Corporate Finance: Asymmetric information and capital structure – the lemons problem

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Akerlof, QJE 1970

The lemons problem

- An entrepreneur establishes a firm
- Cash flow is \( x \sim U[0,1] \)
- The entrepreneur privately observes \( x \)
- How much will investors pay for the firm’s equity?
- Suppose the price is \( p \) – should investors pay it?
- The entrepreneur will agree to sell only if \( x \leq p \)
- Given \( p \), investors should pay \( E(x|x \leq p) \). With uniform dist., \( E(x|x \leq p) = p/2 \).
- It is impossible to sell the firm!
Crucial assumptions for the lemons problem

- The entrepreneur does not have to sell if he does not want to sell
- The entrepreneur knows $x$ but investors do not and investors know this fact
- The value of the firm is the same for the entrepreneur and the investors

$\Rightarrow$ The entrepreneur’s willingness to sell implies that $x \leq p$
Overcoming the lemons problem
additive improvement

- Cash flow $x$ is drawn from a uniform dist. on $[0,100]$

- The buyer can add to the cash flow $b$ (there are “gains from trade”)

- A seller will sell iff $p \geq x \Rightarrow$ the prob. of selling is $p/100$ and if selling occurs, the firm is worth on average $p/2$

- The buyer adds $b$ to the cash flow, so on average it is $p/2 + b$

- The buyer’s expected payoff is

$$\frac{p}{100} \left( \frac{p}{2} + b - p \right) = \frac{p}{100} \left( b - \frac{p}{2} \right)$$

- The value of $p$ that maximizes the payoff is $b$ (but no more than 100)
Overcoming the lemons problem
multiplicative improvement

- Cash flow $x$ is drawn from a uniform dist. on $[0, 100]$

- The buyer can improve the cash flow by a factor of $b \geq 1$

- A seller will sell iff $p \geq x \Rightarrow$ the prob. of selling is $p/100$ and if selling occurs, the firm is worth on average $p/2$

- The buyer improves the firm and makes it worth $bp/2$

- The buyer’s expected payoff is

$$\frac{p}{100} \left( b \frac{p}{2} - p \right) = \frac{p^2}{100} \left( \frac{b}{2} - 1 \right)$$

- If $b > 2$ it’s worth offering 100 for the firm
Arbitrary distribution of cash flows

- Cash flow $x$ is drawn from CDF $F(x)$ on $[0, \infty)$

- The buyer can improve the cash flow by a factor of $b \geq 1$

- A seller will sell iff $p \geq x$, so the expected cash flow conditional on trade is

$$
\hat{x} = \frac{\int_0^p x dF(x)}{F(p)}
$$

- The buyer will agree to buy iff

$$
F(p)(b \hat{x} - p) \geq 0 \quad \Rightarrow \quad b \geq \frac{p}{\hat{x}}
$$
Voluntary disclosure

- Suppose your credit rating ranges from 1 (junk) to 10 (AAA)

- Will you show your credit rating to a potential trading partner (e.g., a supplier)?

- Sure if your rating is 10. But what if it’s 4?

- If you do not show it, he’ll expect it to be 5

  - If your rating were 6-10, you surely disclose it

  - If you do not disclose, your trade partner will think your rating must be 1-5, i.e., 3 on av.

  - Complete unraveling: only 1 may hide!

  - The act of concealing information reveals a lot of information
A simple model of costly disclosure: assumptions

- Cash flow: \( x \) is dist. uniformly on \([0, 100]\)

- Cost of disclosure: \( c \)

- The objective is to maximize the value of the firm
A simple model of costly disclosure: analysis

Disclosure: $x - c$

No disclosure: $\text{Ex}^{\text{ND}}$

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A simple model of costly disclosure: analysis

- Since $x_1$ is the highest $x$ that does not disclose:
  \[ \text{Ex}^{\text{ND}} = x_1/2 \]

- $x_1$ is defined by
  \[ x_1 - c = \text{Eq}^{\text{ND}} = x_1/2 \]

- Solving, we get
  \[ x_1 = 2c \]

- Disclose if $x > 2c$, do not disclose if $x < 2c
“Corporate Financing and Investment Decisions when Firms have Information that Investors Do Not Have”
The model

- The timing:

  - Stage 0: A firm worth $x$ can invest $I$ in a project that yields $R > I$
  - Stage 1: The firm decides whether or not to take the project and how to finance it
  - Stage 2: Cash flow is realized

- Existing value, $x$, is drawn from some dist.

- The mean of $x$ is $\hat{x}$
The full information case

- Suppose that the firm issues equity that gives investors $\alpha$ of the firm

- In order to raise $I$ dollars:

  \[ \alpha(x + R) = I \implies \alpha = \frac{I}{x + R} \]

- The entrepreneur’s payoff:

  \[ U = (1 - \alpha)(x + R) = \frac{x + R - I}{x + R}(x + R) = x + R - I \]
Asymmetric information – binary case: \( x \in \{x_L, x_H\} \)

- Suppose we have a pooling equil. and both type finance the investment

- In order to raise \( I \) dollars:
  \[
  \alpha(\hat{x} + R) = I \quad \Rightarrow \quad \alpha = \frac{I}{\hat{x} + R}
  \]

- The entrepreneur’s payoff:
  \[
  U = (1 - \alpha)(x + R) = \frac{\hat{x} + R - I}{\hat{x} + R}(x + R)
  \]
Will both types finance?

- The entrepreneur will invest iff $U > x$ (value w/o investing)
  
  $$U - x = \frac{\hat{x} + R - I}{\hat{x} + R} (x + R) - x$$
  
  $$= \frac{(\hat{x} + R - I)(x + R) - x(\hat{x} + R)}{\hat{x} + R}$$
  
  $$= \frac{R(\hat{x} + R) - I(x + R)}{\hat{x} + R} = R - I \left( \frac{x + R}{\hat{x} + R} \right)$$

- Type L is subsidized by type H because $x_L < \hat{x} < x_H$
- Both types invest only if $U > x$ for type H
- O/w only type L invests and type H forgoes the investment although it has NPV > 0
Asymmetric information: continuous case: \( x \sim F(x) \) on \([0, \infty)\)

- Let \( \bar{x} \) be the highest \( x \) such that the entrepreneur invests

- The expected value of the firm given \( \bar{x} \):
  \[
  \hat{x}(\bar{x}) = \frac{\int_0^{\bar{x}} x dF(x)}{F(\bar{x})} = E(x| x \leq \bar{x})
  \]

- In order to raise I dollars:
  \[
  \alpha(\hat{x}(\bar{x}) + R) = I \quad \Rightarrow \quad \alpha = \frac{I}{\hat{x}(\bar{x}) + R}
  \]

- The entrepreneur’s payoff:
  \[
  U = (1 - \alpha)(x + R) = \frac{\hat{x}(\bar{x}) + R - I}{\hat{x}(\bar{x}) + R}(x + R)
  \]
How is $\bar{x}$ determined?

- The entrepreneur will invest iff $U > x$ (value w/o investing)

- $\bar{x}$ is determined s.t. the payoff $U = x$:

\[
U = \frac{\hat{x}(\bar{x}) + R - I}{\hat{x}(\bar{x}) + R} (x + R)
\]
How is $\overline{x}$ determined?

- $\overline{x}$ is determined by

$$\frac{\hat{x}(\overline{x}) + R - I}{\hat{x}(\overline{x}) + R} (\overline{x} + R) = \overline{x} \quad \Rightarrow \quad \hat{x}(\overline{x}) + R - I = \frac{I}{R} \overline{x}$$

$$\hat{x}(\overline{x}) = \mathbb{E}(x | x \leq \overline{x})$$

- Solving the two equations yield $\overline{x}$
Implications

- The entrepreneur will invest iff \( x \leq \bar{x} \)

- If the entrepreneur uses safe debt:

\[
U = x + R - D = x + R - I
\]

- This is also the payoff when using RE

- “Pecking order”: RE \( \rightarrow \) Debt \( \rightarrow \) Equity
Price reactions

- Before investing, the expected value of the firm is

\[ E = F(\bar{x}) \left( \int_0^{\bar{x}} \frac{x}{F(\bar{x})} dF(x) + R - I \right) + \left(1 - F(\bar{x})\right) \left( \int_{\bar{x}}^{\infty} \frac{x}{1 - F(\bar{x})} dF(x) \right) = \hat{x} + F(\bar{x})(R - I) \]

- Conditional on passing on the investment project, the market learns that \( x > \bar{x} \) hence, the expected value is:

\[ E(N) = \int_{\bar{x}}^{\infty} \frac{x}{1 - F(\bar{x})} dF(x) = E(x \mid x > \bar{x}) \]

- Conditional on investing, the market learns that \( x \leq \bar{x} \); hence, expected value of the firm is

\[ E(Y) = \int_{0}^{\bar{x}} \frac{x}{F(\bar{x})} dF(x) + R - I = E(x \mid x \leq \bar{x}) + R - I \]
Price reactions to not investing

\[ E(N) - E = \int_{\bar{x}}^{\infty} \frac{x}{1 - F(\bar{x})} \, dF(x) - \hat{x} - F(\bar{x})(R - I) \]

\[ = -\int_{0}^{\bar{x}} xdF(x) + \int_{\bar{x}}^{\infty} \left[ \frac{x}{1 - F(\bar{x})} - x \right] dF(x) - F(\bar{x})(R - I) \]

\[ = F(\bar{x}) \left[ \int_{\bar{x}}^{\infty} \frac{x}{1 - F(\bar{x})} \, dF(x) - \int_{0}^{\bar{x}} \frac{x}{F(\bar{x})} \, dF(x) - (R - I) \right] \]

\[ = F(\bar{x}) \left[ E(x \mid x > \bar{x}) - \left( E(x \mid x \leq \bar{x}) + R - I \right) \right] \]

\[ = F(\bar{x}) \left[ E(x \mid x > \bar{x}) - \frac{I}{R} \bar{x} \right] \]

\[ > F(\bar{x}) \left[ E(x \mid x > \bar{x}) - \bar{x} \right] > 0 \]

☐ Not investing is good news

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Price reactions to investing

\[ E(Y) - E = \int_{0}^{\bar{x}} \frac{x}{F(\bar{x})} dF(x) + R - I - \hat{x} - F(\bar{x})(R - I) \]

\[ = -\int_{\bar{x}}^{\infty} xdF(x) + \int_{0}^{\bar{x}} \left[ \frac{x}{F(\bar{x})} - x \right] dF(x) + (1 - F(\bar{x}))(R - I) \]

\[ = (1 - F(\bar{x})) \left[ \int_{0}^{\bar{x}} \frac{x}{F(\bar{x})} dF(x) - \int_{\bar{x}}^{\infty} \frac{x}{1 - F(\bar{x})} dF(x) + (R - I) \right] \]

\[ = (1 - F(\bar{x})) \left[ E(x \mid x \leq \bar{x}) + (R - I) - E(x \mid x > \bar{x}) \right] \]

\[ = (1 - F(\bar{x})) \left[ \frac{I}{R} \bar{x} - E(x \mid x > \bar{x}) \right] \]

\[ < (1 - F(\bar{x})) \left[ \bar{x} - E(x \mid x > \bar{x}) \right] < 0 \]

Investing is bad news
Yosha, JFI 1995

“Information Disclosure Costs and the Choice of Financing Source”
The model

- The timing:

  - **Stage 0**: An entrepreneur needs to raise funds to invest:
    - Bank
    - Capital market

  - **Stage 1**: A rival observes the entrepreneur’s decision and takes action $x$, that hurts the entrepreneur

  - **Stage 2**: Cash flow is realized

- The entrepreneur’s type is $\theta \sim F(\theta)$

- $\theta$ is disclosed if the entrepreneur raises funds from the capital market but not if he borrows money from a bank

- The rival’s payoff: $\pi^R = \theta R(x) - c(x)$
  - $R'(x) > 0 > R''(x)$, $c'(x) > 0$, $c''(x) > 0$

- The entrepreneur’s payoff: $\pi^E = \pi(x(\theta), \theta)$
  - $\pi^E_x < 0$, $\pi^E_\theta > 0$ (higher $\theta$ means a higher type)
The rival’s action given $\theta$

- The rival’s chooses $x$ to maximize

$$\pi^R = \theta R(x) - c(x)$$

$\theta \uparrow \Rightarrow x(\theta) \uparrow$

The entrepreneur is better off if the rival believes that $\theta$ is low
Bank debt vs. capital market

- Bank debt: the rival does not know $\theta$ and hence uses his belief $\hat{\theta}$; hence the rival takes an action $x(\hat{\theta})$
- Capital market: the rival observes $\theta$ and takes an action $x(\theta)$; the firm incurs an issuing cost $\Delta$

- The effect of $\theta$ on $\pi^E$ with bank debt (holding $\hat{\theta}$ fixed):

\[
\frac{d\pi^E(x(\hat{\theta}), \theta)}{d\theta} = \frac{\partial \pi^E(x(\hat{\theta}), \theta)}{\partial \theta} > 0
\]

- The entrepreneur’s payoff (the positive sign is by assumption):

\[
\frac{d\pi^E(x(\theta), \theta)}{d\theta} = \frac{\partial \pi^E(x(\theta), \theta)}{\partial x} x'(\theta) + \frac{\partial \pi^E(x(\theta), \theta)}{\partial \theta} > 0
\]
Bank debt vs. capital market

- If $\Delta = 0$, then we get complete unraveling: once $\theta \leq \hat{\theta}$ the entrepreneur wants to reveal $\theta$ since then $x$ is lower.

- In particular, when $\theta = 0$, the entrepreneur will always wish to reveal his type.

- Hence, for sufficiently small $\theta$:

$$\pi^E(x(\theta), \theta) < \pi^E(x(\hat{\theta}), \theta)$$
The rival’s action given $\theta$

For small $\theta$, red is above green for small $\Delta$

For high $\theta$, red is below green

$\Rightarrow$ Av. quality of firms that work with banks: 

$$\hat{\theta} = \frac{\int_{\theta^*}^{\infty} \theta dF(\theta)}{1 - F(\theta^*)}$$
Implications

- Good firms work with banks, firms which raise funds in the capital market are less good ⇒ pecking order

- $\Delta \uparrow \Rightarrow \theta^* \downarrow \Rightarrow$ The av. quality of firms which issue in the capital market ↓

- Positive price reactions to private placements

- The literature on info. revelation shows however that info. revelation can lead to either positive or negative reactions by 3rd parties (e.g., rivals)
Rock, JFE 1986

“Why New Issues are Underpriced”
Stylized facts about IPOs

- **Short-run equity underpricing:** The closing price on the first day of trading is on av. above the offering price

  - Weiss, Hanely and Ritter, “Going Public,” *The New Palgrave Dictionary* (Table 1), reports that the av. increase in the stock price shortly after IPOs is
    - 16.4% in the U.S. during the period 1960-1987
    - 31.9% in Japan during the period of 1979-1989
    - 79% in Korea during the period 1984-1990
    - 149.3% in Malaysia during the period 1979-1984

- **Long-run equity overpricing:** For several years following the IPO, the returns on IPOs are on av. lower than those on comparable stocks
  - the av. cumulative matching firm-adjusted return 36 months after the IPO is -15.08% (Ritter, *JF* 1991)

- **Oversubscription:** IPOs are typically oversubscribed and each participant receives just a fraction of his order
The model

- The timing:
  - Stage 0: An entrepreneur goes for an IPO
  - Stage 1: N+1 investors decide whether or not to participate
  - Stage 2: Cash flow is realized

- The firm’s value, $x$, is drawn from CDF $F(x)$ on $[0, \infty)$

- The mean of $x$ is $\hat{x}$

- Each investor has $K$ to invest

- Investors have enough to invest but one investor is not enough: $NK > \hat{x} > K$
All N+1 investors are uninformed

- The value of equity is $E = \hat{x}$

- Each investor has $K$ to invest and hence demands $K / \hat{x}$

- There is oversubscription:
  
  \[
  (N + 1) \frac{K}{\hat{x}} > \frac{NK}{\hat{x}} > 1
  \]

  Aggregate demand
  
  By assumption

- Since total supply is 1, investors are rationed and each one gets:

  \[
  \frac{1}{(N + 1)K} \times \frac{K}{\hat{x}} = \frac{1}{N + 1}
  \]

  Share of each investor
  
  Individual demand
N uninformed investors and 1 informed investor

- The informed investor participates in the IPO iff \( E \leq x \)

- Uninformed investors do not observe the informed investor’s decision (if they could they would infer \( x \))

- If uninformed investors participate, they get:
  - \( 1/(N+1) \) if \( E \leq x \) (the informed investor participates)
  - \( 1/N \) if \( E > x \) (the informed investor does not participate)

- The net expected payoff of an uniformed investor:

  \[
  Y^U = \int_0^E \frac{1}{N} (x - E) dF(x) + \int_E^\infty \frac{1}{N+1} (x - E) dF(x)
  \]

- In a competitive capital market, \( Y^U = 0 \)
Underpricing in IPOs

The condition for a competitive equilibrium:

\[ Y^U = \int_0^E \frac{1}{N} (x - E) dF(x) + \int_E^\infty \frac{1}{N+1} (x - E) dF(x) = 0 \]

Rewriting:

\[ \frac{\hat{x} - E}{N} + \int_E^\infty \left[ \frac{1}{N+1} - \frac{1}{N} \right] (x - E) dF(x) \]

\[ = \frac{\hat{x} - E}{N} - \int_E^\infty \left[ \frac{1}{N(N+1)} \right] (x - E) dF(x) = 0 \]

Multiplying by N and rearranging:

\[ E = \hat{x} - \int_E^\infty \frac{x - E}{N+1} dF(x) < \hat{x} \]

Less underpricing as N gets large
Post IPO share price

- After the IPO the market observes the participation and learns whether the informed investor participated or not.

- If the informed investor did not participate then $x < E \Rightarrow$ new value of equity

  $$E' = \int_{0}^{E} \frac{x}{F(E)} dF(x) < E$$

- If the informed investor participated then $x > E \Rightarrow$ new value of equity

  $$E' = \int_{E}^{\infty} \frac{x}{1 - F(E)} dF(x) > E$$