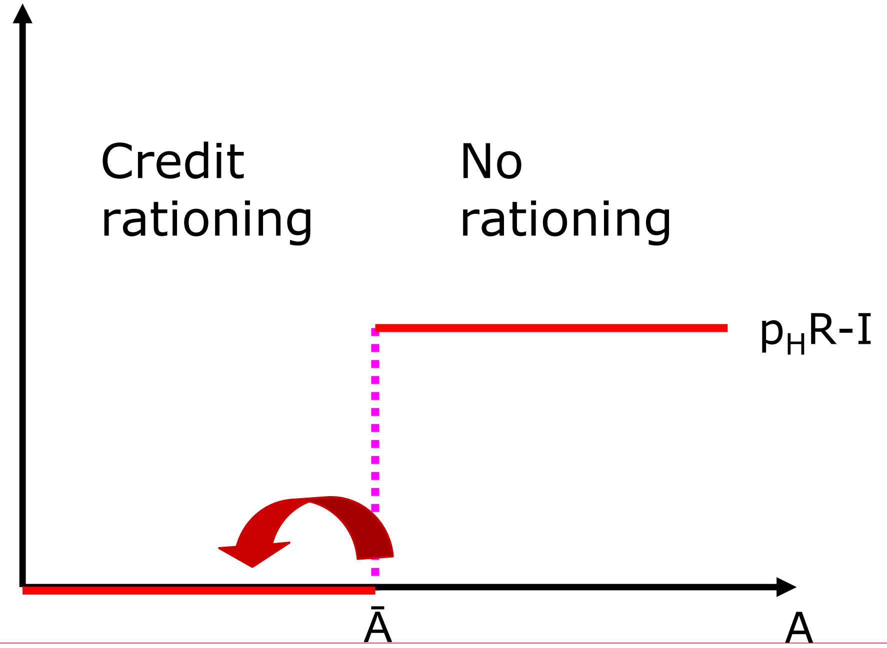
The effect of multiple projects on financing

- The condition for financing:

$$A \geq \overline{A} \equiv I - p_H\left[R - \left(\frac{p_H}{p_H + p_L}\right)\frac{B}{\Delta p}\right]$$

Ā↓

Financing is easier



Multiple projects with perfect correlation

- Entrepreneur's IC:

$$\underbrace{p_H R_2}_{\substack{\text{Entrepreneur's} \\ \text{payoff with effort} \\ \text{on both projects}}} > \underbrace{p_L R_2 + 2B}_{\substack{\text{Entrepreneur's payoff} \\ \text{without effort}}} \Rightarrow R_2 > \frac{2B}{\Delta p}$$

- Creditor's individual rationality (IR):

$$\underbrace{p_H 2R}_{\text{Expected return}} - \underbrace{p_H R_2}_{\substack{\text{Entrepreneur's} \\ \text{payoff}}} = p_H [2R - R_2] \geq 2(I - A)$$

- From entrepreneur's IC:

$$p_H \left[2R - \frac{2B}{\Delta p} \right] \geq 2(I - A) \Rightarrow A \geq \overline{A} \equiv p_H \frac{B}{\Delta p} - (p_H R - I)$$

The creditor's IR under perfect correlation

- Under perfect corr. we are back to the single project case

- Diversification helps because the projects are not perfectly correlated

- Imperfect correlation effectively lowers B to $p_H B/(p_L+p_H)$

Correlation or independence?

- Suppose the entrepreneur can choose whether projects will be correlated or independent but his choice is hidden from the creditor

- Given R_2, the entrepreneur's payoff:
  - Correlation: $p_H R_2$
  - Independence: $p_H^2 R_2$

$\Rightarrow$ The entrepreneur will choose perfect correlation. Why is that?

- Asset substitution: correlation is riskier than independence. The entrepreneur is the residual claimant and likes risk

Continuous investment

- $I \in [0,\infty)$ is a choice variable; the entrepreneur chooses I and whether to exert effort
- Return is RI and private benefit is BI
- IC for the entrepreneur:

$$p_H R_b > p_L R_b + BI \quad \Rightarrow \quad R_b > \frac{BI}{\Delta p}$$

- IR for the creditor:

$$p_H\left(RI - R_b\right) \geq I - A \quad \Rightarrow \quad p_H\left(RI - \frac{BI}{\Delta p}\right) \geq I - A$$

- Rewriting:

$$I \leq \kappa A \quad \Rightarrow \quad \underbrace{\kappa \equiv \frac{1}{1 - p_H R + \frac{p_H B}{\Delta p}}}_{\text{multiplier}}$$

Continuous investment – optimal investment

- In a competitive capital market, the lenders must break even given their anticipation that the entrepreneur will exert effort: $p_H R_l$ = I-A
- The entrepreneur's utility above and beyond A:

$$U = p_H(RI - R_l) - A = p_H\left(RI - \frac{I - A}{p_H}\right) - A = (p_H R - 1)I$$

- Assumption 1: $p_H R > 1$ – investment has a positive NPV with effort
  - Implication: the entrepreneur would like to invest as much as he can
- But if I is high, the IC constraint is violated
- Optimal investment is determined by the multiplier equation: I = κA
- "Invest up to κ times your wealth" or "Borrow κ-1 times your wealth"

Continuous investment - multiplier

- Assumption 1: $p_H R > 1$ – investment has a positive NPV with effort
- Assumption 2: $p_L R + B < 1$ – investment has a negative NPV w/o effort
- Assumption 1 + 2 imply: $p_H R > 1 > p_L R + B \Rightarrow \Delta p R > B \Rightarrow R > B/\Delta p$
- Assumption 3: $p_H R1 - 1 < p_H B/\Delta p$ – NPV is lower than the cost of MH
- Since $R > B/\Delta p$ and given Assumption 3, $\kappa > 1$
- Implication: κ is a "multiplier" – each dollar of equity leads to κ dollars of investment
- κ is smaller if B is large

Continuous investment - leverage

- The optimal investment is κA

- The entrepreneur needs to borrow $(\kappa-1)A$, where

$$\kappa - 1 = \frac{1}{1 - p_H R + \dfrac{p_H B}{\Delta p}} - 1 = \frac{p_H\left(R - \dfrac{B}{\Delta p}\right)}{1 - p_H\left(R - \dfrac{B}{\Delta p}\right)}$$