Corporate Finance: Credit rationing

Yossi Spiegel
Recanati School of Business
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The Theory of Corporate Finance
The model

- The timing:

  **Period 0**
  An entrepreneur has \( A \) dollars and needs to invest in a project that costs \( I > A \)

  **Period 1**
  The entrepreneur exerts effort to boost the prob. of success. If he does not exert effort he gets private benefits \( B \)

  **Period 2**
  If the project succeeds it yields \( R \); if it fails, it yields 0

- Effort raises the prob. of success from \( p_L \) to \( p_H \)
- \( \Delta p = p_H - p_L \)
- The project is viable only if there’s effort:

  \[
  \frac{p_H R - I}{\text{NPV}} > 0 > \frac{p_L R - I + B}{\text{NPV + Benefits}} \quad \Rightarrow \quad \Delta p R > B
  \]
The loan agreement

- The loan can be debt or equity (the model cannot distinguish between them)
- Incentive compatibility (to ensure effort):
  \[ p_H R_b > p_L R_b + B \Rightarrow R_b > \frac{B}{p_H - p_L} = \frac{B}{\Delta p} \]

- Creditor's individual rationality:
  \[ p_H \left( R - R_b \right) \equiv p_H \left( R - \frac{B}{\Delta p} \right) \geq I - A \]
  (Required funds)
Credit rationing

☐ Creditor’s individual rationality:

\[ p_H \left( R - \frac{B}{\Delta p} \right) \geq I - A \quad \Rightarrow \quad A \geq \bar{A} \equiv p_H \frac{B}{\Delta p} - \left( p_H R - I \right) \]

☐ An entrepreneur must have \( \bar{A} \) to get funds

☐ When \( A < \bar{A} \), we get credit rationing: the creditor gets too little ex post to agree to give the entrepreneur \( I - A \)

☐ Credit rationing is “more severe” when \( B \) is large: there’s more agency problem or MH
Entrepreneur’s payoff

- When $A < \bar{A}$, the project is not funded so $U = 0$.

- When $A \geq \bar{A}$, the project is funded; if the entrepreneur has all the bargaining power, the creditor simply breaks even:
  \[
  p_H R_l = I - A \implies R_l = \frac{I - A}{p_H}
  \]

  Creditor's expected payoff

  Min payment to creditor given effort

- The entrepreneur’s net payoff (above and beyond $A$ which he can consume anyway by not investing):
  \[
  U = p_H (R - R_l) - A = p_H \left( R - \frac{I - A}{p_H} \right) - A = \frac{p_H R - I}{\text{NPV with effort}}
  \]

- Since the creditor breaks even, the entrepreneur captures the entire NPV.
The entrepreneur’s net payoff (above and beyond A) - illustration

- The entrepreneur either gets all the NPV or nothing \( \Rightarrow \) the entrepreneur is indifferent to A above \( \bar{A} \)

\[
\begin{align*}
\text{Credit rationing} & \quad \text{No rationing} \\
p_H R - I &
\end{align*}
\]
Overborrowing

- Suppose the firm can \( \uparrow \) the prob. of success by \( \tau \) by investing \( J \) which it borrows from a new creditor.

- Assumption: the investment is inefficient: \( J > \tau R \)

  \[ \Rightarrow \] No point in investing if effort stays the same (investment \( \downarrow \) NPV and hence \( \downarrow \) the entrepreneur’s payoff); the investment’s role is to transfer value from the original creditor.

- The entrepreneur invests \( J \) only if it induces him to exert no effort (the alternative is to forgo \( J \) and exert effort):

  \[
  \left( p_L + \tau \right) R_b - J + B > p_H R_b
  \]

  The entrepreneur's payoff w/o effort when the new creditor breaks even

  No overinvestment and effort
Overborrowing

The condition for overborrowing:

\[
\left( P_L + \tau \right) \left( R - R_l \right) - J + B > p_H \left( R - R_l \right) \Rightarrow \left( p_H - \left( p_L + \tau \right) \right) R_l + B > \Delta pR + J - \pi R
\]

- Overborrowing is worthwhile only if it transfers enough value from the initial creditor to compensate for the resulting inefficiencies.
- If the condition holds, the initial creditor must impose a no-extra investment/loan covenant.
- \( R_l \uparrow \Rightarrow \) overborrowing is more tempting.
- But \( R_l = (I-A)/p_H \); hence, \( A \downarrow \Rightarrow R_l \uparrow \Rightarrow \) overborrowing is more likely when \( A \) is low and hence covenants are needed more.
Debt overhang

- Suppose the firm has initial secured debt with face value $D \leq A$

- The creditor’s IR constraint:

$$p_H \left( R - \frac{B}{\Delta p} \right) - D \geq \frac{I - A}{\text{Size of loan}}$$

- $D$ makes investment less likely
Debt restructuring

- Suppose that $R$ is large enough so the entrepreneur can get a loan without debt but not with the debt:

$$p_H \left( R - \frac{B}{\Delta p} \right) - D < I - A \leq p_H \left( R - \frac{B}{\Delta p} \right)$$

- Absent restructuring, the investment is not made and the creditor gets $A$

- To induce investment $D$ must be lowered to $d$ such that

$$p_H \left( R - \frac{B}{\Delta p} \right) - d = I - A$$
Multiple projects

- 2 identical projects
- Suppose that the entrepreneur gets $R_2$ if both projects succeed and gets 0 otherwise (can also pay $R_1$ if one project succeeds and $R_0$ if none succeeds but $R_2$ is sufficient since the entrepreneur is risk neutral)

- Incentive compatibility:

  \[
  p_H^2 R_2 > p_L^2 R_2 + 2B \quad \Rightarrow \quad \left( \frac{p_H + p_L}{2} \right) (p_H - p_L) R_2 > B \\
  \]

  \[
  p_H^2 R_2 > p_H p_L R_2 + B \quad \Rightarrow \quad p_H \Delta p R_2 > B \\
  \]

- The first IC constraint implies the second
The creditor’s IR

- Creditor’s individual rationality (IR):
  \[
  p_H^2 2R + 2p_H(1 - p_H)R - \frac{p_H^2 R_2}{p_H + p_L \Delta p} = 2p_H R - p_H^2 R_2 \geq 2(I - A)
  \]

- From entrepreneur’s IC:
  \[
  R_2 \geq \frac{1}{p_H + p_L \Delta p} \frac{2B}{2}
  \]

- Substituting from IC into creditor’s IR:
  \[
  2p_H R - p_H^2 \frac{2B}{p_H + p_L \Delta p} \geq 2(I - A) \implies p_H \left[ R - \left( \frac{p_H}{p_H + p_L} \Delta p \right) \frac{B}{\Delta p} \right] \geq I - A
  \]
The effect of multiple projects on financing

- The condition for financing:

\[ A \geq \bar{A} \equiv I - p_H \left[ R - \left( \frac{p_H}{p_H + p_L} \right) \frac{B}{\Delta p} \right] \]

**Credit rationing**

**No rationing**

- Financing is easier

\[ \bar{A} \downarrow \]

\[ p_H R - I \]
Multiple projects with perfect correlation

☐ Entrepreneur’s IC:

\[
\begin{align*}
R_2 > & \frac{2B}{\Delta p} \\
\frac{p_H R_2}{p_L R_2 + 2B} & \Rightarrow R_2 > \frac{2B}{\Delta p}
\end{align*}
\]

☐ Creditor’s individual rationality (IR):

\[
\begin{align*}
\frac{p_H 2R}{p_H R_2} - & = p_H \left[ 2R - R_2 \right] \geq 2(I - A)
\end{align*}
\]

☐ From entrepreneur’s IC:

\[
\begin{align*}
\frac{2R - \frac{2B}{\Delta p}}{p_H} & \geq 2(I - A) \\
A & \geq \bar{A} \equiv p_H \frac{B}{\Delta p} - (p_H R - I)
\end{align*}
\]
The creditor’s IR under perfect correlation

- Under perfect corr. we are back to the single project case

- Diversification helps because the projects are not perfectly correlated

- Imperfect correlation effectively lowers B to $p_H B / (p_L + p_H)$
Correlation or independence?

Suppose the entrepreneur can choose whether projects will be correlated or independent but his choice is hidden from the creditor.

Given $R_2$, the entrepreneur’s payoff:

- Correlation: $p_H R_2$
- Independence: $p_H^2 R_2$

⇒ The entrepreneur will choose perfect correlation. Why is that?

Asset substitution: correlation is riskier than independence. The entrepreneur is the residual claimant and likes risk.
Continuous investment

- \( I \in [0, \infty) \) is a choice variable; the entrepreneur chooses \( I \) and whether to exert effort.

- Return is \( RI \) and private benefit is \( BI \).

- IC for the entrepreneur:

\[
p_H R_b > p_L R_b + BI \quad \Rightarrow \quad R_b > \frac{BI}{\Delta p}
\]

- IR for the creditor:

\[
p_H (RI - R_b) \geq I - A \quad \Rightarrow \quad p_H \left( RI - \frac{BI}{\Delta p} \right) \geq I - A
\]

- Rewriting:

\[
I \leq \kappa A \quad \Rightarrow \quad \kappa = \frac{1}{1 - p_H R + \frac{p_H B}{\Delta p}}
\]

\( \kappa \) is the multiplier.
Continuous investment – optimal investment

- In a competitive capital market, the lenders must break even given their anticipation that the entrepreneur will exert effort: $p_H R_l = I - A$

- The entrepreneur’s utility above and beyond $A$:
  
  
  $$U = p_H (RI - R_l) - A = p_H \left( RI - \frac{I - A}{p_H} \right) - A = (p_H R - 1)I$$

- Assumption 1: $p_H R > 1$ – investment has a positive NPV with effort
  - Implication: the entrepreneur would like to invest as much as he can

- But if $I$ is high, the IC constraint is violated

- Optimal investment is determined by the multiplier equation: $I = \kappa A$

- “Invest up to $\kappa$ times your wealth” or “Borrow $\kappa - 1$ times your wealth”
Continuous investment - multiplier

- Assumption 1: \( p_H R > 1 \) – investment has a positive NPV with effort

- Assumption 2: \( p_L R + B < 1 \) – investment has a negative NPV w/o effort

- Assumption 1 + 2 imply: \( p_H R > 1 > p_L R + B \Rightarrow \Delta p R > B \Rightarrow R > B/\Delta p \)

- Assumption 3: \( p_H R^1 - 1 < p_H B/\Delta p \) – NPV is lower than the cost of MH

- Since \( R > B/\Delta p \) and given Assumption 3, \( \kappa > 1 \)

- Implication: \( \kappa \) is a “multiplier” – each dollar of equity leads to \( \kappa \) dollars of investment

- \( \kappa \) is smaller if \( B \) is large
Continuous investment - leverage

- The optimal investment is $\kappa A$

- The entrepreneur needs to borrow $(\kappa - 1)A$, where

\[
\kappa - 1 = \frac{1}{1 - p_H R + \frac{p_H B}{\Delta p}} - 1 = \frac{p_H \left( R - \frac{B}{\Delta p} \right)}{1 - p_H \left( R - \frac{B}{\Delta p} \right)}
\]